Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.2-Inverse-hyperbolic-cosine/7.2.4-f-x-^m-d+e-x^2-^p-a+b-arccosh-c-x-^n

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May 24, 2020 Compiled on May 24, 2020 at 5:24pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [569]. This is test number [190].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (569)	% 0. (0)
Mathematica	% 98.07 (558)	% 1.93 (11)
Maple	% 83.66 (476)	% 16.34 (93)
Maxima	% 36.73 (209)	% 63.27 (360)
Fricas	% 41.48 (236)	% 58.52 (333)
Sympy	% 19.16 (109)	% 80.84 (460)
Giac	% 32.51 (185)	% 67.49 (384)

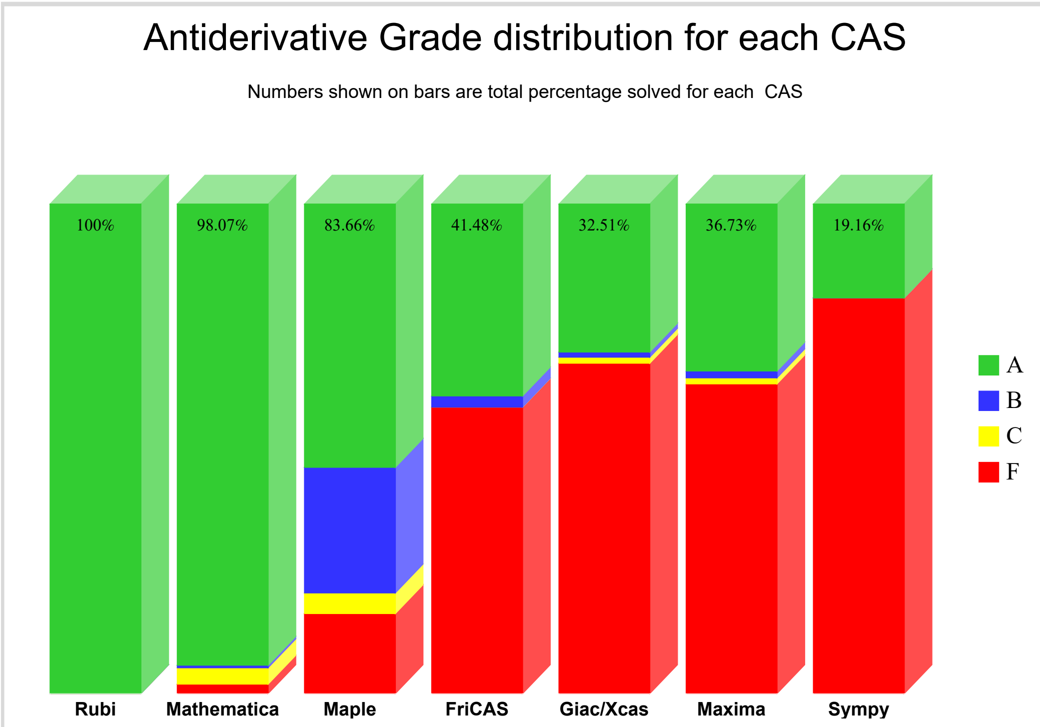
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

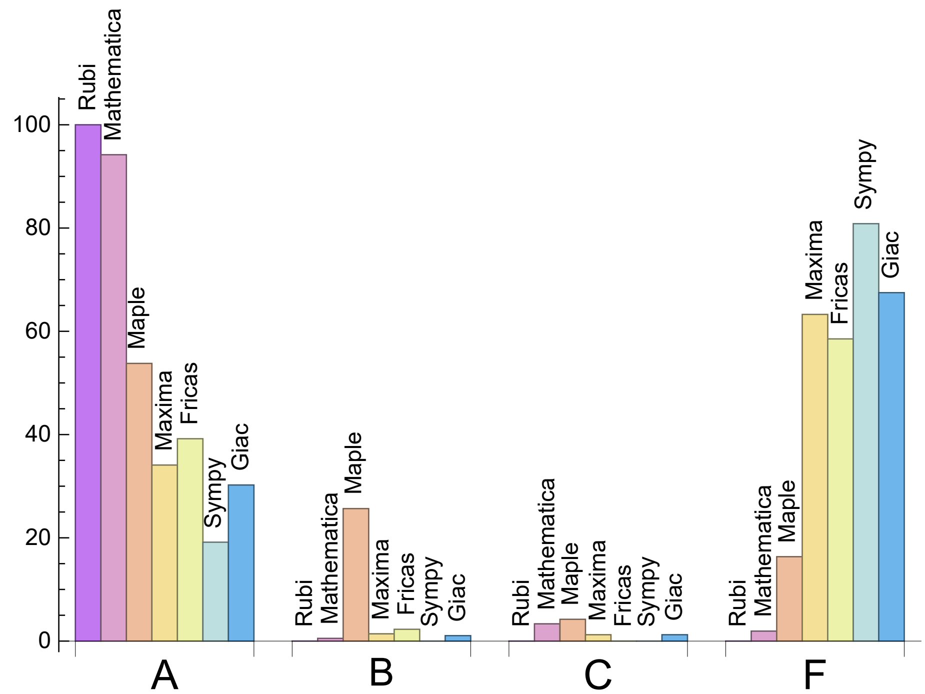
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	94.2	0.53	3.34	1.93
Maple	53.78	25.66	4.22	16.34
Maxima	34.09	1.41	1.23	63.27
Fricas	39.19	2.28	0.	58.52
Sympy	19.16	0.	0.	80.84
Giac	30.23	1.05	1.23	67.49

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.61	231.91	0.81	186.	1.
Mathematica	4.46	194.65	0.71	140.	0.72
Maple	0.28	522.68	1.84	226.	1.43
Maxima	0.47	103.55	0.53	0.	0.
Fricas	1.15	319.17	1.71	0.	0.
Sympy	6.79	111.92	0.47	0.	0.
Giac	0.39	87.88	0.43	0.	0.

1.4 list of integrals that has no closed form antiderivative

{148, 149, 150, 233, 234, 235, 236, 237, 238, 239, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 373, 378, 382, 383, 387, 391, 395, 396, 400, 406, 407, 412, 413, 417, 418, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 514, 515, 522, 523, 524, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {6, 8, 15, 17, 24, 26, 175, 177, 183, 185, 191, 193, 201, 203, 494, 496, 500, 501, 509, 510}

Mathematica {4, 6, 8, 11, 13, 15, 17, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 68, 69, 70, 71, 72, 73, 74, 75, 84, 85, 86, 87, 88, 89, 90, 91, 92, 100, 101, 102, 103, 110, 112, 115, 117, 120, 122, 125, 130, 132, 140, 142, 151, 152, 160, 167, 168, 169, 171, 173, 174, 175, 176, 177, 179, 181, 182, 183, 184, 185, 187, 189, 190, 191, 192, 193, 195, 197, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 230, 231, 232, 243, 244, 245, 246, 247, 250, 251, 252, 258, 259, 260, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 293, 315, 316, 317, 328, 329, 330, 335, 336, 337, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 384, 385, 388, 389, 392, 393, 397, 398, 401, 402, 403, 404, 408, 409, 410, 414, 415, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 440, 441, 442, 462, 464, 466, 468, 471, 473, 475, 477, 480, 482, 484, 486, 489, 490, 491, 492, 494, 495, 496, 497, 498, 502, 503, 504, 505, 506, 511, 512, 513, 543, 544, 545, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

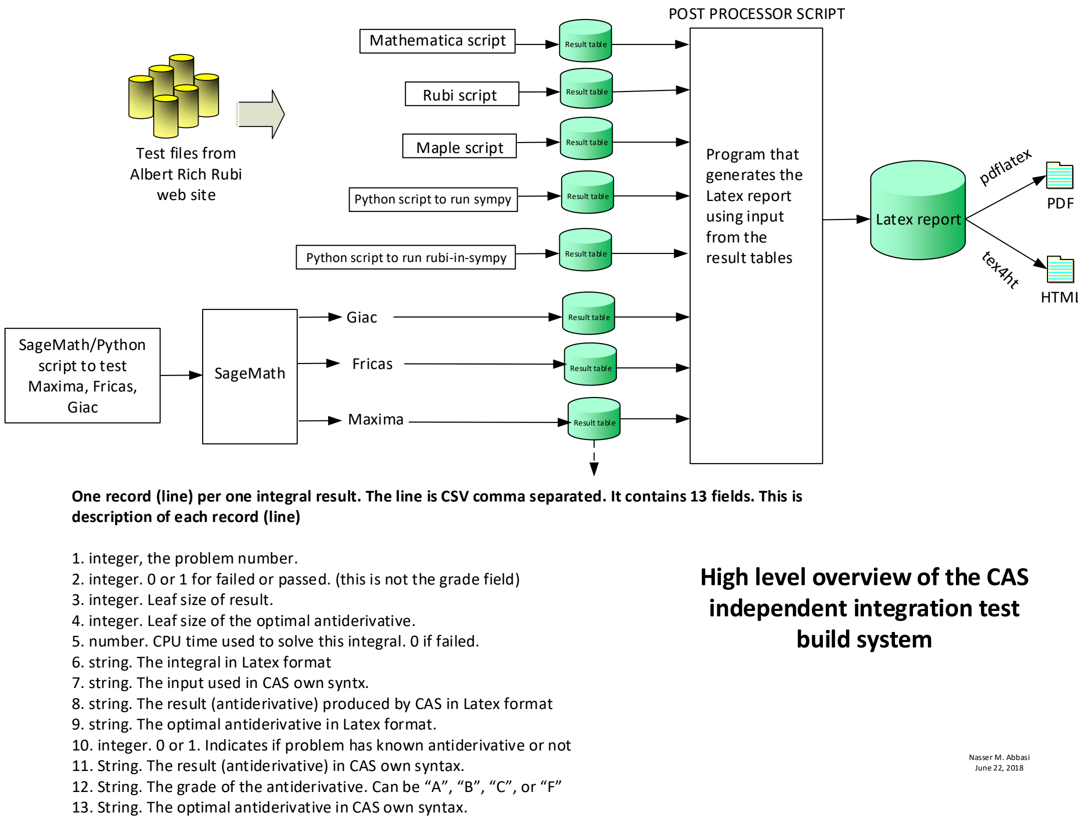
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 492, 493, 495, 497, 499, 507, 508, 514, 515, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 548, 549, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569 }

B grade: { 44, 260, 317 }

C grade: { 160, 251, 252, 489, 491, 494, 496, 498, 502, 503, 504, 505, 506, 511, 512, 513, 516, 517, 518 }

F grade: { 161, 162, 327, 334, 500, 501, 509, 510, 547, 550, 551}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 56, 57, 58, 68, 69, 70, 71, 72, 84, 85, 86, 87, 88, 89, 100, 101, 102, 109, 110, 121, 130, 132, 134, 139, 142, 148, 149, 150, 164, 165, 166, 167, 168, 169, 187, 227, 228, 229, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 257, 259, 261, 262, 263, 264, 265, 266, 267, 269, 271, 272, 273, 274, 275, 277, 279, 280, 281, 282, 283, 287, 288, 289, 290, 291, 296, 297, 298, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 378, 381, 382, 383, 386, 387, 390, 391, 394, 395, 396, 399, 400, 405, 406, 407, 411, 412, 413, 416, 417, 418, 422, 423, 427, 428, 432, 433, 437, 438, 439, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 514, 515, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569 }

B grade: { 35, 42, 51, 53, 59, 60, 61, 62, 63, 64, 65, 66, 67, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 131, 133, 135, 136, 137, 138, 140, 141, 170, 171, 172, 173, 175, 177, 178, 179, 180, 181, 183, 185, 186, 188, 189, 191, 193, 194, 195, 196, 197, 198, 199, 201, 203, 204, 205, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 250, 251, 252, 253, 268, 270, 276, 278, 284, 285, 286, 292, 293, 294, 295, 299, 301, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 499, 507, 508, 543 }

C grade: { 32, 55, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 509, 510, 511, 512, 513 }

F grade: { 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 174, 176, 182, 184, 190, 192, 200, 202, 210, 212, 220, 222, 230, 232, 258, 260, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 384, 385, 388, 389, 392, 393, 397, 398, 401, 402, 403, 404, 408, 409, 410, 414, 415, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 440, 441, 442, 516, 517, 518, 519, 520, 521, 529, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567 }

2.1.4 Maxima

A grade: { 1, 2, 3, 5, 7, 9, 10, 12, 14, 16, 18, 23, 25, 27, 67, 83, 99, 108, 119, 126, 127, 129, 134, 148, 149, 150, 164, 165, 166, 172, 180, 188, 198, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 373, 378, 382, 383, 387, 391, 395, 396, 400, 406, 407, 412, 413, 417, 418, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 470, 471, 472, 473, 474, 476, 478, 479, 480, 481, 482, 483, 485, 487, 488, 522, 523, 524, 525, 526, 527, 528, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 556, 560, 564, 565, 568, 569

B grade: { 4, 11, 13, 19, 20, 21, 22, 40 }

C grade: { 136, 138, 141, 226, 228, 254, 256 }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 128, 130, 131, 132, 133, 135, 137, 139, 140, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 296, 299, 300, 301, 302, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 347, 368, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 381, 384, 385, 386, 388, 389, 390, 392, 393, 394, 397, 398, 399, 401, 402, 403, 404, 405, 408, 409, 410, 411, 414, 415, 416, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 437, 440, 441, 442, 443, 466, 468, 475, 477, 484, 486, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 529, 530, 531, 532, 534, 535, 536, 543, 544, 545, 552, 553, 554, 555, 557, 558, 559, 561, 562, 563, 566, 567 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 63, 64, 65, 66, 67, 76, 77, 78, 79, 80, 81, 82, 83, 93, 94, 95, 96, 97, 98, 99, 104, 106, 108, 111, 113, 114, 116, 118, 124, 126, 128, 136, 138, 148, 149, 150, 164, 165, 166, 170, 172, 178, 180, 186, 188, 194, 196, 198, 226, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 254, 256, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 411, 416, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 470, 471, 472, 473, 474, 476, 478, 479, 480, 481, 482, 483, 485, 487, 488, 514, 515, 516, 522, 523, 524, 525, 526, 527, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551 }

B grade: { 62, 296, 302, 347, 368, 437, 443, 499, 507, 508, 517, 518, 528 }

C grade: { }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 68, 69, 70, 71, 72, 73, 74, 75, 84, 85, 86, 87, 88, 89, 90, 91, 92, 100, 101, 102, 103, 105, 107, 109, 110, 112, 115, 117, 119, 120, 121, 122, 123, 125, 127, 129, 130, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 171, 173, 174, 175, 176, 177, 179, 181, 182, 183, 184, 185, 187, 189, 190, 191, 192, 193, 195, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 299, 300, 301, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 417, 418, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 440, 441, 442, 466, 468, 475, 477, 484, 486, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 509, 510, 511, 512, 513, 519, 520, 521, 529, 534, 535, 536, 543, 544, 545, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 148, 149, 164, 165, 166, 236, 237, 239, 240, 241, 242, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 297, 298, 303, 304, 305, 306, 307, 308, 309, 311, 312, 318, 319, 324, 325, 326, 331, 348, 349, 351, 364, 365, 373, 378, 382, 395, 406, 422, 423, 438, 439, 444, 445, 454, 461, 462, 463, 464, 465, 470, 471, 472, 473, 474, 479, 480, 481, 482, 483, 488, 514, 515, 522, 525, 526, 527, 528, 530, 531, 532, 537, 539, 540, 541, 542, 548, 549, 550, 555, 556, 559, 564, 568 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 238, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 299, 300, 301, 302, 310, 313, 314, 315, 316, 317, 320, 321, 322, 323, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 350, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 446, 447, 448, 449, 450, 451, 452, 453, 455, 456, 457, 458, 459, 460, 466, 467, 468, 469, 475, 476, 477, 478, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 523, 524, 529, 533, 534, 535, 536, 538, 543, 544, 545, 546, 547, 551, 552, 553, 554, 557, 558, 560, 561, 562, 563, 565, 566, 567, 569 }

2.1.7 Giac

A grade: { 1, 2, 3, 5, 10, 11, 12, 14, 21, 23, 134, 148, 149, 150, 164, 165, 166, 236, 237, 238, 239, 240, 241, 242, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 373, 378, 382, 383, 387, 391, 395, 396, 406, 407, 412, 413, 417, 418, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 470, 471, 472, 473, 474, 479, 480, 481, 482, 483, 488, 514, 515, 522, 523, 524, 525, 526, 527, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 555, 556, 564, 565, 568, 569 }

B grade: { 4, 13, 19, 20, 22, 528 }

C grade: { 136, 138, 141, 226, 228, 254, 256 }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 137, 139, 140, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 296, 299, 300, 301, 302, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 347, 368, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 381, 384, 385, 386, 388, 389, 390, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 408, 409, 410, 411, 414, 415, 416, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 437, 440, 441, 442, 443, 466, 467, 468, 469, 475, 476, 477, 478, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 529, 534, 535, 536, 543, 544, 545, 552, 553, 554, 557, 558, 559, 560, 561, 562, 563, 566, 567 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	91	98	248	266	158	238
normalized size	1	1.	0.6	0.65	1.64	1.76	1.05	1.58
time (sec)	N/A	0.15	0.156	0.014	1.162	1.733	8.587	1.999

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	166	160	297	243	144	273
normalized size	1	1.	1.23	1.19	2.2	1.8	1.07	2.02
time (sec)	N/A	0.139	0.092	0.017	1.095	1.83	5.38	1.878

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	89	90	196	235	133	200
normalized size	1	1.	0.74	0.74	1.62	1.94	1.1	1.65
time (sec)	N/A	0.133	0.108	0.01	1.086	1.823	3.122	1.345

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	100	136	243	221	124	243
normalized size	1	1.	1.02	1.39	2.48	2.26	1.27	2.48
time (sec)	N/A	0.042	0.148	0.013	1.109	1.835	1.646	1.497

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	71	73	131	185	97	151
normalized size	1	1.	0.83	0.85	1.52	2.15	1.13	1.76
time (sec)	N/A	0.074	0.085	0.011	1.181	1.823	0.755	1.376

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	116	131	0	0	0	0
normalized size	1	1.	0.99	1.12	0.	0.	0.	0.
time (sec)	N/A	0.116	0.174	0.086	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	110	100	92	289	0	0
normalized size	1	1.	1.45	1.32	1.21	3.8	0.	0.
time (sec)	N/A	0.119	0.18	0.016	1.906	1.96	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	106	140	0	0	0	0
normalized size	1	1.	0.79	1.04	0.	0.	0.	0.
time (sec)	N/A	0.129	0.155	0.157	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	127	108	126	328	0	0
normalized size	1	1.	1.41	1.2	1.4	3.64	0.	0.
time (sec)	N/A	0.125	0.232	0.018	1.83	1.92	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	264	124	128	431	387	236	402
normalized size	1	1.28	0.6	0.62	2.09	1.88	1.15	1.95
time (sec)	N/A	0.293	0.198	0.013	1.18	1.869	26.254	1.5

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	284	194	230	504	373	224	451
normalized size	1	1.42	0.97	1.15	2.52	1.86	1.12	2.25
time (sec)	N/A	0.277	0.226	0.017	1.414	1.823	17.023	1.575

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	223	116	120	352	351	209	347
normalized size	1	1.26	0.66	0.68	1.99	1.98	1.18	1.96
time (sec)	N/A	0.248	0.166	0.011	1.161	1.77	9.92	1.481

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	126	204	424	328	197	406
normalized size	1	1.	0.93	1.5	3.12	2.41	1.45	2.99
time (sec)	N/A	0.067	0.218	0.013	1.216	1.818	6.112	1.66

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	177	99	102	262	296	172	281
normalized size	1	1.24	0.69	0.71	1.83	2.07	1.2	1.97
time (sec)	N/A	0.152	0.154	0.013	1.175	1.862	3.441	1.463

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	162	201	0	0	0	0
normalized size	1	1.	0.88	1.09	0.	0.	0.	0.
time (sec)	N/A	0.204	0.261	0.145	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	182	131	167	196	440	0	0
normalized size	1	1.35	0.97	1.24	1.45	3.26	0.	0.
time (sec)	N/A	0.23	0.158	0.017	1.869	2.07	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	182	220	0	0	0	0
normalized size	1	1.	0.91	1.1	0.	0.	0.	0.
time (sec)	N/A	0.218	0.25	0.261	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	186	135	167	190	452	0	0
normalized size	1	1.31	0.95	1.18	1.34	3.18	0.	0.
time (sec)	N/A	0.234	0.16	0.017	1.745	1.964	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	326	147	158	628	510	296	574
normalized size	1	1.27	0.57	0.62	2.45	1.99	1.16	2.24
time (sec)	N/A	0.439	0.261	0.018	1.257	1.817	65.246	1.55

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	328	162	284	725	475	287	633
normalized size	1	1.43	0.7	1.23	3.15	2.07	1.25	2.75
time (sec)	N/A	0.281	0.343	0.019	1.409	1.856	47.403	1.751

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	285	139	150	524	450	272	501
normalized size	1	1.26	0.61	0.66	2.31	1.98	1.2	2.21
time (sec)	N/A	0.396	0.269	0.013	1.283	1.814	26.399	1.558

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	150	258	620	425	260	574
normalized size	1	1.	0.9	1.55	3.73	2.56	1.57	3.46
time (sec)	N/A	0.079	0.365	0.014	1.199	1.92	17.554	1.823

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	237	123	132	408	389	228	417
normalized size	1	1.24	0.64	0.69	2.14	2.04	1.19	2.18
time (sec)	N/A	0.262	0.259	0.01	1.144	2.119	9.834	1.605

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	207	255	0	0	0	0
normalized size	1	1.	0.87	1.07	0.	0.	0.	0.
time (sec)	N/A	0.3	0.371	0.162	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	239	136	219	315	549	0	0
normalized size	1	1.33	0.76	1.22	1.75	3.05	0.	0.
time (sec)	N/A	0.361	0.286	0.017	1.793	2.435	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	226	275	0	0	0	0
normalized size	1	1.	0.85	1.03	0.	0.	0.	0.
time (sec)	N/A	0.318	0.352	0.306	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	252	142	223	286	554	0	0
normalized size	1	1.29	0.73	1.14	1.47	2.84	0.	0.
time (sec)	N/A	0.39	0.291	0.019	2.189	2.403	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	227	263	0	0	0	0
normalized size	1	1.	1.44	1.66	0.	0.	0.	0.
time (sec)	N/A	0.232	0.307	0.132	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	151	244	0	0	0	0
normalized size	1	1.	1.08	1.74	0.	0.	0.	0.
time (sec)	N/A	0.198	0.297	0.087	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	155	208	0	0	0	0
normalized size	1	1.	1.52	2.04	0.	0.	0.	0.
time (sec)	N/A	0.138	0.149	0.073	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	85	179	0	0	0	0
normalized size	1	1.	1.15	2.42	0.	0.	0.	0.
time (sec)	N/A	0.116	0.086	0.036	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	64	338	0	0	0	0
normalized size	1	1.	1.08	5.73	0.	0.	0.	0.
time (sec)	N/A	0.065	0.067	0.283	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	93	91	0	0	0	0
normalized size	1	1.	1.52	1.49	0.	0.	0.	0.
time (sec)	N/A	0.127	0.148	0.05	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	132	161	0	0	0	0
normalized size	1	1.	1.39	1.69	0.	0.	0.	0.
time (sec)	N/A	0.14	0.358	0.108	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	144	301	0	0	0	0
normalized size	1	1.	1.22	2.55	0.	0.	0.	0.
time (sec)	N/A	0.197	0.527	0.092	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	223	225	0	0	0	0
normalized size	1	1.	1.42	1.43	0.	0.	0.	0.
time (sec)	N/A	0.234	0.342	0.126	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	244	300	0	0	0	0
normalized size	1	1.	1.38	1.69	0.	0.	0.	0.
time (sec)	N/A	0.225	1.046	0.249	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	209	309	0	0	0	0
normalized size	1	1.	1.17	1.73	0.	0.	0.	0.
time (sec)	N/A	0.204	0.621	0.192	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	206	255	0	0	0	0
normalized size	1	1.	1.66	2.06	0.	0.	0.	0.
time (sec)	N/A	0.135	0.74	0.099	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	64	223	136	0	0
normalized size	1	1.	0.87	1.05	3.66	2.23	0.	0.
time (sec)	N/A	0.052	0.134	0.013	1.272	1.765	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	189	252	0	0	0	0
normalized size	1	1.	1.58	2.1	0.	0.	0.	0.
time (sec)	N/A	0.095	1.345	0.062	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	149	339	0	0	0	0
normalized size	1	1.	1.28	2.92	0.	0.	0.	0.
time (sec)	N/A	0.18	0.748	0.094	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	283	259	0	0	0	0
normalized size	1	1.	1.66	1.52	0.	0.	0.	0.
time (sec)	N/A	0.184	0.744	0.139	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	319	371	0	0	0	0
normalized size	1	1.	2.1	2.44	0.	0.	0.	0.
time (sec)	N/A	0.262	0.561	0.112	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	377	352	0	0	0	0
normalized size	1	1.	1.52	1.42	0.	0.	0.	0.
time (sec)	N/A	0.291	1.668	0.188	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	287	383	0	0	0	0
normalized size	1	1.	1.15	1.54	0.	0.	0.	0.
time (sec)	N/A	0.239	1.811	0.425	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	83	136	0	209	0	0
normalized size	1	1.	0.61	1.	0.	1.54	0.	0.
time (sec)	N/A	0.105	0.234	0.023	0.	1.919	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	287	380	0	0	0	0
normalized size	1	1.	1.54	2.04	0.	0.	0.	0.
time (sec)	N/A	0.179	1.733	0.153	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	64	86	0	208	0	0
normalized size	1	1.	0.7	0.95	0.	2.29	0.	0.
time (sec)	N/A	0.058	0.204	0.015	0.	1.863	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	316	378	0	0	0	0
normalized size	1	1.	1.76	2.1	0.	0.	0.	0.
time (sec)	N/A	0.134	1.161	0.106	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	210	508	0	0	0	0
normalized size	1	1.	1.23	2.97	0.	0.	0.	0.
time (sec)	N/A	0.261	1.423	0.182	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	362	392	0	0	0	0
normalized size	1	1.	1.57	1.7	0.	0.	0.	0.
time (sec)	N/A	0.244	1.851	0.18	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	273	641	0	0	0	0
normalized size	1	1.	1.09	2.56	0.	0.	0.	0.
time (sec)	N/A	0.367	2.905	0.238	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	471	504	0	0	0	0
normalized size	1	1.	1.52	1.63	0.	0.	0.	0.
time (sec)	N/A	0.387	1.969	0.237	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	77	321	0	0	0	0
normalized size	1	1.	1.45	6.06	0.	0.	0.	0.
time (sec)	N/A	0.052	0.047	0.017	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	120	184	0	0	0	0
normalized size	1	1.	1.1	1.69	0.	0.	0.	0.
time (sec)	N/A	0.08	0.834	0.078	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	223	276	0	0	0	0
normalized size	1	1.	1.36	1.68	0.	0.	0.	0.
time (sec)	N/A	0.113	2.343	0.088	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	198	449	0	0	0	0
normalized size	1	1.	0.71	1.62	0.	0.	0.	0.
time (sec)	N/A	0.782	1.142	0.514	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	151	346	0	0	0	0
normalized size	1	1.	0.75	1.72	0.	0.	0.	0.
time (sec)	N/A	0.579	1.035	0.269	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	144	239	0	0	0	0
normalized size	1	1.	1.16	1.93	0.	0.	0.	0.
time (sec)	N/A	0.204	0.569	0.161	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	137	286	0	0	0	0
normalized size	1	1.	1.16	2.42	0.	0.	0.	0.
time (sec)	N/A	0.368	0.433	0.241	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	127	88	1017	0	986	0	0
normalized size	1	1.07	0.74	8.55	0.	8.29	0.	0.
time (sec)	N/A	0.285	0.121	0.286	0.	2.265	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	226	128	1741	0	1158	0	0
normalized size	1	1.14	0.64	8.75	0.	5.82	0.	0.
time (sec)	N/A	0.346	0.201	0.335	0.	2.288	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	303	146	2534	0	1320	0	0
normalized size	1	1.09	0.52	9.08	0.	4.73	0.	0.
time (sec)	N/A	0.378	0.257	0.363	0.	2.269	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	302	152	988	0	448	0	0
normalized size	1	1.11	0.56	3.63	0.	1.65	0.	0.
time (sec)	N/A	0.352	0.342	0.421	0.	1.877	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	214	128	640	0	373	0	0
normalized size	1	1.1	0.66	3.28	0.	1.91	0.	0.
time (sec)	N/A	0.323	0.18	0.353	0.	1.833	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	126	98	356	109	301	0	0
normalized size	1	1.07	0.83	3.02	0.92	2.55	0.	0.
time (sec)	N/A	0.212	0.128	0.244	1.139	1.777	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	233	394	0	0	0	0
normalized size	1	1.	1.09	1.85	0.	0.	0.	0.
time (sec)	N/A	0.524	0.834	0.247	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	235	235	307	438	0	0	0	0
normalized size	1	1.	1.31	1.86	0.	0.	0.	0.
time (sec)	N/A	0.52	1.024	0.266	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	290	541	0	0	0	0
normalized size	1	1.	0.92	1.72	0.	0.	0.	0.
time (sec)	N/A	0.743	1.024	0.327	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	360	372	337	561	0	0	0	0
normalized size	1	1.03	0.94	1.56	0.	0.	0.	0.
time (sec)	N/A	1.064	4.513	0.353	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	293	270	456	0	0	0	0
normalized size	1	1.04	0.96	1.62	0.	0.	0.	0.
time (sec)	N/A	0.841	1.85	0.289	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	212	235	344	0	0	0	0
normalized size	1	1.06	1.18	1.72	0.	0.	0.	0.
time (sec)	N/A	0.321	1.195	0.155	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	209	223	427	0	0	0	0
normalized size	1	1.06	1.13	2.17	0.	0.	0.	0.
time (sec)	N/A	0.536	1.011	0.204	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	215	259	1181	0	0	0	0
normalized size	1	1.06	1.28	5.82	0.	0.	0.	0.
time (sec)	N/A	0.634	0.763	0.227	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	179	94	2171	0	1207	0	0
normalized size	1	1.08	0.57	13.08	0.	7.27	0.	0.
time (sec)	N/A	0.354	0.077	0.256	0.	2.698	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	322	136	3144	0	1400	0	0
normalized size	1	1.3	0.55	12.73	0.	5.67	0.	0.
time (sec)	N/A	0.445	0.138	0.311	0.	2.778	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	401	154	4259	0	1624	0	0
normalized size	1	1.22	0.47	12.98	0.	4.95	0.	0.
time (sec)	N/A	0.513	0.313	0.387	0.	2.976	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	480	170	5518	0	1806	0	0
normalized size	1	1.17	0.42	13.49	0.	4.42	0.	0.
time (sec)	N/A	0.606	0.417	0.493	0.	3.08	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	460	182	1846	0	670	0	0
normalized size	1	1.15	0.46	4.63	0.	1.68	0.	0.
time (sec)	N/A	0.496	0.223	0.497	0.	2.304	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	366	164	1376	0	568	0	0
normalized size	1	1.14	0.51	4.29	0.	1.77	0.	0.
time (sec)	N/A	0.437	0.175	0.379	0.	2.211	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	272	150	966	0	482	0	0
normalized size	1	1.12	0.62	3.98	0.	1.98	0.	0.
time (sec)	N/A	0.412	0.221	0.31	0.	2.218	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	178	107	620	138	398	0	0
normalized size	1	1.08	0.65	3.76	0.84	2.41	0.	0.
time (sec)	N/A	0.265	0.208	0.217	1.15	2.163	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	304	336	499	0	0	0	0
normalized size	1	1.04	1.15	1.71	0.	0.	0.	0.
time (sec)	N/A	0.789	1.168	0.218	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	311	323	500	542	0	0	0	0
normalized size	1	1.04	1.61	1.74	0.	0.	0.	0.
time (sec)	N/A	0.801	1.466	0.222	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	321	333	574	570	0	0	0	0
normalized size	1	1.04	1.79	1.78	0.	0.	0.	0.
time (sec)	N/A	0.837	1.184	0.241	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	454	485	581	690	0	0	0	0
normalized size	1	1.07	1.28	1.52	0.	0.	0.	0.
time (sec)	N/A	1.385	6.548	0.49	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	402	415	581	0	0	0	0
normalized size	1	1.08	1.12	1.57	0.	0.	0.	0.
time (sec)	N/A	1.171	4.5	0.335	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	324	347	462	0	0	0	0
normalized size	1	1.11	1.18	1.58	0.	0.	0.	0.
time (sec)	N/A	0.543	2.277	0.204	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	284	315	305	550	0	0	0	0
normalized size	1	1.11	1.07	1.94	0.	0.	0.	0.
time (sec)	N/A	0.678	1.669	0.255	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	324	319	1407	0	0	0	0
normalized size	1	1.11	1.09	4.8	0.	0.	0.	0.
time (sec)	N/A	0.86	1.36	0.265	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	324	400	2429	0	0	0	0
normalized size	1	1.11	1.37	8.29	0.	0.	0.	0.
time (sec)	N/A	0.947	3.187	0.275	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	234	105	3775	0	1473	0	0
normalized size	1	1.07	0.48	17.24	0.	6.73	0.	0.
time (sec)	N/A	0.379	0.095	0.301	0.	2.699	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	448	147	5006	0	1715	0	0
normalized size	1	1.43	0.47	15.94	0.	5.46	0.	0.
time (sec)	N/A	0.526	0.163	0.398	0.	2.923	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	519	165	6379	0	1979	0	0
normalized size	1	1.35	0.43	16.57	0.	5.14	0.	0.
time (sec)	N/A	0.579	0.198	0.543	0.	3.096	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	458	527	193	2374	0	833	0	0
normalized size	1	1.15	0.42	5.18	0.	1.82	0.	0.
time (sec)	N/A	0.516	0.247	0.5	0.	2.292	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	429	175	1840	0	722	0	0
normalized size	1	1.13	0.46	4.87	0.	1.91	0.	0.
time (sec)	N/A	0.468	0.192	0.415	0.	2.125	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	331	160	1102	0	602	0	0
normalized size	1	1.11	0.54	3.7	0.	2.02	0.	0.
time (sec)	N/A	0.424	0.146	0.317	0.	2.179	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	233	117	956	159	502	0	0
normalized size	1	1.07	0.54	4.39	0.73	2.3	0.	0.
time (sec)	N/A	0.282	0.233	0.267	1.223	2.13	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	379	410	471	620	0	0	0	0
normalized size	1	1.08	1.24	1.64	0.	0.	0.	0.
time (sec)	N/A	1.059	3.576	0.258	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	404	435	596	667	0	0	0	0
normalized size	1	1.08	1.48	1.65	0.	0.	0.	0.
time (sec)	N/A	1.064	3.896	0.27	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	407	438	660	691	0	0	0	0
normalized size	1	1.08	1.62	1.7	0.	0.	0.	0.
time (sec)	N/A	1.083	1.391	0.283	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	84	54	152	0	0	0	0
normalized size	1	1.27	0.82	2.3	0.	0.	0.	0.
time (sec)	N/A	0.104	0.111	0.147	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	260	140	670	0	382	0	0
normalized size	1	1.1	0.59	2.84	0.	1.62	0.	0.
time (sec)	N/A	0.709	0.254	0.314	0.	2.137	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	228	171	408	0	0	0	0
normalized size	1	1.08	0.81	1.92	0.	0.	0.	0.
time (sec)	N/A	0.648	0.843	0.355	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	172	113	382	0	308	0	0
normalized size	1	1.1	0.72	2.45	0.	1.97	0.	0.
time (sec)	N/A	0.496	0.212	0.234	0.	2.46	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	140	141	291	0	0	0	0
normalized size	1	1.06	1.07	2.2	0.	0.	0.	0.
time (sec)	N/A	0.398	0.61	0.225	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	80	85	158	85	236	0	0
normalized size	1	1.11	1.18	2.19	1.18	3.28	0.	0.
time (sec)	N/A	0.21	0.159	0.15	1.178	2.087	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	89	0	0	0	0
normalized size	1	1.	1.	1.68	0.	0.	0.	0.
time (sec)	N/A	0.121	0.034	0.043	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	153	327	0	0	0	0
normalized size	1	1.	1.01	2.17	0.	0.	0.	0.
time (sec)	N/A	0.333	0.273	0.195	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	79	71	219	0	586	0	0
normalized size	1	1.11	1.	3.08	0.	8.25	0.	0.
time (sec)	N/A	0.303	0.065	0.167	0.	2.518	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	238	246	309	489	0	0	0	0
normalized size	1	1.03	1.3	2.05	0.	0.	0.	0.
time (sec)	N/A	0.541	1.036	0.243	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	171	174	854	0	999	0	0
normalized size	1	1.1	1.12	5.51	0.	6.45	0.	0.
time (sec)	N/A	0.506	0.317	0.215	0.	2.593	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	262	145	431	0	1046	0	0
normalized size	1	1.12	0.62	1.85	0.	4.49	0.	0.
time (sec)	N/A	0.434	0.102	0.285	0.	2.684	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	237	192	445	0	0	0	0
normalized size	1	1.05	0.85	1.97	0.	0.	0.	0.
time (sec)	N/A	0.681	1.434	0.306	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	163	97	313	0	915	0	0
normalized size	1	1.09	0.65	2.09	0.	6.1	0.	0.
time (sec)	N/A	0.385	0.069	0.218	0.	2.545	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	159	279	0	0	0	0
normalized size	1	1.	1.11	1.95	0.	0.	0.	0.
time (sec)	N/A	0.454	0.655	0.18	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	90	198	0	697	0	0
normalized size	1	1.	1.18	2.61	0.	9.17	0.	0.
time (sec)	N/A	0.248	0.229	0.136	0.	2.812	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	72	180	95	0	0	0
normalized size	1	1.	0.86	2.14	1.13	0.	0.	0.
time (sec)	N/A	0.144	0.029	0.102	1.2	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	301	511	0	0	0	0
normalized size	1	1.	1.31	2.23	0.	0.	0.	0.
time (sec)	N/A	0.591	2.371	0.217	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	159	114	242	0	0	0	0
normalized size	1	1.01	0.72	1.53	0.	0.	0.	0.
time (sec)	N/A	0.397	0.084	0.139	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	405	648	0	0	0	0
normalized size	1	1.	1.23	1.97	0.	0.	0.	0.
time (sec)	N/A	0.862	4.29	0.227	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	161	1050	0	0	0	0
normalized size	1	1.	0.64	4.2	0.	0.	0.	0.
time (sec)	N/A	0.458	0.116	0.211	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	280	167	466	0	1149	0	0
normalized size	1	1.15	0.69	1.92	0.	4.73	0.	0.
time (sec)	N/A	0.443	0.165	0.273	0.	2.675	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	251	225	1519	0	0	0	0
normalized size	1	1.12	1.	6.78	0.	0.	0.	0.
time (sec)	N/A	0.757	0.71	0.308	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	243	122	313	236	1013	0	0
normalized size	1	1.54	0.77	1.98	1.49	6.41	0.	0.
time (sec)	N/A	0.448	0.122	0.208	1.276	2.581	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	160	101	1228	228	0	0	0
normalized size	1	1.2	0.76	9.23	1.71	0.	0.	0.
time (sec)	N/A	0.405	0.199	0.202	1.283	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	154	119	249	0	910	0	0
normalized size	1	1.21	0.94	1.96	0.	7.17	0.	0.
time (sec)	N/A	0.276	0.255	0.164	0.	2.598	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	189	132	1073	212	0	0	0
normalized size	1	1.17	0.81	6.62	1.31	0.	0.	0.
time (sec)	N/A	0.268	0.084	0.143	1.22	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	332	377	619	0	0	0	0
normalized size	1	1.05	1.19	1.95	0.	0.	0.	0.
time (sec)	N/A	0.832	7.011	0.243	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	279	147	1350	0	0	0	0
normalized size	1	1.12	0.59	5.44	0.	0.	0.	0.
time (sec)	N/A	0.442	0.337	0.194	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	479	509	500	801	0	0	0	0
normalized size	1	1.06	1.04	1.67	0.	0.	0.	0.
time (sec)	N/A	1.143	7.207	0.285	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	383	169	1878	0	0	0	0
normalized size	1	1.13	0.5	5.56	0.	0.	0.	0.
time (sec)	N/A	0.541	0.398	0.209	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	276	116	419	258	0	0	192
normalized size	1	1.12	0.47	1.7	1.05	0.	0.	0.78
time (sec)	N/A	0.339	0.088	0.229	1.27	0.	0.	1.456

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	206	93	456	0	0	0	0
normalized size	1	1.42	0.64	3.14	0.	0.	0.	0.
time (sec)	N/A	0.501	0.255	0.299	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	158	74	311	84	209	0	89
normalized size	1	1.44	0.67	2.83	0.76	1.9	0.	0.81
time (sec)	N/A	0.392	0.123	0.207	1.928	2.132	0.	1.223

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	125	75	223	0	0	0	0
normalized size	1	1.42	0.85	2.53	0.	0.	0.	0.
time (sec)	N/A	0.324	0.156	0.186	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	73	55	123	38	151	0	54
normalized size	1	1.49	1.12	2.51	0.78	3.08	0.	1.1
time (sec)	N/A	0.177	0.084	0.12	1.166	2.066	0.	1.186

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	45	45	51	0	0	0	0
normalized size	1	1.41	1.41	1.59	0.	0.	0.	0.
time (sec)	N/A	0.098	0.02	0.034	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	142	113	270	0	0	0	0
normalized size	1	1.38	1.1	2.62	0.	0.	0.	0.
time (sec)	N/A	0.276	0.142	0.142	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	72	57	168	99	0	0	116
normalized size	1	1.5	1.19	3.5	2.06	0.	0.	2.42
time (sec)	N/A	0.254	0.03	0.136	1.788	0.	0.	1.208

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	230	234	349	0	0	0	0
normalized size	1	1.38	1.4	2.09	0.	0.	0.	0.
time (sec)	N/A	0.471	0.29	0.206	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	111	100	0	0	0	0	0
normalized size	1	1.13	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.358	0.106	0.277	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	115	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.383	0.045	0.363	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	387	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	2.755	1.169	3.132	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	290	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.501	0.437	2.434	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	191	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.258	0.23	2.286	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	3.955	0.484	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	160	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	5.898	0.562	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	293	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.335	6.525	0.569	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	723	764	350	0	0	0	0	0
normalized size	1	1.06	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	1.388	1.421	1.507	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	455	477	274	0	0	0	0	0
normalized size	1	1.05	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.904	0.797	1.326	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	288	223	0	0	0	0	0
normalized size	1	1.04	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.577	0.283	1.338	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	147	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.352	0.076	0.45	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	216	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.66	0.242	0.598	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	465	319	0	0	0	0	0
normalized size	1	1.03	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.993	0.719	0.582	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	817	827	387	0	0	0	0	0
normalized size	1	1.01	0.47	0.	0.	0.	0.	0.
time (sec)	N/A	1.58	2.575	2.239	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	513	288	0	0	0	0	0
normalized size	1	1.02	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.991	1.044	1.931	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	312	229	0	0	0	0	0
normalized size	1	1.03	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.557	0.218	1.584	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	264	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.559	6.009	0.505	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	336	336	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.943	2.454	0.659	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	504	504	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.452	2.485	0.677	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	141	124	0	0	0	0	0
normalized size	1	1.1	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.287	0.077	0.368	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	125	188	240	416	243	227
normalized size	1	1.	0.47	0.71	0.9	1.56	0.91	0.85
time (sec)	N/A	0.676	0.28	0.148	1.209	2.037	15.05	1.185

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	101	140	181	321	182	184
normalized size	1	1.	0.52	0.72	0.93	1.65	0.93	0.94
time (sec)	N/A	0.462	0.222	0.047	1.241	2.028	4.988	1.198

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	73	90	103	216	105	127
normalized size	1	1.	0.65	0.8	0.92	1.93	0.94	1.13
time (sec)	N/A	0.263	0.109	0.04	1.155	2.088	1.257	1.192

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	95	201	0	0	0	0
normalized size	1	1.	0.97	2.05	0.	0.	0.	0.
time (sec)	N/A	0.098	0.079	0.043	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	191	288	0	0	0	0
normalized size	1	1.	1.17	1.77	0.	0.	0.	0.
time (sec)	N/A	0.287	0.918	0.084	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	258	258	319	443	0	0	0	0
normalized size	1	1.	1.24	1.72	0.	0.	0.	0.
time (sec)	N/A	0.493	4.804	0.143	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	237	1284	0	753	0	0
normalized size	1	1.	0.64	3.46	0.	2.03	0.	0.
time (sec)	N/A	1.056	0.427	0.544	0.	2.276	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	241	767	0	0	0	0
normalized size	1	1.	0.76	2.4	0.	0.	0.	0.
time (sec)	N/A	0.93	1.991	0.376	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	194	181	726	275	591	0	0
normalized size	1	1.04	0.97	3.9	1.48	3.18	0.	0.
time (sec)	N/A	0.37	0.348	0.366	1.132	2.275	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	235	528	0	0	0	0
normalized size	1	1.	1.15	2.59	0.	0.	0.	0.
time (sec)	N/A	0.349	1.04	0.224	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	449	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.786	1.22	0.349	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	234	234	270	582	0	0	0	0
normalized size	1	1.	1.15	2.49	0.	0.	0.	0.
time (sec)	N/A	0.631	1.624	0.336	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	547	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.877	79.753	0.348	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	336	344	304	2633	0	0	0	0
normalized size	1	1.02	0.9	7.84	0.	0.	0.	0.
time (sec)	N/A	0.583	0.991	0.394	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	507	262	1952	0	996	0	0
normalized size	1	1.02	0.53	3.94	0.	2.01	0.	0.
time (sec)	N/A	1.674	0.588	0.536	0.	2.377	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	441	453	485	1021	0	0	0	0
normalized size	1	1.03	1.1	2.32	0.	0.	0.	0.
time (sec)	N/A	1.477	4.283	0.444	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	361	208	1270	375	801	0	0
normalized size	1	1.04	0.6	3.65	1.08	2.3	0.	0.
time (sec)	N/A	0.554	0.497	0.382	1.176	2.274	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	348	374	775	0	0	0	0
normalized size	1	1.04	1.11	2.31	0.	0.	0.	0.
time (sec)	N/A	0.609	2.924	0.263	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	573	585	650	0	0	0	0	0
normalized size	1	1.02	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	1.255	2.71	0.333	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	453	465	433	942	0	0	0	0
normalized size	1	1.03	0.96	2.08	0.	0.	0.	0.
time (sec)	N/A	0.966	3.985	0.329	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	630	642	1129	0	0	0	0	0
normalized size	1	1.02	1.79	0.	0.	0.	0.	0.
time (sec)	N/A	1.374	168.74	0.35	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	426	438	583	2879	0	0	0	0
normalized size	1	1.03	1.37	6.76	0.	0.	0.	0.
time (sec)	N/A	1.167	2.056	0.342	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	880	911	288	2224	0	1231	0	0
normalized size	1	1.04	0.33	2.53	0.	1.4	0.	0.
time (sec)	N/A	2.345	0.708	0.536	0.	2.556	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	841	872	910	1312	0	0	0	0
normalized size	1	1.04	1.08	1.56	0.	0.	0.	0.
time (sec)	N/A	2.124	5.828	0.533	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	485	234	1958	455	1031	0	0
normalized size	1	1.03	0.5	4.17	0.97	2.19	0.	0.
time (sec)	N/A	0.68	0.592	0.468	1.322	2.568	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	486	517	740	1053	0	0	0	0
normalized size	1	1.06	1.52	2.17	0.	0.	0.	0.
time (sec)	N/A	0.868	3.367	0.339	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	836	867	1031	0	0	0	0	0
normalized size	1	1.04	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	1.776	7.19	0.413	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	607	638	554	1227	0	0	0	0
normalized size	1	1.05	0.91	2.02	0.	0.	0.	0.
time (sec)	N/A	1.29	5.719	0.411	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	890	921	1384	0	0	0	0	0
normalized size	1	1.03	1.56	0.	0.	0.	0.	0.
time (sec)	N/A	1.985	95.217	0.444	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	638	669	803	3431	0	0	0	0
normalized size	1	1.05	1.26	5.38	0.	0.	0.	0.
time (sec)	N/A	1.615	3.358	0.434	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	445	255	1314	0	771	0	0
normalized size	1	1.06	0.61	3.12	0.	1.83	0.	0.
time (sec)	N/A	1.131	0.539	0.477	0.	2.179	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	355	371	295	887	0	0	0	0
normalized size	1	1.05	0.83	2.5	0.	0.	0.	0.
time (sec)	N/A	1.024	1.524	0.48	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	308	201	752	0	598	0	0
normalized size	1	1.05	0.69	2.58	0.	2.05	0.	0.
time (sec)	N/A	0.769	0.445	0.375	0.	2.207	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	234	228	624	0	0	0	0
normalized size	1	1.04	1.01	2.76	0.	0.	0.	0.
time (sec)	N/A	0.626	0.863	0.309	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	163	149	314	196	448	0	0
normalized size	1	1.05	0.96	2.03	1.26	2.89	0.	0.
time (sec)	N/A	0.342	0.38	0.236	1.143	2.198	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	149	0	0	0	0
normalized size	1	1.	1.	2.81	0.	0.	0.	0.
time (sec)	N/A	0.195	0.05	0.066	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	315	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.521	0.629	0.315	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	186	194	179	513	0	0	0	0
normalized size	1	1.04	0.96	2.76	0.	0.	0.	0.
time (sec)	N/A	0.519	0.84	0.257	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	430	438	697	0	0	0	0	0
normalized size	1	1.02	1.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.881	86.907	0.354	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	328	344	346	2198	0	0	0	0
normalized size	1	1.05	1.05	6.7	0.	0.	0.	0.
time (sec)	N/A	0.869	1.58	0.338	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	556	578	358	1099	0	0	0	0
normalized size	1	1.04	0.64	1.98	0.	0.	0.	0.
time (sec)	N/A	1.367	3.784	0.486	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	440	451	343	1141	0	0	0	0
normalized size	1	1.02	0.78	2.59	0.	0.	0.	0.
time (sec)	N/A	1.205	1.954	0.495	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	413	424	302	836	0	0	0	0
normalized size	1	1.03	0.73	2.02	0.	0.	0.	0.
time (sec)	N/A	0.921	1.584	0.394	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	257	270	738	0	0	0	0
normalized size	1	1.	1.05	2.87	0.	0.	0.	0.
time (sec)	N/A	0.795	2.007	0.333	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	210	542	0	0	0	0
normalized size	1	1.	1.07	2.77	0.	0.	0.	0.
time (sec)	N/A	0.445	0.965	0.269	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	126	578	0	0	0	0
normalized size	1	1.	0.64	2.92	0.	0.	0.	0.
time (sec)	N/A	0.351	0.439	0.216	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	471	471	577	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.964	3.526	0.36	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	341	341	315	826	0	0	0	0
normalized size	1	1.	0.92	2.42	0.	0.	0.	0.
time (sec)	N/A	0.936	1.631	0.286	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	650	650	979	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	1.486	91.392	0.372	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	496	496	529	2868	0	0	0	0
normalized size	1	1.	1.07	5.78	0.	0.	0.	0.
time (sec)	N/A	1.461	2.515	0.385	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	568	594	437	1211	0	0	0	0
normalized size	1	1.05	0.77	2.13	0.	0.	0.	0.
time (sec)	N/A	1.476	5.564	0.479	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	482	497	382	4074	0	0	0	0
normalized size	1	1.03	0.79	8.45	0.	0.	0.	0.
time (sec)	N/A	1.333	2.57	0.492	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	351	341	835	0	0	0	0
normalized size	1	1.04	1.01	2.49	0.	0.	0.	0.
time (sec)	N/A	0.969	4.462	0.408	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	389	404	264	3445	0	0	0	0
normalized size	1	1.04	0.68	8.86	0.	0.	0.	0.
time (sec)	N/A	0.731	1.678	0.352	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	298	313	332	720	0	0	0	0
normalized size	1	1.05	1.11	2.42	0.	0.	0.	0.
time (sec)	N/A	0.473	2.494	0.33	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	331	346	289	3050	0	0	0	0
normalized size	1	1.05	0.87	9.21	0.	0.	0.	0.
time (sec)	N/A	0.56	1.489	0.289	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	597	612	806	0	0	0	0	0
normalized size	1	1.03	1.35	0.	0.	0.	0.	0.
time (sec)	N/A	1.33	10.827	0.361	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	476	506	457	3798	0	0	0	0
normalized size	1	1.06	0.96	7.98	0.	0.	0.	0.
time (sec)	N/A	1.191	3.125	0.352	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	796	826	1181	0	0	0	0	0
normalized size	1	1.04	1.48	0.	0.	0.	0.	0.
time (sec)	N/A	1.938	98.	0.449	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	562	607	534	5251	0	0	0	0
normalized size	1	1.08	0.95	9.34	0.	0.	0.	0.
time (sec)	N/A	1.821	3.686	0.392	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	459	220	794	0	0	0	0
normalized size	1	1.07	0.51	1.85	0.	0.	0.	0.
time (sec)	N/A	0.672	1.415	0.255	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	329	116	488	0	0	0	0
normalized size	1	1.35	0.48	2.01	0.	0.	0.	0.
time (sec)	N/A	0.793	0.266	0.28	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	237	123	343	142	324	0	161
normalized size	1	1.34	0.69	1.94	0.8	1.83	0.	0.91
time (sec)	N/A	0.592	0.151	0.192	1.746	2.203	0.	1.192

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	207	87	239	0	0	0	0
normalized size	1	1.37	0.58	1.58	0.	0.	0.	0.
time (sec)	N/A	0.51	0.17	0.162	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	109	54	139	68	246	0	103
normalized size	1	1.38	0.68	1.76	0.86	3.11	0.	1.3
time (sec)	N/A	0.272	0.094	0.13	1.101	2.146	0.	1.174

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	45	45	51	0	0	0	0
normalized size	1	1.41	1.41	1.59	0.	0.	0.	0.
time (sec)	N/A	0.15	0.024	0.038	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	248	151	0	0	0	0	0
normalized size	1	1.36	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.42	0.185	0.158	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	174	111	241	0	0	0	0
normalized size	1	1.4	0.9	1.94	0.	0.	0.	0.
time (sec)	N/A	0.444	0.45	0.151	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	296	398	233	0	0	0	0	0
normalized size	1	1.34	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.723	0.982	0.163	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1153	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.545	1.632	1.329	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	583	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.523	0.513	1.082	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	239	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.448	0.335	1.018	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.474	3.436	0.406	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.561	4.433	0.539	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.557	4.627	0.55	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.37	0.739	0.341	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	505	505	179	294	373	605	367	333
normalized size	1	1.	0.35	0.58	0.74	1.2	0.73	0.66
time (sec)	N/A	1.408	0.413	0.076	1.154	2.177	24.537	1.345

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	147	218	284	467	274	273
normalized size	1	1.	0.38	0.56	0.73	1.2	0.71	0.7
time (sec)	N/A	0.844	0.21	0.056	1.201	2.178	8.767	1.337

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	109	140	167	316	160	196
normalized size	1	1.	0.62	0.8	0.95	1.81	0.91	1.12
time (sec)	N/A	0.477	0.116	0.049	1.227	2.177	2.49	1.318

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	129	273	0	0	0	0
normalized size	1	1.	0.9	1.9	0.	0.	0.	0.
time (sec)	N/A	0.129	0.099	0.043	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	276	464	0	0	0	0
normalized size	1	1.	1.06	1.78	0.	0.	0.	0.
time (sec)	N/A	0.442	2.252	0.095	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	387	387	455	710	0	0	0	0
normalized size	1	1.	1.18	1.83	0.	0.	0.	0.
time (sec)	N/A	0.814	8.253	0.171	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	605	636	189	887	0	0	0	0
normalized size	1	1.05	0.31	1.47	0.	0.	0.	0.
time (sec)	N/A	1.497	1.153	0.259	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	414	148	536	0	0	0	0
normalized size	1	1.03	0.37	1.33	0.	0.	0.	0.
time (sec)	N/A	0.946	0.493	0.174	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	98	256	0	0	0	0
normalized size	1	1.	0.42	1.11	0.	0.	0.	0.
time (sec)	N/A	0.537	0.201	0.204	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	55	0	0	0	0
normalized size	1	1.	1.	1.2	0.	0.	0.	0.
time (sec)	N/A	0.156	0.031	0.039	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	241	241	145	548	0	0	0	0
normalized size	1	1.	0.6	2.27	0.	0.	0.	0.
time (sec)	N/A	0.349	0.24	0.21	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	413	428	258	955	0	0	0	0
normalized size	1	1.04	0.62	2.31	0.	0.	0.	0.
time (sec)	N/A	0.638	0.942	0.315	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	607	637	363	1319	0	0	0	0
normalized size	1	1.05	0.6	2.17	0.	0.	0.	0.
time (sec)	N/A	1.077	1.711	0.359	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	427	136	520	0	0	0	0
normalized size	1	1.36	0.43	1.65	0.	0.	0.	0.
time (sec)	N/A	1.442	0.439	0.28	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	329	140	375	177	447	0	200
normalized size	1	1.35	0.58	1.54	0.73	1.84	0.	0.82
time (sec)	N/A	1.026	0.156	0.198	1.803	2.204	0.	1.34

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	257	98	255	0	0	0	0
normalized size	1	1.37	0.52	1.36	0.	0.	0.	0.
time (sec)	N/A	0.765	0.222	0.165	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	153	101	155	88	348	0	139
normalized size	1	1.39	0.92	1.41	0.8	3.16	0.	1.26
time (sec)	N/A	0.393	0.099	0.121	1.146	2.174	0.	1.28

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	45	45	51	0	0	0	0
normalized size	1	1.41	1.41	1.59	0.	0.	0.	0.
time (sec)	N/A	0.161	0.018	0.035	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	265	356	488	0	0	0	0	0
normalized size	1	1.34	1.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.479	0.647	0.149	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	229	137	313	0	0	0	0
normalized size	1	1.38	0.83	1.89	0.	0.	0.	0.
time (sec)	N/A	0.492	0.512	0.165	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	460	614	1051	0	0	0	0	0
normalized size	1	1.33	2.28	0.	0.	0.	0.	0.
time (sec)	N/A	1.013	6.079	0.181	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.458	3.499	0.379	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	0	0	0
normalized size	1	1.	0.67	0.66	0.	0.	0.	0.
time (sec)	N/A	0.14	0.264	0.039	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	34	33	0	0	0	0
normalized size	1	1.	0.68	0.66	0.	0.	0.	0.
time (sec)	N/A	0.113	0.164	0.032	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	0	0	0	0
normalized size	1	1.	0.86	0.83	0.	0.	0.	0.
time (sec)	N/A	0.081	0.116	0.03	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	1.483	0.136	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	5.889	0.17	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	430	188	591	0	0	0	0
normalized size	1	1.27	0.55	1.74	0.	0.	0.	0.
time (sec)	N/A	0.879	0.501	0.355	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	371	171	543	0	0	0	0
normalized size	1	1.25	0.58	1.83	0.	0.	0.	0.
time (sec)	N/A	0.861	0.434	0.227	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	178	103	227	0	0	0	0
normalized size	1	1.28	0.74	1.63	0.	0.	0.	0.
time (sec)	N/A	0.672	0.297	0.167	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	245	127	361	0	0	0	0
normalized size	1	1.24	0.64	1.83	0.	0.	0.	0.
time (sec)	N/A	0.572	0.31	0.156	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	178	105	227	0	0	0	0
normalized size	1	1.28	0.76	1.63	0.	0.	0.	0.
time (sec)	N/A	0.343	0.202	0.106	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	116	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.077	1.2	0.234	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.931	1.043	0.139	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.447	1.399	0.296	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.438	0.859	0.401	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	397	497	215	725	0	0	0	0
normalized size	1	1.25	0.54	1.83	0.	0.	0.	0.
time (sec)	N/A	0.946	0.924	0.263	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	430	188	591	0	0	0	0
normalized size	1	1.27	0.55	1.74	0.	0.	0.	0.
time (sec)	N/A	0.881	0.716	0.225	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	371	172	543	0	0	0	0
normalized size	1	1.25	0.58	1.83	0.	0.	0.	0.
time (sec)	N/A	0.675	0.663	0.185	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	304	147	409	0	0	0	0
normalized size	1	1.27	0.62	1.71	0.	0.	0.	0.
time (sec)	N/A	0.459	0.442	0.131	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	215	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.752	1.239	0.247	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	163	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.525	1.372	0.253	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.525	1.372	0.309	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.539	0.877	0.382	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	397	497	216	725	0	0	0	0
normalized size	1	1.25	0.54	1.83	0.	0.	0.	0.
time (sec)	N/A	0.997	1.281	0.254	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	439	556	233	773	0	0	0	0
normalized size	1	1.27	0.53	1.76	0.	0.	0.	0.
time (sec)	N/A	0.979	1.169	0.262	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	397	497	216	725	0	0	0	0
normalized size	1	1.25	0.54	1.83	0.	0.	0.	0.
time (sec)	N/A	0.791	1.059	0.213	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	430	191	591	0	0	0	0
normalized size	1	1.27	0.56	1.74	0.	0.	0.	0.
time (sec)	N/A	0.568	0.787	0.167	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	309	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.376	1.27	0.268	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	254	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.093	1.269	0.286	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.546	1.425	0.342	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.551	0.967	0.458	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	137	69	249	0	0	0	0
normalized size	1	1.4	0.7	2.54	0.	0.	0.	0.
time (sec)	N/A	0.466	0.113	0.263	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	91	60	200	0	0	0	0
normalized size	1	1.4	0.92	3.08	0.	0.	0.	0.
time (sec)	N/A	0.45	0.087	0.21	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	91	60	149	0	0	0	0
normalized size	1	1.4	0.92	2.29	0.	0.	0.	0.
time (sec)	N/A	0.434	0.099	0.159	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	41	50	100	0	0	0	0
normalized size	1	1.46	1.79	3.57	0.	0.	0.	0.
time (sec)	N/A	0.301	0.08	0.132	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	41	47	48	0	117	0	0
normalized size	1	1.46	1.68	1.71	0.	4.18	0.	0.
time (sec)	N/A	0.162	0.059	0.079	0.	2.021	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.378	0.55	0.158	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.377	0.685	0.141	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	245	130	349	0	0	0	0
normalized size	1	1.24	0.66	1.77	0.	0.	0.	0.
time (sec)	N/A	0.69	0.279	0.192	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	178	99	232	0	0	0	0
normalized size	1	1.28	0.71	1.67	0.	0.	0.	0.
time (sec)	N/A	0.632	0.261	0.166	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	114	81	173	0	0	0	0
normalized size	1	1.24	0.88	1.88	0.	0.	0.	0.
time (sec)	N/A	0.426	0.202	0.119	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	48	54	55	0	136	0	0
normalized size	1	1.37	1.54	1.57	0.	3.89	0.	0.
time (sec)	N/A	0.221	0.106	0.079	0.	1.9	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.498	0.707	0.221	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.507	1.257	0.227	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.575	4.305	0.165	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.401	6.73	0.223	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.25	0.142	0.19	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.568	3.43	0.281	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.577	2.171	0.218	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.54	1.036	0.753	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.451	0.171	0.764	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.483	0.617	0.315	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.565	1.157	0.46	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.58	1.661	0.458	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	128	107	0	0	0	0
normalized size	1	1.	1.31	1.09	0.	0.	0.	0.
time (sec)	N/A	0.325	0.475	0.049	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	84	87	0	0	0	0
normalized size	1	1.	1.02	1.06	0.	0.	0.	0.
time (sec)	N/A	0.305	0.454	0.04	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	140	61	0	0	0	0
normalized size	1	1.	2.41	1.05	0.	0.	0.	0.
time (sec)	N/A	0.235	0.898	0.036	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	2.978	0.135	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.254	12.51	0.172	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	429	322	1029	0	0	0	0
normalized size	1	1.23	0.92	2.94	0.	0.	0.	0.
time (sec)	N/A	1.085	0.769	0.411	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	185	130	422	0	0	0	0
normalized size	1	1.2	0.84	2.74	0.	0.	0.	0.
time (sec)	N/A	0.879	0.469	0.235	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	418	217	622	0	0	0	0
normalized size	1	1.69	0.88	2.51	0.	0.	0.	0.
time (sec)	N/A	0.689	0.422	0.23	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	177	121	361	0	0	0	0
normalized size	1	1.21	0.83	2.47	0.	0.	0.	0.
time (sec)	N/A	0.335	0.217	0.162	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	181	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.57	31.678	0.434	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.493	9.882	0.145	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.461	154.	0.328	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.456	180.008	0.431	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	354	439	338	1176	0	0	0	0
normalized size	1	1.24	0.95	3.32	0.	0.	0.	0.
time (sec)	N/A	1.135	1.066	0.355	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	348	429	327	1029	0	0	0	0
normalized size	1	1.23	0.94	2.96	0.	0.	0.	0.
time (sec)	N/A	1.056	0.914	0.276	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	305	232	737	0	0	0	0
normalized size	1	1.24	0.94	3.	0.	0.	0.	0.
time (sec)	N/A	0.524	0.591	0.208	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	290	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.872	32.135	0.463	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	156	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.662	60.638	0.555	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.542	149.076	0.648	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	106	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.612	180.005	0.418	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	454	565	446	1676	0	0	0	0
normalized size	1	1.24	0.98	3.69	0.	0.	0.	0.
time (sec)	N/A	1.515	1.703	0.43	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	448	555	436	1499	0	0	0	0
normalized size	1	1.24	0.97	3.35	0.	0.	0.	0.
time (sec)	N/A	1.331	1.363	0.375	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	436	343	1176	0	0	0	0
normalized size	1	1.24	0.98	3.35	0.	0.	0.	0.
time (sec)	N/A	0.652	1.039	0.28	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	385	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.985	8.358	0.553	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	160	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.712	16.579	0.619	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.53	20.44	0.685	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.526	158.216	0.878	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	424	190	1046	0	0	0	0
normalized size	1	1.26	0.56	3.1	0.	0.	0.	0.
time (sec)	N/A	0.86	0.669	0.39	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	301	149	758	0	0	0	0
normalized size	1	1.28	0.63	3.21	0.	0.	0.	0.
time (sec)	N/A	0.787	0.485	0.372	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	298	144	634	0	0	0	0
normalized size	1	1.26	0.61	2.68	0.	0.	0.	0.
time (sec)	N/A	0.768	0.475	0.303	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	175	117	377	0	0	0	0
normalized size	1	1.29	0.86	2.77	0.	0.	0.	0.
time (sec)	N/A	0.626	0.274	0.218	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	169	107	283	0	0	0	0
normalized size	1	1.3	0.82	2.18	0.	0.	0.	0.
time (sec)	N/A	0.431	0.218	0.168	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	50	50	57	0	153	0	0
normalized size	1	1.35	1.35	1.54	0.	4.14	0.	0.
time (sec)	N/A	0.215	0.031	0.04	0.	2.043	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.528	4.603	0.231	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.542	1.505	0.227	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.568	28.811	0.619	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	106	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.646	5.782	0.152	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.403	21.51	0.215	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.32	2.217	0.191	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.549	21.845	0.369	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.56	19.868	0.257	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	108	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.654	5.031	0.394	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.557	48.562	0.533	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.557	7.442	0.517	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.391	43.802	0.418	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.305	3.578	0.316	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.543	39.014	0.662	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.551	14.496	0.749	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.532	1.114	0.875	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.45	0.185	0.865	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.522	0.635	0.358	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.558	1.213	0.5	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.551	1.685	0.509	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	45	45	51	0	120	0	0
normalized size	1	1.41	1.41	1.59	0.	3.75	0.	0.
time (sec)	N/A	0.148	0.025	0.046	0.	1.969	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	259	269	300	0	0	0	0	0
normalized size	1	1.04	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	1.747	2.494	0.237	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	340	350	384	0	0	0	0	0
normalized size	1	1.03	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	1.772	1.488	0.275	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	241	251	331	0	0	0	0	0
normalized size	1	1.04	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	1.154	4.052	0.205	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	246	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.722	1.619	0.178	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	186	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.662	4.125	0.222	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	479	491	527	0	0	0	0	0
normalized size	1	1.03	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	2.133	3.584	0.396	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	462	474	498	0	0	0	0	0
normalized size	1	1.03	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	2.267	2.906	0.487	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	363	375	508	0	0	0	0	0
normalized size	1	1.03	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	1.775	6.963	0.318	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	387	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.924	1.931	0.28	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	288	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.496	3.085	0.326	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	363	154	0	0	0	0	0
normalized size	1	1.03	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.723	0.24	0.354	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	117	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.375	0.132	0.526	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	0	0	0
normalized size	1	1.	1.	0.85	0.	0.	0.	0.
time (sec)	N/A	0.164	0.032	0.05	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	1.584	0.336	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	192	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.432	2.128	0.376	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	511	523	198	0	0	0	0	0
normalized size	1	1.02	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	1.098	0.461	0.309	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	302	302	136	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.618	0.394	0.499	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	0	0	0
normalized size	1	1.	1.	0.85	0.	0.	0.	0.
time (sec)	N/A	0.161	0.044	0.047	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.232	1.674	0.302	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	580	592	213	0	0	0	0	0
normalized size	1	1.02	0.37	0.	0.	0.	0.	0.
time (sec)	N/A	1.526	0.485	0.309	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	148	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.707	0.477	0.51	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	0	0	0
normalized size	1	1.	1.	0.85	0.	0.	0.	0.
time (sec)	N/A	0.153	0.032	0.047	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.228	1.522	0.299	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	368	376	165	0	0	0	0	0
normalized size	1	1.02	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.779	0.27	0.336	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	121	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.392	0.138	0.47	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	44	0	0	0	0
normalized size	1	1.	1.	0.88	0.	0.	0.	0.
time (sec)	N/A	0.175	0.047	0.055	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.258	0.903	0.283	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	198	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.479	2.098	0.321	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	525	533	219	0	0	0	0	0
normalized size	1	1.02	0.42	0.	0.	0.	0.	0.
time (sec)	N/A	1.282	0.442	0.261	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	316	316	144	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.728	0.363	0.448	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	44	0	0	0	0
normalized size	1	1.	1.	0.88	0.	0.	0.	0.
time (sec)	N/A	0.168	0.051	0.054	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	0.966	0.236	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	83	72	0	0	0	0	0
normalized size	1	1.28	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.099	0.312	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	438	438	209	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.445	0.422	0.316	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	153	0	0	0	0	0
normalized size	1	1.	0.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.339	0.248	0.328	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	175	175	114	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.254	0.142	0.504	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	0	0	0	0
normalized size	1	1.	1.	0.89	0.	0.	0.	0.
time (sec)	N/A	0.16	0.037	0.046	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	1.748	0.3	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	2.382	0.378	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	444	411	0	0	0	0	0
normalized size	1	1.03	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.461	1.196	0.322	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	286	295	239	0	0	0	0	0
normalized size	1	1.03	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.349	0.467	0.319	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	176	127	0	0	0	0	0
normalized size	1	1.04	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.239	0.256	0.487	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	0	131	0	0
normalized size	1	1.	1.	0.89	0.	2.85	0.	0.
time (sec)	N/A	0.154	0.034	0.046	0.	2.24	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	109	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.234	1.727	0.289	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	109	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	2.252	0.363	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	337	317	0	0	0	0	0
normalized size	1	1.02	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.745	0.581	0.326	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	207	141	0	0	0	0	0
normalized size	1	1.03	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.223	0.3	0.499	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	134	0	0
normalized size	1	1.	1.	0.85	0.	2.79	0.	0.
time (sec)	N/A	0.154	0.033	0.048	0.	1.962	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	113	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.228	1.74	0.3	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	113	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	2.264	0.377	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	181	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.62	0.995	0.454	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	379	379	241	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.68	1.247	0.391	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	214	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.376	0.688	0.286	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	211	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.082	0.226	0.342	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.902	0.225	0.344	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	658	658	438	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	1.052	3.185	0.319	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	578	578	500	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.825	2.094	0.289	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	450	450	384	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.564	2.087	0.216	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	414	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.836	0.28	0.244	0.	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	291	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.517	0.561	0.272	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	870	870	677	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	1.242	6.881	0.319	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	793	793	633	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	1.054	3.809	0.286	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	674	674	538	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.703	5.366	0.228	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	804	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.555	0.302	0.273	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	485	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.188	0.585	0.27	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	323	375	292	0	0	0	0	0
normalized size	1	1.16	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.732	1.273	0.322	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	250	212	0	0	0	0	0
normalized size	1	1.18	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.616	0.788	0.283	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	180	154	0	0	0	0	0
normalized size	1	1.17	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.422	0.243	0.278	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	56	56	53	0	487	0	0
normalized size	1	1.3	1.3	1.23	0.	11.33	0.	0.
time (sec)	N/A	0.208	0.039	0.034	0.	2.604	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.435	2.482	0.25	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.446	1.293	0.232	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	379	379	291	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.809	1.074	0.335	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.64	0.755	0.346	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	153	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.419	0.224	0.281	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	54	0	509	0	0
normalized size	1	1.	1.	0.95	0.	8.93	0.	0.
time (sec)	N/A	0.194	0.045	0.035	0.	2.685	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.443	0.314	0.304	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.465	0.336	0.282	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.525	0.624	0.313	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.368	0.361	0.28	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.223	0.078	0.22	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.521	0.38	0.276	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.525	0.414	0.267	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.417	0.443	0.311	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	1.113	0.305	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.584	0.209	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.019	0.167	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.587	0.258	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.907	0.271	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.498	0.601	0.326	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.401	0.104	0.401	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.436	0.413	0.331	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.503	0.648	0.298	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	122	133	240	329	230	232
normalized size	1	1.	0.69	0.75	1.36	1.86	1.3	1.31
time (sec)	N/A	0.142	0.106	0.029	1.166	2.497	9.528	1.276

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	140	250	289	308	212	267
normalized size	1	1.	0.87	1.55	1.8	1.91	1.32	1.66
time (sec)	N/A	0.137	0.166	0.019	1.148	2.32	6.395	1.345

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	101	115	188	271	178	194
normalized size	1	1.	0.73	0.83	1.36	1.96	1.29	1.41
time (sec)	N/A	0.121	0.101	0.01	1.143	2.404	3.017	1.254

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	120	202	235	255	160	238
normalized size	1	1.	0.98	1.66	1.93	2.09	1.31	1.95
time (sec)	N/A	0.109	0.132	0.013	1.105	2.423	1.897	1.343

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	76	90	123	208	116	146
normalized size	1	1.	0.81	0.96	1.31	2.21	1.23	1.55
time (sec)	N/A	0.08	0.086	0.008	1.088	2.305	0.911	1.23

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	264	264	119	130	0	0	0	0
normalized size	1	1.	0.45	0.49	0.	0.	0.	0.
time (sec)	N/A	0.676	0.255	0.112	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	105	95	88	298	0	0
normalized size	1	1.	1.4	1.27	1.17	3.97	0.	0.
time (sec)	N/A	0.099	0.132	0.017	1.716	2.617	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	101	126	0	0	0	0
normalized size	1	1.	0.4	0.5	0.	0.	0.	0.
time (sec)	N/A	0.611	0.137	0.147	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	128	146	120	325	0	0
normalized size	1	1.	1.36	1.55	1.28	3.46	0.	0.
time (sec)	N/A	0.104	0.253	0.019	1.703	2.727	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	192	227	412	562	422	383
normalized size	1	1.	0.6	0.71	1.29	1.76	1.32	1.2
time (sec)	N/A	0.411	0.263	0.014	1.026	2.385	31.74	1.368

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	341	341	214	440	485	539	389	432
normalized size	1	1.	0.63	1.29	1.42	1.58	1.14	1.27
time (sec)	N/A	0.36	0.335	0.019	1.146	2.546	19.748	1.446

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	163	195	333	471	340	328
normalized size	1	1.	0.63	0.75	1.28	1.81	1.31	1.26
time (sec)	N/A	0.318	0.212	0.013	1.146	2.384	23.233	1.353

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	183	363	405	436	306	387
normalized size	1	1.	0.68	1.35	1.51	1.62	1.14	1.44
time (sec)	N/A	0.249	0.306	0.016	1.133	2.574	11.707	1.452

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	130	157	243	370	246	262
normalized size	1	1.	0.66	0.8	1.24	1.89	1.26	1.34
time (sec)	N/A	0.203	0.192	0.012	1.183	2.427	3.76	1.292

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	369	217	225	0	0	0	0
normalized size	1	1.08	0.63	0.66	0.	0.	0.	0.
time (sec)	N/A	0.786	0.415	0.239	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	185	128	177	184	512	0	0
normalized size	1	1.16	0.8	1.11	1.15	3.2	0.	0.
time (sec)	N/A	0.302	0.231	0.018	1.725	2.962	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	321	321	173	198	0	0	0	0
normalized size	1	1.	0.54	0.62	0.	0.	0.	0.
time (sec)	N/A	0.814	0.437	0.316	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	133	196	176	487	0	0
normalized size	1	1.	0.72	1.07	0.96	2.65	0.	0.
time (sec)	N/A	0.279	0.237	0.02	1.689	3.348	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	276	335	609	868	638	552
normalized size	1	1.	0.63	0.77	1.4	2.	1.47	1.27
time (sec)	N/A	0.617	0.389	0.013	1.198	2.586	130.21	1.468

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	494	494	294	659	706	798	604	612
normalized size	1	1.	0.6	1.33	1.43	1.62	1.22	1.24
time (sec)	N/A	0.648	0.529	0.02	1.191	2.531	50.657	1.573

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	236	289	505	701	532	479
normalized size	1	1.	0.65	0.79	1.38	1.92	1.46	1.31
time (sec)	N/A	0.541	0.312	0.012	1.165	2.409	36.968	1.416

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	358	358	256	553	601	657	490	552
normalized size	1	1.	0.72	1.54	1.68	1.84	1.37	1.54
time (sec)	N/A	0.364	0.385	0.017	1.177	2.452	19.446	1.584

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	193	235	387	559	396	396
normalized size	1	1.	0.67	0.82	1.35	1.95	1.38	1.38
time (sec)	N/A	0.376	0.272	0.012	1.13	2.353	13.365	1.37

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	509	509	314	351	0	0	0	0
normalized size	1	1.	0.62	0.69	0.	0.	0.	0.
time (sec)	N/A	1.092	0.744	0.139	0.	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	182	282	302	716	0	0
normalized size	1	1.	0.69	1.06	1.14	2.7	0.	0.
time (sec)	N/A	0.418	0.337	0.019	1.685	3.127	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	476	476	267	296	0	0	0	0
normalized size	1	1.	0.56	0.62	0.	0.	0.	0.
time (sec)	N/A	1.76	0.658	0.165	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	184	278	271	683	0	0
normalized size	1	1.	0.71	1.07	1.04	2.63	0.	0.
time (sec)	N/A	0.462	0.375	0.021	1.686	4.279	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	265	331	560	803	600	547
normalized size	1	1.	0.67	0.84	1.42	2.03	1.52	1.38
time (sec)	N/A	0.475	0.391	0.013	1.115	2.694	41.694	1.462

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	627	627	524	364	0	0	0	0
normalized size	1	1.	0.84	0.58	0.	0.	0.	0.
time (sec)	N/A	1.052	1.381	3.506	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	521	521	512	2912	0	0	0	0
normalized size	1	1.	0.98	5.59	0.	0.	0.	0.
time (sec)	N/A	0.911	0.522	0.29	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	544	544	457	284	0	0	0	0
normalized size	1	1.	0.84	0.52	0.	0.	0.	0.
time (sec)	N/A	0.902	0.698	0.617	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	449	449	447	2805	0	0	0	0
normalized size	1	1.	1.	6.25	0.	0.	0.	0.
time (sec)	N/A	0.738	0.13	0.194	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	397	232	0	0	0	0
normalized size	1	1.	0.79	0.46	0.	0.	0.	0.
time (sec)	N/A	0.734	0.32	0.065	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	489	472	418	393	0	0	0	0
normalized size	1	0.97	0.85	0.8	0.	0.	0.	0.
time (sec)	N/A	0.925	0.775	0.159	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	543	543	549	329	0	0	0	0
normalized size	1	1.	1.01	0.61	0.	0.	0.	0.
time (sec)	N/A	0.906	1.397	0.774	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	550	531	479	462	0	0	0	0
normalized size	1	0.97	0.87	0.84	0.	0.	0.	0.
time (sec)	N/A	0.955	1.332	0.202	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	624	624	641	410	0	0	0	0
normalized size	1	1.	1.03	0.66	0.	0.	0.	0.
time (sec)	N/A	0.981	1.484	0.811	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	562	562	693	2964	0	0	0	0
normalized size	1	1.	1.23	5.27	0.	0.	0.	0.
time (sec)	N/A	0.991	1.915	0.338	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	123	638	0	1122	0	0
normalized size	1	1.	1.09	5.65	0.	9.93	0.	0.
time (sec)	N/A	0.094	0.308	0.047	0.	2.332	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD
size	598	581	0	529	0	0	0	0
normalized size	1	0.97	0.	0.88	0.	0.	0.	0.
time (sec)	N/A	1.062	4.87	0.203	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD
size	634	616	0	723	0	0	0	0
normalized size	1	0.97	0.	1.14	0.	0.	0.	0.
time (sec)	N/A	1.087	5.71	0.24	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	839	839	776	1749	0	0	0	0
normalized size	1	1.	0.92	2.08	0.	0.	0.	0.
time (sec)	N/A	2.188	2.225	2.26	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	792	792	719	1689	0	0	0	0
normalized size	1	1.	0.91	2.13	0.	0.	0.	0.
time (sec)	N/A	1.969	1.797	0.957	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	804	804	733	1695	0	0	0	0
normalized size	1	1.	0.91	2.11	0.	0.	0.	0.
time (sec)	N/A	1.024	1.893	0.884	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	846	846	820	1821	0	0	0	0
normalized size	1	1.	0.97	2.15	0.	0.	0.	0.
time (sec)	N/A	2.022	2.527	2.61	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	737	737	1155	5196	0	0	0	0
normalized size	1	1.	1.57	7.05	0.	0.	0.	0.
time (sec)	N/A	1.182	7.11	0.79	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	241	192	2499	0	2473	0	0
normalized size	1	1.04	0.83	10.82	0.	10.71	0.	0.
time (sec)	N/A	0.362	0.89	0.038	0.	3.333	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	183	2443	0	2522	0	0
normalized size	1	1.	1.03	13.8	0.	14.25	0.	0.
time (sec)	N/A	0.135	0.98	0.03	0.	3.394	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD
size	772	755	0	1478	0	0	0	0
normalized size	1	0.98	0.	1.91	0.	0.	0.	0.
time (sec)	N/A	1.246	8.131	0.26	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD
size	834	815	0	1928	0	0	0	0
normalized size	1	0.98	0.	2.31	0.	0.	0.	0.
time (sec)	N/A	1.295	12.01	0.403	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1224	1224	1185	3125	0	0	0	0
normalized size	1	1.	0.97	2.55	0.	0.	0.	0.
time (sec)	N/A	3.962	6.901	2.02	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1234	1234	1193	2269	0	0	0	0
normalized size	1	1.	0.97	1.84	0.	0.	0.	0.
time (sec)	N/A	2.918	6.738	1.105	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1234	1234	1184	3128	0	0	0	0
normalized size	1	1.	0.96	2.53	0.	0.	0.	0.
time (sec)	N/A	1.46	6.387	1.283	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	5.763	0.862	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	3.742	0.434	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	556	0	0	736	0	0
normalized size	1	1.	5.5	0.	0.	7.29	0.	0.
time (sec)	N/A	0.192	3.118	0.536	0.	1.948	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	190	633	0	0	1503	0	0
normalized size	1	1.04	3.48	0.	0.	8.26	0.	0.
time (sec)	N/A	0.183	2.242	0.806	0.	2.525	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	685	0	0	2782	0	0
normalized size	1	1.	2.41	0.	0.	9.8	0.	0.
time (sec)	N/A	0.801	3.79	1.055	0.	3.404	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	529	397	0	0	0	0	0
normalized size	1	0.95	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	2.809	1.387	5.19	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	332	293	0	0	0	0	0
normalized size	1	0.94	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.56	0.497	4.069	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	187	186	0	0	0	0	0
normalized size	1	0.94	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	0.619	3.49	0.	0.	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	9.651	0.619	0.	0.	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	8.439	0.583	0.	0.	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	15.822	0.589	0.	0.	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	609	609	453	632	923	1338	996	986
normalized size	1	1.	0.74	1.04	1.52	2.2	1.64	1.62
time (sec)	N/A	2.099	0.843	0.085	1.159	1.931	20.429	2.571

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	299	402	579	845	602	656
normalized size	1	1.	0.83	1.12	1.61	2.35	1.68	1.83
time (sec)	N/A	1.199	0.536	0.067	1.115	1.898	7.01	2.16

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	174	217	294	454	286	373
normalized size	1	1.	1.04	1.29	1.75	2.7	1.7	2.22
time (sec)	N/A	0.573	0.277	0.047	1.093	1.87	1.862	1.779

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	84	78	97	212	88	150
normalized size	1	1.	1.65	1.53	1.9	4.16	1.73	2.94
time (sec)	N/A	0.157	0.087	0.04	1.174	1.716	0.364	1.326

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	763	763	623	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	1.308	0.57	0.402	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	15.351	0.299	0.	0.	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	11.039	0.289	0.	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	3.595	0.24	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	7.258	0.24	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	380	254	380	0	0	0	0
normalized size	1	0.98	0.65	0.98	0.	0.	0.	0.
time (sec)	N/A	0.793	0.506	0.123	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	176	125	178	0	0	0	0
normalized size	1	1.27	0.9	1.28	0.	0.	0.	0.
time (sec)	N/A	0.382	0.222	0.096	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	56	0	0	0	0
normalized size	1	1.	0.85	1.04	0.	0.	0.	0.
time (sec)	N/A	0.07	0.063	0.032	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.568	0.359	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	3.136	0.361	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	1.096	0.281	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	1.05	0.256	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	1.488	0.217	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	3.74	0.222	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	510	498	456	1102	0	0	0	0
normalized size	1	0.98	0.89	2.16	0.	0.	0.	0.
time (sec)	N/A	0.951	3.061	0.226	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	249	225	465	0	0	0	0
normalized size	1	0.97	0.88	1.81	0.	0.	0.	0.
time (sec)	N/A	0.595	1.529	0.144	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	86	80	125	0	0	0	0
normalized size	1	0.96	0.89	1.39	0.	0.	0.	0.
time (sec)	N/A	0.323	0.322	0.049	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	172.089	0.33	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	180.005	0.422	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	26.441	0.29	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	23.629	0.258	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	180.001	0.223	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	180.004	0.219	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	672	672	536	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	2.402	6.368	0.256	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	322	322	317	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	1.287	2.672	0.119	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	100	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.42	0.188	0.	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	4.663	0.25	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	20.744	0.396	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	442	442	812	0	0	0	0	0
normalized size	1	1.	1.84	0.	0.	0.	0.	0.
time (sec)	N/A	1.732	3.329	0.122	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	269	0	0	0	0	0
normalized size	1	1.	1.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.402	0.643	0.	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	2.068	0.245	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	12.723	0.426	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	608	608	530	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	1.148	1.092	0.246	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	213	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.535	0.608	0.127	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	100	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.111	0.001	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.151	0.251	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.275	0.38	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	358	358	268	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.831	1.914	0.118	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	132	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.416	0.253	0.	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.165	0.25	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.289	0.392	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [486] had the largest ratio of [0.9048]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.	23	0.261
2	A	7	7	1.	23	0.304
3	A	6	6	1.	23	0.261
4	A	4	3	1.	21	0.143
5	A	4	4	1.	20	0.2
6	A	8	8	1.	23	0.348
7	A	5	6	1.	23	0.261
8	A	9	9	1.	23	0.391
9	A	5	6	1.	23	0.261
10	A	7	7	1.28	25	0.28
11	A	9	10	1.42	25	0.4
12	A	6	6	1.26	25	0.24
13	A	5	3	1.	23	0.13
14	A	6	6	1.24	22	0.273
15	A	12	8	1.	25	0.32
16	A	8	8	1.35	25	0.32
17	A	13	11	1.	25	0.44
18	A	8	9	1.31	25	0.36
19	A	6	6	1.27	25	0.24
20	A	11	10	1.43	25	0.4
21	A	6	6	1.26	25	0.24
22	A	6	3	1.	23	0.13
23	A	6	6	1.24	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	17	8	1.	25	0.32
25	A	8	8	1.33	25	0.32
26	A	18	11	1.	25	0.44
27	A	9	9	1.29	25	0.36
28	A	12	8	1.	25	0.32
29	A	8	8	1.	25	0.32
30	A	8	6	1.	25	0.24
31	A	5	5	1.	23	0.217
32	A	6	4	1.	22	0.182
33	A	7	5	1.	25	0.2
34	A	9	7	1.	25	0.28
35	A	9	7	1.	25	0.28
36	A	14	9	1.	25	0.36
37	A	12	9	1.	25	0.36
38	A	10	10	1.	25	0.4
39	A	8	6	1.	25	0.24
40	A	2	2	1.	23	0.087
41	A	8	6	1.	22	0.273
42	A	9	7	1.	25	0.28
43	A	13	11	1.	25	0.44
44	A	13	10	1.	25	0.4
45	A	20	13	1.	25	0.52
46	A	13	10	1.	25	0.4
47	A	7	7	1.	25	0.28
48	A	10	7	1.	25	0.28
49	A	3	3	1.	23	0.13
50	A	10	6	1.	22	0.273
51	A	12	8	1.	25	0.32
52	A	17	11	1.	25	0.44
53	A	17	11	1.	25	0.44
54	A	26	13	1.	25	0.52
55	A	6	4	1.	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	8	6	1.	18	0.333
57	A	10	6	1.	18	0.333
58	A	8	5	1.	27	0.185
59	A	6	5	1.	27	0.185
60	A	4	4	1.	24	0.167
61	A	4	4	1.	27	0.148
62	A	4	3	1.07	27	0.111
63	A	5	7	1.14	27	0.259
64	A	5	7	1.09	27	0.259
65	A	4	5	1.11	27	0.185
66	A	4	5	1.1	27	0.185
67	A	3	2	1.07	25	0.08
68	A	9	7	1.	27	0.259
69	A	9	7	1.	27	0.259
70	A	11	8	1.	27	0.296
71	A	11	7	1.03	27	0.259
72	A	9	7	1.04	27	0.259
73	A	7	6	1.06	24	0.25
74	A	7	6	1.06	27	0.222
75	A	7	6	1.06	27	0.222
76	A	5	4	1.08	27	0.148
77	A	6	8	1.3	27	0.296
78	A	6	8	1.22	27	0.296
79	A	6	8	1.17	27	0.296
80	A	5	6	1.15	27	0.222
81	A	5	6	1.14	27	0.222
82	A	5	6	1.12	27	0.222
83	A	4	3	1.08	25	0.12
84	A	11	8	1.04	27	0.296
85	A	12	9	1.04	27	0.333
86	A	12	9	1.04	27	0.333
87	A	15	9	1.07	27	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	13	9	1.08	27	0.333
89	A	9	7	1.11	24	0.292
90	A	11	9	1.11	27	0.333
91	A	11	8	1.11	27	0.296
92	A	11	8	1.11	27	0.296
93	A	5	4	1.07	27	0.148
94	A	7	9	1.43	27	0.333
95	A	6	8	1.35	27	0.296
96	A	5	6	1.15	27	0.222
97	A	5	6	1.13	27	0.222
98	A	5	6	1.11	27	0.222
99	A	4	3	1.07	25	0.12
100	A	14	9	1.08	27	0.333
101	A	14	10	1.08	27	0.37
102	A	15	10	1.08	27	0.37
103	A	4	4	1.27	14	0.286
104	A	7	5	1.1	27	0.185
105	A	6	4	1.08	27	0.148
106	A	5	5	1.1	27	0.185
107	A	4	4	1.06	27	0.148
108	A	3	3	1.11	25	0.12
109	A	2	2	1.	24	0.083
110	A	7	5	1.	27	0.185
111	A	3	3	1.11	27	0.111
112	A	9	7	1.03	27	0.259
113	A	5	5	1.1	27	0.185
114	A	5	9	1.12	27	0.333
115	A	8	7	1.05	27	0.259
116	A	4	7	1.09	27	0.259
117	A	4	4	1.	27	0.148
118	A	3	3	1.	25	0.12
119	A	3	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	9	7	1.	27	0.259
121	A	5	7	1.01	27	0.259
122	A	12	9	1.	27	0.333
123	A	6	7	1.	27	0.259
124	A	6	9	1.15	27	0.333
125	A	8	6	1.12	27	0.222
126	A	5	9	1.54	27	0.333
127	A	5	4	1.2	27	0.148
128	A	4	4	1.21	25	0.16
129	A	5	5	1.17	24	0.208
130	A	12	8	1.05	27	0.296
131	A	6	8	1.12	27	0.296
132	A	16	11	1.06	27	0.407
133	A	6	8	1.13	27	0.296
134	A	7	5	1.12	20	0.25
135	A	6	4	1.42	22	0.182
136	A	5	5	1.44	22	0.227
137	A	4	4	1.42	22	0.182
138	A	3	3	1.49	20	0.15
139	A	2	2	1.41	19	0.105
140	A	7	5	1.38	22	0.227
141	A	3	3	1.5	22	0.136
142	A	9	7	1.38	22	0.318
143	A	2	2	1.13	30	0.067
144	A	2	2	1.	31	0.065
145	A	8	9	1.	27	0.333
146	A	7	8	1.	27	0.296
147	A	6	7	1.	25	0.28
148	A	0	0	0.	0	0.
149	A	0	0	0.	0	0.
150	A	0	0	0.	0	0.
151	A	10	7	1.06	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	7	6	1.05	29	0.207
153	A	4	4	1.04	29	0.138
154	A	2	2	1.	29	0.069
155	A	4	4	1.	29	0.138
156	A	6	4	1.03	29	0.138
157	A	9	6	1.01	35	0.171
158	A	6	5	1.02	35	0.143
159	A	3	3	1.03	35	0.086
160	A	2	2	1.	35	0.057
161	A	4	4	1.	35	0.114
162	A	6	4	1.	35	0.114
163	A	2	2	1.1	24	0.083
164	A	14	5	1.	20	0.25
165	A	10	5	1.	20	0.25
166	A	6	4	1.	18	0.222
167	A	8	5	1.	20	0.25
168	A	11	8	1.	20	0.4
169	A	15	9	1.	20	0.45
170	A	17	9	1.	29	0.31
171	A	12	9	1.	29	0.31
172	A	6	6	1.04	27	0.222
173	A	6	6	1.	26	0.231
174	A	13	9	1.	29	0.31
175	A	8	8	1.	29	0.276
176	A	13	10	1.	29	0.345
177	A	11	11	1.02	29	0.379
178	A	26	13	1.02	29	0.448
179	A	20	13	1.03	29	0.448
180	A	8	8	1.04	27	0.296
181	A	11	9	1.04	26	0.346
182	A	18	13	1.02	29	0.448
183	A	15	14	1.03	29	0.483

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	18	15	1.02	29	0.517
185	A	18	13	1.03	29	0.448
186	A	34	18	1.04	29	0.621
187	A	30	21	1.04	29	0.724
188	A	8	8	1.03	27	0.296
189	A	17	9	1.06	26	0.346
190	A	25	17	1.04	29	0.586
191	A	24	16	1.05	29	0.552
192	A	27	21	1.03	29	0.724
193	A	29	17	1.05	29	0.586
194	A	17	8	1.06	29	0.276
195	A	12	8	1.05	29	0.276
196	A	10	8	1.05	29	0.276
197	A	6	6	1.04	29	0.207
198	A	5	4	1.05	27	0.148
199	A	2	2	1.	26	0.077
200	A	9	6	1.	29	0.207
201	A	7	7	1.04	29	0.241
202	A	13	10	1.02	29	0.345
203	A	10	10	1.05	29	0.345
204	A	23	14	1.04	29	0.483
205	A	15	13	1.02	29	0.448
206	A	14	10	1.03	29	0.345
207	A	8	8	1.	29	0.276
208	A	8	6	1.	27	0.222
209	A	7	7	1.	26	0.269
210	A	16	11	1.	29	0.379
211	A	15	11	1.	29	0.379
212	A	26	15	1.	29	0.517
213	A	25	13	1.	29	0.448
214	A	27	13	1.05	29	0.448
215	A	19	13	1.03	29	0.448

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	17	9	1.04	29	0.31
217	A	12	12	1.04	29	0.414
218	A	10	8	1.05	27	0.296
219	A	10	10	1.05	26	0.385
220	A	25	13	1.03	29	0.448
221	A	20	15	1.06	29	0.517
222	A	39	19	1.04	29	0.655
223	A	34	18	1.08	29	0.621
224	A	14	11	1.07	22	0.5
225	A	12	8	1.35	24	0.333
226	A	9	8	1.34	24	0.333
227	A	6	6	1.37	24	0.25
228	A	4	4	1.38	22	0.182
229	A	2	2	1.41	21	0.095
230	A	9	6	1.36	24	0.25
231	A	7	7	1.4	24	0.292
232	A	13	10	1.34	24	0.417
233	A	0	0	0.	0	0.
234	A	0	0	0.	0	0.
235	A	0	0	0.	0	0.
236	A	0	0	0.	0	0.
237	A	0	0	0.	0	0.
238	A	0	0	0.	0	0.
239	A	0	0	0.	0	0.
240	A	26	14	1.	20	0.7
241	A	18	11	1.	20	0.55
242	A	10	7	1.	18	0.389
243	A	10	6	1.	20	0.3
244	A	18	10	1.	20	0.5
245	A	28	11	1.	20	0.55
246	A	25	10	1.05	22	0.454
247	A	15	9	1.03	22	0.409

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
248	A	7	6	1.	22	0.273
249	A	2	2	1.	22	0.091
250	A	8	8	1.	22	0.364
251	A	12	11	1.04	22	0.5
252	A	18	12	1.05	22	0.546
253	A	14	5	1.36	24	0.208
254	A	11	7	1.35	24	0.292
255	A	7	5	1.37	24	0.208
256	A	5	4	1.39	22	0.182
257	A	2	2	1.41	21	0.095
258	A	11	7	1.34	24	0.292
259	A	8	8	1.38	24	0.333
260	A	19	11	1.33	24	0.458
261	A	0	0	0.	0	0.
262	A	7	3	1.	20	0.15
263	A	6	3	1.	20	0.15
264	A	5	3	1.	18	0.167
265	A	0	0	0.	0	0.
266	A	0	0	0.	0	0.
267	A	13	6	1.27	28	0.214
268	A	13	6	1.25	28	0.214
269	A	7	6	1.28	28	0.214
270	A	10	6	1.24	26	0.231
271	A	7	6	1.28	25	0.24
272	A	0	0	0.	0	0.
273	A	0	0	0.	0	0.
274	A	0	0	0.	0	0.
275	A	0	0	0.	0	0.
276	A	16	6	1.25	28	0.214
277	A	13	6	1.27	28	0.214
278	A	13	6	1.25	26	0.231
279	A	10	6	1.27	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	0	0	0.	0	0.
281	A	0	0	0.	0	0.
282	A	0	0	0.	0	0.
283	A	0	0	0.	0	0.
284	A	16	6	1.25	28	0.214
285	A	16	6	1.27	28	0.214
286	A	16	6	1.25	26	0.231
287	A	13	6	1.27	25	0.24
288	A	0	0	0.	0	0.
289	A	0	0	0.	0	0.
290	A	0	0	0.	0	0.
291	A	0	0	0.	0	0.
292	A	6	4	1.4	24	0.167
293	A	6	4	1.4	24	0.167
294	A	5	4	1.4	24	0.167
295	A	3	3	1.46	22	0.136
296	A	2	2	1.46	21	0.095
297	A	0	0	0.	0	0.
298	A	0	0	0.	0	0.
299	A	10	6	1.24	28	0.214
300	A	7	6	1.28	28	0.214
301	A	5	5	1.24	26	0.192
302	A	2	2	1.37	25	0.08
303	A	0	0	0.	0	0.
304	A	0	0	0.	0	0.
305	A	0	0	0.	0	0.
306	A	0	0	0.	0	0.
307	A	0	0	0.	0	0.
308	A	0	0	0.	0	0.
309	A	0	0	0.	0	0.
310	A	0	0	0.	0	0.
311	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
312	A	0	0	0.	0	0.
313	A	0	0	0.	0	0.
314	A	0	0	0.	0	0.
315	A	8	4	1.	20	0.2
316	A	7	4	1.	20	0.2
317	A	6	4	1.	18	0.222
318	A	0	0	0.	0	0.
319	A	0	0	0.	0	0.
320	A	23	7	1.23	28	0.25
321	A	17	8	1.2	28	0.286
322	A	15	8	1.69	26	0.308
323	A	8	8	1.21	25	0.32
324	A	0	0	0.	0	0.
325	A	0	0	0.	0	0.
326	A	0	0	0.	0	0.
327	A	0	0	0.	0	0.
328	A	20	7	1.24	28	0.25
329	A	23	9	1.23	26	0.346
330	A	11	7	1.24	25	0.28
331	A	0	0	0.	0	0.
332	A	0	0	0.	0	0.
333	A	0	0	0.	0	0.
334	A	0	0	0.	0	0.
335	A	29	7	1.24	28	0.25
336	A	29	9	1.24	26	0.346
337	A	14	7	1.24	25	0.28
338	A	0	0	0.	0	0.
339	A	0	0	0.	0	0.
340	A	0	0	0.	0	0.
341	A	0	0	0.	0	0.
342	A	14	7	1.26	28	0.25
343	A	11	7	1.28	28	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
344	A	11	7	1.26	28	0.25
345	A	8	8	1.29	28	0.286
346	A	6	6	1.3	26	0.231
347	A	2	2	1.35	25	0.08
348	A	0	0	0.	0	0.
349	A	0	0	0.	0	0.
350	A	0	0	0.	0	0.
351	A	0	0	0.	0	0.
352	A	0	0	0.	0	0.
353	A	0	0	0.	0	0.
354	A	0	0	0.	0	0.
355	A	0	0	0.	0	0.
356	A	0	0	0.	0	0.
357	A	0	0	0.	0	0.
358	A	0	0	0.	0	0.
359	A	0	0	0.	0	0.
360	A	0	0	0.	0	0.
361	A	0	0	0.	0	0.
362	A	0	0	0.	0	0.
363	A	0	0	0.	0	0.
364	A	0	0	0.	0	0.
365	A	0	0	0.	0	0.
366	A	0	0	0.	0	0.
367	A	0	0	0.	0	0.
368	A	2	2	1.41	21	0.095
369	A	27	7	1.04	27	0.259
370	A	32	7	1.03	27	0.259
371	A	17	9	1.04	25	0.36
372	A	14	7	1.	24	0.292
373	A	0	0	0.	0	0.
374	A	32	7	1.03	29	0.241
375	A	42	7	1.03	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	32	9	1.03	27	0.333
377	A	19	7	1.	26	0.269
378	A	0	0	0.	0	0.
379	A	25	12	1.03	24	0.5
380	A	11	10	1.	24	0.417
381	A	2	2	1.	24	0.083
382	A	0	0	0.	0	0.
383	A	0	0	0.	0	0.
384	A	27	13	1.02	24	0.542
385	A	12	10	1.	24	0.417
386	A	2	2	1.	24	0.083
387	A	0	0	0.	0	0.
388	A	40	15	1.02	24	0.625
389	A	14	12	1.	24	0.5
390	A	2	2	1.	24	0.083
391	A	0	0	0.	0	0.
392	A	25	12	1.02	24	0.5
393	A	11	10	1.	24	0.417
394	A	2	2	1.	24	0.083
395	A	0	0	0.	0	0.
396	A	0	0	0.	0	0.
397	A	27	13	1.02	24	0.542
398	A	12	10	1.	24	0.417
399	A	2	2	1.	24	0.083
400	A	0	0	0.	0	0.
401	A	7	6	1.28	19	0.316
402	A	19	7	1.	24	0.292
403	A	14	7	1.	24	0.292
404	A	9	7	1.	24	0.292
405	A	2	2	1.	24	0.083
406	A	0	0	0.	0	0.
407	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
408	A	20	8	1.03	24	0.333
409	A	15	8	1.03	24	0.333
410	A	10	9	1.04	24	0.375
411	A	2	2	1.	24	0.083
412	A	0	0	0.	0	0.
413	A	0	0	0.	0	0.
414	A	19	11	1.02	24	0.458
415	A	8	7	1.03	24	0.292
416	A	2	2	1.	24	0.083
417	A	0	0	0.	0	0.
418	A	0	0	0.	0	0.
419	A	7	5	1.	29	0.172
420	A	10	5	1.	27	0.185
421	A	7	5	1.	26	0.192
422	A	0	0	0.	0	0.
423	A	0	0	0.	0	0.
424	A	13	5	1.	29	0.172
425	A	13	5	1.	27	0.185
426	A	10	5	1.	26	0.192
427	A	0	0	0.	0	0.
428	A	0	0	0.	0	0.
429	A	16	5	1.	29	0.172
430	A	16	5	1.	27	0.185
431	A	13	5	1.	26	0.192
432	A	0	0	0.	0	0.
433	A	0	0	0.	0	0.
434	A	10	5	1.16	28	0.179
435	A	7	5	1.18	28	0.179
436	A	5	4	1.17	26	0.154
437	A	2	2	1.3	25	0.08
438	A	0	0	0.	0	0.
439	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	10	5	1.	29	0.172
441	A	7	5	1.	29	0.172
442	A	5	4	1.	27	0.148
443	A	2	2	1.	26	0.077
444	A	0	0	0.	0	0.
445	A	0	0	0.	0	0.
446	A	0	0	0.	0	0.
447	A	0	0	0.	0	0.
448	A	0	0	0.	0	0.
449	A	0	0	0.	0	0.
450	A	0	0	0.	0	0.
451	A	0	0	0.	0	0.
452	A	0	0	0.	0	0.
453	A	0	0	0.	0	0.
454	A	0	0	0.	0	0.
455	A	0	0	0.	0	0.
456	A	0	0	0.	0	0.
457	A	0	0	0.	0	0.
458	A	0	0	0.	0	0.
459	A	0	0	0.	0	0.
460	A	0	0	0.	0	0.
461	A	7	5	1.	19	0.263
462	A	6	6	1.	19	0.316
463	A	5	5	1.	19	0.263
464	A	4	4	1.	17	0.235
465	A	3	3	1.	16	0.188
466	A	13	13	1.	19	0.684
467	A	4	4	1.	19	0.21
468	A	11	11	1.	19	0.579
469	A	4	4	1.	19	0.21
470	A	7	7	1.	21	0.333
471	A	9	10	1.	21	0.476

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
472	A	6	6	1.	21	0.286
473	A	7	7	1.	19	0.368
474	A	6	6	1.	18	0.333
475	A	16	15	1.08	21	0.714
476	A	7	7	1.16	21	0.333
477	A	14	15	1.	21	0.714
478	A	7	8	1.	21	0.381
479	A	6	6	1.	21	0.286
480	A	10	9	1.	21	0.429
481	A	6	6	1.	21	0.286
482	A	8	7	1.	19	0.368
483	A	6	6	1.	18	0.333
484	A	23	15	1.	21	0.714
485	A	7	7	1.	21	0.333
486	A	18	19	1.	21	0.905
487	A	9	9	1.	21	0.429
488	A	6	6	1.	18	0.333
489	A	27	12	1.	21	0.571
490	A	23	9	1.	21	0.429
491	A	23	9	1.	21	0.429
492	A	18	6	1.	19	0.316
493	A	18	6	1.	18	0.333
494	A	25	8	0.97	21	0.381
495	A	23	10	1.	21	0.476
496	A	27	10	0.97	21	0.476
497	A	28	12	1.	21	0.571
498	A	24	10	1.	21	0.476
499	A	4	4	1.	19	0.21
500	A	29	12	0.97	21	0.571
501	A	31	14	0.97	21	0.667
502	A	49	12	1.	21	0.571
503	A	46	10	1.	21	0.476

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
504	A	26	9	1.	18	0.5
505	A	49	13	1.	21	0.619
506	A	29	11	1.	21	0.524
507	A	9	10	1.04	21	0.476
508	A	5	5	1.	19	0.263
509	A	34	13	0.98	21	0.619
510	A	36	15	0.98	21	0.714
511	A	80	11	1.	21	0.524
512	A	62	11	1.	21	0.524
513	A	34	10	1.	18	0.556
514	A	0	0	0.	0	0.
515	A	0	0	0.	0	0.
516	A	7	8	1.	20	0.4
517	A	8	10	1.04	20	0.5
518	A	9	11	1.	20	0.55
519	A	8	9	0.95	23	0.391
520	A	7	8	0.94	23	0.348
521	A	5	5	0.94	21	0.238
522	A	0	0	0.	0	0.
523	A	0	0	0.	0	0.
524	A	0	0	0.	0	0.
525	A	26	7	1.	20	0.35
526	A	17	7	1.	20	0.35
527	A	10	7	1.	18	0.389
528	A	3	3	1.	10	0.3
529	A	22	7	1.	20	0.35
530	A	0	0	0.	0	0.
531	A	0	0	0.	0	0.
532	A	0	0	0.	0	0.
533	A	0	0	0.	0	0.
534	A	27	7	0.98	20	0.35
535	A	15	7	1.27	18	0.389

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
536	A	4	4	1.	10	0.4
537	A	0	0	0.	0	0.
538	A	0	0	0.	0	0.
539	A	0	0	0.	0	0.
540	A	0	0	0.	0	0.
541	A	0	0	0.	0	0.
542	A	0	0	0.	0	0.
543	A	26	7	0.98	20	0.35
544	A	15	7	0.97	18	0.389
545	A	5	5	0.96	10	0.5
546	A	0	0	0.	0	0.
547	A	0	0	0.	0	0.
548	A	0	0	0.	0	0.
549	A	0	0	0.	0	0.
550	A	0	0	0.	0	0.
551	A	0	0	0.	0	0.
552	A	42	9	1.	22	0.409
553	A	23	9	1.	20	0.45
554	A	7	6	1.	12	0.5
555	A	0	0	0.	0	0.
556	A	0	0	0.	0	0.
557	A	32	12	1.	20	0.6
558	A	8	7	1.	12	0.583
559	A	0	0	0.	0	0.
560	A	0	0	0.	0	0.
561	A	39	8	1.	22	0.364
562	A	21	8	1.	20	0.4
563	A	6	5	1.	12	0.417
564	A	0	0	0.	0	0.
565	A	0	0	0.	0	0.
566	A	21	8	1.	20	0.4
567	A	7	6	1.	12	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
568	A	0	0	0.	0	0.
569	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int x^4 \left(d-c^2 d x^2\right) \left(a+b \cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=151

$$-\frac{1}{7} c^2 d x^7 \left(a+b \cosh^{-1}(cx)\right)+\frac{1}{5} d x^5 \left(a+b \cosh^{-1}(cx)\right)-\frac{76 b d x^2 \sqrt{cx-1} \sqrt{cx+1}}{3675 c^3}-\frac{152 b d \sqrt{cx-1} \sqrt{cx+1}}{3675 c^5}+\frac{1}{49} b c d x^6 \sqrt{}$$

```
[Out] (-152*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^5) - (76*b*d*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(3675*c^3) - (19*b*d*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/
(1225*c) + (b*c*d*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/49 + (d*x^5*(a + b*ArcC
osh[c*x]))/5 - (c^2*d*x^7*(a + b*ArcCosh[c*x]))/7
```

Rubi [A] time = 0.150428, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {14, 5731, 12, 460, 100, 74}

$$-\frac{1}{7} c^2 d x^7 \left(a+b \cosh^{-1}(cx)\right)+\frac{1}{5} d x^5 \left(a+b \cosh^{-1}(cx)\right)-\frac{76 b d x^2 \sqrt{cx-1} \sqrt{cx+1}}{3675 c^3}-\frac{152 b d \sqrt{cx-1} \sqrt{cx+1}}{3675 c^5}+\frac{1}{49} b c d x^6 \sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-152*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^5) - (76*b*d*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(3675*c^3) - (19*b*d*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/
(1225*c) + (b*c*d*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/49 + (d*x^5*(a + b*ArcC
```

```
osh[c*x]))/5 - (c^2*d*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^4 \left(d - c^2 d x^2\right) \left(a + b \cosh^{-1}(cx)\right) dx = \frac{1}{5} d x^5 \left(a + b \cosh^{-1}(cx)\right) - \frac{1}{7} c^2 d x^7 \left(a + b \cosh^{-1}(cx)\right) - (bc) \int \frac{d x^5 \left(7 - 5c^2\right.}{35\sqrt{-1 + cx}\sqrt{}}$$

$$= \frac{1}{5} d x^5 \left(a + b \cosh^{-1}(cx)\right) - \frac{1}{7} c^2 d x^7 \left(a + b \cosh^{-1}(cx)\right) - \frac{1}{35}(bcd) \int \frac{x^5 \left(7 - \right.}{\sqrt{-1 + cx}}$$

$$= \frac{1}{49} bcd x^6 \sqrt{-1 + cx}\sqrt{1 + cx} + \frac{1}{5} d x^5 \left(a + b \cosh^{-1}(cx)\right) - \frac{1}{7} c^2 d x^7 \left(a + b \cosh^{-1}\right.$$

$$= -\frac{19bdx^4\sqrt{-1 + cx}\sqrt{1 + cx}}{1225c} + \frac{1}{49} bcd x^6 \sqrt{-1 + cx}\sqrt{1 + cx} + \frac{1}{5} d x^5 \left(a + b \cosh^{-}\right.$$

$$= -\frac{19bdx^4\sqrt{-1 + cx}\sqrt{1 + cx}}{1225c} + \frac{1}{49} bcd x^6 \sqrt{-1 + cx}\sqrt{1 + cx} + \frac{1}{5} d x^5 \left(a + b \cosh^{-}\right.$$

$$= -\frac{76bdx^2\sqrt{-1 + cx}\sqrt{1 + cx}}{3675c^3} - \frac{19bdx^4\sqrt{-1 + cx}\sqrt{1 + cx}}{1225c} + \frac{1}{49} bcd x^6 \sqrt{-1 + cx}$$

$$= -\frac{76bdx^2\sqrt{-1 + cx}\sqrt{1 + cx}}{3675c^3} - \frac{19bdx^4\sqrt{-1 + cx}\sqrt{1 + cx}}{1225c} + \frac{1}{49} bcd x^6 \sqrt{-1 + cx}$$

$$= -\frac{152bd\sqrt{-1 + cx}\sqrt{1 + cx}}{3675c^5} - \frac{76bdx^2\sqrt{-1 + cx}\sqrt{1 + cx}}{3675c^3} - \frac{19bdx^4\sqrt{-1 + cx}\sqrt{1}}{1225c}$$

Mathematica [A] time = 0.156148, size = 91, normalized size = 0.6

$$\frac{d\left(-105ax^5\left(5c^2x^2 - 7\right) + \frac{b\sqrt{cx-1}\sqrt{cx+1}\left(75c^6x^6 - 57c^4x^4 - 76c^2x^2 - 152\right)}{c^5} - 105bx^5\left(5c^2x^2 - 7\right)\cosh^{-1}(cx)\right)}{3675}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(-105*a*x^5*(-7 + 5*c^2*x^2) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-152 - 7
6*c^2*x^2 - 57*c^4*x^4 + 75*c^6*x^6))/c^5 - 105*b*x^5*(-7 + 5*c^2*x^2)*ArcC
osh[c*x]))/3675
```

Maple [A] time = 0.014, size = 98, normalized size = 0.7

$$\frac{1}{c^5}\left(-da\left(\frac{c^7x^7}{7}-\frac{c^5x^5}{5}\right)-db\left(\frac{\operatorname{arccosh}(cx)\,c^7x^7}{7}-\frac{\operatorname{arccosh}(cx)\,c^5x^5}{5}-\frac{75\,c^6x^6-57\,c^4x^4-76\,c^2x^2-152}{3675}\sqrt{cx-1}\sqrt{cx+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(-d*a*(1/7*c^7*x^7-1/5*c^5*x^5)-d*b*(1/7*arccosh(c*x)*c^7*x^7-1/5*arc
cosh(c*x)*c^5*x^5-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*x^6-57*c^4*x^4
-76*c^2*x^2-152)))
```

Maxima [A] time = 1.16226, size = 248, normalized size = 1.64

$$-\frac{1}{7}\,ac^2dx^7+\frac{1}{5}\,adx^5-\frac{1}{245}\left(35\,x^7\operatorname{arcosh}(cx)-\left(\frac{5\,\sqrt{c^2x^2-1}x^6}{c^2}+\frac{6\,\sqrt{c^2x^2-1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2-1}x^2}{c^6}+\frac{16\,\sqrt{c^2x^2-1}}{c^8}\right)c\right)bc^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 - 1/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x
^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6
 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arccosh(c*x) - (3*sq
rt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)
/c^6)*c)*b*d
```

Fricas [A] time = 1.7331, size = 266, normalized size = 1.76

$$-\frac{525\,ac^7dx^7-735\,ac^5dx^5+105\left(5\,bc^7dx^7-7\,bc^5dx^5\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(75\,bc^6dx^6-57\,bc^4dx^4-76\,bc^2dx^2-152\,b\right.}{3675\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/3675*(525*a*c^7*d*x^7 - 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 - 7*b*c^5*d
*x^5)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*d*x^6 - 57*b*c^4*d*x^4 - 76*
```

```
b*c^2*d*x^2 - 152*b*d)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] time = 8.58666, size = 158, normalized size = 1.05

$$\begin{cases} -\frac{ac^2dx^7}{7} + \frac{adx^5}{5} - \frac{bc^2dx^7\operatorname{acosh}(cx)}{7} + \frac{bcdx^6\sqrt{c^2x^2-1}}{49} + \frac{bdx^5\operatorname{acosh}(cx)}{5} - \frac{19bdx^4\sqrt{c^2x^2-1}}{1225c} - \frac{76bdx^2\sqrt{c^2x^2-1}}{3675c^3} - \frac{152bd\sqrt{c^2x^2-1}}{3675c^5} & \text{for } c \neq 0 \\ \frac{dx^5\left(a+\frac{i\pi b}{2}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**7/7 + a*d*x**5/5 - b*c**2*d*x**7*acosh(c*x)/7 + b*c
*d*x**6*sqrt(c**2*x**2 - 1)/49 + b*d*x**5*acosh(c*x)/5 - 19*b*d*x**4*sqrt(c
**2*x**2 - 1)/(1225*c) - 76*b*d*x**2*sqrt(c**2*x**2 - 1)/(3675*c**3) - 152*
b*d*sqrt(c**2*x**2 - 1)/(3675*c**5), Ne(c, 0)), (d*x**5*(a + I*pi*b/2)/5, T
rue))
```

Giac [A] time = 1.99931, size = 238, normalized size = 1.58

$$-\frac{1}{7}\,ac^2dx^7 + \frac{1}{5}\,adx^5 - \frac{1}{245}\left(35\,x^7\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{5\left(c^2x^2-1\right)^{\frac{7}{2}} + 21\left(c^2x^2-1\right)^{\frac{5}{2}} + 35\left(c^2x^2-1\right)^{\frac{3}{2}} + 35\sqrt{c^2x^2-}}{c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] -1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 - 1/245*(35*x^7*log(c*x + sqrt(c^2*x^2 - 1))
 - (5*(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5/2) + 35*(c^2*x^2 - 1)^(3/2)
 + 35*sqrt(c^2*x^2 - 1))/c^7)*b*c^2*d + 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2
 - 1)) - (3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2
- 1))/c^5)*b*d
```

3.2 $\int x^3\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=135

$$-\frac{1}{6}c^2dx^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{bdx\sqrt{cx-1}\sqrt{cx+1}}{24c^3}-\frac{bd\cosh^{-1}(cx)}{24c^4}+\frac{1}{36}bcdx^5\sqrt{cx-1}\sqrt{cx+1}$$

```
[Out] -(b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^3) - (b*d*x^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])/(36*c) + (b*c*d*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/36 - (b*d*Ar
cCosh[c*x])/(24*c^4) + (d*x^4*(a + b*ArcCosh[c*x]))/4 - (c^2*d*x^6*(a + b*A
rcCosh[c*x]))/6
```

Rubi [A] time = 0.139109, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {14, 5731, 12, 460, 100, 90, 52}

$$-\frac{1}{6}c^2dx^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{bdx\sqrt{cx-1}\sqrt{cx+1}}{24c^3}-\frac{bd\cosh^{-1}(cx)}{24c^4}+\frac{1}{36}bcdx^5\sqrt{cx-1}\sqrt{cx+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^3) - (b*d*x^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])/(36*c) + (b*c*d*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/36 - (b*d*Ar
cCosh[c*x])/(24*c^4) + (d*x^4*(a + b*ArcCosh[c*x]))/4 - (c^2*d*x^6*(a + b*A
rcCosh[c*x]))/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x^3 \left(d - c^2dx^2\right) \left(a + b\cosh^{-1}(cx)\right) dx = \frac{1}{4}dx^4 \left(a + b\cosh^{-1}(cx)\right) - \frac{1}{6}c^2dx^6 \left(a + b\cosh^{-1}(cx)\right) - (bc) \int \frac{dx^4 \left(3 - 2c^2x\right.}{12\sqrt{-1 + cx}\sqrt{1}}$$

$$= \frac{1}{4}dx^4 \left(a + b\cosh^{-1}(cx)\right) - \frac{1}{6}c^2dx^6 \left(a + b\cosh^{-1}(cx)\right) - \frac{1}{12}(bcd) \int \frac{x^4 \left(3 - 2c\right.}{\sqrt{-1 + cx}\sqrt{}}$$

$$= \frac{1}{36}bcdx^5\sqrt{-1 + cx}\sqrt{1 + cx} + \frac{1}{4}dx^4 \left(a + b\cosh^{-1}(cx)\right) - \frac{1}{6}c^2dx^6 \left(a + b\cosh^{-1}(c\right.$$

$$= -\frac{bdx^3\sqrt{-1 + cx}\sqrt{1 + cx}}{36c} + \frac{1}{36}bcdx^5\sqrt{-1 + cx}\sqrt{1 + cx} + \frac{1}{4}dx^4 \left(a + b\cosh^{-1}(cx\right.$$

$$= -\frac{bdx^3\sqrt{-1 + cx}\sqrt{1 + cx}}{36c} + \frac{1}{36}bcdx^5\sqrt{-1 + cx}\sqrt{1 + cx} + \frac{1}{4}dx^4 \left(a + b\cosh^{-1}(cx\right.$$

$$= -\frac{bdx\sqrt{-1 + cx}\sqrt{1 + cx}}{24c^3} - \frac{bdx^3\sqrt{-1 + cx}\sqrt{1 + cx}}{36c} + \frac{1}{36}bcdx^5\sqrt{-1 + cx}\sqrt{1 + cx}$$

$$= -\frac{bdx\sqrt{-1 + cx}\sqrt{1 + cx}}{24c^3} - \frac{bdx^3\sqrt{-1 + cx}\sqrt{1 + cx}}{36c} + \frac{1}{36}bcdx^5\sqrt{-1 + cx}\sqrt{1 + cx}$$

Mathematica [A] time = 0.0920071, size = 166, normalized size = 1.23

$$-\frac{1}{6}ac^2dx^6 + \frac{1}{4}adx^4 - \frac{1}{6}bc^2dx^6\cosh^{-1}(cx) - \frac{bdx\sqrt{cx - 1}\sqrt{cx + 1}}{24c^3} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)}{12c^4} + \frac{1}{36}bcdx^5\sqrt{cx - 1}\sqrt{cx + 1} - \frac{bdx^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (a*d*x^4)/4 - (a*c^2*d*x^6)/6 - (b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^
3) - (b*d*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(36*c) + (b*c*d*x^5*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/36 + (b*d*x^4*ArcCosh[c*x])/4 - (b*c^2*d*x^6*ArcCosh[c*x
])/6 - (b*d*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(12*c^4)
```

Maple [A] time = 0.017, size = 160, normalized size = 1.2

$$-\frac{c^2dax^6}{6} + \frac{dax^4}{4} - \frac{c^2db\mathrm{arccosh}\,(cx)\,x^6}{6} + \frac{db\mathrm{arccosh}\,(cx)\,x^4}{4} + \frac{dbcx^5}{36}\sqrt{cx - 1}\sqrt{cx + 1} - \frac{dbx^3}{36\,c}\sqrt{cx - 1}\sqrt{cx + 1} - \frac{dbx}{24\,c^3}\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/6*c^2*d*a*x^6+1/4*d*a*x^4-1/6*c^2*d*b*arccosh(c*x)*x^6+1/4*d*b*arccosh(c
*x)*x^4+1/36*b*c*d*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/36*b*d*x^3*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c-1/24*b*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/24/c^4*d*b*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.09547, size = 297, normalized size = 2.2

$$-\frac{1}{6}ac^2dx^6 + \frac{1}{4}adx^4 - \frac{1}{288}\left(48x^6\operatorname{arcosh}(cx) - \left(\frac{8\sqrt{c^2x^2-1}x^5}{c^2} + \frac{10\sqrt{c^2x^2-1}x^3}{c^4} + \frac{15\sqrt{c^2x^2-1}x}{c^6} + \frac{15\log\left(2c^2x+2\right.}{\sqrt{c^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 - 1/288*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x
^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6
 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^6))*c)*b*c^
2*d + 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*
x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*
c^4))*c)*b*d
```

Fricas [A] time = 1.82995, size = 243, normalized size = 1.8

$$-\frac{12ac^6dx^6 - 18ac^4dx^4 + 3\left(4bc^6dx^6 - 6bc^4dx^4 + bd\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(2bc^5dx^5 - 2bc^3dx^3 - 3bcdx\right)\sqrt{c^2x^2-1}}{72c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/72*(12*a*c^6*d*x^6 - 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 - 6*b*c^4*d*x^4 +
 b*d)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^5*d*x^5 - 2*b*c^3*d*x^3 - 3*b*c
*d*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] time = 5.38028, size = 144, normalized size = 1.07

$$\begin{cases} -\frac{ac^2dx^6}{6} + \frac{adx^4}{4} - \frac{bc^2dx^6\operatorname{acosh}(cx)}{6} + \frac{bcdx^5\sqrt{c^2x^2-1}}{36} + \frac{bdx^4\operatorname{acosh}(cx)}{4} - \frac{bdx^3\sqrt{c^2x^2-1}}{36c} - \frac{bdx\sqrt{c^2x^2-1}}{24c^3} - \frac{bd\operatorname{acosh}(cx)}{24c^4} & \text{for } c \neq 0 \\ \frac{dx^4\left(a+\frac{i\pi b}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**6/6 + a*d*x**4/4 - b*c**2*d*x**6*acosh(c*x)/6 + b*c
*d*x**5*sqrt(c**2*x**2 - 1)/36 + b*d*x**4*acosh(c*x)/4 - b*d*x**3*sqrt(c**2
*x**2 - 1)/(36*c) - b*d*x*sqrt(c**2*x**2 - 1)/(24*c**3) - b*d*acosh(c*x)/(2
4*c**4), Ne(c, 0)), (d*x**4*(a + I*pi*b/2)/4, True))
```

Giac [A] time = 1.87797, size = 273, normalized size = 2.02

$$-\frac{1}{6}ac^2dx^6 + \frac{1}{4}adx^4 - \frac{1}{288}\left(48x^6\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(\sqrt{c^2x^2-1}\left(2x^2\left(\frac{4x^2}{c^2} + \frac{5}{c^4}\right) + \frac{15}{c^6}\right)x - \frac{15\log\left(\left|-x|c| + \sqrt{c^2x^2 -\right.\right.}{c^6|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] -1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 - 1/288*(48*x^6*log(c*x + sqrt(c^2*x^2 - 1))
 - (sqrt(c^2*x^2 - 1)*(2*x^2*(4*x^2/c^2 + 5/c^4) + 15/c^6)*x - 15*log(abs(-
x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^6*abs(c)))*c)*b*c^2*d + 1/32*(8*x^4*log(c
*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*x*(2*x^2/c^2 + 3/c^4) - 3*log(
abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*abs(c)))*c)*b*d
```

3.3 $\int x^2 \left(d-c^2 d x^2\right) \left(a+b \cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=121

$$-\frac{1}{5}c^2dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{26bd\sqrt{cx-1}\sqrt{cx+1}}{225c^3}+\frac{1}{25}bcdx^4\sqrt{cx-1}\sqrt{cx+1}-\frac{13bdx^2\sqrt{c}}{2}$$

```
[Out] (-26*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^3) - (13*b*d*x^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])/(225*c) + (b*c*d*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/25 + (
d*x^3*(a + b*ArcCosh[c*x]))/3 - (c^2*d*x^5*(a + b*ArcCosh[c*x]))/5
```

Rubi [A] time = 0.132875, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {14, 5731, 12, 460, 100, 74}

$$-\frac{1}{5}c^2dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{26bd\sqrt{cx-1}\sqrt{cx+1}}{225c^3}+\frac{1}{25}bcdx^4\sqrt{cx-1}\sqrt{cx+1}-\frac{13bdx^2\sqrt{c}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-26*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^3) - (13*b*d*x^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])/(225*c) + (b*c*d*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/25 + (
d*x^3*(a + b*ArcCosh[c*x]))/3 - (c^2*d*x^5*(a + b*ArcCosh[c*x]))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^2 \left(d - c^2 d x^2\right) \left(a + b \cosh^{-1}(cx)\right) dx = \frac{1}{3} d x^3 \left(a + b \cosh^{-1}(cx)\right) - \frac{1}{5} c^2 d x^5 \left(a + b \cosh^{-1}(cx)\right) - (bc) \int \frac{d x^3 \left(5 - 3c^2\right.}{15\sqrt{-1+cx}\sqrt{}}$$

$$= \frac{1}{3} d x^3 \left(a + b \cosh^{-1}(cx)\right) - \frac{1}{5} c^2 d x^5 \left(a + b \cosh^{-1}(cx)\right) - \frac{1}{15}(bcd) \int \frac{x^3 \left(5 - \right.}{\sqrt{-1+cx}}$$

$$= \frac{1}{25} b c d x^4 \sqrt{-1+cx}\sqrt{1+cx} + \frac{1}{3} d x^3 \left(a + b \cosh^{-1}(cx)\right) - \frac{1}{5} c^2 d x^5 \left(a + b \cosh^{-1}\right.$$

$$= -\frac{13 b d x^2 \sqrt{-1+cx}\sqrt{1+cx}}{225c} + \frac{1}{25} b c d x^4 \sqrt{-1+cx}\sqrt{1+cx} + \frac{1}{3} d x^3 \left(a + b \cosh^{-}\right.$$

$$= -\frac{13 b d x^2 \sqrt{-1+cx}\sqrt{1+cx}}{225c} + \frac{1}{25} b c d x^4 \sqrt{-1+cx}\sqrt{1+cx} + \frac{1}{3} d x^3 \left(a + b \cosh^{-}\right.$$

$$= -\frac{26 b d \sqrt{-1+cx}\sqrt{1+cx}}{225c^3} - \frac{13 b d x^2 \sqrt{-1+cx}\sqrt{1+cx}}{225c} + \frac{1}{25} b c d x^4 \sqrt{-1+cx}\sqrt{1}$$

Mathematica [A] time = 0.107724, size = 89, normalized size = 0.74

$$-\frac{d\left(15 a c^3 x^3 \left(3c^2x^2 - 5\right) + b\sqrt{cx-1}\sqrt{cx+1}\left(-9c^4x^4 + 13c^2x^2 + 26\right) + 15 b c^3 x^3 \left(3c^2x^2 - 5\right)\cosh^{-1}(cx)\right)}{225c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d*(15*a*c^3*x^3*(-5 + 3*c^2*x^2) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(26 + 1
3*c^2*x^2 - 9*c^4*x^4) + 15*b*c^3*x^3*(-5 + 3*c^2*x^2)*ArcCosh[c*x]))/(225*
c^3)
```

Maple [A] time = 0.01, size = 90, normalized size = 0.7

$$\frac{1}{c^3}\left(-da\left(\frac{c^5x^5}{5} - \frac{c^3x^3}{3}\right) - db\left(\frac{\operatorname{arccosh}(cx)\,c^5x^5}{5} - \frac{c^3x^3\operatorname{arccosh}(cx)}{3} - \frac{9\,c^4x^4 - 13\,c^2x^2 - 26}{225}\sqrt{cx-1}\sqrt{cx+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(-d*a*(1/5*c^5*x^5-1/3*c^3*x^3)-d*b*(1/5*arccosh(c*x)*c^5*x^5-1/3*c^3
*x^3*arccosh(c*x)-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*x^4-13*c^2*x^2-2
```

```
6)))
```

Maxima [A] time = 1.08639, size = 196, normalized size = 1.62

$$-\frac{1}{5}\,ac^2dx^5 - \frac{1}{75}\left(15\,x^5\,\mathrm{arcosh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2-1}x^4}{c^2} + \frac{4\,\sqrt{c^2x^2-1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^2d + \frac{1}{3}\,adx^3 + \frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/5*a*c^2*d*x^5 - 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2
 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^2*d + 1/3*
a*d*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c
^2*x^2 - 1)/c^4))*b*d
```

Fricas [A] time = 1.82329, size = 235, normalized size = 1.94

$$-\frac{45\,ac^5dx^5 - 75\,ac^3dx^3 + 15\left(3\,bc^5dx^5 - 5\,bc^3dx^3\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(9\,bc^4dx^4 - 13\,bc^2dx^2 - 26\,bd\right)\sqrt{c^2x^2-1}}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/225*(45*a*c^5*d*x^5 - 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 - 5*b*c^3*d*x^3
)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^4*d*x^4 - 13*b*c^2*d*x^2 - 26*b*d)*
sqrt(c^2*x^2 - 1))/c^3
```

Sympy [A] time = 3.12227, size = 133, normalized size = 1.1

$$\begin{cases} -\frac{ac^2dx^5}{5} + \frac{adx^3}{3} - \frac{bc^2dx^5\,\mathrm{acosh}\,(cx)}{5} + \frac{bcdx^4\sqrt{c^2x^2-1}}{25} + \frac{bdx^3\,\mathrm{acosh}\,(cx)}{3} - \frac{13bdx^2\sqrt{c^2x^2-1}}{225c} - \frac{26bd\sqrt{c^2x^2-1}}{225c^3} & \text{for}\ c \neq 0 \\ \frac{dx^3\left(a+\frac{i\pi b}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**5/5 + a*d*x**3/3 - b*c**2*d*x**5*acosh(c*x)/5 + b*c
*d*x**4*sqrt(c**2*x**2 - 1)/25 + b*d*x**3*acosh(c*x)/3 - 13*b*d*x**2*sqrt(c
**2*x**2 - 1)/(225*c) - 26*b*d*sqrt(c**2*x**2 - 1)/(225*c**3), Ne(c, 0)), (
d*x**3*(a + I*pi*b/2)/3, True))
```

Giac [A] time = 1.34541, size = 200, normalized size = 1.65

$$-\frac{1}{5}\,ac^2dx^5-\frac{1}{75}\left(15\,x^5\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{3\left(c^2x^2-1\right)^{\frac{5}{2}}+10\left(c^2x^2-1\right)^{\frac{3}{2}}+15\,\sqrt{c^2x^2-1}}{c^5}\right)bc^2d+\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\,\mathrm{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] -1/5*a*c^2*d*x^5 - 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2 - 1)) - (3*(c^2*x^2
- 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))/c^5)*b*c^2*d +
1/3*a*d*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3/2
) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*d
```

3.4 $\int x\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=98

$$-\frac{d\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}+\frac{3bd\cosh^{-1}(cx)}{32c^2}+\frac{bdx(cx-1)^{3/2}(cx+1)^{3/2}}{16c}-\frac{3bdx\sqrt{cx-1}\sqrt{cx+1}}{32c}$$

```
[Out] (-3*b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c) + (b*d*x*(-1 + c*x)^(3/2)*(1
 + c*x)^(3/2))/(16*c) + (3*b*d*ArcCosh[c*x])/(32*c^2) - (d*(1 - c^2*x^2)^2*
(a + b*ArcCosh[c*x]))/(4*c^2)
```

Rubi [A] time = 0.0416606, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5716, 38, 52}

$$-\frac{d\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}+\frac{3bd\cosh^{-1}(cx)}{32c^2}+\frac{bdx(cx-1)^{3/2}(cx+1)^{3/2}}{16c}-\frac{3bdx\sqrt{cx-1}\sqrt{cx+1}}{32c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c) + (b*d*x*(-1 + c*x)^(3/2)*(1
 + c*x)^(3/2))/(16*c) + (3*b*d*ArcCosh[c*x])/(32*c^2) - (d*(1 - c^2*x^2)^2*
(a + b*ArcCosh[c*x]))/(4*c^2)
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)dx=-\frac{d\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}+\frac{(bd)\int(-1+cx)^{3/2}(1+cx)^{3/2}\,dx}{4c}$$
$$=\frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c}-\frac{d\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}-\frac{(3bd)\int\sqrt{-1+\ldots}}{16\ldots}$$
$$=-\frac{3bdx\sqrt{-1+cx}\sqrt{1+cx}}{32c}+\frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c}-\frac{d\left(1-c^2x^2\right)^2\left(a+b\,\mathrm{co}\ldots\right.}{4c^2}$$
$$=-\frac{3bdx\sqrt{-1+cx}\sqrt{1+cx}}{32c}+\frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c}+\frac{3bd\cosh^{-1}(cx)}{32c^2}-\frac{d(1\ldots}{\ldots}$$

Mathematica [A] time = 0.147541, size = 100, normalized size = 1.02

$$-\frac{d\left(cx\left(8acx\left(c^2x^2-2\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(5-2c^2x^2\right)\right)+8bc^2x^2\left(c^2x^2-2\right)\cosh^{-1}(cx)+10b\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)\right)}{32c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5 - 2*c^2*x^2) + 8*a*c*x*(-2 + c^
2*x^2)) + 8*b*c^2*x^2*(-2 + c^2*x^2)*ArcCosh[c*x] + 10*b*ArcTanh[Sqrt[(-1 +
 c*x)/(1 + c*x)]]))/(32*c^2)
```

Maple [A] time = 0.013, size = 136, normalized size = 1.4

$$-\frac{c^2dax^4}{4}+\frac{dax^2}{2}-\frac{c^2db\operatorname{arccosh}(cx)\,x^4}{4}+\frac{db\operatorname{arccosh}(cx)\,x^2}{2}+\frac{dbcx^3}{16}\sqrt{cx-1}\sqrt{cx+1}-\frac{5\,dbx}{32\,c}\sqrt{cx-1}\sqrt{cx+1}-\frac{5\,db}{32\,c^2}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/4*c^2*d*a*x^4+1/2*d*a*x^2-1/4*c^2*d*b*arccosh(c*x)*x^4+1/2*d*b*arccosh(c
*x)*x^2+1/16*c*d*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-5/32*b*d*x*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c-5/32/c^2*d*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)
*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [B] time = 1.10897, size = 243, normalized size = 2.48

$$-\frac{1}{4}ac^2dx^4 - \frac{1}{32}\left(8x^4\operatorname{arcosh}(cx) - \left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2} + \frac{3\sqrt{c^2x^2-1}x}{c^4} + \frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^4}\right)c\right)bc^2d + \frac{1}{2}adx^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/4*a*c^2*d*x^4 - 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2
+ 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2)
)/(sqrt(c^2)*c^4))*c)*b*c^2*d + 1/2*a*d*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(
sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqr
t(c^2)*c^2)))*b*d
```

Fricas [A] time = 1.835, size = 221, normalized size = 2.26

$$-\frac{8ac^4dx^4 - 16ac^2dx^2 + \left(8bc^4dx^4 - 16bc^2dx^2 + 5bd\right)\log\left(cx+\sqrt{c^2x^2-1}\right) - \left(2bc^3dx^3 - 5bcdx\right)\sqrt{c^2x^2-1}}{32c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/32*(8*a*c^4*d*x^4 - 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 - 16*b*c^2*d*x^2 + 5
*b*d)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^3*d*x^3 - 5*b*c*d*x)*sqrt(c^2*x
^2 - 1))/c^2
```

Sympy [A] time = 1.64605, size = 124, normalized size = 1.27

$$\begin{cases} -\frac{ac^2dx^4}{4} + \frac{adx^2}{2} - \frac{bc^2dx^4 \operatorname{acosh}(cx)}{4} + \frac{bcdx^3\sqrt{c^2x^2-1}}{16} + \frac{bdx^2 \operatorname{acosh}(cx)}{2} - \frac{5bdx\sqrt{c^2x^2-1}}{32c} - \frac{5bd \operatorname{acosh}(cx)}{32c^2} & \text{for } c \neq 0 \\ \frac{dx^2\left(a+\frac{i\pi b}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**4/4 + a*d*x**2/2 - b*c**2*d*x**4*acosh(c*x)/4 + b*c
*d*x**3*sqrt(c**2*x**2 - 1)/16 + b*d*x**2*acosh(c*x)/2 - 5*b*d*x*sqrt(c**2*
x**2 - 1)/(32*c) - 5*b*d*acosh(c*x)/(32*c**2), Ne(c, 0)), (d*x**2*(a + I*pi
*b/2)/2, True))
```

Giac [B] time = 1.49737, size = 243, normalized size = 2.48

$$-\frac{1}{4}\,ac^2dx^4 - \frac{1}{32}\left(8\,x^4\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(\sqrt{c^2x^2-1}x\left(\frac{2\,x^2}{c^2} + \frac{3}{c^4}\right) - \frac{3\,\log\left(\left|-x|c| + \sqrt{c^2x^2-1}\right|\right)}{c^4|c|}\right)c\right)bc^2d + \frac{1}{2}\,adx^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] -1/4*a*c^2*d*x^4 - 1/32*(8*x^4*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2
 - 1)*x*(2*x^2/c^2 + 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^
4*abs(c)))*c)*b*c^2*d + 1/2*a*d*x^2 + 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 - 1
)) - c*(sqrt(c^2*x^2 - 1)*x/c^2 - log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(
c^2*abs(c))))*b*d
```

3.5 $\int \left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=86

$$-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)+dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}bcdx^2\sqrt{cx-1}\sqrt{cx+1}-\frac{7bd\sqrt{cx-1}\sqrt{cx+1}}{9c}$$

```
[Out] (-7*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c) + (b*c*d*x^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/9 + d*x*(a + b*ArcCosh[c*x]) - (c^2*d*x^3*(a + b*ArcCosh[c*x]))
/3
```

Rubi [A] time = 0.074217, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5680, 12, 460, 74}

$$-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)+dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}bcdx^2\sqrt{cx-1}\sqrt{cx+1}-\frac{7bd\sqrt{cx-1}\sqrt{cx+1}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-7*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c) + (b*c*d*x^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/9 + d*x*(a + b*ArcCosh[c*x]) - (c^2*d*x^3*(a + b*ArcCosh[c*x]))
/3
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
```

```
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx = dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)-(bc)\int\frac{dx\left(1-\frac{c^2x^2}{3}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,d$$

$$= dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)-(bcd)\int\frac{x\left(1-\frac{c^2x^2}{3}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{9}bcdx^2\sqrt{-1+cx}\sqrt{1+cx}+dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)-$$

$$= -\frac{7bd\sqrt{-1+cx}\sqrt{1+cx}}{9c}+\frac{1}{9}bcdx^2\sqrt{-1+cx}\sqrt{1+cx}+dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}$$

Mathematica [A] time = 0.0845899, size = 71, normalized size = 0.83

$$\frac{d\left(a\left(9cx-3c^3x^3\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2-7\right)-3bcx\left(c^2x^2-3\right)\cosh^{-1}(cx)\right)}{9c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-7 + c^2*x^2) + a*(9*c*x - 3*c^3*x^3) -
 3*b*c*x*(-3 + c^2*x^2)*ArcCosh[c*x]))/(9*c)
```

Maple [A] time = 0.011, size = 73, normalized size = 0.9

$$\frac{1}{c}\left(-da\left(\frac{c^3x^3}{3}-cx\right)-db\left(\frac{c^3x^3\mathrm{arccosh}\,(cx)}{3}-cx\mathrm{arccosh}\,(cx)-\frac{c^2x^2-7}{9}\sqrt{cx-1}\sqrt{cx+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-d*a*(1/3*c^3*x^3-c*x)-d*b*(1/3*c^3*x^3*arccosh(c*x)-c*x*arccosh(c*x)-
1/9*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*x^2-7)))
```

Maxima [A] time = 1.18129, size = 131, normalized size = 1.52

$$-\frac{1}{3}\,ac^2dx^3-\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^2d+adx+\frac{\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/3*a*c^2*d*x^3 - 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 +
 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d + a*d*x + (c*x*arccosh(c*x) - sqrt(c^2*x
^2 - 1))*b*d/c
```

Fricas [A] time = 1.82291, size = 185, normalized size = 2.15

$$-\frac{3\,ac^3dx^3-9\,acdx+3\left(bc^3dx^3-3\,bcdx\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^2dx^2-7\,bd\right)\sqrt{c^2x^2-1}}{9\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/9*(3*a*c^3*d*x^3 - 9*a*c*d*x + 3*(b*c^3*d*x^3 - 3*b*c*d*x)*log(c*x + sqr
t(c^2*x^2 - 1)) - (b*c^2*d*x^2 - 7*b*d)*sqrt(c^2*x^2 - 1))/c
```

Sympy [A] time = 0.755015, size = 97, normalized size = 1.13

$$\begin{cases} -\frac{ac^2dx^3}{3} + adx - \frac{bc^2dx^3 \operatorname{acosh}(cx)}{3} + \frac{bcdx^2\sqrt{c^2x^2-1}}{9} + bdx \operatorname{acosh}(cx) - \frac{7bd\sqrt{c^2x^2-1}}{9c} & \text{for } c \neq 0 \\ dx\left(a + \frac{i\pi b}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**3/3 + a*d*x - b*c**2*d*x**3*acosh(c*x)/3 + b*c*d*x*
*2*sqrt(c**2*x**2 - 1)/9 + b*d*x*acosh(c*x) - 7*b*d*sqrt(c**2*x**2 - 1)/(9*
c), Ne(c, 0)), (d*x*(a + I*pi*b/2), True))
```

Giac [A] time = 1.37603, size = 151, normalized size = 1.76

$$-\frac{1}{3}ac^2dx^3 - \frac{1}{9}\left(3x^3\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{\left(c^2x^2-1\right)^{\frac{3}{2}} + 3\sqrt{c^2x^2-1}}{c^3}\right)bc^2d + \left(x\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{\sqrt{c^2x^2-1}}{c}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] -1/3*a*c^2*d*x^3 - 1/9*(3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)
^(3/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*c^2*d + (x*log(c*x + sqrt(c^2*x^2 - 1)
) - sqrt(c^2*x^2 - 1)/c)*b*d + a*d*x
```

3.6 $\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=117

$$-\frac{1}{2}bd\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+\frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{d\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+d\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a\right.$$

```
[Out] (b*c*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/4 - (b*d*ArcCosh[c*x])/4 + (d*(1 - c
^2*x^2)*(a + b*ArcCosh[c*x]))/2 + (d*(a + b*ArcCosh[c*x])^2)/(2*b) + d*(a +
 b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])] - (b*d*PolyLog[2, -E^(-2*ArcC
osh[c*x])])/2
```

Rubi [A] time = 0.115941, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5727, 5660, 3718, 2190, 2279, 2391, 38, 52}

$$\frac{1}{2}bd\mathrm{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)+\frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{d\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+d\log\left(e^{2\cosh^{-1}(cx)}+1\right)\left(a+b\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (b*c*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/4 - (b*d*ArcCosh[c*x])/4 + (d*(1 - c
^2*x^2)*(a + b*ArcCosh[c*x]))/2 - (d*(a + b*ArcCosh[c*x])^2)/(2*b) + d*(a +
 b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])] + (b*d*PolyLog[2, -E^(2*ArcCos
h[c*x])])/2
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
```

```
0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x}\,dx = \frac{1}{2}d(1-c^2x^2)(a+b\cosh^{-1}(cx)) + d\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx + \frac{1}{2}(bcd)\int \sqrt{-1+c}$$
$$= \frac{1}{4}bcdx\sqrt{-1+cx}\sqrt{1+cx} + \frac{1}{2}d(1-c^2x^2)(a+b\cosh^{-1}(cx)) + d\,\mathrm{Subst}\left(\int (a+bx\right.$$
$$= \frac{1}{4}bcdx\sqrt{-1+cx}\sqrt{1+cx} - \frac{1}{4}bd\cosh^{-1}(cx) + \frac{1}{2}d(1-c^2x^2)(a+b\cosh^{-1}(cx)) -$$
$$= \frac{1}{4}bcdx\sqrt{-1+cx}\sqrt{1+cx} - \frac{1}{4}bd\cosh^{-1}(cx) + \frac{1}{2}d(1-c^2x^2)(a+b\cosh^{-1}(cx)) -$$
$$= \frac{1}{4}bcdx\sqrt{-1+cx}\sqrt{1+cx} - \frac{1}{4}bd\cosh^{-1}(cx) + \frac{1}{2}d(1-c^2x^2)(a+b\cosh^{-1}(cx)) -$$
$$= \frac{1}{4}bcdx\sqrt{-1+cx}\sqrt{1+cx} - \frac{1}{4}bd\cosh^{-1}(cx) + \frac{1}{2}d(1-c^2x^2)(a+b\cosh^{-1}(cx)) -$$

Mathematica [A] time = 0.174166, size = 116, normalized size = 0.99

$$-\frac{1}{4}d\left(2b\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right) + 2ac^2x^2 - 4a\log(x) + 2b\cosh^{-1}(cx)\left(c^2x^2 - 2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\right) - bcx\sqrt{cx-1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -(d*(2*a*c^2*x^2 - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*ArcCosh[c*x]^2
- 2*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]] + 2*b*ArcCosh[c*x]*(c^2*x^2 - 2*L
og[1 + E^(-2*ArcCosh[c*x])]) - 4*a*Log[x] + 2*b*PolyLog[2, -E^(-2*ArcCosh[c
*x])]))/4
```

Maple [A] time = 0.086, size = 131, normalized size = 1.1

$$-\frac{dac^2x^2}{2} + da\ln(cx) - \frac{db(\mathrm{arccosh}(cx))^2}{2} - \frac{db\,\mathrm{arccosh}(cx)\,c^2x^2}{2} + \frac{dbcx}{4}\sqrt{cx-1}\sqrt{cx+1} + \frac{bd\,\mathrm{arccosh}(cx)}{4} + db\,\mathrm{arccosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x)
```

```
[Out] -1/2*d*a*c^2*x^2+d*a*ln(c*x)-1/2*d*b*arccosh(c*x)^2-1/2*d*b*arccosh(c*x)*c^
2*x^2+1/4*b*c*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/4*b*d*arccosh(c*x)+d*b*arcc
osh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)+1/2*d*b*polylog(2,-(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,ac^2dx^2 + ad\log\left(x\right) - \int bc^2dx\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) - \frac{bd\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*d*x^2 + a*d*log(x) - integrate(b*c^2*d*x*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1)) - b*d*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\operatorname{arcosh}\left(cx\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d\left(\int -\frac{a}{x}\,dx + \int ac^2x\,dx + \int -\frac{b\operatorname{acosh}\left(cx\right)}{x}\,dx + \int bc^2x\operatorname{acosh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x,x)
```

```
[Out] -d*(Integral(-a/x, x) + Integral(a*c**2*x, x) + Integral(-b*acosh(c*x)/x, x
) + Integral(b*c**2*x*acosh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(c^2dx^2 - d)(b\,\text{arcosh}\,(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)*(b*arccosh(c*x) + a)/x, x)
```

3.7 $\int \frac{(d-c^2dx^2)\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=76

$$c^2(-d)x\left(a+b\cosh^{-1}(cx)\right)-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+bcd\sqrt{cx-1}\sqrt{cx+1}+bcd\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

```
[Out] b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - (d*(a + b*ArcCosh[c*x]))/x - c^2*d*x*(
a + b*ArcCosh[c*x]) + b*c*d*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rubi [A] time = 0.119375, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {14, 5731, 12, 460, 92, 205}

$$c^2(-d)x\left(a+b\cosh^{-1}(cx)\right)-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+bcd\sqrt{cx-1}\sqrt{cx+1}+bcd\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - (d*(a + b*ArcCosh[c*x]))/x - c^2*d*x*(
a + b*ArcCosh[c*x]) + b*c*d*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}-c^2dx\left(a+b\cosh^{-1}(cx)\right)-(bc)\int \frac{d\left(-1-c^2x^2\right)}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}-c^2dx\left(a+b\cosh^{-1}(cx)\right)-(bcd)\int \frac{-1-c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= bcd\sqrt{-1+cx}\sqrt{1+cx}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}-c^2dx\left(a+b\cosh^{-1}(cx)\right)+(bcd)\int$$

$$= bcd\sqrt{-1+cx}\sqrt{1+cx}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}-c^2dx\left(a+b\cosh^{-1}(cx)\right)+\left(bc^2d\right)\text{S}$$

$$= bcd\sqrt{-1+cx}\sqrt{1+cx}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}-c^2dx\left(a+b\cosh^{-1}(cx)\right)+bcd\tan$$

Mathematica [A] time = 0.179708, size = 110, normalized size = 1.45

$$-ac^2dx-\frac{ad}{x}+\frac{bcd\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-bc^2dx\cosh^{-1}(cx)+bcd\sqrt{cx-1}\sqrt{cx+1}-\frac{bd\cosh^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) - a*c^2*d*x + b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - (b*d*ArcCosh[
c*x])/x - b*c^2*d*x*ArcCosh[c*x] + (b*c*d*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1
 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [A] time = 0.016, size = 100, normalized size = 1.3

$$-dac^2x-\frac{da}{x}-db\mathrm{arccosh}\,(cx)\,c^2x-\frac{bd\mathrm{arccosh}\,(cx)}{x}+bcd\sqrt{cx-1}\sqrt{cx+1}-dbc\sqrt{cx-1}\sqrt{cx+1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] -d*a*c^2*x-d*a/x-d*b*arccosh(c*x)*c^2*x-d*b*arccosh(c*x)/x+b*c*d*(c*x-1)^(1
/2)*(c*x+1)^(1/2)-c*d*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arcta
n(1/(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.9058, size = 92, normalized size = 1.21

$$-ac^2dx-\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bcd-\left(c\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bd-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -a*c^2*d*x - (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*c*d - (c*arcsin(1/(sq
rt(c^2)*abs(x))) + arccosh(c*x)/x)*b*d - a*d/x
```

Fricas [A] time = 1.9603, size = 289, normalized size = 3.8

$$-\frac{ac^2dx^2 - 2\,bcdx\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) - \sqrt{c^2x^2 - 1}bcdx - \left(bc^2 + b\right)dx\log\left(-cx + \sqrt{c^2x^2 - 1}\right) + ad + \left(bc^2dx^2 - \left(bc^2\right.\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*d*x^2 - 2*b*c*d*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 -
 1)*b*c*d*x - (b*c^2 + b)*d*x*log(-c*x + sqrt(c^2*x^2 - 1)) + a*d + (b*c^2*
d*x^2 - (b*c^2 + b)*d*x + b*d)*log(c*x + sqrt(c^2*x^2 - 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d\left(\int ac^2\,dx + \int -\frac{a}{x^2}\,dx + \int bc^2\operatorname{acosh}(cx)\,dx + \int -\frac{b\operatorname{acosh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] -d*(Integral(a*c**2, x) + Integral(-a/x**2, x) + Integral(b*c**2*acosh(c*x)
, x) + Integral(-b*acosh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(c^2dx^2 - d\right)(b\operatorname{arcosh}(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)*(b*arccosh(c*x) + a)/x^2, x)
```

3.8 $\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=135

$$\frac{1}{2}bc^2d\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-\frac{d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)^2}{2b}-c^2d\log\left(e^{-2\cosh^{-1}(cx)}+\right.$$

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*c^2*d*ArcCosh[c*x])/2 - (d*
(1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/(2*x^2) - (c^2*d*(a + b*ArcCosh[c*x])^2
)/(2*b) - c^2*d*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])] + (b*c^2*
d*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

Rubi [A] time = 0.128991, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5729, 97, 12, 52, 5660, 3718, 2190, 2279, 2391}

$$-\frac{1}{2}bc^2d\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)-\frac{d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)^2}{2b}-c^2d\log\left(e^{2\cosh^{-1}(cx)}+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*c^2*d*ArcCosh[c*x])/2 - (d*
(1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/(2*x^2) + (c^2*d*(a + b*ArcCosh[c*x])^2
)/(2*b) - c^2*d*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])] - (b*c^2*d
*PolyLog[2, -E^(2*ArcCosh[c*x])])/2
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x^3}\,dx = -\frac{d(1-c^2x^2)(a+b\cosh^{-1}(cx))}{2x^2} - \frac{1}{2}(bcd)\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^2}\,dx - (c^2d)\int \frac{a}{}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{d(1-c^2x^2)(a+b\cosh^{-1}(cx))}{2x^2} - \frac{1}{2}(bcd)\int \frac{c^2}{\sqrt{-1+cx}\sqrt{}}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{d(1-c^2x^2)(a+b\cosh^{-1}(cx))}{2x^2} + \frac{c^2d(a+b\cosh^{-1}(cx)}{2b}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{1}{2}bc^2d\cosh^{-1}(cx) - \frac{d(1-c^2x^2)(a+b\cosh^{-1}(cx))}{2x^2} + \frac{c}{}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{1}{2}bc^2d\cosh^{-1}(cx) - \frac{d(1-c^2x^2)(a+b\cosh^{-1}(cx))}{2x^2} + \frac{c}{}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{1}{2}bc^2d\cosh^{-1}(cx) - \frac{d(1-c^2x^2)(a+b\cosh^{-1}(cx))}{2x^2} + \frac{c}{}$$

Mathematica [A] time = 0.154572, size = 106, normalized size = 0.79

$$-\frac{d\left(-bc^2x^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+2ac^2x^2\log(x)+a+bc^2x^2\cosh^{-1}(cx)^2+b\cosh^{-1}(cx)\left(2c^2x^2\log\left(e^{-2\cosh^{-1}(cx)}\right.\right.\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -(d*(a - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*c^2*x^2*ArcCosh[c*x]^2 + b*
ArcCosh[c*x]*(1 + 2*c^2*x^2*Log[1 + E^(-2*ArcCosh[c*x])]) + 2*a*c^2*x^2*Log
[x] - b*c^2*x^2*PolyLog[2, -E^(-2*ArcCosh[c*x])]))/(2*x^2)
```

Maple [A] time = 0.157, size = 140, normalized size = 1.

$$-c^2da\ln(cx) - \frac{da}{2x^2} + \frac{c^2db(\mathrm{arccosh}(cx))^2}{2} + \frac{bcd}{2x}\sqrt{cx-1}\sqrt{cx+1} - \frac{c^2db}{2} - \frac{bd\mathrm{arccosh}(cx)}{2x^2} - c^2db\,\mathrm{arccosh}(cx)\ln\left(\left(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] -c^2*d*a*ln(c*x)-1/2*d*a/x^2+1/2*c^2*d*b*arccosh(c*x)^2+1/2*b*c*d*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/x-1/2*c^2*d*b-1/2*d*b*arccosh(c*x)/x^2-c^2*d*b*arccosh(c
*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)-1/2*c^2*d*b*polylog(2,-(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-bc^2d\int \frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{x}\,dx - ac^2d\log(x) + \frac{1}{2}\,bd\left(\frac{\sqrt{c^2x^2-1}c}{x} - \frac{\operatorname{arcosh}(cx)}{x^2}\right) - \frac{ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -b*c^2*d*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) - a*c^2*d*l
og(x) + 1/2*b*d*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) - 1/2*a*d/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\operatorname{arcosh}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d\left(\int -\frac{a}{x^3}\,dx + \int \frac{ac^2}{x}\,dx + \int -\frac{b\operatorname{acosh}(cx)}{x^3}\,dx + \int \frac{bc^2\operatorname{acosh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] -d*(Integral(-a/x**3, x) + Integral(a*c**2/x, x) + Integral(-b*acosh(c*x)/x
**3, x) + Integral(b*c**2*acosh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(c^2dx^2 - d\right)(b \operatorname{arcosh}(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)*(b*arccosh(c*x) + a)/x^3, x)
```

3.9 $\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=90

$$\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{5}{6}bc^3d\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (d*(a + b*ArcCosh[c*x]))/(3*
x^3) + (c^2*d*(a + b*ArcCosh[c*x]))/x - (5*b*c^3*d*ArcTan[Sqrt[-1 + c*x]*Sq
rt[1 + c*x]])/6
```

Rubi [A] time = 0.125307, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {14, 5731, 12, 454, 92, 205}

$$\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{5}{6}bc^3d\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (d*(a + b*ArcCosh[c*x]))/(3*
x^3) + (c^2*d*(a + b*ArcCosh[c*x]))/x - (5*b*c^3*d*ArcTan[Sqrt[-1 + c*x]*Sq
rt[1 + c*x]])/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx &= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-(bc)\int \frac{d\left(-1+3c^2x^2\right)}{3x^3\sqrt{-1+cx}\sqrt{1+cx}\\
&= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{1}{3}(bcd)\int \frac{-1+3c^2x^2}{x^3\sqrt{-1+cx}\sqrt{1+c\\
&= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{1}{6}\left(5bc\\
&= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{1}{6}\left(5bc\\
&= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{5}{6}bc^3d\end{aligned}$$

Mathematica [A] time = 0.231506, size = 127, normalized size = 1.41

$$\frac{ac^2d}{x}-\frac{ad}{3x^3}-\frac{5bc^3d\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{6\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^2d\cosh^{-1}(cx)}{x}+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{6x^2}-\frac{bd\cosh^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -(a*d)/(3*x^3) + (a*c^2*d)/x + (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2)
 - (b*d*ArcCosh[c*x])/(3*x^3) + (b*c^2*d*ArcCosh[c*x])/x - (5*b*c^3*d*Sqrt[
-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [A] time = 0.018, size = 108, normalized size = 1.2

$$\frac{c^2da}{x}-\frac{da}{3\,x^3}+\frac{bc^2d\mathrm{arccosh}\,(cx)}{x}-\frac{bd\mathrm{arccosh}\,(cx)}{3\,x^3}+\frac{5\,c^3db}{6}\sqrt{cx-1}\sqrt{cx+1}\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\frac{1}{\sqrt{c^2x^2-1}}+\frac{bcd}{6\,x^2}\sqrt{cx-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] c^2*d*a/x-1/3*d*a/x^3+c^2*d*b*arccosh(c*x)/x-1/3*d*b*arccosh(c*x)/x^3+5/6*c
^3*d*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(
1/2))+1/6*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

Maxima [A] time = 1.8301, size = 126, normalized size = 1.4

$$\left(c\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bc^2d-\frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\,\mathrm{arcosh}\,(cx)}{x^3}\right)bd+\frac{ac^2d}{x}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] (c*arcsin(1/(sqrt(c^2)*abs(x))) + arccosh(c*x)/x)*b*c^2*d - 1/6*((c^2*arcsi
n(1/(sqrt(c^2)*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)/x^3)*b*
```

```
d + a*c^2*d/x - 1/3*a*d/x^3
```

Fricas [A] time = 1.91985, size = 328, normalized size = 3.64

$$-\frac{10\,bc^3dx^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-6\,ac^2dx^2+2\left(3\,bc^2-b\right)dx^3\log\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}bcdx+2\,ad-2\left(\right.}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(10*b*c^3*d*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 6*a*c^2*d*x^2 + 2*(
3*b*c^2 - b)*d*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)*b*c*d*
x + 2*a*d - 2*(3*b*c^2*d*x^2 - (3*b*c^2 - b)*d*x^3 - b*d)*log(c*x + sqrt(c^
2*x^2 - 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d\left(\int -\frac{a}{x^4}\,dx+\int \frac{ac^2}{x^2}\,dx+\int -\frac{b\operatorname{acosh}(cx)}{x^4}\,dx+\int \frac{bc^2\operatorname{acosh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] -d*(Integral(-a/x**4, x) + Integral(a*c**2/x**2, x) + Integral(-b*acosh(c*x
)/x**4, x) + Integral(b*c**2*acosh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(c^2dx^2-d\right)\left(b\operatorname{arcosh}(cx)+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)*(b*arccosh(c*x) + a)/x^4, x)
```

3.10 $\int x^4 \left(d-c^2 d x^2\right)^2 \left(a+b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=206

$$\frac{1}{9}c^4d^2x^9\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{bd^2(cx-1)^{9/2}(cx+1)^{9/2}}{81c^5}-\frac{10bd^2}{}$$

```
[Out] (-8*b*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(315*c^5) + (4*b*d^2*(-1 + c*x)^(3/
2)*(1 + c*x)^(3/2))/(945*c^5) - (b*d^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(5
25*c^5) - (10*b*d^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/(441*c^5) - (b*d^2*(-
1 + c*x)^(9/2)*(1 + c*x)^(9/2))/(81*c^5) + (d^2*x^5*(a + b*ArcCosh[c*x]))/5
 - (2*c^2*d^2*x^7*(a + b*ArcCosh[c*x]))/7 + (c^4*d^2*x^9*(a + b*ArcCosh[c*x
]))/9
```

Rubi [A] time = 0.293386, antiderivative size = 264, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {270, 5731, 12, 520, 1251, 897, 1153}

$$\frac{1}{9}c^4d^2x^9\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{bd^2\left(1-c^2x^2\right)^5}{81c^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{10bd^2\left(\right.}{441c^5\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*(1 - c^2*x^2))/(315*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*d^2*(
1 - c^2*x^2)^2)/(945*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^
2)^3)/(525*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (10*b*d^2*(1 - c^2*x^2)^4)/(
441*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^2)^5)/(81*c^5*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^5*(a + b*ArcCosh[c*x]))/5 - (2*c^2*d^2*
x^7*(a + b*ArcCosh[c*x]))/7 + (c^4*d^2*x^9*(a + b*ArcCosh[c*x]))/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
```

```
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^4 \left(d - c^2dx^2\right)^2 \left(a + b\cosh^{-1}(cx)\right) dx = \frac{1}{5}d^2x^5\left(a + b\cosh^{-1}(cx)\right) - \frac{2}{7}c^2d^2x^7\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{9}c^4d^2x^9\left(a + b\cosh\right.$$

$$= \frac{1}{5}d^2x^5\left(a + b\cosh^{-1}(cx)\right) - \frac{2}{7}c^2d^2x^7\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{9}c^4d^2x^9\left(a + b\cosh\right.$$

$$= \frac{1}{5}d^2x^5\left(a + b\cosh^{-1}(cx)\right) - \frac{2}{7}c^2d^2x^7\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{9}c^4d^2x^9\left(a + b\cosh\right.$$

$$= \frac{1}{5}d^2x^5\left(a + b\cosh^{-1}(cx)\right) - \frac{2}{7}c^2d^2x^7\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{9}c^4d^2x^9\left(a + b\cosh\right.$$

$$= \frac{1}{5}d^2x^5\left(a + b\cosh^{-1}(cx)\right) - \frac{2}{7}c^2d^2x^7\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{9}c^4d^2x^9\left(a + b\cosh\right.$$

$$= \frac{1}{5}d^2x^5\left(a + b\cosh^{-1}(cx)\right) - \frac{2}{7}c^2d^2x^7\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{9}c^4d^2x^9\left(a + b\cosh\right.$$

$$= \frac{8bd^2\left(1 - c^2x^2\right)}{315c^5\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{4bd^2\left(1 - c^2x^2\right)^2}{945c^5\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{bd^2\left(1 - c^2x^2\right)^3}{525c^5\sqrt{-1 + cx}\sqrt{1 + cx}}$$

Mathematica [A] time = 0.19838, size = 124, normalized size = 0.6

$$\frac{d^2\left(315ac^5x^5\left(35c^4x^4 - 90c^2x^2 + 63\right) - b\sqrt{cx - 1}\sqrt{cx + 1}\left(1225c^8x^8 - 2650c^6x^6 + 789c^4x^4 + 1052c^2x^2 + 2104\right) + 315bc^5x\right.}{99225c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(315*a*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4) - b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(2104 + 1052*c^2*x^2 + 789*c^4*x^4 - 2650*c^6*x^6 + 1225*c^8*x^8)
+ 315*b*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4)*ArcCosh[c*x]))/(99225*c^5)
```

Maple [A] time = 0.013, size = 128, normalized size = 0.6

$$\frac{1}{c^5}\left(d^2a\left(\frac{c^9x^9}{9} - \frac{2\,c^7x^7}{7} + \frac{c^5x^5}{5}\right) + d^2b\left(\frac{\operatorname{arccosh}(cx)\,c^9x^9}{9} - \frac{2\operatorname{arccosh}(cx)\,c^7x^7}{7} + \frac{\operatorname{arccosh}(cx)\,c^5x^5}{5} - \frac{1225\,c^8x^8 - 2650}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(d^2*a*(1/9*c^9*x^9-2/7*c^7*x^7+1/5*c^5*x^5)+d^2*b*(1/9*arccosh(c*x)*
c^9*x^9-2/7*arccosh(c*x)*c^7*x^7+1/5*arccosh(c*x)*c^5*x^5-1/99225*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*(1225*c^8*x^8-2650*c^6*x^6+789*c^4*x^4+1052*c^2*x^2+2104
)))
```

Maxima [A] time = 1.18024, size = 431, normalized size = 2.09

$$\frac{1}{9}\,ac^4d^2x^9 - \frac{2}{7}\,ac^2d^2x^7 + \frac{1}{2835}\left(315\,x^9\,\operatorname{arcosh}(cx) - \left(\frac{35\,\sqrt{c^2x^2-1}x^8}{c^2} + \frac{40\,\sqrt{c^2x^2-1}x^6}{c^4} + \frac{48\,\sqrt{c^2x^2-1}x^4}{c^6} + \frac{64\,\sqrt{c^2x^2}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^4*d^2*x^9 - 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arccosh(c*x) - (35*
sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2
- 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c
)*b*c^4*d^2 + 1/5*a*d^2*x^5 - 2/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2
- 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 +
16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arccosh(c*x) - (3*sqr
t(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/
c^6)*c)*b*d^2
```

Fricas [A] time = 1.86919, size = 387, normalized size = 1.88

$$\frac{11025\,ac^9d^2x^9 - 28350\,ac^7d^2x^7 + 19845\,ac^5d^2x^5 + 315\left(35\,bc^9d^2x^9 - 90\,bc^7d^2x^7 + 63\,bc^5d^2x^5\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(\right.}{99225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^2*x^9 - 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 +
315*(35*b*c^9*d^2*x^9 - 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*log(c*x + sqrt
```

```
(c^2*x^2 - 1)) - (1225*b*c^8*d^2*x^8 - 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^2*x
^4 + 1052*b*c^2*d^2*x^2 + 2104*b*d^2)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] time = 26.2538, size = 236, normalized size = 1.15

$$\begin{cases} \frac{ac^4d^2x^9}{9} - \frac{2ac^2d^2x^7}{7} + \frac{ad^2x^5}{5} + \frac{bc^4d^2x^9\operatorname{acosh}(cx)}{9} - \frac{bc^3d^2x^8\sqrt{c^2x^2-1}}{81} - \frac{2bc^2d^2x^7\operatorname{acosh}(cx)}{7} + \frac{106bcd^2x^6\sqrt{c^2x^2-1}}{3969} + \frac{bd^2x^5\operatorname{acosh}(cx)}{5} - \frac{263bd^2x^4\cdots}{3307\cdots} \\ \frac{d^2x^5\left(a+\frac{i\pi b}{2}\right)}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**9/9 - 2*a*c**2*d**2*x**7/7 + a*d**2*x**5/5 + b*c*
*4*d**2*x**9*acosh(c*x)/9 - b*c**3*d**2*x**8*sqrt(c**2*x**2 - 1)/81 - 2*b*c
**2*d**2*x**7*acosh(c*x)/7 + 106*b*c*d**2*x**6*sqrt(c**2*x**2 - 1)/3969 + b
*d**2*x**5*acosh(c*x)/5 - 263*b*d**2*x**4*sqrt(c**2*x**2 - 1)/(33075*c) - 1
052*b*d**2*x**2*sqrt(c**2*x**2 - 1)/(99225*c**3) - 2104*b*d**2*sqrt(c**2*x*
*2 - 1)/(99225*c**5), Ne(c, 0)), (d**2*x**5*(a + I*pi*b/2)/5, True))
```

Giac [A] time = 1.50041, size = 402, normalized size = 1.95

$$\frac{1}{9}ac^4d^2x^9 - \frac{2}{7}ac^2d^2x^7 + \frac{1}{2835}\left(315x^9\log\left(cx+\sqrt{c^2x^2-1}\right) - \frac{35\left(c^2x^2-1\right)^{\frac{9}{2}} + 180\left(c^2x^2-1\right)^{\frac{7}{2}} + 378\left(c^2x^2-1\right)^{\frac{5}{2}} + 420\left(\cdots\right.}{c^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/9*a*c^4*d^2*x^9 - 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*log(c*x + sqrt(c^2*
x^2 - 1)) - (35*(c^2*x^2 - 1)^(9/2) + 180*(c^2*x^2 - 1)^(7/2) + 378*(c^2*x^
2 - 1)^(5/2) + 420*(c^2*x^2 - 1)^(3/2) + 315*sqrt(c^2*x^2 - 1))/c^9)*b*c^4*
d^2 + 1/5*a*d^2*x^5 - 2/245*(35*x^7*log(c*x + sqrt(c^2*x^2 - 1)) - (5*(c^2*
x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5/2) + 35*(c^2*x^2 - 1)^(3/2) + 35*sqrt(
c^2*x^2 - 1))/c^7)*b*c^2*d^2 + 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2 - 1)) -
(3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))/c^5
)*b*d^2
```

3.11 $\int x^3 \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=200

$$\frac{1}{8}c^4d^2x^8\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{64}bc^3d^2x^7\sqrt{cx-1}\sqrt{cx+1}-\frac{73}{\ldots}$$

```
[Out] (-73*b*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3072*c^3) - (73*b*d^2*x^3*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(4608*c) + (43*b*c*d^2*x^5*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])/1152 - (b*c^3*d^2*x^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/64 - (73*b*d^2*A
rcCosh[c*x])/(3072*c^4) + (d^2*x^4*(a + b*ArcCosh[c*x]))/4 - (c^2*d^2*x^6*(
a + b*ArcCosh[c*x]))/3 + (c^4*d^2*x^8*(a + b*ArcCosh[c*x]))/8
```

Rubi [A] time = 0.276914, antiderivative size = 284, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 43, 5731, 12, 520, 1267, 459, 321, 217, 206}

$$\frac{1}{8}c^4d^2x^8\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{bc^3d^2x^7\left(1-c^2x^2\right)}{64\sqrt{cx-1}\sqrt{cx+1}}-\frac{43bcd^2x^5}{1152\sqrt{cx\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (73*b*d^2*x*(1 - c^2*x^2))/(3072*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (73*b*
d^2*x^3*(1 - c^2*x^2))/(4608*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (43*b*c*d^2*
x^5*(1 - c^2*x^2))/(1152*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*x^7*(1
- c^2*x^2))/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^4*(a + b*ArcCosh[c*x
]))/4 - (c^2*d^2*x^6*(a + b*ArcCosh[c*x]))/3 + (c^4*d^2*x^8*(a + b*ArcCosh[
c*x]))/8 - (73*b*d^2*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/
(3072*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3 \left(d - c^2dx^2\right)^2 \left(a + b\cosh^{-1}(cx)\right) dx = \frac{1}{4}d^2x^4\left(a + b\cosh^{-1}(cx)\right) - \frac{1}{3}c^2d^2x^6\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{8}c^4d^2x^8\left(a + b\cosh\right.$$

$$= \frac{1}{4}d^2x^4\left(a + b\cosh^{-1}(cx)\right) - \frac{1}{3}c^2d^2x^6\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{8}c^4d^2x^8\left(a + b\cosh\right.$$

$$= \frac{1}{4}d^2x^4\left(a + b\cosh^{-1}(cx)\right) - \frac{1}{3}c^2d^2x^6\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{8}c^4d^2x^8\left(a + b\cosh\right.$$

$$= \frac{bc^3d^2x^7\left(1 - c^2x^2\right)}{64\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{1}{4}d^2x^4\left(a + b\cosh^{-1}(cx)\right) - \frac{1}{3}c^2d^2x^6\left(a + b\cosh^{-1}(cx)\right.$$

$$= -\frac{43bcd^2x^5\left(1 - c^2x^2\right)}{1152\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{bc^3d^2x^7\left(1 - c^2x^2\right)}{64\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{1}{4}d^2x^4\left(a + b\cosh^{-1}(cx)\right) -$$

$$= \frac{73bd^2x^3\left(1 - c^2x^2\right)}{4608c\sqrt{-1 + cx}\sqrt{1 + cx}} - \frac{43bcd^2x^5\left(1 - c^2x^2\right)}{1152\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{bc^3d^2x^7\left(1 - c^2x^2\right)}{64\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{1}{4}$$

$$= \frac{73bd^2x\left(1 - c^2x^2\right)}{3072c^3\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{73bd^2x^3\left(1 - c^2x^2\right)}{4608c\sqrt{-1 + cx}\sqrt{1 + cx}} - \frac{43bcd^2x^5\left(1 - c^2x^2\right)}{1152\sqrt{-1 + cx}\sqrt{1 + cx}}$$

$$= \frac{73bd^2x\left(1 - c^2x^2\right)}{3072c^3\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{73bd^2x^3\left(1 - c^2x^2\right)}{4608c\sqrt{-1 + cx}\sqrt{1 + cx}} - \frac{43bcd^2x^5\left(1 - c^2x^2\right)}{1152\sqrt{-1 + cx}\sqrt{1 + cx}}$$

$$= \frac{73bd^2x\left(1 - c^2x^2\right)}{3072c^3\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{73bd^2x^3\left(1 - c^2x^2\right)}{4608c\sqrt{-1 + cx}\sqrt{1 + cx}} - \frac{43bcd^2x^5\left(1 - c^2x^2\right)}{1152\sqrt{-1 + cx}\sqrt{1 + cx}}$$

Mathematica [A] time = 0.226475, size = 194, normalized size = 0.97

$$\frac{d^2\left(1152ac^8x^8 - 3072ac^6x^6 + 2304ac^4x^4 - 144bc^7x^7\sqrt{cx-1}\sqrt{cx+1} + 344bc^5x^5\sqrt{cx-1}\sqrt{cx+1} - 146bc^3x^3\sqrt{cx-1}\sqrt{cx-\right.}{9216c^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(2304*a*c^4*x^4 - 3072*a*c^6*x^6 + 1152*a*c^8*x^8 - 219*b*c*x*Sqrt[-1
+ c*x]*Sqrt[1 + c*x] - 146*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 344*b*c
^5*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 144*b*c^7*x^7*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x] + 384*b*c^4*x^4*(6 - 8*c^2*x^2 + 3*c^4*x^4)*ArcCosh[c*x] - 438*b*ArcT
```

```
anh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(9216*c^4)
```

Maple [A] time = 0.017, size = 230, normalized size = 1.2

$$\frac{c^4d^2ax^8}{8}-\frac{c^2d^2ax^6}{3}+\frac{d^2ax^4}{4}+\frac{c^4d^2b\mathrm{arccosh}\,(cx)\,x^8}{8}-\frac{c^2d^2b\mathrm{arccosh}\,(cx)\,x^6}{3}+\frac{d^2b\mathrm{arccosh}\,(cx)\,x^4}{4}-\frac{d^2bc^3x^7}{64}\sqrt{cx-1}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/8*c^4*d^2*a*x^8-1/3*c^2*d^2*a*x^6+1/4*d^2*a*x^4+1/8*c^4*d^2*b*arccosh(c*x
)*x^8-1/3*c^2*d^2*b*arccosh(c*x)*x^6+1/4*d^2*b*arccosh(c*x)*x^4-1/64*b*c^3*
d^2*x^7*(c*x-1)^(1/2)*(c*x+1)^(1/2)+43/1152*b*c*d^2*x^5*(c*x-1)^(1/2)*(c*x+
1)^(1/2)-73/4608*b*d^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-73/3072*b*d^2*x*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-73/3072/c^4*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [B] time = 1.41441, size = 504, normalized size = 2.52

$$\frac{1}{8}\,ac^4d^2x^8-\frac{1}{3}\,ac^2d^2x^6+\frac{1}{3072}\left(384\,x^8\,\mathrm{arcosh}\,(cx)-\left(\frac{48\,\sqrt{c^2x^2-1}x^7}{c^2}+\frac{56\,\sqrt{c^2x^2-1}x^5}{c^4}+\frac{70\,\sqrt{c^2x^2-1}x^3}{c^6}+\frac{105\,\sqrt{c^2x^2}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^2*x^8 - 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arccosh(c*x) - (48*
sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2
- 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x
^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^8))*c)*b*c^4*d^2 + 1/4*a*d^2*x^4 - 1/144*(4
8*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^
3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*s
qrt(c^2))/(sqrt(c^2)*c^6))*c)*b*c^2*d^2 + 1/32*(8*x^4*arccosh(c*x) - (2*sqr
t(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt
(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*d^2
```

Fricas [A] time = 1.82347, size = 373, normalized size = 1.86

$$\frac{1152\,ac^8d^2x^8-3072\,ac^6d^2x^6+2304\,ac^4d^2x^4+3\left(384\,bc^8d^2x^8-1024\,bc^6d^2x^6+768\,bc^4d^2x^4-73\,bd^2\right)\log\left(cx+\sqrt{c^2x^2-\cdots}\right.}{9216\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9216*(1152*a*c^8*d^2*x^8 - 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(3
84*b*c^8*d^2*x^8 - 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*log(c
*x + sqrt(c^2*x^2 - 1)) - (144*b*c^7*d^2*x^7 - 344*b*c^5*d^2*x^5 + 146*b*c^
3*d^2*x^3 + 219*b*c*d^2*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] time = 17.0226, size = 224, normalized size = 1.12

$$\begin{cases}\frac{ac^4d^2x^8}{8}-\frac{ac^2d^2x^6}{3}+\frac{ad^2x^4}{4}+\frac{bc^4d^2x^8\operatorname{acosh}(cx)}{8}-\frac{bc^3d^2x^7\sqrt{c^2x^2-1}}{64}-\frac{bc^2d^2x^6\operatorname{acosh}(cx)}{3}+\frac{43bcd^2x^5\sqrt{c^2x^2-1}}{1152}+\frac{bd^2x^4\operatorname{acosh}(cx)}{4}-\frac{73bd^2x^3\sqrt{c^2x^2\cdots}}{4608c}\cdots \\ \frac{d^2x^4\left(a+\frac{i\pi b}{2}\right)}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**8/8 - a*c**2*d**2*x**6/3 + a*d**2*x**4/4 + b*c**4
*d**2*x**8*acosh(c*x)/8 - b*c**3*d**2*x**7*sqrt(c**2*x**2 - 1)/64 - b*c**2*
d**2*x**6*acosh(c*x)/3 + 43*b*c*d**2*x**5*sqrt(c**2*x**2 - 1)/1152 + b*d**2
*x**4*acosh(c*x)/4 - 73*b*d**2*x**3*sqrt(c**2*x**2 - 1)/(4608*c) - 73*b*d**
2*x*sqrt(c**2*x**2 - 1)/(3072*c**3) - 73*b*d**2*acosh(c*x)/(3072*c**4), Ne(
c, 0)), (d**2*x**4*(a + I*pi*b/2)/4, True))
```

Giac [A] time = 1.57489, size = 451, normalized size = 2.25

$$\frac{1}{8}\,ac^4d^2x^8-\frac{1}{3}\,ac^2d^2x^6+\frac{1}{3072}\left(384\,x^8\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(\sqrt{c^2x^2-1}\left(2\left(4\,x^2\left(\frac{6\,x^2}{c^2}+\frac{7}{c^4}\right)+\frac{35}{c^6}\right)x^2+\frac{105}{c^8}\right)x-\frac{105\,\mathrm{lo}\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*a*c^4*d^2*x^8 - 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*log(c*x + sqrt(c^2*
x^2 - 1)) - (sqrt(c^2*x^2 - 1)*(2*(4*x^2*(6*x^2/c^2 + 7/c^4) + 35/c^6)*x^2
+ 105/c^8)*x - 105*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^8*abs(c)))*c)
*b*c^4*d^2 + 1/4*a*d^2*x^4 - 1/144*(48*x^6*log(c*x + sqrt(c^2*x^2 - 1)) - (
sqrt(c^2*x^2 - 1)*(2*x^2*(4*x^2/c^2 + 5/c^4) + 15/c^6)*x - 15*log(abs(-x*ab
s(c) + sqrt(c^2*x^2 - 1)))/(c^6*abs(c)))*c)*b*c^2*d^2 + 1/32*(8*x^4*log(c*x
 + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*x*(2*x^2/c^2 + 3/c^4) - 3*log(ab
s(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*abs(c)))*c)*b*d^2
```

3.12 $\int x^2 \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=177

$$\frac{1}{7}c^4d^2x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{bd^2(cx-1)^{7/2}(cx+1)^{7/2}}{49c^3}-\frac{bd^2(cx$$

```
[Out] (-8*b*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(105*c^3) + (4*b*d^2*(-1 + c*x)^(3/
2)*(1 + c*x)^(3/2))/(315*c^3) - (b*d^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(1
75*c^3) - (b*d^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/(49*c^3) + (d^2*x^3*(a +
 b*ArcCosh[c*x]))/3 - (2*c^2*d^2*x^5*(a + b*ArcCosh[c*x]))/5 + (c^4*d^2*x^7
*(a + b*ArcCosh[c*x]))/7
```

Rubi [A] time = 0.248498, antiderivative size = 223, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {270, 5731, 12, 520, 1251, 771}

$$\frac{1}{7}c^4d^2x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{bd^2\left(1-c^2x^2\right)^4}{49c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{bd^2\left(1}{175c^3\sqrt{cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*(1 - c^2*x^2))/(105*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*d^2*(
1 - c^2*x^2)^2)/(315*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^
2)^3)/(175*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d^2*(1 - c^2*x^2)^4)/(49*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^3*(a + b*ArcCosh[c*x]))/3 - (2*c
^2*d^2*x^5*(a + b*ArcCosh[c*x]))/5 + (c^4*d^2*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
```

```
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int x^2 \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a+b\cosh\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a+b\cosh\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a+b\cosh\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a+b\cosh\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a+b\cosh\right.$$

$$= \frac{8bd^2\left(1-c^2x^2\right)}{105c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4bd^2\left(1-c^2x^2\right)^2}{315c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd^2\left(1-c^2x^2\right)^3}{175c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.1657, size = 116, normalized size = 0.66

$$\frac{d^2\left(105ac^3x^3\left(15c^4x^4-42c^2x^2+35\right)-b\sqrt{cx-1}\sqrt{cx+1}\left(225c^6x^6-612c^4x^4+409c^2x^2+818\right)+105bc^3x^3\left(15c^4x^4-42c^2\right.}{11025c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(105*a*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4) - b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(818 + 409*c^2*x^2 - 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*(3
5 - 42*c^2*x^2 + 15*c^4*x^4)*ArcCosh[c*x]))/(11025*c^3)
```

Maple [A] time = 0.011, size = 120, normalized size = 0.7

$$\frac{1}{c^3}\left(d^2a\left(\frac{c^7x^7}{7}-\frac{2\,c^5x^5}{5}+\frac{c^3x^3}{3}\right)+d^2b\left(\frac{\mathrm{arccosh}\,(cx)\,c^7x^7}{7}-\frac{2\,\mathrm{arccosh}\,(cx)\,c^5x^5}{5}+\frac{c^3x^3\mathrm{arccosh}\,(cx)}{3}-\frac{225\,c^6x^6-612\,c^4x}{11}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(d^2*a*(1/7*c^7*x^7-2/5*c^5*x^5+1/3*c^3*x^3)+d^2*b*(1/7*arccosh(c*x)*
c^7*x^7-2/5*arccosh(c*x)*c^5*x^5+1/3*c^3*x^3*arccosh(c*x)-1/11025*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*(225*c^6*x^6-612*c^4*x^4+409*c^2*x^2+818)))
```

Maxima [A] time = 1.16108, size = 352, normalized size = 1.99

$$\frac{1}{7}\,ac^4d^2x^7 - \frac{2}{5}\,ac^2d^2x^5 + \frac{1}{245}\left(35\,x^7\,\operatorname{arcosh}(cx) - \left(\frac{5\,\sqrt{c^2x^2-1}x^6}{c^2} + \frac{6\,\sqrt{c^2x^2-1}x^4}{c^4} + \frac{8\,\sqrt{c^2x^2-1}x^2}{c^6} + \frac{16\,\sqrt{c^2x^2-1}}{c^8}\right)c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^2*x^7 - 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arccosh(c*x) - (5*sqr
t(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*
x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^4*d^2 - 2/75*(15*x^5*arccosh(c*x
) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 - 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(
sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2
```

Fricas [A] time = 1.77013, size = 351, normalized size = 1.98

$$\frac{1575\,ac^7d^2x^7 - 4410\,ac^5d^2x^5 + 3675\,ac^3d^2x^3 + 105\left(15\,bc^7d^2x^7 - 42\,bc^5d^2x^5 + 35\,bc^3d^2x^3\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(225\right.}{11025\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/11025*(1575*a*c^7*d^2*x^7 - 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 105
*(15*b*c^7*d^2*x^7 - 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*log(c*x + sqrt(c^
2*x^2 - 1)) - (225*b*c^6*d^2*x^6 - 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2 +
818*b*d^2)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [A] time = 9.92045, size = 209, normalized size = 1.18

$$\begin{cases} \frac{ac^4d^2x^7}{7} - \frac{2ac^2d^2x^5}{5} + \frac{ad^2x^3}{3} + \frac{bc^4d^2x^7\operatorname{acosh}(cx)}{7} - \frac{bc^3d^2x^6\sqrt{c^2x^2-1}}{49} - \frac{2bc^2d^2x^5\operatorname{acosh}(cx)}{5} + \frac{68bcd^2x^4\sqrt{c^2x^2-1}}{1225} + \frac{bd^2x^3\operatorname{acosh}(cx)}{3} - \frac{409bd^2x^2\sqrt{}}{11025} \\ \frac{d^2x^3\left(a+\frac{i\pi b}{2}\right)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**7/7 - 2*a*c**2*d**2*x**5/5 + a*d**2*x**3/3 + b*c*
*4*d**2*x**7*acosh(c*x)/7 - b*c**3*d**2*x**6*sqrt(c**2*x**2 - 1)/49 - 2*b*c
**2*d**2*x**5*acosh(c*x)/5 + 68*b*c*d**2*x**4*sqrt(c**2*x**2 - 1)/1225 + b*
d**2*x**3*acosh(c*x)/3 - 409*b*d**2*x**2*sqrt(c**2*x**2 - 1)/(11025*c) - 81
8*b*d**2*sqrt(c**2*x**2 - 1)/(11025*c**3), Ne(c, 0)), (d**2*x**3*(a + I*pi*
b/2)/3, True))
```

Giac [A] time = 1.48075, size = 347, normalized size = 1.96

$$\frac{1}{7}ac^4d^2x^7 - \frac{2}{5}ac^2d^2x^5 + \frac{1}{245}\left(35x^7\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{5\left(c^2x^2-1\right)^{\frac{7}{2}} + 21\left(c^2x^2-1\right)^{\frac{5}{2}} + 35\left(c^2x^2-1\right)^{\frac{3}{2}} + 35\sqrt{c^2x^2-}}{c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/7*a*c^4*d^2*x^7 - 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*log(c*x + sqrt(c^2*x^
2 - 1)) - (5*(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5/2) + 35*(c^2*x^2 - 1
)^(3/2) + 35*sqrt(c^2*x^2 - 1))/c^7)*b*c^4*d^2 - 2/75*(15*x^5*log(c*x + sqr
t(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt
(c^2*x^2 - 1))/c^5)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c
^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*d^2
```

3.13 $\int x\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=136

$$-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6c^2}+\frac{5bd^2\cosh^{-1}(cx)}{96c^2}-\frac{bd^2x(cx-1)^{5/2}(cx+1)^{5/2}}{36c}+\frac{5bd^2x(cx-1)^{3/2}(cx+1)^{3/2}}{144c}-\frac{5bd^2}{}$$

```
[Out] (-5*b*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96*c) + (5*b*d^2*x*(-1 + c*x)^(3
/2)*(1 + c*x)^(3/2))/(144*c) - (b*d^2*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(
36*c) + (5*b*d^2*ArcCosh[c*x])/(96*c^2) - (d^2*(1 - c^2*x^2)^3*(a + b*ArcCo
sh[c*x]))/(6*c^2)
```

Rubi [A] time = 0.0673495, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5716, 38, 52}

$$-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6c^2}+\frac{5bd^2\cosh^{-1}(cx)}{96c^2}-\frac{bd^2x(cx-1)^{5/2}(cx+1)^{5/2}}{36c}+\frac{5bd^2x(cx-1)^{3/2}(cx+1)^{3/2}}{144c}-\frac{5bd^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5*b*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96*c) + (5*b*d^2*x*(-1 + c*x)^(3
/2)*(1 + c*x)^(3/2))/(144*c) - (b*d^2*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(
36*c) + (5*b*d^2*ArcCosh[c*x])/(96*c^2) - (d^2*(1 - c^2*x^2)^3*(a + b*ArcCo
sh[c*x]))/(6*c^2)
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)dx=-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6c^2}-\frac{\left(bd^2\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\,dx}{6c}$$

$$=-\frac{bd^2x(-1+cx)^{5/2}(1+cx)^{5/2}}{36c}-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6c^2}+\frac{\left(5bd^2\right)\int(-$$

$$=\frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c}-\frac{bd^2x(-1+cx)^{5/2}(1+cx)^{5/2}}{36c}-\frac{d^2\left(1-c^2x^2\right)^3\left(a\right.}{6c^2}$$

$$=-\frac{5bd^2x\sqrt{-1+cx}\sqrt{1+cx}}{96c}+\frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c}-\frac{bd^2x(-1+cx)^{5/2}(1}{36c}$$

$$=-\frac{5bd^2x\sqrt{-1+cx}\sqrt{1+cx}}{96c}+\frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c}-\frac{bd^2x(-1+cx)^{5/2}(1}{36c}$$

Mathematica [A] time = 0.217832, size = 126, normalized size = 0.93

$$\frac{d^2\left(cx\left(48acx\left(c^4x^4-3c^2x^2+3\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-8c^4x^4+26c^2x^2-33\right)\right)+48bc^2x^2\left(c^4x^4-3c^2x^2+3\right)\cosh^{-1}(cx)-\right.}{288c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-33 + 26*c^2*x^2 - 8*c^4*x^4) +
48*a*c*x*(3 - 3*c^2*x^2 + c^4*x^4)) + 48*b*c^2*x^2*(3 - 3*c^2*x^2 + c^4*x^4
)*ArcCosh[c*x] - 66*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(288*c^2)
```

Maple [A] time = 0.013, size = 204, normalized size = 1.5

$$\frac{c^4d^2ax^6}{6}-\frac{c^2d^2ax^4}{2}+\frac{d^2ax^2}{2}+\frac{c^4d^2b\operatorname{arccosh}(cx)\,x^6}{6}-\frac{c^2d^2b\operatorname{arccosh}(cx)\,x^4}{2}+\frac{d^2b\operatorname{arccosh}(cx)\,x^2}{2}-\frac{d^2bc^3x^5}{36}\sqrt{cx-1}\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/6*c^4*d^2*a*x^6-1/2*c^2*d^2*a*x^4+1/2*d^2*a*x^2+1/6*c^4*d^2*b*arccosh(c*x
)*x^6-1/2*c^2*d^2*b*arccosh(c*x)*x^4+1/2*d^2*b*arccosh(c*x)*x^2-1/36*c^3*d^
2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5+13/144*c*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*x^3-11/96*b*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-11/96/c^2*d^2*b*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [B] time = 1.21624, size = 424, normalized size = 3.12

$$\frac{1}{6}\,ac^4d^2x^6 - \frac{1}{2}\,ac^2d^2x^4 + \frac{1}{288}\left(48\,x^6\,\mathrm{arcosh}\,(cx) - \left(\frac{8\,\sqrt{c^2x^2-1}x^5}{c^2} + \frac{10\,\sqrt{c^2x^2-1}x^3}{c^4} + \frac{15\,\sqrt{c^2x^2-1}x}{c^6} + \frac{15\,\log\left(2\,c^2x +\right.}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^2*x^6 - 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arccosh(c*x) - (8*sqr
t(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1
)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^6))*
c)*b*c^4*d^2 - 1/16*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*
sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(s
qrt(c^2)*c^4))*c)*b*c^2*d^2 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(
sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqr
t(c^2)*c^2)))*b*d^2
```

Fricas [A] time = 1.81814, size = 328, normalized size = 2.41

$$\frac{48\,ac^6d^2x^6 - 144\,ac^4d^2x^4 + 144\,ac^2d^2x^2 + 3\left(16\,bc^6d^2x^6 - 48\,bc^4d^2x^4 + 48\,bc^2d^2x^2 - 11\,bd^2\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(8\,b\right.}{288\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*d^2*x^6 - 144*a*c^4*d^2*x^4 + 144*a*c^2*d^2*x^2 + 3*(16*b*c
^6*d^2*x^6 - 48*b*c^4*d^2*x^4 + 48*b*c^2*d^2*x^2 - 11*b*d^2)*log(c*x + sqrt
(c^2*x^2 - 1)) - (8*b*c^5*d^2*x^5 - 26*b*c^3*d^2*x^3 + 33*b*c*d^2*x)*sqrt(c
^2*x^2 - 1))/c^2
```

Sympy [A] time = 6.11155, size = 197, normalized size = 1.45

$$\begin{cases} \frac{ac^4d^2x^6}{6} - \frac{ac^2d^2x^4}{2} + \frac{ad^2x^2}{2} + \frac{bc^4d^2x^6\operatorname{acosh}(cx)}{6} - \frac{bc^3d^2x^5\sqrt{c^2x^2-1}}{36} - \frac{bc^2d^2x^4\operatorname{acosh}(cx)}{2} + \frac{13bcd^2x^3\sqrt{c^2x^2-1}}{144} + \frac{bd^2x^2\operatorname{acosh}(cx)}{2} - \frac{11bd^2x\sqrt{c^2x^2-1}}{96c} \\ \frac{d^2x^2\left(a+\frac{i\pi b}{2}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**6/6 - a*c**2*d**2*x**4/2 + a*d**2*x**2/2 + b*c**4
*d**2*x**6*acosh(c*x)/6 - b*c**3*d**2*x**5*sqrt(c**2*x**2 - 1)/36 - b*c**2*
d**2*x**4*acosh(c*x)/2 + 13*b*c*d**2*x**3*sqrt(c**2*x**2 - 1)/144 + b*d**2*
x**2*acosh(c*x)/2 - 11*b*d**2*x*sqrt(c**2*x**2 - 1)/(96*c) - 11*b*d**2*acos
h(c*x)/(96*c**2), Ne(c, 0)), (d**2*x**2*(a + I*pi*b/2)/2, True))
```

Giac [B] time = 1.66007, size = 406, normalized size = 2.99

$$\frac{1}{6}\,ac^4d^2x^6 - \frac{1}{2}\,ac^2d^2x^4 + \frac{1}{288}\left(48\,x^6\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(\sqrt{c^2x^2-1}\left(2\,x^2\left(\frac{4\,x^2}{c^2} + \frac{5}{c^4}\right) + \frac{15}{c^6}\right)x - \frac{15\,\log\left(\left|-x|c| + \sqrt{c^2x^2\right.\right.}{c^6|c|}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*a*c^4*d^2*x^6 - 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*log(c*x + sqrt(c^2*x^
2 - 1)) - (sqrt(c^2*x^2 - 1)*(2*x^2*(4*x^2/c^2 + 5/c^4) + 15/c^6)*x - 15*lo
g(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^6*abs(c)))*c)*b*c^4*d^2 - 1/16*(8*
x^4*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*x*(2*x^2/c^2 + 3/c^4)
 - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*abs(c)))*c)*b*c^2*d^2 + 1
/2*a*d^2*x^2 + 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 - 1)) - c*(sqrt(c^2*x^2 -
1)*x/c^2 - log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^2*abs(c))))*b*d^2
```

3.14 $\int \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=143

$$\frac{1}{5}c^4d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{bd^2(cx-1)^{5/2}(cx+1)^{5/2}}{25c}+\frac{4bd^2(c$$

```
[Out] (-8*b*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c) + (4*b*d^2*(-1 + c*x)^(3/2)*
(1 + c*x)^(3/2))/(45*c) - (b*d^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(25*c) +
 d^2*x*(a + b*ArcCosh[c*x]) - (2*c^2*d^2*x^3*(a + b*ArcCosh[c*x]))/3 + (c^4
*d^2*x^5*(a + b*ArcCosh[c*x]))/5
```

Rubi [A] time = 0.151904, antiderivative size = 177, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {194, 5680, 12, 520, 1247, 698}

$$\frac{1}{5}c^4d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{bd^2\left(1-c^2x^2\right)^3}{25c\sqrt{cx-1}\sqrt{cx+1}}+\frac{4bd^2\left(1-\right.}{45c\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*(1 - c^2*x^2))/(15*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*d^2*(1 -
 c^2*x^2)^2)/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^2)^3)/
(25*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*x*(a + b*ArcCosh[c*x]) - (2*c^2*d
^2*x^3*(a + b*ArcCosh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcCosh[c*x]))/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx = d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\cosh^{-1}(\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\cosh^{-1}(\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\cosh^{-1}(\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\cosh^{-1}(\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\cosh^{-1}(\right.$$

$$= \frac{8bd^2\left(1-c^2x^2\right)}{15c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4bd^2\left(1-c^2x^2\right)^2}{45c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd^2\left(1-c^2x^2\right)^3}{25c\sqrt{-1+cx}\sqrt{1+cx}}+d^2x\left(\right.$$

<u>Mathematica</u> [A] time = 0.15438, size = 99, normalized size = 0.69

$$\frac{d^2\left(15acx\left(3c^4x^4-10c^2x^2+15\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-9c^4x^4+38c^2x^2-149\right)+15bcx\left(3c^4x^4-10c^2x^2+15\right)\cosh^{-1}(cx)\right.}{225c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-149 + 38*c^2*x^2 - 9*c^4*x^4) + 15*a
*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 15*b*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4)
*ArcCosh[c*x]))/(225*c)
```

<u>Maple</u> [A] time = 0.013, size = 102, normalized size = 0.7

$$\frac{1}{c}\left(d^2a\left(\frac{c^5x^5}{5}-\frac{2\,c^3x^3}{3}+cx\right)+d^2b\left(\frac{\mathrm{arccosh}\,(cx)\,c^5x^5}{5}-\frac{2\,c^3x^3\mathrm{arccosh}\,(cx)}{3}+cx\mathrm{arccosh}\,(cx)-\frac{9\,c^4x^4-38\,c^2x^2+149}{225}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(d^2*a*(1/5*c^5*x^5-2/3*c^3*x^3+c*x)+d^2*b*(1/5*arccosh(c*x)*c^5*x^5-2/
3*c^3*x^3*arccosh(c*x)+c*x*arccosh(c*x)-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(
9*c^4*x^4-38*c^2*x^2+149)))
```

Maxima [A] time = 1.17515, size = 262, normalized size = 1.83

$$\frac{1}{5}\,ac^4d^2x^5+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}\left(cx\right)-\left(\frac{3\,\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^4d^2-\frac{2}{3}\,ac^2d^2x^3-\frac{2}{9}\left(3\,x^3\operatorname{arcos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^
2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^4*d^2 - 2
/3*a*c^2*d^2*x^3 - 2/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 +
 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arccosh(c*x) - sqrt(c
^2*x^2 - 1))*b*d^2/c
```

Fricas [A] time = 1.86214, size = 296, normalized size = 2.07

$$\frac{45\,ac^5d^2x^5-150\,ac^3d^2x^3+225\,acd^2x+15\left(3\,bc^5d^2x^5-10\,bc^3d^2x^3+15\,bcd^2x\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,bc^4d^2x^4-38\,bc\right.}{225\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*d^2*x^5 - 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*d
^2*x^5 - 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (9
*b*c^4*d^2*x^4 - 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(c^2*x^2 - 1))/c
```

Sympy [A] time = 3.44075, size = 172, normalized size = 1.2

$$\begin{cases}\frac{ac^4d^2x^5}{5}-\frac{2ac^2d^2x^3}{3}+ad^2x+\frac{bc^4d^2x^5\operatorname{acosh}(cx)}{5}-\frac{bc^3d^2x^4\sqrt{c^2x^2-1}}{25}-\frac{2bc^2d^2x^3\operatorname{acosh}(cx)}{3}+\frac{38bcd^2x^2\sqrt{c^2x^2-1}}{225}+bd^2x\operatorname{acosh}(cx)-\frac{149bd^2}{22} \\ d^2x\left(a+\frac{i\pi b}{2}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**5/5 - 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d*
*2*x**5*acosh(c*x)/5 - b*c**3*d**2*x**4*sqrt(c**2*x**2 - 1)/25 - 2*b*c**2*d
**2*x**3*acosh(c*x)/3 + 38*b*c*d**2*x**2*sqrt(c**2*x**2 - 1)/225 + b*d**2*x
*acosh(c*x) - 149*b*d**2*sqrt(c**2*x**2 - 1)/(225*c), Ne(c, 0)), (d**2*x*(a
 + I*pi*b/2), True))
```

Giac [A] time = 1.46325, size = 281, normalized size = 1.97

$$\frac{1}{5}ac^4d^2x^5 + \frac{1}{75}\left(15x^5\log\left(cx+\sqrt{c^2x^2-1}\right) - \frac{3\left(c^2x^2-1\right)^{\frac{5}{2}} + 10\left(c^2x^2-1\right)^{\frac{3}{2}} + 15\sqrt{c^2x^2-1}}{c^5}\right)bc^4d^2 - \frac{2}{3}ac^2d^2x^3 - \frac{2}{9}\left(3x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2 - 1)) - (3*(c^2*x^2
 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))/c^5)*b*c^4*d^2
 - 2/3*a*c^2*d^2*x^3 - 2/9*(3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2
- 1)^(3/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*c^2*d^2 + (x*log(c*x + sqrt(c^2*x^
2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d^2 + a*d^2*x
```

3.15 $\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=184

$$-\frac{1}{2}bd^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+\frac{1}{4}d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{d^2\left(a+b\,c\right.}{2}$$

```
[Out] (11*b*c*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/32 - (b*c*d^2*x*(-1 + c*x)^(3/2
)*(1 + c*x)^(3/2))/16 - (11*b*d^2*ArcCosh[c*x])/32 + (d^2*(1 - c^2*x^2)*(a
+ b*ArcCosh[c*x]))/2 + (d^2*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 + (d^2*
(a + b*ArcCosh[c*x])^2)/(2*b) + d^2*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcC
osh[c*x])] - (b*d^2*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

Rubi [A] time = 0.204203, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5727, 5660, 3718, 2190, 2279, 2391, 38, 52}

$$\frac{1}{2}bd^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)+\frac{1}{4}d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(a+b\cosh\right.}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (11*b*c*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/32 - (b*c*d^2*x*(-1 + c*x)^(3/2
)*(1 + c*x)^(3/2))/16 - (11*b*d^2*ArcCosh[c*x])/32 + (d^2*(1 - c^2*x^2)*(a
+ b*ArcCosh[c*x]))/2 + (d^2*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 - (d^2*
(a + b*ArcCosh[c*x])^2)/(2*b) + d^2*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCo
sh[c*x])] + (b*d^2*PolyLog[2, -E^(2*ArcCosh[c*x])])/2
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{1}{4}d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+d\int \frac{\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx-\frac{1}{4}$$

$$= -\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{2}d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^2\left(1-\right.$$

$$= \frac{11}{32}bcd^2x\sqrt{-1+cx}\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{2}d^2\left(1-c^2x^2\right)\left(a\right.$$

$$= \frac{11}{32}bcd^2x\sqrt{-1+cx}\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{11}{32}bd^2\cosh^{-1}(cx)$$

$$= \frac{11}{32}bcd^2x\sqrt{-1+cx}\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{11}{32}bd^2\cosh^{-1}(cx)$$

$$= \frac{11}{32}bcd^2x\sqrt{-1+cx}\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{11}{32}bd^2\cosh^{-1}(cx)$$

$$= \frac{11}{32}bcd^2x\sqrt{-1+cx}\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{11}{32}bd^2\cosh^{-1}(cx)$$

Mathematica [A] time = 0.260816, size = 162, normalized size = 0.88

$$\frac{1}{32}d^2\left(-16b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+8ac^4x^4-32ac^2x^2+32a\log(x)-2bc^3x^3\sqrt{cx-1}\sqrt{cx+1}+8b\cosh^{-1}(cx)\left(c^4x^4\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (d^2*(-32*a*c^2*x^2 + 8*a*c^4*x^4 + 13*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] -
 2*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 16*b*ArcCosh[c*x]^2 + 26*b*ArcT
anh[Sqrt[(-1 + c*x)/(1 + c*x)]] + 8*b*ArcCosh[c*x]*(-4*c^2*x^2 + c^4*x^4 +
4*Log[1 + E^(-2*ArcCosh[c*x])]) + 32*a*Log[x] - 16*b*PolyLog[2, -E^(-2*ArcC
osh[c*x])]))/32
```

Maple [A] time = 0.145, size = 201, normalized size = 1.1

$$\frac{d^2ac^4x^4}{4}-d^2ac^2x^2+d^2a\ln(cx)+\frac{13\,bd^2\text{arccosh}\,(cx)}{32}+\frac{d^2b\text{arccosh}\,(cx)\,c^4x^4}{4}+d^2b\text{arccosh}\,(cx)\ln\left(\left(cx+\sqrt{cx-1}\sqrt{cx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/4*d^2*a*c^4*x^4-d^2*a*c^2*x^2+d^2*a*ln(c*x)+13/32*b*d^2*arccosh(c*x)+1/4*
d^2*b*arccosh(c*x)*c^4*x^4+d^2*b*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2+1)-1/16*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3*x^3+13/32*b*c*d^2*x
*(c*x-1)^(1/2)*(c*x+1)^(1/2)-d^2*b*arccosh(c*x)*c^2*x^2-1/2*d^2*b*arccosh(c
*x)^2+1/2*d^2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,ac^4d^2x^4 - ac^2d^2x^2 + ad^2\log\left(x\right) + \int bc^4d^2x^3\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) - 2\,bc^2d^2x\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \frac{bd^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^2*x^4 - a*c^2*d^2*x^2 + a*d^2*log(x) + integrate(b*c^4*d^2*x^3*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 2*b*c^2*d^2*x*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1)) + b*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\text{arcosh}\left(cx\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{a}{x}\,dx + \int -2ac^2x\,dx + \int ac^4x^3\,dx + \int \frac{b\,\text{acosh}\left(cx\right)}{x}\,dx + \int -2bc^2x\,\text{acosh}\left(cx\right)dx + \int bc^4x^3\,\text{acosh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x,x)
```

```
[Out] d**2*(Integral(a/x, x) + Integral(-2*a*c**2*x, x) + Integral(a*c**4*x**3, x
) + Integral(b*acosh(c*x)/x, x) + Integral(-2*b*c**2*x*acosh(c*x), x) + Int
egral(b*c**4*x**3*acosh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 - d\right)^2 (b \operatorname{arcosh}(cx) + a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*(b*arccosh(c*x) + a)/x, x)
```

3.16 $\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=135

$$\frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(cx)\right)-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{1}{9}bcd^2(cx-1)^{3/2}(cx+1)^{3/2}+\frac{5}{3}bcd^2\sqrt{}$$

```
[Out] (5*b*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/3 - (b*c*d^2*(-1 + c*x)^(3/2)*(1 +
 c*x)^(3/2))/9 - (d^2*(a + b*ArcCosh[c*x]))/x - 2*c^2*d^2*x*(a + b*ArcCosh[
c*x]) + (c^4*d^2*x^3*(a + b*ArcCosh[c*x]))/3 + b*c*d^2*ArcTan[Sqrt[-1 + c*x
]*Sqrt[1 + c*x]]
```

Rubi [A] time = 0.229905, antiderivative size = 182, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {270, 5731, 12, 520, 1251, 897, 1153, 205}

$$\frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(cx)\right)-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{bcd^2\left(1-c^2x^2\right)^2}{9\sqrt{cx-1}\sqrt{cx+1}}-\frac{5bcd^2\left(1-c^2x^2\right)}{3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (-5*b*c*d^2*(1 - c^2*x^2))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2*(1 -
 c^2*x^2)^2)/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*ArcCosh[c*x]))/
x - 2*c^2*d^2*x*(a + b*ArcCosh[c*x]) + (c^4*d^2*x^3*(a + b*ArcCosh[c*x]))/3
 + (b*c*d^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
```

```
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x} - 2c^2d^2x\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x} - 2c^2d^2x\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x} - 2c^2d^2x\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x} - 2c^2d^2x\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x} - 2c^2d^2x\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x} - 2c^2d^2x\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= -\frac{5bcd^2\left(1-c^2x^2\right)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2\left(1-c^2x^2\right)^2}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x} - 2c^2d^2x\left(a\right.$$

$$= -\frac{5bcd^2\left(1-c^2x^2\right)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2\left(1-c^2x^2\right)^2}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x} - 2c^2d^2x\left(a\right.$$

Mathematica [A] time = 0.158013, size = 131, normalized size = 0.97

$$\frac{d^2\left(3ac^4x^4-18ac^2x^2-9a-bc^3x^3\sqrt{cx-1}\sqrt{cx+1}+3b\left(c^4x^4-6c^2x^2-3\right)\cosh^{-1}(cx)+16bcx\sqrt{cx-1}\sqrt{cx+1}-9bcx\,\text{ta}\right.}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (d^2*(-9*a - 18*a*c^2*x^2 + 3*a*c^4*x^4 + 16*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 +
c*x] - b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 3*b*(-3 - 6*c^2*x^2 + c^4*x
```

```
^4)*ArcCosh[c*x] - 9*b*c*x*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])]))/(9*x)
```

Maple [A] time = 0.017, size = 167, normalized size = 1.2

$$\frac{d^2ac^4x^3}{3}-2\,d^2ac^2x-\frac{d^2a}{x}+\frac{d^2b\mathrm{arccosh}\,(cx)\,c^4x^3}{3}-2\,d^2b\mathrm{arccosh}\,(cx)\,c^2x-\frac{bd^2\mathrm{arccosh}\,(cx)}{x}-\frac{d^2bc^3x^2}{9}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] 1/3*d^2*a*c^4*x^3-2*d^2*a*c^2*x-d^2*a/x+1/3*d^2*b*arccosh(c*x)*c^4*x^3-2*d^
2*b*arccosh(c*x)*c^2*x-d^2*b*arccosh(c*x)/x-1/9*d^2*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*c^3*x^2+16/9*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-c*d^2*b*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.86869, size = 196, normalized size = 1.45

$$\frac{1}{3}\,ac^4d^2x^3+\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^4d^2-2\,ac^2d^2x-2\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bcd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2
+ 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^4*d^2 - 2*a*c^2*d^2*x - 2*(c*x*arccosh(c*x)
 - sqrt(c^2*x^2 - 1))*b*c*d^2 - (c*arcsin(1/(sqrt(c^2)*abs(x))) + arccosh(c
*x)/x)*b*d^2 - a*d^2/x
```

Fricas [A] time = 2.06974, size = 440, normalized size = 3.26

$$\frac{3\,ac^4d^2x^4-18\,ac^2d^2x^2+18\,bcd^2x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-3\left(bc^4-6\,bc^2-3\,b\right)d^2x\log\left(-cx+\sqrt{c^2x^2-1}\right)-9\,ad^2+3}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^4*d^2*x^4 - 18*a*c^2*d^2*x^2 + 18*b*c*d^2*x*arctan(-c*x + sqrt(c
^2*x^2 - 1)) - 3*(b*c^4 - 6*b*c^2 - 3*b)*d^2*x*log(-c*x + sqrt(c^2*x^2 - 1)
) - 9*a*d^2 + 3*(b*c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - (b*c^4 - 6*b*c^2 - 3*b)*
d^2*x - 3*b*d^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^3*d^2*x^3 - 16*b*c*d^2
*x)*sqrt(c^2*x^2 - 1))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int -2ac^2\,dx + \int \frac{a}{x^2}\,dx + \int ac^4x^2\,dx + \int -2bc^2\operatorname{acosh}(cx)\,dx + \int \frac{b\operatorname{acosh}(cx)}{x^2}\,dx + \int bc^4x^2\operatorname{acosh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] d**2*(Integral(-2*a*c**2, x) + Integral(a/x**2, x) + Integral(a*c**4*x**2,
x) + Integral(-2*b*c**2*acosh(c*x), x) + Integral(b*acosh(c*x)/x**2, x) + I
ntegral(b*c**4*x**2*acosh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 - d\right)^2 (b\operatorname{arcosh}(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*(b*arccosh(c*x) + a)/x^2, x)
```

3.17 $\int \frac{(d-c^2dx^2)^2(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=200

$$bc^2d^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-c^2d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{c^2d^2\left(a+b\cosh^{-1}(cx)\right)^2}{b}$$

```
[Out] (b*c^3*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/4 - (b*c*d^2*(-1 + c*x)^(3/2)*(1
 + c*x)^(3/2))/(2*x) - (b*c^2*d^2*ArcCosh[c*x])/4 - c^2*d^2*(1 - c^2*x^2)*(
a + b*ArcCosh[c*x]) - (d^2*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/(2*x^2) -
(c^2*d^2*(a + b*ArcCosh[c*x])^2)/b - 2*c^2*d^2*(a + b*ArcCosh[c*x])*Log[1 +
 E^(-2*ArcCosh[c*x])] + b*c^2*d^2*PolyLog[2, -E^(-2*ArcCosh[c*x])]
```

Rubi [A] time = 0.217591, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5729, 97, 12, 38, 52, 5727, 5660, 3718, 2190, 2279, 2391}

$$-bc^2d^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)-c^2d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{c^2d^2\left(a+b\cosh^{-1}(cx)\right)^2}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c^3*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/4 - (b*c*d^2*(-1 + c*x)^(3/2)*(1
 + c*x)^(3/2))/(2*x) - (b*c^2*d^2*ArcCosh[c*x])/4 - c^2*d^2*(1 - c^2*x^2)*(
a + b*ArcCosh[c*x]) - (d^2*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/(2*x^2) +
(c^2*d^2*(a + b*ArcCosh[c*x])^2)/b - 2*c^2*d^2*(a + b*ArcCosh[c*x])*Log[1 +
 E^(2*ArcCosh[c*x])] - b*c^2*d^2*PolyLog[2, -E^(2*ArcCosh[c*x])]
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\left(2c^2d\right)\int \frac{\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx$$

$$= -\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-c^2d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(1-c^2x^2\right)}{}$$

$$= -\frac{1}{2}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-c^2d^2\left(1-c^2x^2\right)\left(a+\right.$$

$$= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}+\frac{1}{2}bc^2d^2\cosh^{-1}(cx)-$$

$$= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-\frac{1}{4}bc^2d^2\cosh^{-1}(cx)-$$

$$= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-\frac{1}{4}bc^2d^2\cosh^{-1}(cx)-$$

$$= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-\frac{1}{4}bc^2d^2\cosh^{-1}(cx)-$$

Mathematica [A] time = 0.250183, size = 182, normalized size = 0.91

$$\frac{d^2\left(4bc^2x^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+2ac^4x^4-8ac^2x^2\log(x)-2a-bc^3x^3\sqrt{cx-1}\sqrt{cx+1}-4bc^2x^2\cosh^{-1}(cx)^2-2b\right.}{4x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (d^2*(-2*a + 2*a*c^4*x^4 + 2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - b*c^3*x^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 4*b*c^2*x^2*ArcCosh[c*x]^2 - 2*b*c^2*x^2*Ar
cTanh[Sqrt[(-1 + c*x)/(1 + c*x)]] + 2*b*ArcCosh[c*x]*(-1 + c^4*x^4 - 4*c^2*
x^2*Log[1 + E^(-2*ArcCosh[c*x])]) - 8*a*c^2*x^2*Log[x] + 4*b*c^2*x^2*PolyLo
g[2, -E^(-2*ArcCosh[c*x])]))/(4*x^2)
```

Maple [A] time = 0.261, size = 220, normalized size = 1.1

$$\frac{c^4d^2ax^2}{2}-2c^2d^2a\ln(cx)-\frac{d^2a}{2x^2}+c^2d^2b\left(\operatorname{arccosh}(cx)\right)^2+\frac{c^4d^2b\operatorname{arccosh}(cx)x^2}{2}-\frac{bc^3d^2x}{4}\sqrt{cx-1}\sqrt{cx+1}-\frac{bc^2d^2\operatorname{arcc}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] 1/2*c^4*d^2*a*x^2-2*c^2*d^2*a*ln(c*x)-1/2*d^2*a/x^2+c^2*d^2*b*arccosh(c*x)^
2+1/2*c^4*d^2*b*arccosh(c*x)*x^2-1/4*b*c^3*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2
)-1/4*b*c^2*d^2*arccosh(c*x)-1/2*d^2*b*c^2+1/2*b*c*d^2*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/x-1/2*d^2*b*arccosh(c*x)/x^2-2*c^2*d^2*b*arccosh(c*x)*ln((c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2+1)-c^2*d^2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}ac^4d^2x^2-2ac^2d^2\log(x)+\frac{1}{2}bd^2\left(\frac{\sqrt{c^2x^2-1}c}{x}-\frac{\operatorname{arcosh}(cx)}{x^2}\right)-\frac{ad^2}{2x^2}+\int bc^4d^2x\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)-\frac{2bc^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^4*d^2*x^2 - 2*a*c^2*d^2*log(x) + 1/2*b*d^2*(sqrt(c^2*x^2 - 1)*c/x -
 arccosh(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate(b*c^4*d^2*x*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1)) - 2*b*c^2*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arcosh}\left(cx\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{a}{x^3}\,dx + \int -\frac{2ac^2}{x}\,dx + \int ac^4x\,dx + \int \frac{b\operatorname{acosh}\left(cx\right)}{x^3}\,dx + \int -\frac{2bc^2\operatorname{acosh}\left(cx\right)}{x}\,dx + \int bc^4x\operatorname{acosh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(-2*a*c**2/x, x) + Integral(a*c**4*x, x
) + Integral(b*acosh(c*x)/x**3, x) + Integral(-2*b*c**2*acosh(c*x)/x, x) +
Integral(b*c**4*x*acosh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 - d\right)^2\left(b\operatorname{arcosh}\left(cx\right) + a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*(b*arccosh(c*x) + a)/x^3, x)
```

3.18 $\int \frac{(d-c^2dx^2)^2(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=142

$$c^4d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-bc^3d^2\sqrt{cx-1}\sqrt{cx+1}-\frac{11}{6}bc^3d^2\tan^{-1}\left(\sqrt{\right.$$

```
[Out] -(b*c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(6*x^2) - (d^2*(a + b*ArcCosh[c*x]))/(3*x^3) + (2*c^2*d^2*(a + b*Ar
cCosh[c*x]))/x + c^4*d^2*x*(a + b*ArcCosh[c*x]) - (11*b*c^3*d^2*ArcTan[Sqrt
[-1 + c*x]*Sqrt[1 + c*x]])/6
```

Rubi [A] time = 0.234374, antiderivative size = 186, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {270, 5731, 12, 520, 1251, 897, 1157, 388, 205}

$$c^4d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{bc^3d^2\left(1-c^2x^2\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{cx-1}\sqrt{cx+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*c^3*d^2*(1 - c^2*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2*(1 - c^
2*x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*ArcCosh[c*x]))/(
3*x^3) + (2*c^2*d^2*(a + b*ArcCosh[c*x]))/x + c^4*d^2*x*(a + b*ArcCosh[c*x]
) - (11*b*c^3*d^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
```

```
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\cosh^{-1}(cx)\right)-$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\cosh^{-1}(cx)\right)-$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\cosh^{-1}(cx)\right)-$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\cosh^{-1}(cx)\right)-$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\cosh^{-1}(cx)\right)-$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4$$

$$= \frac{bc^3d^2\left(1-c^2x^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+\right.}{}$$

$$= \frac{bc^3d^2\left(1-c^2x^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+\right.}{}$$

Mathematica [A] time = 0.160193, size = 135, normalized size = 0.95

$$\frac{d^2\left(6ac^4x^4+12ac^2x^2-2a-6bc^3x^3\sqrt{cx-1}\sqrt{cx+1}+11bc^3x^3\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+2b\left(3c^4x^4+6c^2x^2-1\right)\cosh^{-1}(cx)\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (d^2*(-2*a + 12*a*c^2*x^2 + 6*a*c^4*x^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x
] - 6*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*(-1 + 6*c^2*x^2 + 3*c^4*
x^4)*ArcCosh[c*x] + 11*b*c^3*x^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])]))
/(6*x^3)
```

Maple [A] time = 0.017, size = 167, normalized size = 1.2

$$c^4d^2ax+2\,\frac{c^2d^2a}{x}-\frac{d^2a}{3\,x^3}+c^4d^2b\mathrm{arccosh}\,(cx)\,x+2\,\frac{bc^2d^2\mathrm{arccosh}\,(cx)}{x}-\frac{bd^2\mathrm{arccosh}\,(cx)}{3\,x^3}-bc^3d^2\sqrt{cx-1}\sqrt{cx+1}+\frac{11}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] c^4*d^2*a*x+2*c^2*d^2*a/x-1/3*d^2*a/x^3+c^4*d^2*b*arccosh(c*x)*x+2*c^2*d^2*
b*arccosh(c*x)/x-1/3*d^2*b*arccosh(c*x)/x^3-b*c^3*d^2*(c*x-1)^(1/2)*(c*x+1)
^(1/2)+11/6*c^3*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(
1/(c^2*x^2-1)^(1/2))+1/6*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

Maxima [A] time = 1.74458, size = 190, normalized size = 1.34

$$ac^4d^2x+\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bc^3d^2+2\left(c\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bc^2d^2-\frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^2*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*c^3*d^2 + 2*(c*arcsi
n(1/(sqrt(c^2)*abs(x))) + arccosh(c*x)/x)*b*c^2*d^2 - 1/6*((c^2*arcsin(1/(s
```

```
qrt(c^2)*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)/x^3)*b*d^2 +
2*a*c^2*d^2/x - 1/3*a*d^2/x^3
```

Fricas [A] time = 1.96439, size = 452, normalized size = 3.18

$$\frac{6\,ac^4d^2x^4 - 22\,bc^3d^2x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right) + 12\,ac^2d^2x^2 - 2\left(3\,bc^4+6\,bc^2-b\right)d^2x^3\log\left(-cx+\sqrt{c^2x^2-1}\right) - 2\,ad^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c^4*d^2*x^4 - 22*b*c^3*d^2*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) +
12*a*c^2*d^2*x^2 - 2*(3*b*c^4 + 6*b*c^2 - b)*d^2*x^3*log(-c*x + sqrt(c^2*x^
2 - 1)) - 2*a*d^2 + 2*(3*b*c^4*d^2*x^4 + 6*b*c^2*d^2*x^2 - (3*b*c^4 + 6*b*c
^2 - b)*d^2*x^3 - b*d^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^3*d^2*x^3 -
b*c*d^2*x)*sqrt(c^2*x^2 - 1))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int ac^4\,dx + \int \frac{a}{x^4}\,dx + \int -\frac{2ac^2}{x^2}\,dx + \int bc^4\operatorname{acosh}(cx)\,dx + \int \frac{b\operatorname{acosh}(cx)}{x^4}\,dx + \int -\frac{2bc^2\operatorname{acosh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] d**2*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(-2*a*c**2/x**2,
x) + Integral(b*c**4*acosh(c*x), x) + Integral(b*acosh(c*x)/x**4, x) + Inte
gral(-2*b*c**2*acosh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2-d\right)^2\left(b\operatorname{arcosh}(cx)+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*(b*arccosh(c*x) + a)/x^4, x)
```

3.19 $\int x^4 \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=256

$$-\frac{1}{11}c^6d^3x^{11}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+$$

```
[Out] (-16*b*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1155*c^5) + (8*b*d^3*(-1 + c*x)^(
3/2)*(1 + c*x)^(3/2))/(3465*c^5) - (2*b*d^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2
))/(1925*c^5) + (b*d^3*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/(1617*c^5) + (4*b*
d^3*(-1 + c*x)^(9/2)*(1 + c*x)^(9/2))/(297*c^5) + (b*d^3*(-1 + c*x)^(11/2)*
(1 + c*x)^(11/2))/(121*c^5) + (d^3*x^5*(a + b*ArcCosh[c*x]))/5 - (3*c^2*d^3
*x^7*(a + b*ArcCosh[c*x]))/7 + (c^4*d^3*x^9*(a + b*ArcCosh[c*x]))/3 - (c^6*
d^3*x^11*(a + b*ArcCosh[c*x]))/11
```

Rubi [A] time = 0.438654, antiderivative size = 326, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {270, 5731, 12, 1610, 1799, 1620}

$$-\frac{1}{11}c^6d^3x^{11}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^3*(1 - c^2*x^2))/(1155*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*d^3
*(1 - c^2*x^2)^2)/(3465*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d^3*(1 - c
^2*x^2)^3)/(1925*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^4
)/(1617*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*d^3*(1 - c^2*x^2)^5)/(297*
c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^6)/(121*c^5*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^5*(a + b*ArcCosh[c*x]))/5 - (3*c^2*d^3*x^7
*(a + b*ArcCosh[c*x]))/7 + (c^4*d^3*x^9*(a + b*ArcCosh[c*x]))/3 - (c^6*d^3*
x^11*(a + b*ArcCosh[c*x]))/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^4 \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\cosh\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\cosh\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\cosh\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\cosh\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\cosh\right.$$

$$=\frac{16bd^3\left(1-c^2x^2\right)}{1155c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{8bd^3\left(1-c^2x^2\right)^2}{3465c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bd^3\left(1-c^2x^2\right)^3}{1925c^5\sqrt{-1+cx}\sqrt{1+\right.}}$$

Mathematica [A] time = 0.260505, size = 147, normalized size = 0.57

$$-\frac{d^3\left(3465ac^5x^5\left(105c^6x^6-385c^4x^4+495c^2x^2-231\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-33075c^{10}x^{10}+111475c^8x^8-117625c^6x^6+1\right.\right.}{4002075c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d^3*(3465*a*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^4 + 105*c^6*x^6) + b*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(50488 + 25244*c^2*x^2 + 18933*c^4*x^4 - 11762
5*c^6*x^6 + 111475*c^8*x^8 - 33075*c^10*x^10) + 3465*b*c^5*x^5*(-231 + 495*
c^2*x^2 - 385*c^4*x^4 + 105*c^6*x^6)*ArcCosh[c*x]))/(4002075*c^5)
```

Maple [A] time = 0.018, size = 158, normalized size = 0.6

$$\frac{1}{c^5}\left(-d^3a\left(\frac{c^{11}x^{11}}{11}-\frac{c^9x^9}{3}+\frac{3\,c^7x^7}{7}-\frac{c^5x^5}{5}\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)\,c^{11}x^{11}}{11}-\frac{\operatorname{arccosh}(cx)\,c^9x^9}{3}+\frac{3\operatorname{arccosh}(cx)\,c^7x^7}{7}-\frac{\operatorname{arcc}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(-d^3*a*(1/11*c^11*x^11-1/3*c^9*x^9+3/7*c^7*x^7-1/5*c^5*x^5)-d^3*b*(1
/11*arccosh(c*x)*c^11*x^11-1/3*arccosh(c*x)*c^9*x^9+3/7*arccosh(c*x)*c^7*x^
7-1/5*arccosh(c*x)*c^5*x^5-1/4002075*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(33075*c^1
0*x^10-111475*c^8*x^8+117625*c^6*x^6-18933*c^4*x^4-25244*c^2*x^2-50488)))
```

Maxima [B] time = 1.25723, size = 628, normalized size = 2.45

$$-\frac{1}{11}\,ac^6d^3x^{11}+\frac{1}{3}\,ac^4d^3x^9-\frac{3}{7}\,ac^2d^3x^7-\frac{1}{7623}\left(693\,x^{11}\,\mathrm{arcosh}\,(cx)-\left(\frac{63\,\sqrt{c^2x^2-1}x^{10}}{c^2}+\frac{70\,\sqrt{c^2x^2-1}x^8}{c^4}+\frac{80\,\sqrt{c^2x^2-}}{c^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 - 3/7*a*c^2*d^3*x^7 - 1/7623*(693*
x^11*arccosh(c*x) - (63*sqrt(c^2*x^2 - 1)*x^10/c^2 + 70*sqrt(c^2*x^2 - 1)*x
^8/c^4 + 80*sqrt(c^2*x^2 - 1)*x^6/c^6 + 96*sqrt(c^2*x^2 - 1)*x^4/c^8 + 128*
sqrt(c^2*x^2 - 1)*x^2/c^10 + 256*sqrt(c^2*x^2 - 1)/c^12)*c)*b*c^6*d^3 + 1/9
45*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2
- 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8
+ 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5 - 3/245*(35*x^7*
arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 +
 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d^3 + 1/7
5*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)
*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^3
```

Fricas [A] time = 1.81655, size = 510, normalized size = 1.99

$$-\frac{363825\,ac^{11}d^3x^{11}-1334025\,ac^9d^3x^9+1715175\,ac^7d^3x^7-800415\,ac^5d^3x^5+3465\left(105\,bc^{11}d^3x^{11}-385\,bc^9d^3x^9+495\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/4002075*(363825*a*c^11*d^3*x^11 - 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7*
d^3*x^7 - 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 - 385*b*c^9*d^3*
```

```
x^9 + 495*b*c^7*d^3*x^7 - 231*b*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 - 1)) -
 (33075*b*c^10*d^3*x^10 - 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 - 189
33*b*c^4*d^3*x^4 - 25244*b*c^2*d^3*x^2 - 50488*b*d^3)*sqrt(c^2*x^2 - 1))/c^
5
```

Sympy [A] time = 65.2464, size = 296, normalized size = 1.16

$$\begin{cases} -\frac{ac^6d^3x^{11}}{11} + \frac{ac^4d^3x^9}{3} - \frac{3ac^2d^3x^7}{7} + \frac{ad^3x^5}{5} - \frac{bc^6d^3x^{11}\operatorname{acosh}(cx)}{11} + \frac{bc^5d^3x^{10}\sqrt{c^2x^2-1}}{121} + \frac{bc^4d^3x^9\operatorname{acosh}(cx)}{3} - \frac{91bc^3d^3x^8\sqrt{c^2x^2-1}}{3267} - \frac{3bc^2d^3x^7\operatorname{acosh}}{7} \\ \frac{d^3x^5\left(a+\frac{i\pi b}{2}\right)}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**11/11 + a*c**4*d**3*x**9/3 - 3*a*c**2*d**3*x**7/
7 + a*d**3*x**5/5 - b*c**6*d**3*x**11*acosh(c*x)/11 + b*c**5*d**3*x**10*sqr
t(c**2*x**2 - 1)/121 + b*c**4*d**3*x**9*acosh(c*x)/3 - 91*b*c**3*d**3*x**8*
sqrt(c**2*x**2 - 1)/3267 - 3*b*c**2*d**3*x**7*acosh(c*x)/7 + 4705*b*c*d**3*
x**6*sqrt(c**2*x**2 - 1)/160083 + b*d**3*x**5*acosh(c*x)/5 - 6311*b*d**3*x*
*4*sqrt(c**2*x**2 - 1)/(1334025*c) - 25244*b*d**3*x**2*sqrt(c**2*x**2 - 1)/
(4002075*c**3) - 50488*b*d**3*sqrt(c**2*x**2 - 1)/(4002075*c**5), Ne(c, 0))
, (d**3*x**5*(a + I*pi*b/2)/5, True))
```

Giac [B] time = 1.54981, size = 574, normalized size = 2.24

$$-\frac{1}{11}ac^6d^3x^{11} + \frac{1}{3}ac^4d^3x^9 - \frac{3}{7}ac^2d^3x^7 - \frac{1}{7623}\left(693\,x^{11}\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{63\left(c^2x^2-1\right)^{\frac{11}{2}} + 385\left(c^2x^2-1\right)^{\frac{9}{2}} + 990\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] -1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 - 3/7*a*c^2*d^3*x^7 - 1/7623*(693*
x^11*log(c*x + sqrt(c^2*x^2 - 1)) - (63*(c^2*x^2 - 1)^(11/2) + 385*(c^2*x^2
 - 1)^(9/2) + 990*(c^2*x^2 - 1)^(7/2) + 1386*(c^2*x^2 - 1)^(5/2) + 1155*(c^
2*x^2 - 1)^(3/2) + 693*sqrt(c^2*x^2 - 1))/c^11)*b*c^6*d^3 + 1/945*(315*x^9*
log(c*x + sqrt(c^2*x^2 - 1)) - (35*(c^2*x^2 - 1)^(9/2) + 180*(c^2*x^2 - 1)^
```

```
(7/2) + 378*(c^2*x^2 - 1)^(5/2) + 420*(c^2*x^2 - 1)^(3/2) + 315*sqrt(c^2*x^
2 - 1))/c^9)*b*c^4*d^3 + 1/5*a*d^3*x^5 - 3/245*(35*x^7*log(c*x + sqrt(c^2*x
^2 - 1)) - (5*(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5/2) + 35*(c^2*x^2 -
1)^(3/2) + 35*sqrt(c^2*x^2 - 1))/c^7)*b*c^2*d^3 + 1/75*(15*x^5*log(c*x + sq
rt(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqr
t(c^2*x^2 - 1))/c^5)*b*d^3
```

3.20 $\int x^3 \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=230

$$-\frac{d^3(cx-1)^5(cx+1)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}-\frac{d^3(cx-1)^4(cx+1)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{bd^3x(cx-1)^{9/2}(cx+1)^{9/2}}{100c^3}+\frac{7bd^3x(\cdots}{}$$

```
[Out] (-49*b*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5120*c^3) + (49*b*d^3*x*(-1 + c
*x)^(3/2)*(1 + c*x)^(3/2))/(7680*c^3) - (49*b*d^3*x*(-1 + c*x)^(5/2)*(1 + c
*x)^(5/2))/(9600*c^3) + (7*b*d^3*x*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/(1600*
c^3) + (b*d^3*x*(-1 + c*x)^(9/2)*(1 + c*x)^(9/2))/(100*c^3) + (49*b*d^3*Arc
Cosh[c*x])/(5120*c^4) - (d^3*(-1 + c*x)^4*(1 + c*x)^4*(a + b*ArcCosh[c*x]))
/(8*c^4) - (d^3*(-1 + c*x)^5*(1 + c*x)^5*(a + b*ArcCosh[c*x]))/(10*c^4)
```

Rubi [A] time = 0.281196, antiderivative size = 328, normalized size of antiderivative = 1.43, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 43, 5731, 12, 566, 21, 388, 195, 217, 206}

$$\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}-\frac{bd^3x\left(1-c^2x^2\right)^5}{100c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{7bd^3x\left(1-c^2x^2\right)^4}{1600c^3\sqrt{cx-1}\sqrt{cx+1}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (49*b*d^3*x*(1 - c^2*x^2))/(5120*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (49*b*
d^3*x*(1 - c^2*x^2)^2)/(7680*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (49*b*d^3*
x*(1 - c^2*x^2)^3)/(9600*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*b*d^3*x*(1
- c^2*x^2)^4)/(1600*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d^3*x*(1 - c^2*x
^2)^5)/(100*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(1 - c^2*x^2)^4*(a + b
*ArcCosh[c*x]))/(8*c^4) + (d^3*(1 - c^2*x^2)^5*(a + b*ArcCosh[c*x]))/(10*c^
4) + (49*b*d^3*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(5120*
c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 566

```
Int[((e1_) + (f1_.)*(x_)^(n2_.))^(r_.)*((e2_) + (f2_.)*(x_)^(n2_.))^(r_.)*(
(a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :>
Dist[((e1 + f1*x^(n/2))^FracPart[r]*(e2 + f2*x^(n/2))^FracPart[r])/(e1*e2 +
 f1*f2*x^n)^FracPart[r], Int[(a + b*x^n)^p*(c + d*x^n)^q*(e1*e2 + f1*f2*x^n
)^r, x], x] /; FreeQ[{a, b, c, d, e1, f1, e2, f2, n, p, q, r}, x] && EqQ[n2
, n/2] && EqQ[e2*f1 + e1*f2, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}-(bc$$

$$= -\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}-\frac{(bc}{}$$

$$= -\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}-\frac{(bc}{}$$

$$= -\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}-\frac{(bc}{}$$

$$= -\frac{bd^3x\left(1-c^2x^2\right)^5}{100c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1-c^2x^2\right.}{}$$

$$= \frac{7bd^3x\left(1-c^2x^2\right)^4}{1600c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bd^3x\left(1-c^2x^2\right)^5}{100c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\,c\right.}{8c^4}$$

$$= \frac{49bd^3x\left(1-c^2x^2\right)^3}{9600c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{7bd^3x\left(1-c^2x^2\right)^4}{1600c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bd^3x\left(1-c^2x^2\right)^5}{100c^3\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{49bd^3x\left(1-c^2x^2\right)^2}{7680c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)^3}{9600c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{7bd^3x\left(1-c^2x^2\right)}{1600c^3\sqrt{-1+cx}\sqrt{1}}$$

$$= \frac{49bd^3x\left(1-c^2x^2\right)}{5120c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)^2}{7680c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)}{9600c^3\sqrt{-1+cx}\sqrt{1}}$$

$$= \frac{49bd^3x\left(1-c^2x^2\right)}{5120c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)^2}{7680c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)}{9600c^3\sqrt{-1+cx}\sqrt{1}}$$

$$= \frac{49bd^3x\left(1-c^2x^2\right)}{5120c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)^2}{7680c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)}{9600c^3\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A] time = 0.342736, size = 162, normalized size = 0.7

$$-\frac{d^3\left(1920ac^4x^4\left(4c^6x^6-15c^4x^4+20c^2x^2-10\right)+bcx\sqrt{cx-1}\sqrt{cx+1}\left(-768c^8x^8+2736c^6x^6-3208c^4x^4+790c^2x^2+1\right.\right.}{76800c^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d^3*(1920*a*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^4 + 4*c^6*x^6) + b*c*x*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(1185 + 790*c^2*x^2 - 3208*c^4*x^4 + 2736*c^6*x
^6 - 768*c^8*x^8) + 1920*b*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^4 + 4*c^6*x
^6)*ArcCosh[c*x] + 2370*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(76800*c^4)
```

Maple [A] time = 0.019, size = 284, normalized size = 1.2

$$-\frac{c^6d^3ax^{10}}{10}+\frac{3\,c^4d^3ax^8}{8}-\frac{c^2d^3ax^6}{2}+\frac{d^3ax^4}{4}-\frac{c^6d^3b\mathrm{arccosh}\,(cx)\,x^{10}}{10}+\frac{3\,c^4d^3b\mathrm{arccosh}\,(cx)\,x^8}{8}-\frac{c^2d^3b\mathrm{arccosh}\,(cx)\,x^6}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/10*c^6*d^3*a*x^10+3/8*c^4*d^3*a*x^8-1/2*c^2*d^3*a*x^6+1/4*d^3*a*x^4-1/10
*c^6*d^3*b*arccosh(c*x)*x^10+3/8*c^4*d^3*b*arccosh(c*x)*x^8-1/2*c^2*d^3*b*a
rccosh(c*x)*x^6+1/4*d^3*b*arccosh(c*x)*x^4+1/100*c^5*d^3*b*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*x^9-57/1600*c^3*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^7+401/9600*
c*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5-79/7680/c*d^3*b*(c*x-1)^(1/2)*(c*x+
1)^(1/2)*x^3-79/5120*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-79/5120/c^4*d^
3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [B] time = 1.40942, size = 725, normalized size = 3.15

$$-\frac{1}{10}\,ac^6d^3x^{10}+\frac{3}{8}\,ac^4d^3x^8-\frac{1}{2}\,ac^2d^3x^6-\frac{1}{12800}\left(1280\,x^{10}\,\mathrm{arcosh}\,(cx)-\left(\frac{128\,\sqrt{c^2x^2-1}x^9}{c^2}+\frac{144\,\sqrt{c^2x^2-1}x^7}{c^4}+\frac{168\,\sqrt{c^2}}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 - 1/2*a*c^2*d^3*x^6 - 1/12800*(128
0*x^10*arccosh(c*x) - (128*sqrt(c^2*x^2 - 1)*x^9/c^2 + 144*sqrt(c^2*x^2 - 1
)*x^7/c^4 + 168*sqrt(c^2*x^2 - 1)*x^5/c^6 + 210*sqrt(c^2*x^2 - 1)*x^3/c^8 +
 315*sqrt(c^2*x^2 - 1)*x/c^10 + 315*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(
c^2))/(sqrt(c^2)*c^10))*c)*b*c^6*d^3 + 1/1024*(384*x^8*arccosh(c*x) - (48*s
```

```
qrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2 -
 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x^
2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^8))*c)*b*c^4*d^3 + 1/4*a*d^3*x^4 - 1/96*(48*
x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/
c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqr
t(c^2))/(sqrt(c^2)*c^6))*c)*b*c^2*d^3 + 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(
c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c
^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*d^3
```

Fricas [A] time = 1.85556, size = 475, normalized size = 2.07

$$-\frac{7680\,ac^{10}d^3x^{10} - 28800\,ac^8d^3x^8 + 38400\,ac^6d^3x^6 - 19200\,ac^4d^3x^4 + 15\left(512\,bc^{10}d^3x^{10} - 1920\,bc^8d^3x^8 + 2560\,bc^6d^3x^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/76800*(7680*a*c^10*d^3*x^10 - 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6
- 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 - 1920*b*c^8*d^3*x^8 + 2560
*b*c^6*d^3*x^6 - 1280*b*c^4*d^3*x^4 + 79*b*d^3)*log(c*x + sqrt(c^2*x^2 - 1)
) - (768*b*c^9*d^3*x^9 - 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 - 790*b*c^
3*d^3*x^3 - 1185*b*c*d^3*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] time = 47.4026, size = 287, normalized size = 1.25

$$\begin{cases} -\frac{ac^6d^3x^{10}}{10} + \frac{3ac^4d^3x^8}{8} - \frac{ac^2d^3x^6}{2} + \frac{ad^3x^4}{4} - \frac{bc^6d^3x^{10}\operatorname{acosh}(cx)}{10} + \frac{bc^5d^3x^9\sqrt{c^2x^2-1}}{100} + \frac{3bc^4d^3x^8\operatorname{acosh}(cx)}{8} - \frac{57bc^3d^3x^7\sqrt{c^2x^2-1}}{1600} - \frac{bc^2d^3x^6\operatorname{acos}}{2} \\ \frac{d^3x^4\left(a+\frac{i\pi b}{2}\right)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**10/10 + 3*a*c**4*d**3*x**8/8 - a*c**2*d**3*x**6/
2 + a*d**3*x**4/4 - b*c**6*d**3*x**10*acosh(c*x)/10 + b*c**5*d**3*x**9*sqrt
(c**2*x**2 - 1)/100 + 3*b*c**4*d**3*x**8*acosh(c*x)/8 - 57*b*c**3*d**3*x**7
*sqrt(c**2*x**2 - 1)/1600 - b*c**2*d**3*x**6*acosh(c*x)/2 + 401*b*c*d**3*x*
*5*sqrt(c**2*x**2 - 1)/9600 + b*d**3*x**4*acosh(c*x)/4 - 79*b*d**3*x**3*sqr
```

```
t(c**2*x**2 - 1)/(7680*c) - 79*b*d**3*x*sqrt(c**2*x**2 - 1)/(5120*c**3) - 7
9*b*d**3*acosh(c*x)/(5120*c**4), Ne(c, 0)), (d**3*x**4*(a + I*pi*b/2)/4, Tr
ue))
```

Giac [B] time = 1.75086, size = 633, normalized size = 2.75

$$-\frac{1}{10}\,ac^6d^3x^{10}+\frac{3}{8}\,ac^4d^3x^8-\frac{1}{2}\,ac^2d^3x^6-\frac{1}{12800}\left(1280\,x^{10}\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(\sqrt{c^2x^2-1}\left(2\left(4\left(2\,x^2\left(\frac{8\,x^2}{c^2}+\frac{9}{c^4}\right)+\frac{21}{c^6}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] -1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 - 1/2*a*c^2*d^3*x^6 - 1/12800*(128
0*x^10*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*(2*(4*(2*x^2*(8*x^
2/c^2 + 9/c^4) + 21/c^6)*x^2 + 105/c^8)*x^2 + 315/c^10)*x - 315*log(abs(-x*
abs(c) + sqrt(c^2*x^2 - 1)))/(c^10*abs(c)))*c)*b*c^6*d^3 + 1/1024*(384*x^8*
log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*(2*(4*x^2*(6*x^2/c^2 + 7/
c^4) + 35/c^6)*x^2 + 105/c^8)*x - 105*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)
))/(c^8*abs(c)))*c)*b*c^4*d^3 + 1/4*a*d^3*x^4 - 1/96*(48*x^6*log(c*x + sqrt
(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*(2*x^2*(4*x^2/c^2 + 5/c^4) + 15/c^6)*x
- 15*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^6*abs(c)))*c)*b*c^2*d^3 + 1
/32*(8*x^4*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*x*(2*x^2/c^2 +
 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*abs(c)))*c)*b*d^3
```

3.21 $\int x^2 \left(d-c^2 d x^2\right)^3 \left(a+b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=227

$$-\frac{1}{9}c^6d^3x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+$$

```
[Out] (-16*b*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(315*c^3) + (8*b*d^3*(-1 + c*x)^(3
/2)*(1 + c*x)^(3/2))/(945*c^3) - (2*b*d^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))
/(525*c^3) + (b*d^3*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/(441*c^3) + (b*d^3*(-
1 + c*x)^(9/2)*(1 + c*x)^(9/2))/(81*c^3) + (d^3*x^3*(a + b*ArcCosh[c*x]))/3
 - (3*c^2*d^3*x^5*(a + b*ArcCosh[c*x]))/5 + (3*c^4*d^3*x^7*(a + b*ArcCosh[c
*x]))/7 - (c^6*d^3*x^9*(a + b*ArcCosh[c*x]))/9
```

Rubi [A] time = 0.395913, antiderivative size = 285, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {270, 5731, 12, 1610, 1799, 1620}

$$-\frac{1}{9}c^6d^3x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^3*(1 - c^2*x^2))/(315*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*d^3*
(1 - c^2*x^2)^2)/(945*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d^3*(1 - c^2
*x^2)^3)/(525*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^4)/(
441*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d^3*(1 - c^2*x^2)^5)/(81*c^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^3*(a + b*ArcCosh[c*x]))/3 - (3*c^2*d^3*
x^5*(a + b*ArcCosh[c*x]))/5 + (3*c^4*d^3*x^7*(a + b*ArcCosh[c*x]))/7 - (c^6
*d^3*x^9*(a + b*ArcCosh[c*x]))/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)dx=\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\cos\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\cos\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\cos\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\cos\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\cos\right.$$

$$=\frac{16bd^3\left(1-c^2x^2\right)}{315c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{8bd^3\left(1-c^2x^2\right)^2}{945c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bd^3\left(1-c^2x^2\right)^3}{525c^3\sqrt{-1+cx}\sqrt{1+c}}$$

Mathematica [A] time = 0.268577, size = 139, normalized size = 0.61

$$-\frac{d^3\left(315ac^3x^3\left(35c^6x^6-135c^4x^4+189c^2x^2-105\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-1225c^8x^8+4675c^6x^6-6297c^4x^4+2629c^2x^2\right.\right.}{99225c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d^3*(315*a*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^6) + b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(5258 + 2629*c^2*x^2 - 6297*c^4*x^4 + 4675*c^6*x
^6 - 1225*c^8*x^8) + 315*b*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c
^6*x^6)*ArcCosh[c*x]))/(99225*c^3)
```

Maple [A] time = 0.013, size = 150, normalized size = 0.7

$$\frac{1}{c^3}\left(-d^3a\left(\frac{c^9x^9}{9}-\frac{3\,c^7x^7}{7}+\frac{3\,c^5x^5}{5}-\frac{c^3x^3}{3}\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)\,c^9x^9}{9}-\frac{3\operatorname{arccosh}(cx)\,c^7x^7}{7}+\frac{3\operatorname{arccosh}(cx)\,c^5x^5}{5}-\frac{c^3x^3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(-d^3*a*(1/9*c^9*x^9-3/7*c^7*x^7+3/5*c^5*x^5-1/3*c^3*x^3)-d^3*b*(1/9*
arccosh(c*x)*c^9*x^9-3/7*arccosh(c*x)*c^7*x^7+3/5*arccosh(c*x)*c^5*x^5-1/3*
c^3*x^3*arccosh(c*x)-1/99225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*x^8-4675
*c^6*x^6+6297*c^4*x^4-2629*c^2*x^2-5258)))
```

Maxima [B] time = 1.28282, size = 524, normalized size = 2.31

$$-\frac{1}{9}\,ac^{6}d^{3}x^{9}+\frac{3}{7}\,ac^{4}d^{3}x^{7}-\frac{1}{2835}\left(315\,x^{9}\,\operatorname{arcosh}\left(cx\right)-\left(\frac{35\,\sqrt{c^{2}x^{2}-1}x^{8}}{c^{2}}+\frac{40\,\sqrt{c^{2}x^{2}-1}x^{6}}{c^{4}}+\frac{48\,\sqrt{c^{2}x^{2}-1}x^{4}}{c^{6}}+\frac{64\,\sqrt{c^{2}x^{2}}}{c^{8}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 - 1/2835*(315*x^9*arccosh(c*x) - (35
*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2
 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*
c)*b*c^6*d^3 - 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2
*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c
^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^4*d^3 - 1/25*(15*x^5*arccosh(c*x) - (
3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2
- 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(
c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^3
```

Fricas [A] time = 1.81398, size = 450, normalized size = 1.98

$$-\frac{11025\,ac^{9}d^{3}x^{9}-42525\,ac^{7}d^{3}x^{7}+59535\,ac^{5}d^{3}x^{5}-33075\,ac^{3}d^{3}x^{3}+315\left(35\,bc^{9}d^{3}x^{9}-135\,bc^{7}d^{3}x^{7}+189\,bc^{5}d^{3}x^{5}-105\right.}{99225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/99225*(11025*a*c^9*d^3*x^9 - 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5 -
 33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 - 135*b*c^7*d^3*x^7 + 189*b*c^
5*d^3*x^5 - 105*b*c^3*d^3*x^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (1225*b*c^8*d
```

```
^3*x^8 - 4675*b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 - 2629*b*c^2*d^3*x^2 - 525
8*b*d^3)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [A] time = 26.3989, size = 272, normalized size = 1.2

$$\begin{cases} -\frac{ac^6d^3x^9}{9} + \frac{3ac^4d^3x^7}{7} - \frac{3ac^2d^3x^5}{5} + \frac{ad^3x^3}{3} - \frac{bc^6d^3x^9\operatorname{acosh}(cx)}{9} + \frac{bc^5d^3x^8\sqrt{c^2x^2-1}}{81} + \frac{3bc^4d^3x^7\operatorname{acosh}(cx)}{7} - \frac{187bc^3d^3x^6\sqrt{c^2x^2-1}}{3969} - \frac{3bc^2d^3x^5\operatorname{acosh}(cx)}{5} \\ \frac{d^3x^3\left(a+\frac{i\pi b}{2}\right)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**9/9 + 3*a*c**4*d**3*x**7/7 - 3*a*c**2*d**3*x**5/
5 + a*d**3*x**3/3 - b*c**6*d**3*x**9*acosh(c*x)/9 + b*c**5*d**3*x**8*sqrt(c
**2*x**2 - 1)/81 + 3*b*c**4*d**3*x**7*acosh(c*x)/7 - 187*b*c**3*d**3*x**6*s
qrt(c**2*x**2 - 1)/3969 - 3*b*c**2*d**3*x**5*acosh(c*x)/5 + 2099*b*c*d**3*x
**4*sqrt(c**2*x**2 - 1)/33075 + b*d**3*x**3*acosh(c*x)/3 - 2629*b*d**3*x**2
*sqrt(c**2*x**2 - 1)/(99225*c) - 5258*b*d**3*sqrt(c**2*x**2 - 1)/(99225*c**
3), Ne(c, 0)), (d**3*x**3*(a + I*pi*b/2)/3, True))
```

Giac [A] time = 1.55787, size = 501, normalized size = 2.21

$$-\frac{1}{9}ac^6d^3x^9 + \frac{3}{7}ac^4d^3x^7 - \frac{1}{2835}\left(315x^9\log\left(cx+\sqrt{c^2x^2-1}\right) - \frac{35\left(c^2x^2-1\right)^{\frac{9}{2}} + 180\left(c^2x^2-1\right)^{\frac{7}{2}} + 378\left(c^2x^2-1\right)^{\frac{5}{2}} + 4}{c^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] -1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 - 1/2835*(315*x^9*log(c*x + sqrt(c^2
*x^2 - 1)) - (35*(c^2*x^2 - 1)^(9/2) + 180*(c^2*x^2 - 1)^(7/2) + 378*(c^2*x
^2 - 1)^(5/2) + 420*(c^2*x^2 - 1)^(3/2) + 315*sqrt(c^2*x^2 - 1))/c^9)*b*c^6
*d^3 - 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*log(c*x + sqrt(c^2*x^2 - 1)) - (5*
(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5/2) + 35*(c^2*x^2 - 1)^(3/2) + 35*
sqrt(c^2*x^2 - 1))/c^7)*b*c^4*d^3 - 1/25*(15*x^5*log(c*x + sqrt(c^2*x^2 - 1
)) - (3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1)
)/c^5)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c^2*x^2 - 1))
```

```
- ((c^2*x^2 - 1)^(3/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*d^3
```

3.22 $\int x\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=166

$$-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{35bd^3\cosh^{-1}(cx)}{1024c^2}+\frac{bd^3x(cx-1)^{7/2}(cx+1)^{7/2}}{64c}-\frac{7bd^3x(cx-1)^{5/2}(cx+1)^{5/2}}{384c}+\frac{35b}{\ldots}$$

```
[Out] (-35*b*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1024*c) + (35*b*d^3*x*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2))/(1536*c) - (7*b*d^3*x*(-1 + c*x)^(5/2)*(1 + c*x)^(
5/2))/(384*c) + (b*d^3*x*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/(64*c) + (35*b*d
^3*ArcCosh[c*x])/(1024*c^2) - (d^3*(1 - c^2*x^2)^4*(a + b*ArcCosh[c*x]))/(8
*c^2)
```

Rubi [A] time = 0.0787097, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5716, 38, 52}

$$-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{35bd^3\cosh^{-1}(cx)}{1024c^2}+\frac{bd^3x(cx-1)^{7/2}(cx+1)^{7/2}}{64c}-\frac{7bd^3x(cx-1)^{5/2}(cx+1)^{5/2}}{384c}+\frac{35b}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (-35*b*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1024*c) + (35*b*d^3*x*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2))/(1536*c) - (7*b*d^3*x*(-1 + c*x)^(5/2)*(1 + c*x)^(
5/2))/(384*c) + (b*d^3*x*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/(64*c) + (35*b*d
^3*ArcCosh[c*x])/(1024*c^2) - (d^3*(1 - c^2*x^2)^4*(a + b*ArcCosh[c*x]))/(8
*c^2)
```


Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{\left(bd^3\right)\int(-1+cx)^{7/2}(1+cx)^{7/2}\,dx}{8c}$$

$$=\frac{bd^3x(-1+cx)^{7/2}(1+cx)^{7/2}}{64c}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}-\frac{\left(7bd^3\right)\int(-1$$

$$=-\frac{7bd^3x(-1+cx)^{5/2}(1+cx)^{5/2}}{384c}+\frac{bd^3x(-1+cx)^{7/2}(1+cx)^{7/2}}{64c}-\frac{d^3\left(1-c^2x^2\right)^4(a}{8$$

$$=\frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c}-\frac{7bd^3x(-1+cx)^{5/2}(1+cx)^{5/2}}{384c}+\frac{bd^3x(-1+cx)^7}{64c}$$

$$=-\frac{35bd^3x\sqrt{-1+cx}\sqrt{1+cx}}{1024c}+\frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c}-\frac{7bd^3x(-1+cx)^{5}}{384c}$$

$$=-\frac{35bd^3x\sqrt{-1+cx}\sqrt{1+cx}}{1024c}+\frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c}-\frac{7bd^3x(-1+cx)^{5}}{384c}$$

Mathematica [A] time = 0.365045, size = 150, normalized size = 0.9

$$-\frac{d^3\left(cx\left(384acx\left(c^6x^6-4c^4x^4+6c^2x^2-4\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-48c^6x^6+200c^4x^4-326c^2x^2+279\right)\right)+384bc^2x^2\left(c^6x^6\right.\right.}{3072c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d^3*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(279 - 326*c^2*x^2 + 200*c^4*x^4
 - 48*c^6*x^6) + 384*a*c*x*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6)) + 384*b*
c^2*x^2*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6)*ArcCosh[c*x] + 558*b*ArcTanh
```

```
[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(3072*c^2)
```

Maple [A] time = 0.014, size = 258, normalized size = 1.6

$$-\frac{c^6d^3ax^8}{8}+\frac{c^4d^3ax^6}{2}-\frac{3\,c^2d^3ax^4}{4}+\frac{d^3ax^2}{2}-\frac{c^6d^3b\mathrm{arccosh}\,(cx)\,x^8}{8}+\frac{c^4d^3b\mathrm{arccosh}\,(cx)\,x^6}{2}-\frac{3\,c^2d^3b\mathrm{arccosh}\,(cx)\,x^4}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/8*c^6*d^3*a*x^8+1/2*c^4*d^3*a*x^6-3/4*c^2*d^3*a*x^4+1/2*d^3*a*x^2-1/8*c^
6*d^3*b*arccosh(c*x)*x^8+1/2*c^4*d^3*b*arccosh(c*x)*x^6-3/4*c^2*d^3*b*arcco
sh(c*x)*x^4+1/2*d^3*b*arccosh(c*x)*x^2+1/64*c^5*d^3*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*x^7-25/384*c^3*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5+163/1536*c*d^3*
b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-93/1024*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c-93/1024/c^2*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x
+(c^2*x^2-1)^(1/2))
```

Maxima [B] time = 1.19907, size = 620, normalized size = 3.73

$$-\frac{1}{8}\,ac^6d^3x^8+\frac{1}{2}\,ac^4d^3x^6-\frac{1}{3072}\left(384\,x^8\,\mathrm{arcosh}\,(cx)-\left(\frac{48\,\sqrt{c^2x^2-1}x^7}{c^2}+\frac{56\,\sqrt{c^2x^2-1}x^5}{c^4}+\frac{70\,\sqrt{c^2x^2-1}x^3}{c^6}+\frac{105\,\sqrt{c^2}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 - 1/3072*(384*x^8*arccosh(c*x) - (48
*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2
 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*
x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^8))*c)*b*c^6*d^3 - 3/4*a*c^2*d^3*x^4 + 1/9
6*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1
)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 -
1)*sqrt(c^2))/(sqrt(c^2)*c^6))*c)*b*c^4*d^3 - 3/32*(8*x^4*arccosh(c*x) - (2
*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*
sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*c^2*d^3 + 1/2*a*d^3*x^2
+ 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sq
```

```
rt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^2)))*b*d^3
```

Fricas [A] time = 1.91986, size = 425, normalized size = 2.56

$$-\frac{384\,ac^8d^3x^8 - 1536\,ac^6d^3x^6 + 2304\,ac^4d^3x^4 - 1536\,ac^2d^3x^2 + 3\left(128\,bc^8d^3x^8 - 512\,bc^6d^3x^6 + 768\,bc^4d^3x^4 - 512\,bc^2d^3x^2\right.}{3072\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/3072*(384*a*c^8*d^3*x^8 - 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 - 1536
*a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 - 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3*x
^4 - 512*b*c^2*d^3*x^2 + 93*b*d^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (48*b*c^7
*d^3*x^7 - 200*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 - 279*b*c*d^3*x)*sqrt(c^2*
x^2 - 1))/c^2
```

Sympy [A] time = 17.5542, size = 260, normalized size = 1.57

$$\begin{cases} -\frac{ac^6d^3x^8}{8} + \frac{ac^4d^3x^6}{2} - \frac{3ac^2d^3x^4}{4} + \frac{ad^3x^2}{2} - \frac{bc^6d^3x^8\operatorname{acosh}(cx)}{8} + \frac{bc^5d^3x^7\sqrt{c^2x^2-1}}{64} + \frac{bc^4d^3x^6\operatorname{acosh}(cx)}{2} - \frac{25bc^3d^3x^5\sqrt{c^2x^2-1}}{384} - \frac{3bc^2d^3x^4\operatorname{acosh}(c\right. \\ \frac{d^3x^2\left(a+\frac{i\pi b}{2}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**8/8 + a*c**4*d**3*x**6/2 - 3*a*c**2*d**3*x**4/4
+ a*d**3*x**2/2 - b*c**6*d**3*x**8*acosh(c*x)/8 + b*c**5*d**3*x**7*sqrt(c**
2*x**2 - 1)/64 + b*c**4*d**3*x**6*acosh(c*x)/2 - 25*b*c**3*d**3*x**5*sqrt(c
**2*x**2 - 1)/384 - 3*b*c**2*d**3*x**4*acosh(c*x)/4 + 163*b*c*d**3*x**3*sqr
t(c**2*x**2 - 1)/1536 + b*d**3*x**2*acosh(c*x)/2 - 93*b*d**3*x*sqrt(c**2*x*
*2 - 1)/(1024*c) - 93*b*d**3*acosh(c*x)/(1024*c**2), Ne(c, 0)), (d**3*x**2*
(a + I*pi*b/2)/2, True))
```

Giac [B] time = 1.82256, size = 574, normalized size = 3.46

$$-\frac{1}{8}ac^6d^3x^8+\frac{1}{2}ac^4d^3x^6-\frac{1}{3072}\left(384x^8\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(\sqrt{c^2x^2-1}\left(2\left(4x^2\left(\frac{6x^2}{c^2}+\frac{7}{c^4}\right)+\frac{35}{c^6}\right)x^2+\frac{105}{c^8}\right)x-\frac{105}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] -1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 - 1/3072*(384*x^8*log(c*x + sqrt(c^2
*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*(2*(4*x^2*(6*x^2/c^2 + 7/c^4) + 35/c^6)*x^2
 + 105/c^8)*x - 105*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^8*abs(c)))*c
)*b*c^6*d^3 - 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*log(c*x + sqrt(c^2*x^2 - 1))
 - (sqrt(c^2*x^2 - 1)*(2*x^2*(4*x^2/c^2 + 5/c^4) + 15/c^6)*x - 15*log(abs(-
x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^6*abs(c)))*c)*b*c^4*d^3 - 3/32*(8*x^4*log
(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*x*(2*x^2/c^2 + 3/c^4) - 3*lo
g(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*abs(c)))*c)*b*c^2*d^3 + 1/2*a*d^
3*x^2 + 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 - 1)) - c*(sqrt(c^2*x^2 - 1)*x/c^
2 - log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^2*abs(c))))*b*d^3
```

3.23 $\int \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\, dx$

Optimal. Leaf size=191

$$-\frac{1}{7}c^6d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+d^3x\left(a+b\cosh^{-1}(cx)\right)+\frac{bd^3(c$$

```
[Out] (-16*b*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(35*c) + (8*b*d^3*(-1 + c*x)^(3/2)
*(1 + c*x)^(3/2))/(105*c) - (6*b*d^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(175
*c) + (b*d^3*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/(49*c) + d^3*x*(a + b*ArcCos
h[c*x]) - c^2*d^3*x^3*(a + b*ArcCosh[c*x]) + (3*c^4*d^3*x^5*(a + b*ArcCosh[
c*x]))/5 - (c^6*d^3*x^7*(a + b*ArcCosh[c*x]))/7
```

Rubi [A] time = 0.26202, antiderivative size = 237, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {194, 5680, 12, 1610, 1799, 1850}

$$-\frac{1}{7}c^6d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+d^3x\left(a+b\cosh^{-1}(cx)\right)+\frac{bd^3}{49c\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^3*(1 - c^2*x^2))/(35*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*d^3*(1
- c^2*x^2)^2)/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*b*d^3*(1 - c^2*x^2)
^3)/(175*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^4)/(49*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*x*(a + b*ArcCosh[c*x]) - c^2*d^3*x^3*(a +
 b*ArcCosh[c*x]) + (3*c^4*d^3*x^5*(a + b*ArcCosh[c*x]))/5 - (c^6*d^3*x^7*(a
 + b*ArcCosh[c*x]))/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
```

```
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cosh^{-1}(cx)\right)$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cosh^{-1}(cx)\right)$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cosh^{-1}(cx)\right)$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cosh^{-1}(cx)\right)$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cosh^{-1}(cx)\right)$$

$$= \frac{16bd^3\left(1-c^2x^2\right)}{35c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{8bd^3\left(1-c^2x^2\right)^2}{105c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{6bd^3\left(1-c^2x^2\right)^3}{175c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc}{49c\sqrt{}}$$

Mathematica [A] time = 0.258576, size = 123, normalized size = 0.64

$$-\frac{d^3\left(105acx\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-75c^6x^6+351c^4x^4-757c^2x^2+2161\right)+105bcx\left(5c^6x^6\right.\right.}{3675c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d^3*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2161 - 757*c^2*x^2 + 351*c^4*x^4 - 7
5*c^6*x^6) + 105*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + 105*b*
c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcCosh[c*x]))/(3675*c)
```

Maple [A] time = 0.01, size = 132, normalized size = 0.7

$$\frac{1}{c}\left(-d^3a\left(\frac{c^7x^7}{7}-\frac{3\,c^5x^5}{5}+c^3x^3-cx\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)\,c^7x^7}{7}-\frac{3\operatorname{arccosh}(cx)\,c^5x^5}{5}+c^3x^3\operatorname{arccosh}(cx)-cx\operatorname{arccosh}(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-d^3*a*(1/7*c^7*x^7-3/5*c^5*x^5+c^3*x^3-c*x)-d^3*b*(1/7*arccosh(c*x)*c
^7*x^7-3/5*arccosh(c*x)*c^5*x^5+c^3*x^3*arccosh(c*x)-c*x*arccosh(c*x)-1/367
5*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*x^6-351*c^4*x^4+757*c^2*x^2-2161)))
```

Maxima [A] time = 1.14356, size = 408, normalized size = 2.14

$$-\frac{1}{7}\,ac^6d^3x^7+\frac{3}{5}\,ac^4d^3x^5-\frac{1}{245}\left(35\,x^7\operatorname{arcosh}(cx)-\left(\frac{5\,\sqrt{c^2x^2-1}x^6}{c^2}+\frac{6\,\sqrt{c^2x^2-1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2-1}x^2}{c^6}+\frac{16\,\sqrt{c^2x^2-1}}{c^8}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 - 1/245*(35*x^7*arccosh(c*x) - (5*sq
rt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)
*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arccosh(c*
x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^
2*x^2 - 1)/c^6)*c)*b*c^4*d^3 - a*c^2*d^3*x^3 - 1/3*(3*x^3*arccosh(c*x) - c*
(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d^3 + a*d^3*x
+ (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^3/c
```

Fricas [A] time = 2.11889, size = 389, normalized size = 2.04

$$-\frac{525\,ac^7d^3x^7-2205\,ac^5d^3x^5+3675\,ac^3d^3x^3-3675\,acd^3x+105\left(5\,bc^7d^3x^7-21\,bc^5d^3x^5+35\,bc^3d^3x^3-35\,bcd^3x\right)\log\left(\cdots\right.}{3675\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/3675*(525*a*c^7*d^3*x^7 - 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 - 3675
*a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 - 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 - 3
5*b*c*d^3*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*d^3*x^6 - 351*b*c^4*d
^3*x^4 + 757*b*c^2*d^3*x^2 - 2161*b*d^3)*sqrt(c^2*x^2 - 1))/c
```

Sympy [A] time = 9.83384, size = 228, normalized size = 1.19

$$\begin{cases} -\frac{ac^6d^3x^7}{7} + \frac{3ac^4d^3x^5}{5} - ac^2d^3x^3 + ad^3x - \frac{bc^6d^3x^7\operatorname{acosh}(cx)}{7} + \frac{bc^5d^3x^6\sqrt{c^2x^2-1}}{49} + \frac{3bc^4d^3x^5\operatorname{acosh}(cx)}{5} - \frac{117bc^3d^3x^4\sqrt{c^2x^2-1}}{1225} - bc^2d^3x^3\operatorname{aco} \\ d^3x\left(a + \frac{i\pi b}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 - a*c**2*d**3*x**3 +
a*d**3*x - b*c**6*d**3*x**7*acosh(c*x)/7 + b*c**5*d**3*x**6*sqrt(c**2*x**2
- 1)/49 + 3*b*c**4*d**3*x**5*acosh(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(c**2*
x**2 - 1)/1225 - b*c**2*d**3*x**3*acosh(c*x) + 757*b*c*d**3*x**2*sqrt(c**2*
x**2 - 1)/3675 + b*d**3*x*acosh(c*x) - 2161*b*d**3*sqrt(c**2*x**2 - 1)/(367
5*c), Ne(c, 0)), (d**3*x*(a + I*pi*b/2), True))
```

Giac [A] time = 1.60529, size = 417, normalized size = 2.18

$$-\frac{1}{7}ac^6d^3x^7 + \frac{3}{5}ac^4d^3x^5 - \frac{1}{245}\left(35x^7\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{5\left(c^2x^2-1\right)^{\frac{7}{2}} + 21\left(c^2x^2-1\right)^{\frac{5}{2}} + 35\left(c^2x^2-1\right)^{\frac{3}{2}} + 35\sqrt{c^2x^2}}{c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] -1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 - 1/245*(35*x^7*log(c*x + sqrt(c^2*x
^2 - 1)) - (5*(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5/2) + 35*(c^2*x^2 -
1)^(3/2) + 35*sqrt(c^2*x^2 - 1))/c^7)*b*c^6*d^3 + 1/25*(15*x^5*log(c*x + sq
rt(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqr
t(c^2*x^2 - 1))/c^5)*b*c^4*d^3 - a*c^2*d^3*x^3 - 1/3*(3*x^3*log(c*x + sqrt(
c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*c^2*d^3
+ (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d^3 + a*d^3*x
```

3.24 $\int \frac{(d-c^2dx^2)^3(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=239

$$-\frac{1}{2}bd^3\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+\frac{1}{6}d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d^3\left(1-\right.$$

```
[Out] (19*b*c*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/48 - (7*b*c*d^3*x*(-1 + c*x)^(3
/2)*(1 + c*x)^(3/2))/72 + (b*c*d^3*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/36 -
 (19*b*d^3*ArcCosh[c*x])/48 + (d^3*(1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/2 +
(d^3*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 + (d^3*(1 - c^2*x^2)^3*(a + b*
ArcCosh[c*x]))/6 + (d^3*(a + b*ArcCosh[c*x])^2)/(2*b) + d^3*(a + b*ArcCosh[
c*x])*Log[1 + E^(-2*ArcCosh[c*x])] - (b*d^3*PolyLog[2, -E^(-2*ArcCosh[c*x])
])/2
```

Rubi [A] time = 0.300358, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5727, 5660, 3718, 2190, 2279, 2391, 38, 52}

$$\frac{1}{2}bd^3\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)+\frac{1}{6}d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d^3\left(1-c^2\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (19*b*c*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/48 - (7*b*c*d^3*x*(-1 + c*x)^(3
/2)*(1 + c*x)^(3/2))/72 + (b*c*d^3*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/36 -
 (19*b*d^3*ArcCosh[c*x])/48 + (d^3*(1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/2 +
(d^3*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 + (d^3*(1 - c^2*x^2)^3*(a + b*
ArcCosh[c*x]))/6 - (d^3*(a + b*ArcCosh[c*x])^2)/(2*b) + d^3*(a + b*ArcCosh[
c*x])*Log[1 + E^(2*ArcCosh[c*x])] + (b*d^3*PolyLog[2, -E^(2*ArcCosh[c*x])])
/2
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
```

```
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{1}{6}d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)+d\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx +$$

$$= \frac{1}{36}bcd^3x(-1+cx)^{5/2}(1+cx)^{5/2}+\frac{1}{4}d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}d^3\left(1-\right.$$

$$= -\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x(-1+cx)^{5/2}(1+cx)^{5/2}+\frac{1}{2}d^3\left(1-c\right.$$

$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x(-1+c$$

$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x(-1+c$$

$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x(-1+c$$

$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x(-1+c$$

$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x(-1+c$$

Mathematica [A] time = 0.370836, size = 207, normalized size = 0.87

$$-\frac{1}{144}d^3\left(72b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+24ac^6x^6-108ac^4x^4+216ac^2x^2-144a\log(x)-4bc^5x^5\sqrt{cx-1}\sqrt{cx+1}+22\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -(d^3*(216*a*c^2*x^2 - 108*a*c^4*x^4 + 24*a*c^6*x^6 - 75*b*c*x*Sqrt[-1 + c*
x]*Sqrt[1 + c*x] + 22*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 4*b*c^5*x^5*
Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 72*b*ArcCosh[c*x]^2 - 150*b*ArcTanh[Sqrt[(-1
 + c*x)/(1 + c*x)]] + 12*b*ArcCosh[c*x]*(18*c^2*x^2 - 9*c^4*x^4 + 2*c^6*x^6
 - 12*Log[1 + E^(-2*ArcCosh[c*x])]) - 144*a*Log[x] + 72*b*PolyLog[2, -E^(-2
*ArcCosh[c*x])]))/144
```

Maple [A] time = 0.162, size = 255, normalized size = 1.1

$$-\frac{d^3ac^6x^6}{6}+\frac{3\,d^3ac^4x^4}{4}-\frac{3\,d^3ac^2x^2}{2}+d^3a\ln(cx)+\frac{25\,bd^3\mathrm{arccosh}(cx)}{48}-\frac{3\,d^3b\mathrm{arccosh}(cx)\,c^2x^2}{2}-\frac{d^3b\,(\mathrm{arccosh}(cx))^2}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x)
```

```
[Out] -1/6*d^3*a*c^6*x^6+3/4*d^3*a*c^4*x^4-3/2*d^3*a*c^2*x^2+d^3*a*ln(c*x)+25/48*
b*d^3*arccosh(c*x)-3/2*d^3*b*arccosh(c*x)*c^2*x^2-1/2*d^3*b*arccosh(c*x)^2-
1/6*d^3*b*arccosh(c*x)*c^6*x^6+3/4*d^3*b*arccosh(c*x)*c^4*x^4+1/36*d^3*b*(c
*x-1)^(1/2)*(c*x+1)^(1/2)*c^5*x^5-11/72*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c
^3*x^3+25/48*b*c*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/2*d^3*b*polylog(2,-(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+d^3*b*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\,ac^6d^3x^6+\frac{3}{4}\,ac^4d^3x^4-\frac{3}{2}\,ac^2d^3x^2+ad^3\log(x)-\int bc^6d^3x^5\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)-3\,bc^4d^3x^3\log\left(cx+\sqrt{cx+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 - 3/2*a*c^2*d^3*x^2 + a*d^3*log(x) -
 integrate(b*c^6*d^3*x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*b*c^4*d
^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*c^2*d^3*x*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) - b*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{ac^6d^3x^6-3\,ac^4d^3x^4+3\,ac^2d^3x^2-ad^3+\left(bc^6d^3x^6-3\,bc^4d^3x^4+3\,bc^2d^3x^2-bd^3\right)\mathrm{arcosh}(cx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c
^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d^3\left(\int -\frac{a}{x}\,dx + \int 3ac^2x\,dx + \int -3ac^4x^3\,dx + \int ac^6x^5\,dx + \int -\frac{b\,\mathrm{acosh}\,(cx)}{x}\,dx + \int 3bc^2x\,\mathrm{acosh}\,(cx)\,dx + \int -3bc^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x,x)
```

```
[Out] -d**3*(Integral(-a/x, x) + Integral(3*a*c**2*x, x) + Integral(-3*a*c**4*x**
3, x) + Integral(a*c**6*x**5, x) + Integral(-b*acosh(c*x)/x, x) + Integral(
3*b*c**2*x*acosh(c*x), x) + Integral(-3*b*c**4*x**3*acosh(c*x), x) + Integr
al(b*c**6*x**5*acosh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(c^2dx^2 - d\right)^3 (b\,\mathrm{arcosh}\,(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)^3*(b*arccosh(c*x) + a)/x, x)
```

3.25 $\int \frac{(d-c^2dx^2)^3(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=180

$$-\frac{1}{5}c^6d^3x^5\left(a+b\cosh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\cosh^{-1}(cx)\right)-3c^2d^3x\left(a+b\cosh^{-1}(cx)\right)-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{1}{25}bcd^3($$

```
[Out] (11*b*c*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/5 - (b*c*d^3*(-1 + c*x)^(3/2)*(1
+ c*x)^(3/2))/5 + (b*c*d^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/25 - (d^3*(a +
 b*ArcCosh[c*x]))/x - 3*c^2*d^3*x*(a + b*ArcCosh[c*x]) + c^4*d^3*x^3*(a + b
*ArcCosh[c*x]) - (c^6*d^3*x^5*(a + b*ArcCosh[c*x]))/5 + b*c*d^3*ArcTan[Sqrt
[-1 + c*x]*Sqrt[1 + c*x]]
```

Rubi [A] time = 0.360977, antiderivative size = 239, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {270, 5731, 12, 1610, 1799, 1620, 63, 205}

$$-\frac{1}{5}c^6d^3x^5\left(a+b\cosh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\cosh^{-1}(cx)\right)-3c^2d^3x\left(a+b\cosh^{-1}(cx)\right)-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{bcd^3\left(1\right.}{25\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (-11*b*c*d^3*(1 - c^2*x^2))/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^3*(1
- c^2*x^2)^2)/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^3*(1 - c^2*x^2)^3)/
(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh[c*x]))/x - 3*c^2*d^
3*x*(a + b*ArcCosh[c*x]) + c^4*d^3*x^3*(a + b*ArcCosh[c*x]) - (c^6*d^3*x^5*
(a + b*ArcCosh[c*x]))/5 + (b*c*d^3*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*
x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{11bcd^3\left(1-c^2x^2\right)}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^2}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^3}{25\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3\left(a+b\,\mathrm{co}\right.}{x}$$

$$= -\frac{11bcd^3\left(1-c^2x^2\right)}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^2}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^3}{25\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3\left(a+b\,\mathrm{co}\right.}{x}$$

$$= -\frac{11bcd^3\left(1-c^2x^2\right)}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^2}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^3}{25\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3\left(a+b\,\mathrm{co}\right.}{x}$$

Mathematica [A] time = 0.286109, size = 136, normalized size = 0.76

$$\frac{1}{25}d^3\left(-5ac^6x^5+25ac^4x^3-75ac^2x-\frac{25a}{x}+bc\sqrt{cx-1}\sqrt{cx+1}\left(c^4x^4-7c^2x^2+61\right)-\frac{5b\left(c^6x^6-5c^4x^4+15c^2x^2+5\right)\cosh}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (d^3*((-25*a)/x - 75*a*c^2*x + 25*a*c^4*x^3 - 5*a*c^6*x^5 + b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(61 - 7*c^2*x^2 + c^4*x^4) - (5*b*(5 + 15*c^2*x^2 - 5*c^4
*x^4 + c^6*x^6)*ArcCosh[c*x])/x - 25*b*c*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])]))/25
```

Maple [A] time = 0.017, size = 219, normalized size = 1.2

$$-\frac{d^3ac^6x^5}{5}+d^3ac^4x^3-3\,d^3ac^2x-\frac{d^3a}{x}-\frac{d^3b\mathrm{arccosh}\,(cx)\,c^6x^5}{5}+d^3b\mathrm{arccosh}\,(cx)\,c^4x^3-3\,d^3b\mathrm{arccosh}\,(cx)\,c^2x-\frac{bd^3\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] -1/5*d^3*a*c^6*x^5+d^3*a*c^4*x^3-3*d^3*a*c^2*x-d^3*a/x-1/5*d^3*b*arccosh(c*
x)*c^6*x^5+d^3*b*arccosh(c*x)*c^4*x^3-3*d^3*b*arccosh(c*x)*c^2*x-d^3*b*arcc
osh(c*x)/x+1/25*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5*x^4-7/25*d^3*b*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*c^3*x^2+61/25*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-c*d
^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/
2))
```

Maxima [A] time = 1.79264, size = 315, normalized size = 1.75

$$-\frac{1}{5}\,ac^6d^3x^5-\frac{1}{75}\left(15\,x^5\,\mathrm{arcosh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^6d^3+ac^4d^3x^3+\frac{1}{3}\left(3\,x^3\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/5*a*c^6*d^3*x^5 - 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c
^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^6*d^3 +
a*c^4*d^3*x^3 + 1/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*
sqrt(c^2*x^2 - 1)/c^4))*b*c^4*d^3 - 3*a*c^2*d^3*x - 3*(c*x*arccosh(c*x) - s
qrt(c^2*x^2 - 1))*b*c*d^3 - (c*arcsin(1/(sqrt(c^2)*abs(x))) + arccosh(c*x)/
x)*b*d^3 - a*d^3/x
```

Fricas [A] time = 2.43489, size = 549, normalized size = 3.05

$$-\frac{5\,ac^6d^3x^6-25\,ac^4d^3x^4+75\,ac^2d^3x^2-50\,bcd^3x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-5\left(bc^6-5\,bc^4+15\,bc^2+5\,b\right)d^3x\log\left(-cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/25*(5*a*c^6*d^3*x^6 - 25*a*c^4*d^3*x^4 + 75*a*c^2*d^3*x^2 - 50*b*c*d^3*x
*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 5*(b*c^6 - 5*b*c^4 + 15*b*c^2 + 5*b)*d^
3*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 25*a*d^3 + 5*(b*c^6*d^3*x^6 - 5*b*c^4*d
^3*x^4 + 15*b*c^2*d^3*x^2 - (b*c^6 - 5*b*c^4 + 15*b*c^2 + 5*b)*d^3*x + 5*b*
d^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^5*d^3*x^5 - 7*b*c^3*d^3*x^3 + 61*b
*c*d^3*x)*sqrt(c^2*x^2 - 1))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d^3\left(\int 3ac^2\,dx + \int -\frac{a}{x^2}\,dx + \int -3ac^4x^2\,dx + \int ac^6x^4\,dx + \int 3bc^2\operatorname{acosh}(cx)\,dx + \int -\frac{b\operatorname{acosh}(cx)}{x^2}\,dx + \int -3bc^4x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] -d**3*(Integral(3*a*c**2, x) + Integral(-a/x**2, x) + Integral(-3*a*c**4*x*
*2, x) + Integral(a*c**6*x**4, x) + Integral(3*b*c**2*acosh(c*x), x) + Inte
gral(-b*acosh(c*x)/x**2, x) + Integral(-3*b*c**4*x**2*acosh(c*x), x) + Inte
gral(b*c**6*x**4*acosh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(c^2dx^2 - d\right)^3 (b\operatorname{arcosh}(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)^3*(b*arccosh(c*x) + a)/x^2, x)
```

3.26 $\int \frac{(d-c^2dx^2)^3(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=267

$$\frac{3}{2}bc^2d^3\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-\frac{d^3(1-c^2x^2)^3(a+b\cosh^{-1}(cx))}{2x^2}-\frac{3}{4}c^2d^3(1-c^2x^2)^2(a+b\cosh^{-1}(cx))-\frac{3}{2}c^2d^3$$

```
[Out] (-3*b*c^3*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/32 - (7*b*c^3*d^3*x*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2))/16 + (b*c*d^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(2
*x) + (3*b*c^2*d^3*ArcCosh[c*x])/32 - (3*c^2*d^3*(1 - c^2*x^2)*(a + b*ArcCo
sh[c*x]))/2 - (3*c^2*d^3*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 - (d^3*(1
- c^2*x^2)^3*(a + b*ArcCosh[c*x]))/(2*x^2) - (3*c^2*d^3*(a + b*ArcCosh[c*x]
)^2)/(2*b) - 3*c^2*d^3*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])] +
(3*b*c^2*d^3*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

Rubi [A] time = 0.317803, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5729, 97, 12, 38, 52, 5727, 5660, 3718, 2190, 2279, 2391}

$$-\frac{3}{2}bc^2d^3\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)-\frac{d^3(1-c^2x^2)^3(a+b\cosh^{-1}(cx))}{2x^2}-\frac{3}{4}c^2d^3(1-c^2x^2)^2(a+b\cosh^{-1}(cx))-\frac{3}{2}c^2d^3$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-3*b*c^3*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/32 - (7*b*c^3*d^3*x*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2))/16 + (b*c*d^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(2
*x) + (3*b*c^2*d^3*ArcCosh[c*x])/32 - (3*c^2*d^3*(1 - c^2*x^2)*(a + b*ArcCo
sh[c*x]))/2 - (3*c^2*d^3*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 - (d^3*(1
- c^2*x^2)^3*(a + b*ArcCosh[c*x]))/(2*x^2) + (3*c^2*d^3*(a + b*ArcCosh[c*x]
)^2)/(2*b) - 3*c^2*d^3*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])] - (
3*b*c^2*d^3*PolyLog[2, -E^(2*ArcCosh[c*x])])/2
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
```

```
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \left(3c^2d\right)\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x}\,d$$

$$= \frac{bcd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{2x} - \frac{3}{4}c^2d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right) - \frac{d^3\left(1-c^2x^2\right.}{}$$

$$= \frac{3}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bcd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{2x} - \frac{3}{2}c^2d^3\left(1-c^2x^2\right)$$

$$= -\frac{33}{32}bc^3d^3x\sqrt{-1+cx}\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bcd^3(-1+cx)^{}}{2x}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bcd^3(-1+cx)^{}}{2x}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bcd^3(-1+cx)^{}}{2x}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bcd^3(-1+cx)^{}}{2x}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bcd^3(-1+cx)^{}}{2x}$$

Mathematica [A] time = 0.351571, size = 226, normalized size = 0.85

$$-\frac{d^3\left(-48bc^2x^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+8ac^6x^6-48ac^4x^4+96ac^2x^2\log(x)+16a-2bc^5x^5\sqrt{cx-1}\sqrt{cx+1}+21bc^3x\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -(d^3*(16*a - 48*a*c^4*x^4 + 8*a*c^6*x^6 - 16*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x] + 21*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*c^5*x^5*Sqrt[-1 + c
*x]*Sqrt[1 + c*x] + 48*b*c^2*x^2*ArcCosh[c*x]^2 + 42*b*c^2*x^2*ArcTanh[Sqrt
[(-1 + c*x)/(1 + c*x)]] + 8*b*ArcCosh[c*x]*(2 - 6*c^4*x^4 + c^6*x^6 + 12*c^
2*x^2*Log[1 + E^(-2*ArcCosh[c*x])]) + 96*a*c^2*x^2*Log[x] - 48*b*c^2*x^2*Po
lyLog[2, -E^(-2*ArcCosh[c*x])]))/(32*x^2)
```

Maple [A] time = 0.306, size = 275, normalized size = 1.

$$-\frac{c^6 d^3 a x^4}{4}+\frac{3 c^4 d^3 a x^2}{2}-3 c^2 d^3 a \ln (c x)-\frac{d^3 a}{2 x^2}-\frac{d^3 b c^2}{2}+\frac{3 c^2 d^3 b (\operatorname{arccosh}(c x))^2}{2}-\frac{21 b c^2 d^3 \operatorname{arccosh}(c x)}{32}-\frac{3 d^3 b c^2}{2}\mathrm{pol}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] -1/4*c^6*d^3*a*x^4+3/2*c^4*d^3*a*x^2-3*c^2*d^3*a*ln(c*x)-1/2*d^3*a/x^2-1/2*
d^3*b*c^2+3/2*c^2*d^3*b*arccosh(c*x)^2-21/32*b*c^2*d^3*arccosh(c*x)-3/2*c^2
*d^3*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*c*d^3*b/x*(c*x+1
)^(1/2)*(c*x-1)^(1/2)+3/2*c^4*d^3*b*arccosh(c*x)*x^2-1/2*d^3*b*arccosh(c*x)
/x^2-1/4*c^6*d^3*b*arccosh(c*x)*x^4+1/16*c^5*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*x^3-21/32*b*c^3*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-3*c^2*d^3*b*arccosh(c
*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4} a c^6 d^3 x^4+\frac{3}{2} a c^4 d^3 x^2-3 a c^2 d^3 \log (x)+\frac{1}{2} b d^3\left(\frac{\sqrt{c^2 x^2-1} c}{x}-\frac{\operatorname{arcosh}(c x)}{x^2}\right)-\frac{a d^3}{2 x^2}-\int b c^6 d^3 x^3 \log \left(c x+\sqrt{c x+1} \sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 - 3*a*c^2*d^3*log(x) + 1/2*b*d^3*(sq
rt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) - 1/2*a*d^3/x^2 - integrate(b*c^6*d
^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*b*c^4*d^3*x*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) + 3*b*c^2*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{a c^6 d^3 x^6-3 a c^4 d^3 x^4+3 a c^2 d^3 x^2-a d^3+\left(b c^6 d^3 x^6-3 b c^4 d^3 x^4+3 b c^2 d^3 x^2-b d^3\right) \operatorname{arcosh}(c x)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c
^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))/x^3,
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d^3\left(\int -\frac{a}{x^3}\,dx + \int \frac{3ac^2}{x}\,dx + \int -3ac^4x\,dx + \int ac^6x^3\,dx + \int -\frac{b\operatorname{acosh}(cx)}{x^3}\,dx + \int \frac{3bc^2\operatorname{acosh}(cx)}{x}\,dx + \int -3bc^4x\operatorname{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] -d**3*(Integral(-a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(-3*a*c**4*
x, x) + Integral(a*c**6*x**3, x) + Integral(-b*acosh(c*x)/x**3, x) + Integr
al(3*b*c**2*acosh(c*x)/x, x) + Integral(-3*b*c**4*x*acosh(c*x), x) + Integr
al(b*c**6*x**3*acosh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(c^2dx^2 - d\right)^3 (b\operatorname{arcosh}(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)^3*(b*arccosh(c*x) + a)/x^3, x)
```

3.27 $\int \frac{(d-c^2dx^2)^3(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=195

$$-\frac{1}{3}c^6d^3x^3\left(a+b\cosh^{-1}(cx)\right)+3c^4d^3x\left(a+b\cosh^{-1}(cx)\right)+\frac{3c^2d^3\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{1}{9}bc^3d^3$$

```
[Out] (-8*b*c^3*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/3 + (b*c*d^3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/(6*x^2) + (b*c^3*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/9 - (d^3
*(a + b*ArcCosh[c*x]))/(3*x^3) + (3*c^2*d^3*(a + b*ArcCosh[c*x]))/x + 3*c^4
*d^3*x*(a + b*ArcCosh[c*x]) - (c^6*d^3*x^3*(a + b*ArcCosh[c*x]))/3 - (17*b*
c^3*d^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/6
```

Rubi [A] time = 0.389749, antiderivative size = 252, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {270, 5731, 12, 1610, 1799, 1621, 897, 1153, 205}

$$-\frac{1}{3}c^6d^3x^3\left(a+b\cosh^{-1}(cx)\right)+3c^4d^3x\left(a+b\cosh^{-1}(cx)\right)+\frac{3c^2d^3\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{bc^3d^3(}{9\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (8*b*c^3*d^3*(1 - c^2*x^2))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^3*(1
- c^2*x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^3*(1 - c^2*x^2)
^2)/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh[c*x]))/(3*x^3) +
 (3*c^2*d^3*(a + b*ArcCosh[c*x]))/x + 3*c^4*d^3*x*(a + b*ArcCosh[c*x]) - (c
^6*d^3*x^3*(a + b*ArcCosh[c*x]))/3 - (17*b*c^3*d^3*Sqrt[-1 + c^2*x^2]*ArcTa
n[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^3(a+b\cosh^{-1}(cx))}{x^4}\,dx = -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c^4d^3x(a+b\cosh^{-1}(cx))$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c^4d^3x(a+b\cosh^{-1}(cx))$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c^4d^3x(a+b\cosh^{-1}(cx))$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c^4d^3x(a+b\cosh^{-1}(cx))$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c$$

$$= \frac{8bc^3d^3(1-c^2x^2)}{3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3d^3(1-c^2x^2)^2}{9\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3(a+b\,\mathrm{co}}{3x}$$

$$= \frac{8bc^3d^3(1-c^2x^2)}{3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3d^3(1-c^2x^2)^2}{9\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3(a+b\,\mathrm{co}}{3x}$$

Mathematica [A] time = 0.291029, size = 142, normalized size = 0.73

$$\frac{d^3\left(-6ac^6x^6+54ac^4x^4+54ac^2x^2-6a+bcx\sqrt{cx-1}\sqrt{cx+1}\left(2c^4x^4-50c^2x^2+3\right)+51bc^3x^3\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)-6b\left(c^6\right.\right.}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (d^3*(-6*a + 54*a*c^2*x^2 + 54*a*c^4*x^4 - 6*a*c^6*x^6 + b*c*x*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(3 - 50*c^2*x^2 + 2*c^4*x^4) - 6*b*(1 - 9*c^2*x^2 - 9*c^4*
x^4 + c^6*x^6)*ArcCosh[c*x] + 51*b*c^3*x^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x])]))/(18*x^3)
```

Maple [A] time = 0.019, size = 223, normalized size = 1.1

$$-\frac{c^6d^3ax^3}{3}+3\,c^4d^3ax+3\,\frac{c^2d^3a}{x}-\frac{d^3a}{3\,x^3}-\frac{c^6d^3b\mathrm{arccosh}\,(cx)\,x^3}{3}+3\,c^4d^3b\mathrm{arccosh}\,(cx)\,x+3\,\frac{bc^2d^3\mathrm{arccosh}\,(cx)}{x}-\frac{bd^3\mathrm{arc}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] -1/3*c^6*d^3*a*x^3+3*c^4*d^3*a*x+3*c^2*d^3*a/x-1/3*d^3*a/x^3-1/3*c^6*d^3*b*
arccosh(c*x)*x^3+3*c^4*d^3*b*arccosh(c*x)*x+3*c^2*d^3*b*arccosh(c*x)/x-1/3*
d^3*b*arccosh(c*x)/x^3+1/9*c^5*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-25/9*b
*c^3*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+17/6*c^3*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))+1/6*b*c*d^3*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/x^2
```

Maxima [A] time = 2.18894, size = 286, normalized size = 1.47

$$-\frac{1}{3}\,ac^6d^3x^3-\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^6d^3+3\,ac^4d^3x+3\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*a*c^6*d^3*x^3 - 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2
 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arccosh(c*x
) - sqrt(c^2*x^2 - 1))*b*c^3*d^3 + 3*(c*arcsin(1/(sqrt(c^2)*abs(x))) + arcc
osh(c*x)/x)*b*c^2*d^3 - 1/6*((c^2*arcsin(1/(sqrt(c^2)*abs(x))) - sqrt(c^2*x
^2 - 1)/x^2)*c + 2*arccosh(c*x)/x^3)*b*d^3 + 3*a*c^2*d^3/x - 1/3*a*d^3/x^3
```

Fricas [A] time = 2.40329, size = 554, normalized size = 2.84

$$-\frac{6\,ac^6d^3x^6 - 54\,ac^4d^3x^4 + 102\,bc^3d^3x^3\arctan\left(-cx + \sqrt{c^2x^2-1}\right) - 54\,ac^2d^3x^2 - 6\left(bc^6 - 9\,bc^4 - 9\,bc^2 + b\right)d^3x^3\log\left(-cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(6*a*c^6*d^3*x^6 - 54*a*c^4*d^3*x^4 + 102*b*c^3*d^3*x^3*arctan(-c*x +
 sqrt(c^2*x^2 - 1)) - 54*a*c^2*d^3*x^2 - 6*(b*c^6 - 9*b*c^4 - 9*b*c^2 + b)*
d^3*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) + 6*a*d^3 + 6*(b*c^6*d^3*x^6 - 9*b*c^
4*d^3*x^4 - 9*b*c^2*d^3*x^2 - (b*c^6 - 9*b*c^4 - 9*b*c^2 + b)*d^3*x^3 + b*d
^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^5*d^3*x^5 - 50*b*c^3*d^3*x^3 + 3*
b*c*d^3*x)*sqrt(c^2*x^2 - 1))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d^3\left(\int -3ac^4\,dx + \int -\frac{a}{x^4}\,dx + \int \frac{3ac^2}{x^2}\,dx + \int ac^6x^2\,dx + \int -3bc^4\operatorname{acosh}(cx)\,dx + \int -\frac{b\operatorname{acosh}(cx)}{x^4}\,dx + \int \frac{3bc^2\operatorname{acos}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] -d**3*(Integral(-3*a*c**4, x) + Integral(-a/x**4, x) + Integral(3*a*c**2/x*
*2, x) + Integral(a*c**6*x**2, x) + Integral(-3*b*c**4*acosh(c*x), x) + Int
egral(-b*acosh(c*x)/x**4, x) + Integral(3*b*c**2*acosh(c*x)/x**2, x) + Inte
gral(b*c**6*x**2*acosh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(c^2dx^2 - d\right)^3 (b\operatorname{arcosh}(cx) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)^3*(b*arccosh(c*x) + a)/x^4, x)
```

3.28 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=158

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^5d}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^5d}-\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d}+\frac{2\tanh^{-1}\left(e^{\cos\cdots}\right.}{}$$

```
[Out] (11*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^5*d) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/(9*c^3*d) - (x*(a + b*ArcCosh[c*x]))/(c^4*d) - (x^3*(a + b*ArcCos
h[c*x]))/(3*c^2*d) + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^5*
d) + (b*PolyLog[2, -E^ArcCosh[c*x]])/(c^5*d) - (b*PolyLog[2, E^ArcCosh[c*x]
])/(c^5*d)
```

Rubi [A] time = 0.232191, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5766, 100, 12, 74, 5694, 4182, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^5d}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^5d}-\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d}+\frac{2\tanh^{-1}\left(e^{\cos\cdots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```


```
[Out] (11*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^5*d) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/(9*c^3*d) - (x*(a + b*ArcCosh[c*x]))/(c^4*d) - (x^3*(a + b*ArcCos
h[c*x]))/(3*c^2*d) + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^5*
d) + (b*PolyLog[2, -E^ArcCosh[c*x]])/(c^5*d) - (b*PolyLog[2, E^ArcCosh[c*x]
])/(c^5*d)
```


Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
```

```
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx = -\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}+\frac{\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{c^2}+\frac{b\int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3cd}$$

$$= \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d}-\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx}{c^4}$$

$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^5d}+\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d}-\frac{x^3\left(a+b\cosh^{-1}(cx\right.}{3c^2d}$$

$$= \frac{11b\sqrt{-1+cx}\sqrt{1+cx}}{9c^5d}+\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d}-\frac{x^3\left(a+b\cosh^{-1}\right.}{3c^2d}$$

$$= \frac{11b\sqrt{-1+cx}\sqrt{1+cx}}{9c^5d}+\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d}-\frac{x^3\left(a+b\cosh^{-1}\right.}{3c^2d}$$

$$= \frac{11b\sqrt{-1+cx}\sqrt{1+cx}}{9c^5d}+\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d}-\frac{x^3\left(a+b\cosh^{-1}\right.}{3c^2d}$$

Mathematica [A] time = 0.307293, size = 227, normalized size = 1.44

$$-\frac{18b\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)+18b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)+6ac^3x^3+18acx+9a\log(1-cx)-9a\log(cx+1)-2bc}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] -(18*a*c*x + 6*a*c^3*x^3 - 18*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 18*b*c*x*Sqrt[
(-1 + c*x)/(1 + c*x)] - 4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*c^2*x^2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x] + 18*b*c*x*ArcCosh[c*x] + 6*b*c^3*x^3*ArcCosh[c*x]
 - 9*b*ArcCosh[c*x]^2 - 18*b*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 18*b
*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 9*a*Log[1 - c*x] - 9*a*Log[1 + c*x]
 + 18*b*PolyLog[2, -E^(-ArcCosh[c*x])] + 18*b*PolyLog[2, E^ArcCosh[c*x]])/(
18*c^5*d)
```

Maple [A] time = 0.132, size = 263, normalized size = 1.7

$$-\frac{x^3 a}{3\,c^2 d}-\frac{ax}{dc^4}-\frac{a\ln\left(cx-1\right)}{2\,c^5 d}+\frac{a\ln\left(cx+1\right)}{2\,c^5 d}-\frac{b\mathrm{arccosh}\left(cx\right)}{c^5 d}\ln\left(1-cx-\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{b\mathrm{arccosh}\left(cx\right)}{c^5 d}\ln\left(1+cx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] -1/3/c^2*a/d*x^3-1/c^4*a/d*x-1/2/c^5*a/d*ln(c*x-1)+1/2/c^5*a/d*ln(c*x+1)-1/
c^5*b/d*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/c^5*b/d*arccos
h(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/3/c^2*b/d*arccosh(c*x)*x^3-1
/c^4*b/d*arccosh(c*x)*x+1/9*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d+11/9*b*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d-b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))/c^5/d+b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{72}\left(4\,c^4\left(\frac{2\left(c^2x^3+3\,x\right)}{c^8 d}-\frac{3\,\log\left(cx+1\right)}{c^9 d}+\frac{3\,\log\left(cx-1\right)}{c^9 d}\right)+36\,c^2\left(\frac{2\,x}{c^6 d}-\frac{\log\left(cx+1\right)}{c^7 d}+\frac{\log\left(cx-1\right)}{c^7 d}\right)+648\,c\int\frac{x\log\left(\right.}{12\left(c^6 dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/72*(4*c^4*(2*(c^2*x^3 + 3*x)/(c^8*d) - 3*log(c*x + 1)/(c^9*d) + 3*log(c*x
 - 1)/(c^9*d)) + 36*c^2*(2*x/(c^6*d) - log(c*x + 1)/(c^7*d) + log(c*x - 1)/
(c^7*d)) + 648*c*integrate(1/12*x*log(c*x - 1)/(c^6*d*x^2 - c^4*d), x) - 3*
(4*(2*c^3*x^3 + 6*c*x - 3*log(c*x + 1) + 3*log(c*x - 1))*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + 3*log(c*x + 1)^2 + 6*log(c*x + 1)*log(c*x - 1))/(c^5
*d) + 72*integrate(-1/6*(2*c^3*x^3 + 6*c*x - 3*log(c*x + 1) + 3*log(c*x - 1
))/(c^7*d*x^3 - c^5*d*x + (c^6*d*x^2 - c^4*d)*sqrt(c*x + 1)*sqrt(c*x - 1)),
 x) - 216*integrate(1/12*log(c*x - 1)/(c^6*d*x^2 - c^4*d), x))*b - 1/6*a*(2
*(c^2*x^3 + 3*x)/(c^4*d) - 3*log(c*x + 1)/(c^5*d) + 3*log(c*x - 1)/(c^5*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{bx^4\,\mathrm{arcosh}\left(cx\right)+ax^4}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax^4}{c^2x^2-1}\,dx + \int \frac{bx^4\operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x**4/(c**2*x**2 - 1), x) + Integral(b*x**4*acosh(c*x)/(c**2*x*
*2 - 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(b\operatorname{arcosh}(cx) + a)x^4}{c^2dx^2 - d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x^4/(c^2*d*x^2 - d), x)
```

3.29 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=140

$$-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2c^4d}-\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d}-\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^4d}+$$

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^3*d) + (b*ArcCosh[c*x])/(4*c^4*d) -
 (x^2*(a + b*ArcCosh[c*x]))/(2*c^2*d) + (a + b*ArcCosh[c*x])^2/(2*b*c^4*d)
- ((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c^4*d) - (b*PolyLog[2
, E^(2*ArcCosh[c*x])])/(2*c^4*d)
```

Rubi [A] time = 0.197875, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5766, 90, 52, 5715, 3716, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2c^4d}-\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d}-\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^4d}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```


```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^3*d) + (b*ArcCosh[c*x])/(4*c^4*d) -
 (x^2*(a + b*ArcCosh[c*x]))/(2*c^2*d) + (a + b*ArcCosh[c*x])^2/(2*b*c^4*d)
- ((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c^4*d) - (b*PolyLog[2
, E^(2*ArcCosh[c*x])])/(2*c^4*d)
```


Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx = -\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}+\frac{\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{c^2}+\frac{b\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2cd}$$

$$=\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d}-\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}-\frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx,x,\cosh^{-1}(cx\right.}{c^4d}$$

$$=\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d}+\frac{b\cosh^{-1}(cx)}{4c^4d}-\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d}+\frac{2\,\text{Su}}{}$$

$$=\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d}+\frac{b\cosh^{-1}(cx)}{4c^4d}-\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d}-\frac{(a+}{}$$

$$=\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d}+\frac{b\cosh^{-1}(cx)}{4c^4d}-\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d}-\frac{(a+}{}$$

$$=\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d}+\frac{b\cosh^{-1}(cx)}{4c^4d}-\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d}-\frac{(a+}{}$$

Mathematica [A] time = 0.296799, size = 151, normalized size = 1.08

$$-\frac{4b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)+4b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)+2c^2x^2\left(a+b\cosh^{-1}(cx)\right)-\frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{b}+4\log\left(1-e^{\cosh}\right.}{4c^4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] -(2*c^2*x^2*(a + b*ArcCosh[c*x]) - (2*(a + b*ArcCosh[c*x])^2)/b - b*(c*x*Sq
rt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]) + 4*(a
+ b*ArcCosh[c*x])*Log[1 - E^ArcCosh[c*x]] + 4*(a + b*ArcCosh[c*x])*Log[1 +
E^ArcCosh[c*x]] + 4*b*PolyLog[2, -E^ArcCosh[c*x]] + 4*b*PolyLog[2, E^ArcCos
h[c*x]])/(4*c^4*d)
```

Maple [A] time = 0.087, size = 244, normalized size = 1.7

$$-\frac{ax^2}{2\,c^2d}-\frac{a\ln(cx-1)}{2\,dc^4}-\frac{a\ln(cx+1)}{2\,dc^4}+\frac{b\,(\mathrm{arccosh}\,(cx))^2}{2\,dc^4}-\frac{bx^2\mathrm{arccosh}\,(cx)}{2\,c^2d}+\frac{bx}{4\,c^3d}\sqrt{cx-1}\sqrt{cx+1}+\frac{b\mathrm{arccosh}\,(cx)}{4\,dc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] -1/2/c^2*a/d*x^2-1/2/c^4*a/d*ln(c*x-1)-1/2/c^4*a/d*ln(c*x+1)+1/2/c^4*b/d*ar
ccosh(c*x)^2-1/2/c^2*b/d*arccosh(c*x)*x^2+1/4*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c^3/d+1/4*b*arccosh(c*x)/d/c^4-1/c^4*b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))-1/c^4*b/d*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1
/c^4*b/d*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/c^4*b/d*polyl
og(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{x^2}{c^2d}+\frac{\log\left(c^2x^2-1\right)}{c^4d}\right)+\frac{1}{8}\,b\left(\frac{2\,c^2x^2-4\left(c^2x^2+\log(cx+1)+\log(cx-1)\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+2\,(\log(cx}{c^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(x^2/(c^2*d) + log(c^2*x^2 - 1)/(c^4*d)) + 1/8*b*((2*c^2*x^2 - 4*(c^
2*x^2 + log(c*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
 + 2*(log(c*x - 1) + 1)*log(c*x + 1) + log(c*x + 1)^2 + log(c*x - 1)^2 + 2*
log(c*x - 1))/(c^4*d) - 8*integrate(1/2*(c^2*x^2 + log(c*x + 1) + log(c*x -
 1))/(c^6*d*x^3 - c^4*d*x + (c^5*d*x^2 - c^3*d)*e^(1/2*log(c*x + 1) + 1/2*l
og(c*x - 1))), x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{bx^3\,\mathrm{arcosh}\,(cx)+ax^3}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^3*arccosh(c*x) + a*x^3)/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax^3}{c^2x^2-1}\,dx + \int \frac{bx^3 \operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x**3/(c**2*x**2 - 1), x) + Integral(b*x**3*acosh(c*x)/(c**2*x*
*2 - 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(b \operatorname{arcosh}(cx) + a)x^3}{c^2dx^2 - d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x^3/(c^2*d*x^2 - d), x)
```

3.30 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=102

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^3d}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx\right.}{c^3d}$$

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d) - (x*(a + b*ArcCosh[c*x]))/(c^2*d)
 + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^3*d) + (b*PolyLog[2,
 -E^ArcCosh[c*x]])/(c^3*d) - (b*PolyLog[2, E^ArcCosh[c*x]])/(c^3*d)
```

Rubi [A] time = 0.13777, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {5766, 74, 5694, 4182, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^3d}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx\right.}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```



```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d) - (x*(a + b*ArcCosh[c*x]))/(c^2*d)
 + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^3*d) + (b*PolyLog[2,
 -E^ArcCosh[c*x]])/(c^3*d) - (b*PolyLog[2, E^ArcCosh[c*x]])/(c^3*d)
```


Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx = -\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d} + \frac{\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx}{c^2} + \frac{b\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{cd}$$
$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d} - \frac{\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^3d}$$
$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d} + \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^3d} +$$
$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d} + \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^3d} +$$
$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d} + \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^3d} +$$

Mathematica [A] time = 0.149289, size = 155, normalized size = 1.52

$$\frac{-2b\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right) - 2b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right) - 2acx - a\log(1-cx) + a\log(cx+1) + 2b\sqrt{\frac{cx-1}{cx+1}} + 2bcx\sqrt{}}{2c^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (-2*a*c*x + 2*b*Sqrt[(-1 + c*x)/(1 + c*x)] + 2*b*c*x*Sqrt[(-1 + c*x)/(1 + c
*x)] - 2*b*c*x*ArcCosh[c*x] + b*ArcCosh[c*x]^2 + 2*b*ArcCosh[c*x]*Log[1 + E
^(-ArcCosh[c*x])] - 2*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] - a*Log[1 - c*
x] + a*Log[1 + c*x] - 2*b*PolyLog[2, -E^(-ArcCosh[c*x])] - 2*b*PolyLog[2, E
^ArcCosh[c*x]])/(2*c^3*d)
```

Maple [A] time = 0.073, size = 208, normalized size = 2.

$$-\frac{ax}{c^2d} - \frac{a\ln(cx-1)}{2c^3d} + \frac{a\ln(cx+1)}{2c^3d} + \frac{b\text{arccosh}(cx)}{c^3d}\ln\left(1+cx+\sqrt{cx-1}\sqrt{cx+1}\right) - \frac{b\text{arccosh}(cx)}{c^3d}\ln\left(1-cx-\sqrt{cx-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] -1/c^2*a/d*x-1/2/c^3*a/d*ln(c*x-1)+1/2/c^3*a/d*ln(c*x+1)+1/c^3*b/d*arccosh(
c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/c^3*b/d*arccosh(c*x)*ln(1-c*x-
(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/c^2*b/d*arccosh(c*x)*x+b*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/c^3/d+b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d-b*polylo
g(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(4\,c^2\left(\frac{2\,x}{c^4d}-\frac{\log\left(cx+1\right)}{c^5d}+\frac{\log\left(cx-1\right)}{c^5d}\right)+24\,c\int\frac{x\log\left(cx-1\right)}{4\left(c^4dx^2-c^2d\right)}\,dx-\frac{4\left(2\,cx-\log\left(cx+1\right)+\log\left(cx-1\right)\right)\log\left(cx+\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*(4*c^2*(2*x/(c^4*d) - log(c*x + 1)/(c^5*d) + log(c*x - 1)/(c^5*d)) + 24
*c*integrate(1/4*x*log(c*x - 1)/(c^4*d*x^2 - c^2*d), x) - (4*(2*c*x - log(c
*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + log(c*x +
1)^2 + 2*log(c*x + 1)*log(c*x - 1))/(c^3*d) + 8*integrate(-1/2*(2*c*x - log
(c*x + 1) + log(c*x - 1))/(c^5*d*x^3 - c^3*d*x + (c^4*d*x^2 - c^2*d)*sqrt(c
*x + 1)*sqrt(c*x - 1)), x) - 8*integrate(1/4*log(c*x - 1)/(c^4*d*x^2 - c^2*
d), x))*b - 1/2*a*(2*x/(c^2*d) - log(c*x + 1)/(c^3*d) + log(c*x - 1)/(c^3*d
))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{bx^2\,\mathrm{arcosh}\left(cx\right)+ax^2}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2*arccosh(c*x) + a*x^2)/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax^2}{c^2x^2-1}\,dx + \int \frac{bx^2 \operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x**2/(c**2*x**2 - 1), x) + Integral(b*x**2*acosh(c*x)/(c**2*x*
*2 - 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(b \operatorname{arcosh}(cx) + a)x^2}{c^2dx^2 - d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d), x)
```

3.31 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=74

$$-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2c^2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d}-\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d}$$

```
[Out] (a + b*ArcCosh[c*x])^2/(2*b*c^2*d) - ((a + b*ArcCosh[c*x])*Log[1 - E^(2*Arc
Cosh[c*x])])/(c^2*d) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*c^2*d)
```

Rubi [A] time = 0.116287, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5715, 3716, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2c^2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d}-\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (a + b*ArcCosh[c*x])^2/(2*b*c^2*d) - ((a + b*ArcCosh[c*x])*Log[1 - E^(2*Arc
Cosh[c*x])])/(c^2*d) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*c^2*d)
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx &= -\frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^2d}\\
&= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{c^2d}\\
&= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{2\cosh^{-1}(cx)}\right)}{c^2d} + \frac{b\,\text{Subst}\left(\int \log\left(1-e^{2x}\right)\right.}{c^2d}\\
&= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{2\cosh^{-1}(cx)}\right)}{c^2d} + \frac{b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x\right.}{2c^2d}\\
&= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{2\cosh^{-1}(cx)}\right)}{c^2d} - \frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2c^2d}
\end{aligned}$$

Mathematica [A] time = 0.0858123, size = 85, normalized size = 1.15

$$\frac{-2b^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-2b^2\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)+\left(a+b\cosh^{-1}(cx)\right)\left(a+b\cosh^{-1}(cx)-2b\log\left(1-e^{\cosh^{-1}}\right.\right.}{2bc^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] ((a + b*ArcCosh[c*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 - E^ArcCosh[c*x]] - 2
*b*Log[1 + E^ArcCosh[c*x]]) - 2*b^2*PolyLog[2, -E^ArcCosh[c*x]] - 2*b^2*Pol
yLog[2, E^ArcCosh[c*x]])/(2*b*c^2*d)
```

Maple [A] time = 0.036, size = 179, normalized size = 2.4

$$-\frac{a\ln(cx-1)}{2c^2d}-\frac{a\ln(cx+1)}{2c^2d}+\frac{b(\operatorname{arccosh}(cx))^2}{2c^2d}-\frac{b\operatorname{arccosh}(cx)}{c^2d}\ln\left(1+cx+\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{b}{c^2d}\operatorname{polylog}\left(2,-cx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] -1/2/c^2*a/d*ln(c*x-1)-1/2/c^2*a/d*ln(c*x+1)+1/2/c^2*b/d*arccosh(c*x)^2-1/c
^2*b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/c^2*b/d*polylog
(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/c^2*b/d*arccosh(c*x)*ln(1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))-1/c^2*b/d*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}b\left(\frac{4(\log(cx+1)+\log(cx-1))\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)-\log(cx+1)^2-2\log(cx+1)\log(cx-1)-\log(cx-1)^2}{c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*b*((4*(log(c*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) - log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) - log(c*x - 1)^2)/(c^2*
d) + 8*integrate(1/2*(log(c*x + 1) + log(c*x - 1))/(c^4*d*x^3 - c^2*d*x + (
c^3*d*x^2 - c*d)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)) - 1/2*a*log(
c^2*d*x^2 - d)/(c^2*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{bx\operatorname{arcosh}(cx)+ax}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x*arccosh(c*x) + a*x)/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax}{c^2x^2-1}\,dx + \int \frac{bx\operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x/(c**2*x**2 - 1), x) + Integral(b*x*acosh(c*x)/(c**2*x**2 - 1
), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(b\operatorname{arcosh}(cx) + a)x}{c^2dx^2 - d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d), x)
```

3.32 $\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=59

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{cd}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{cd}+\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{cd}$$

```
[Out] (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c*d) + (b*PolyLog[2, -E^A
rcCosh[c*x]])/(c*d) - (b*PolyLog[2, E^ArcCosh[c*x]])/(c*d)
```

Rubi [A] time = 0.0653781, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5694, 4182, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{cd}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{cd}+\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2),x]
```


```
[Out] (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c*d) + (b*PolyLog[2, -E^A
rcCosh[c*x]])/(c*d) - (b*PolyLog[2, E^ArcCosh[c*x]])/(c*d)
```


Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx = -\frac{\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\,dx, x, \cosh^{-1}(cx)\right)}{cd}$$

$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd} + \frac{b\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \cosh^{-1}(cx)\right)}{cd} - \frac{b\,\text{Sub}\ldots}{}$$

$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd} + \frac{b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\cosh^{-1}(cx)}\right)}{cd} - \frac{b\,\text{Subst}\left(\int\ldots\right.}{}$$

$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd} + \frac{b\mathbf{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{cd} - \frac{b\mathbf{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{cd}$$

Mathematica [A] time = 0.0668811, size = 64, normalized size = 1.08

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)+\left(\log\left(1-e^{\cosh^{-1}(cx)}\right)-\log\left(e^{\cosh^{-1}(cx)}+1\right)\right)\left(-\left(a+b\cosh^{-1}(c\ldots\right.\right.}{cd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])*(Log[1 - E^ArcCosh[c*x]] - Log[1 + E^ArcCosh[c*x]])
) + b*PolyLog[2, -E^ArcCosh[c*x]] - b*PolyLog[2, E^ArcCosh[c*x]])/(c*d)
```

Maple [C] time = 0.283, size = 338, normalized size = 5.7

$$\frac{a\text{Artanh}\,(cx)}{cd}+\frac{b\text{Artanh}\,(cx)\,\text{arccosh}\,(cx)}{cd}+\frac{2\,ib\text{Artanh}\,(cx)}{cd\,(cx-1)\,(cx+1)}\sqrt{\frac{1}{2}+\frac{cx}{2}}\sqrt{-c^2x^2+1}\sqrt{-\frac{1}{2}+\frac{cx}{2}}\ln\left(1+i\,(cx+1)\,\frac{}{\sqrt{\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] 1/c*a/d*arctanh(c*x)+1/c*b/d*arctanh(c*x)*arccosh(c*x)+2*I/c*b/d*(1/2+1/2*c
*x)^(1/2)*(-c^2*x^2+1)^(1/2)*(-1/2+1/2*c*x)^(1/2)/(c*x-1)/(c*x+1)*arctanh(c
*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*I/c*b/d*(1/2+1/2*c*x)^(1/2)*(-c^2*
x^2+1)^(1/2)*(-1/2+1/2*c*x)^(1/2)/(c*x-1)/(c*x+1)*arctanh(c*x)*ln(1-I*(c*x+
1)/(-c^2*x^2+1)^(1/2))+2*I/c*b/d*(1/2+1/2*c*x)^(1/2)*(-c^2*x^2+1)^(1/2)*(-1
/2+1/2*c*x)^(1/2)/(c*x-1)/(c*x+1)*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*I
/c*b/d*(1/2+1/2*c*x)^(1/2)*(-c^2*x^2+1)^(1/2)*(-1/2+1/2*c*x)^(1/2)/(c*x-1)/
(c*x+1)*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}b\left(\frac{4\left(\log\left(cx+1\right)-\log\left(cx-1\right)\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)-\log\left(cx+1\right)^{2}-2\,\log\left(cx+1\right)\log\left(cx-1\right)}{cd}+8\int\frac{(3\,cx-1}{4\left(c^{2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*b*((4*(log(c*x + 1) - log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1)) - log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1))/(c*d) + 8*integrate(1/4
*(3*c*x - 1)*log(c*x - 1)/(c^2*d*x^2 - d), x) + 8*integrate(1/2*(log(c*x +
1) - log(c*x - 1))/(c^3*d*x^3 - c*d*x + (c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(
c*x - 1)), x)) + 1/2*a*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{b\,\mathrm{arcosh}\left(cx\right)+a}{c^{2}dx^{2}-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{a}{c^2x^2-1}\,dx + \int \frac{b\,\mathrm{acosh}\,(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**2 - 1), x) + Integral(b*acosh(c*x)/(c**2*x**2 - 1), x
))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\,\mathrm{arcosh}\,(cx) + a}{c^2dx^2 - d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

3.33 $\int \frac{a+b\cosh^{-1}(cx)}{x(d-c^2dx^2)}\,dx$

Optimal. Leaf size=61

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}$$

```
[Out] (2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d + (b*PolyLog[2, -E^(
2*ArcCosh[c*x])])/(2*d) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*d)
```

Rubi [A] time = 0.126603, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5721, 5461, 4182, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)),x]
```

```
[Out] (2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d + (b*PolyLog[2, -E^(
2*ArcCosh[c*x])])/(2*d) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*d)
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)}\,dx &= -\frac{\text{Subst}\left(\int (a+bx)\operatorname{csch}(x)\operatorname{sech}(x)\,dx, x, \cosh^{-1}(cx)\right)}{d}\\
&= -\frac{2\,\text{Subst}\left(\int (a+bx)\operatorname{csch}(2x)\,dx, x, \cosh^{-1}(cx)\right)}{d}\\
&= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}+\frac{b\,\text{Subst}\left(\int \log\left(1-e^{2x}\right)dx, x, \cosh^{-1}(cx)\right)}{d}-\frac{b\,\text{S}
\\
&= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}+\frac{b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{b\,\text{Subst}
\\
&= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}+\frac{b\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.148326, size = 93, normalized size = 1.52

$$
-\frac{b\left(\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)+2\cosh^{-1}(cx)\left(\log\left(1-e^{-2\cosh^{-1}(cx)}\right)-\log\left(e^{-2\cosh^{-1}(cx)}\right.\right.\right.
}{2d}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)),x]
```

```
[Out] (a*Log[x])/d - (a*Log[1 - c^2*x^2])/(2*d) - (b*(2*ArcCosh[c*x]*(Log[1 - E^(
-2*ArcCosh[c*x])] - Log[1 + E^(-2*ArcCosh[c*x])]) + PolyLog[2, -E^(-2*ArcCo
sh[c*x])] - PolyLog[2, E^(-2*ArcCosh[c*x])]))/(2*d)
```

Maple [A] time = 0.05, size = 91, normalized size = 1.5

$$-\frac{a\ln(cx-1)}{2d}+\frac{a\ln(cx)}{d}-\frac{a\ln(cx+1)}{2d}-\frac{b}{d}\text{dilog}\left(\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)^{-2}\right)+\frac{b}{4d}\text{dilog}\left(\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)^{-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x)
```

```
[Out] -1/2*a/d*ln(c*x-1)+a/d*ln(c*x)-1/2*a/d*ln(c*x+1)-b/d*dilog(1/(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2)+1/4*b/d*dilog(1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a\left(\frac{\log(cx+1)}{d}+\frac{\log(cx-1)}{d}-\frac{2\log(x)}{d}\right)-b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{c^2dx^3-dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(c*x + 1)/d + log(c*x - 1)/d - 2*log(x)/d) - b*integrate(log(c*x
 + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*d*x^3 - d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\,\text{arcosh}(cx)+a}{c^2dx^3-dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^3 - d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{a}{c^2x^3-x}\,dx + \int \frac{b\,\mathrm{acosh}\,(cx)}{c^2x^3-x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**3 - x), x) + Integral(b*acosh(c*x)/(c**2*x**3 - x), x
))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\,\mathrm{arcosh}\,(cx) + a}{\left(c^2dx^2 - d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x), x)
```

3.34 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)}\,dx$

Optimal. Leaf size=95

$$\frac{bc\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{d}-\frac{bc\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{d}-\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{2c\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}$$

```
[Out] -((a + b*ArcCosh[c*x])/(d*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d + (2*c*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d + (b*c*PolyLog[2,
-E^ArcCosh[c*x]])/d - (b*c*PolyLog[2, E^ArcCosh[c*x]])/d
```

Rubi [A] time = 0.139802, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5746, 92, 205, 5694, 4182, 2279, 2391}

$$\frac{bc\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{d}-\frac{bc\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{d}-\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{2c\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d + (2*c*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d + (b*c*PolyLog[2,
-E^ArcCosh[c*x]])/d - (b*c*PolyLog[2, E^ArcCosh[c*x]])/d
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)}\,dx = -\frac{a+b\cosh^{-1}(cx)}{dx} + c^2\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx + \frac{(bc)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} - \frac{c\,\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\,dx, x, \cosh^{-1}(cx)\right)}{d} + \frac{\left(bc^2\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x\right.}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} + \frac{2c\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} + \frac{2c\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} + \frac{2c\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d}$$

Mathematica [A] time = 0.358219, size = 132, normalized size = 1.39

$$\frac{bc\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right) - bc\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right) - \frac{a+b\cosh^{-1}(cx)}{x} - c\log\left(1-e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right) + c\log}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])/x) + (b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x
^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - c*(a + b*ArcCosh[c*x])*Log[1 - E^Arc
Cosh[c*x]] + c*(a + b*ArcCosh[c*x])*Log[1 + E^ArcCosh[c*x]] + b*c*PolyLog[2
, -E^ArcCosh[c*x]] - b*c*PolyLog[2, E^ArcCosh[c*x]])/d
```

Maple [A] time = 0.108, size = 161, normalized size = 1.7

$$-\frac{a}{dx} - \frac{ca\ln(cx-1)}{2d} + \frac{ca\ln(cx+1)}{2d} - \frac{b\text{arccosh}(cx)}{dx} + 2\frac{bc\arctan\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)}{d} + \frac{bc}{d}\text{dilog}\left(cx+\sqrt{cx-1}\sqrt{c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x)
```

```
[Out] -a/d/x-1/2*c*a/d*ln(c*x-1)+1/2*c*a/d*ln(c*x+1)-b/d*arccosh(c*x)/x+2*c*b/d*a
rctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+c*b/d*dilog(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))+c*b/d*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+c*b/d*arccosh(c*x)*
ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(24\,c^3\int\frac{x\log\left(cx-1\right)}{4\left(c^2dx^2-d\right)}\,dx-4\,c^2\left(\frac{\log\left(cx+1\right)}{cd}-\frac{\log\left(cx-1\right)}{cd}\right)-8\,c^2\int\frac{\log\left(cx-1\right)}{4\left(c^2dx^2-d\right)}\,dx-\frac{cx\log\left(cx+1\right)^2+2\,cx\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*(24*c^3*integrate(1/4*x*log(c*x - 1)/(c^2*d*x^2 - d), x) - 4*c^2*(log(c
*x + 1)/(c*d) - log(c*x - 1)/(c*d)) - 8*c^2*integrate(1/4*log(c*x - 1)/(c^2
*d*x^2 - d), x) - (c*x*log(c*x + 1)^2 + 2*c*x*log(c*x + 1)*log(c*x - 1) - 4
*(c*x*log(c*x + 1) - c*x*log(c*x - 1) - 2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)))/(d*x) + 8*integrate(1/2*(c^2*x*log(c*x + 1) - c^2*x*log(c*x - 1) -
2*c)/(c^3*d*x^4 - c*d*x^2 + (c^2*d*x^3 - d*x)*sqrt(c*x + 1)*sqrt(c*x - 1)),
 x))*b + 1/2*a*(c*log(c*x + 1)/d - c*log(c*x - 1)/d - 2/(d*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\,\text{arcosh}\left(cx\right)+a}{c^2dx^4-dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^4 - d*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{a}{c^2x^4-x^2}\,dx+\int\frac{b\,\text{acosh}\left(cx\right)}{c^2x^4-x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**4 - x**2), x) + Integral(b*acosh(c*x)/(c**2*x**4 - x*
*2), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^2), x)
```

3.35 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d-c^2dx^2)}\,dx$

Optimal. Leaf size=118

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{bc^2\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d}+\frac{2c^2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}-\frac{a+b\cos}{2d}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x) - (a + b*ArcCosh[c*x])/(2*d*x^2)
 + (2*c^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d + (b*c^2*Poly
Log[2, -E^(2*ArcCosh[c*x])])/(2*d) - (b*c^2*PolyLog[2, E^(2*ArcCosh[c*x])])
/(2*d)
```

Rubi [A] time = 0.19727, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5746, 95, 5721, 5461, 4182, 2279, 2391}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{bc^2\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d}+\frac{2c^2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}-\frac{a+b\cos}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x) - (a + b*ArcCosh[c*x])/(2*d*x^2)
 + (2*c^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d + (b*c^2*Poly
Log[2, -E^(2*ArcCosh[c*x])])/(2*d) - (b*c^2*PolyLog[2, E^(2*ArcCosh[c*x])])
/(2*d)
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2dx^2}+c^2\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)}\,dx+\frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2d}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}-\frac{c^2\,\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\mathbf{sech}(x)\,dx,x,\cosh^{-1}(c$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}-\frac{\left(2c^2\right)\text{Subst}\left(\int (a+bx)\mathbf{csch}(2x)\,dx,x,\cosh^{-1}(cx)\right)}{d}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}+\frac{\left(bc^2\right.}{}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}+\frac{\left(bc^2\right.}{}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}+\frac{bc^2\mathbf{I}}{}$$

Mathematica [A] time = 0.527178, size = 144, normalized size = 1.22

$$-\frac{bc^2\left(\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)+\frac{\cosh^{-1}(cx)}{c^2x^2}-\frac{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}{cx}+2\cosh^{-1}(cx)\log\left(1-e^{-2\cosh^{-1}}\right.\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)),x]
```

```
[Out] -(a/x^2 - 2*a*c^2*Log[x] + a*c^2*Log[1 - c^2*x^2] + b*c^2*(-((Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x))/(c*x)) + ArcCosh[c*x]/(c^2*x^2) + 2*ArcCosh[c*x]*L
og[1 - E^(-2*ArcCosh[c*x])] - 2*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] +
 PolyLog[2, -E^(-2*ArcCosh[c*x])] - PolyLog[2, E^(-2*ArcCosh[c*x])]))/(2*d)
```

Maple [B] time = 0.092, size = 301, normalized size = 2.6

$$-\frac{c^2a\ln(cx-1)}{2d}-\frac{a}{2dx^2}+\frac{c^2a\ln(cx)}{d}-\frac{c^2a\ln(cx+1)}{2d}+\frac{bc}{2dx}\sqrt{cx-1}\sqrt{cx+1}-\frac{c^2b}{2d}-\frac{b\,\text{arccosh}(cx)}{2dx^2}+\frac{c^2b\,\text{arccosh}(c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x)
```

```
[Out] -1/2*c^2*a/d*ln(c*x-1)-1/2*a/d/x^2+c^2*a/d*ln(c*x)-1/2*c^2*a/d*ln(c*x+1)+1/
2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x-1/2*c^2*b/d-1/2*b/d*arccosh(c*x)/x^2+
c^2*b/d*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)+1/2*b*c^2*po
lylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d-c^2*b/d*arccosh(c*x)*ln(1+c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-c^2*b/d*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))-c^2*b/d*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-c^2*b/d
*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(\frac{c^2\log\left(cx+1\right)}{d}+\frac{c^2\log\left(cx-1\right)}{d}-\frac{2\,c^2\log\left(x\right)}{d}+\frac{1}{dx^2}\right)a-b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{c^2dx^5-dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*log(c*x + 1)/d + c^2*log(c*x - 1)/d - 2*c^2*log(x)/d + 1/(d*x^2))
*a - b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*d*x^5 - d*x^3)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{b\,\mathrm{arcosh}\left(cx\right)+a}{c^2dx^5-dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{a}{c^2x^5-x^3}\,dx+\int\frac{b\,\mathrm{acosh}\left(cx\right)}{c^2x^5-x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**5 - x**3), x) + Integral(b*acosh(c*x)/(c**2*x**5 - x*
*3), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\,\mathrm{arcosh}\,(cx) + a}{\left(c^2dx^2 - d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^3), x)
```

3.36 $\int \frac{a+b\cosh^{-1}(cx)}{x^4(d-c^2dx^2)}\,dx$

Optimal. Leaf size=157

$$\frac{bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{d}-\frac{bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{d}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+\frac{2c^3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh\right.}{d}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2) - (a + b*ArcCosh[c*x])/(3*d*x^
3) - (c^2*(a + b*ArcCosh[c*x]))/(d*x) + (7*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt
[1 + c*x]])/(6*d) + (2*c^3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d
+ (b*c^3*PolyLog[2, -E^ArcCosh[c*x]])/d - (b*c^3*PolyLog[2, E^ArcCosh[c*x]]
)/d
```

Rubi [A] time = 0.234406, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5746, 103, 12, 92, 205, 5694, 4182, 2279, 2391}

$$\frac{bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{d}-\frac{bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{d}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+\frac{2c^3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2) - (a + b*ArcCosh[c*x])/(3*d*x^
3) - (c^2*(a + b*ArcCosh[c*x]))/(d*x) + (7*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt
[1 + c*x]])/(6*d) + (2*c^3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d
+ (b*c^3*PolyLog[2, -E^ArcCosh[c*x]])/d - (b*c^3*PolyLog[2, E^ArcCosh[c*x]]
)/d
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
```

```
IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3dx^3}+c^2\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)}\,dx+\frac{(bc)\int \frac{1}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3d}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+c^4\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx+$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}-\frac{c^3\,\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\right.}{d}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+\frac{bc^3\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+c\right.}{d}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+\frac{7bc^3\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+\right.}{6d}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+\frac{7bc^3\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+\right.}{6d}$$

Mathematica [A] time = 0.342144, size = 223, normalized size = 1.42

$$\frac{6bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-6bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)-\frac{6ac^2}{x}-6ac^3\log\left(1-e^{\cosh^{-1}(cx)}\right)+6ac^3\log\left(e^{\cosh^{-1}(cx)}+1\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)),x]
```

```
[Out] ((-2*a)/x^3 - (6*a*c^2)/x + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x^2 - (2*b*A
rcCosh[c*x])/x^3 - (6*b*c^2*ArcCosh[c*x])/x + (7*b*c^3*Sqrt[-1 + c^2*x^2]*A
rcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - 6*a*c^3*Log[1 -
```

```
 E^ArcCosh[c*x]] - 6*b*c^3*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 6*a*c^3*L
og[1 + E^ArcCosh[c*x]] + 6*b*c^3*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + 6*b
*c^3*PolyLog[2, -E^ArcCosh[c*x]] - 6*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/(6*d
)
```

Maple [A] time = 0.126, size = 225, normalized size = 1.4

$$-\frac{c^3 a \ln(cx-1)}{2d} - \frac{a}{3dx^3} - \frac{c^2 a}{dx} + \frac{c^3 a \ln(cx+1)}{2d} - \frac{c^2 b \operatorname{arccosh}(cx)}{dx} + \frac{bc}{6dx^2}\sqrt{cx-1}\sqrt{cx+1} - \frac{b \operatorname{arccosh}(cx)}{3dx^3} + \frac{7bc^3}{3d} \text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x)
```

```
[Out] -1/2*c^3*a/d*ln(c*x-1)-1/3*a/d/x^3-c^2*a/d/x+1/2*c^3*a/d*ln(c*x+1)-c^2*b/d*
arccosh(c*x)/x+1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2-1/3*b/d*arccosh(c*
x)/x^3+7/3*c^3*b/d*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+c^3*b/d*dilog(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+c^3*b/d*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))+c^3*b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\left(\frac{3\,c^3 \log(cx+1)}{d} - \frac{3\,c^3 \log(cx-1)}{d} - \frac{2\left(3\,c^2x^2+1\right)}{dx^3}\right)a + \frac{1}{24}\left(216\,c^5 \int \frac{x^3 \log(cx-1)}{12\left(c^2dx^4 - dx^2\right)}\,dx - 12\,c^4\left(\frac{\log(cx+1)}{cd} - \text{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/6*(3*c^3*log(c*x + 1)/d - 3*c^3*log(c*x - 1)/d - 2*(3*c^2*x^2 + 1)/(d*x^3
))*a + 1/24*(216*c^5*integrate(1/12*x^3*log(c*x - 1)/(c^2*d*x^4 - d*x^2), x
) - 12*c^4*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) - 72*c^4*integrate(1/1
2*x^2*log(c*x - 1)/(c^2*d*x^4 - d*x^2), x) - 4*c^2*(c*log(c*x + 1)/d - c*lo
g(c*x - 1)/d - 2/(d*x)) - (3*c^3*x^3*log(c*x + 1)^2 + 6*c^3*x^3*log(c*x + 1
)*log(c*x - 1) - 4*(3*c^3*x^3*log(c*x + 1) - 3*c^3*x^3*log(c*x - 1) - 6*c^2
*x^2 - 2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(d*x^3) + 24*integrate(1/
6*(3*c^4*x^3*log(c*x + 1) - 3*c^4*x^3*log(c*x - 1) - 6*c^3*x^2 - 2*c)/(c^3*
d*x^6 - c*d*x^4 + (c^2*d*x^5 - d*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x))*b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\,\text{arcosh}\left(cx\right)+a}{c^{2}dx^{6}-dx^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^6 - d*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{a}{c^{2}x^{6}-x^{4}}\,dx+\int \frac{b\,\text{acosh}\left(cx\right)}{c^{2}x^{6}-x^{4}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**6 - x**4), x) + Integral(b*acosh(c*x)/(c**2*x**6 - x*
*4), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\,\text{arcosh}\left(cx\right)+a}{\left(c^{2}dx^{2}-d\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^4), x)
```

3.37 $\int \frac{x^4(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal. Leaf size=177

$$-\frac{3b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2c^5d^2}+\frac{3b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2c^5d^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}-\frac{3\tanh^{-1}}{}$$

```
[Out] -(b*x^2)/(2*c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/(2*c^5*d^2) + (3*x*(a + b*ArcCosh[c*x]))/(2*c^4*d^2) + (x^3*(a +
b*ArcCosh[c*x]))/(2*c^2*d^2*(1 - c^2*x^2)) - (3*(a + b*ArcCosh[c*x])*ArcTan
h[E^ArcCosh[c*x]])/(c^5*d^2) - (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c^5*d^2
) + (3*b*PolyLog[2, E^ArcCosh[c*x]])/(2*c^5*d^2)
```

Rubi [A] time = 0.225377, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5750, 98, 21, 74, 5766, 5694, 4182, 2279, 2391}

$$-\frac{3b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2c^5d^2}+\frac{3b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2c^5d^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}-\frac{3\tanh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```


```
[Out] -(b*x^2)/(2*c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/(2*c^5*d^2) + (3*x*(a + b*ArcCosh[c*x]))/(2*c^4*d^2) + (x^3*(a +
b*ArcCosh[c*x]))/(2*c^2*d^2*(1 - c^2*x^2)) - (3*(a + b*ArcCosh[c*x])*ArcTan
h[E^ArcCosh[c*x]])/(c^5*d^2) - (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c^5*d^2
) + (3*b*PolyLog[2, E^ArcCosh[c*x]])/(2*c^5*d^2)
```


Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx=\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\int\frac{x^3}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2}-\frac{3\int\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{2c^2d}$$

$$=-\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{b\int\frac{x(-2-}{\sqrt{-1+cx}(1}}{2c^3}$$

$$=-\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3b\sqrt{-1+cx}\sqrt{1+cx}}{2c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}+\frac{x^3\left(a+b\cos\right.}{2c^2d^2\left(1-\right.}$$

$$=-\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{2c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}+\frac{x^3\left(a+b\cosh\right.}{2c^2d^2\left(1-c\right.}$$

$$=-\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{2c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}+\frac{x^3\left(a+b\cosh\right.}{2c^2d^2\left(1-c\right.}$$

$$=-\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{2c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}+\frac{x^3\left(a+b\cosh\right.}{2c^2d^2\left(1-c\right.}$$

Mathematica [A] time = 1.04641, size = 244, normalized size = 1.38

$$-6b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)+6b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)-\frac{2acx}{c^2x^2-1}+4acx+3a\log(1-cx)-3a\log(cx+1)-4bcx\sqrt{\frac{cx-1}{cx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (4*a*c*x - 3*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*b*c*x*Sqrt[(-1 + c*x)/(1 + c*
x)] + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + (b*c*x*Sqrt[(-1 + c*x)/(1
+ c*x)])/(1 - c*x) - (2*a*c*x)/(-1 + c^2*x^2) + 4*b*c*x*ArcCosh[c*x] + (b*A
rcCosh[c*x])/(1 - c*x) - (b*ArcCosh[c*x])/(1 + c*x) + 6*b*ArcCosh[c*x]*Log[
1 - E^ArcCosh[c*x]] - 6*b*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + 3*a*Log[1
- c*x] - 3*a*Log[1 + c*x] - 6*b*PolyLog[2, -E^ArcCosh[c*x]] + 6*b*PolyLog[2
, E^ArcCosh[c*x]])/(4*c^5*d^2)
```

Maple [A] time = 0.249, size = 300, normalized size = 1.7

$$\frac{ax}{d^2c^4}-\frac{a}{4\,c^5d^2\,(cx-1)}+\frac{3\,a\ln(cx-1)}{4\,c^5d^2}-\frac{a}{4\,c^5d^2\,(cx+1)}-\frac{3\,a\ln(cx+1)}{4\,c^5d^2}+\frac{b\text{arccosh}(cx)\,x}{d^2c^4}-\frac{b}{c^5d^2}\sqrt{cx-1}\sqrt{cx+1}-\frac{ba}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] 1/c^4*a/d^2*x-1/4/c^5*a/d^2/(c*x-1)+3/4/c^5*a/d^2*ln(c*x-1)-1/4/c^5*a/d^2/(
c*x+1)-3/4/c^5*a/d^2*ln(c*x+1)+1/c^4*b/d^2*arccosh(c*x)*x-b*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^5/d^2-1/2/c^4*b/d^2/(c^2*x^2-1)*arccosh(c*x)*x-1/2/c^5*b/d^2
/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)-3/2/c^5*b/d^2*arccosh(c*x)*ln(1+c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-3/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(
1/2))/c^5/d^2+3/2/c^5*b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))+3/2*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(16*c^4*(2*x/(c^10*d^2*x^2 - c^8*d^2) - 4*x/(c^8*d^2) + 3*log(c*x + 1)
/(c^9*d^2) - 3*log(c*x - 1)/(c^9*d^2)) - 576*c^3*integrate(1/8*x^3*log(c*x
- 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x) - 24*c^2*(2*x/(c^8*d^2*x^2
 - c^6*d^2) + log(c*x + 1)/(c^7*d^2) - log(c*x - 1)/(c^7*d^2)) + 192*c^2*in
tegrate(1/8*x^2*log(c*x - 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x) -
9*(c*(2/(c^8*d^2*x - c^7*d^2) - log(c*x + 1)/(c^7*d^2) + log(c*x - 1)/(c^7*
d^2)) + 4*log(c*x - 1)/(c^8*d^2*x^2 - c^6*d^2))*c + 4*(3*(c^2*x^2 - 1)*log(
c*x + 1)^2 + 6*(c^2*x^2 - 1)*log(c*x + 1)*log(c*x - 1) + 4*(4*c^3*x^3 - 6*c
*x - 3*(c^2*x^2 - 1)*log(c*x + 1) + 3*(c^2*x^2 - 1)*log(c*x - 1))*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^7*d^2*x^2 - c^5*d^2) - 64*integrate(-1/4*
(4*c^3*x^3 - 6*c*x - 3*(c^2*x^2 - 1)*log(c*x + 1) + 3*(c^2*x^2 - 1)*log(c*x
 - 1))/(c^9*d^2*x^5 - 2*c^7*d^2*x^3 + c^5*d^2*x + (c^8*d^2*x^4 - 2*c^6*d^2*
x^2 + c^4*d^2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - 192*integrate(1/8*log(c*x
 - 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x))*b - 1/4*a*(2*x/(c^6*d^2*
x^2 - c^4*d^2) - 4*x/(c^4*d^2) + 3*log(c*x + 1)/(c^5*d^2) - 3*log(c*x - 1)/
(c^5*d^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4 \operatorname{arcosh}(cx) + ax^4}{c^4d^2x^4 - 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^4}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^4 \operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**4/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**4*acosh(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^4}{\left(c^2dx^2 - d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(c^2*d*x^2 - d)^2, x)
```

3.38 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=179

$$\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2c^4d^2}+\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2}-$$

```
[Out] -b/(2*c^4*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-1 + c*x])/(2*c^4*d^2
*Sqrt[1 + c*x]) + (b*ArcCosh[c*x])/(2*c^4*d^2) + (x^2*(a + b*ArcCosh[c*x]))
/(2*c^2*d^2*(1 - c^2*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*c^4*d^2) + ((a + b
*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c^4*d^2) + (b*PolyLog[2, E^(2*
ArcCosh[c*x])])/(2*c^4*d^2)
```

Rubi [A] time = 0.204298, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5750, 89, 12, 78, 52, 5715, 3716, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2c^4d^2}+\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```



```
[Out] -b/(2*c^4*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-1 + c*x])/(2*c^4*d^2
*Sqrt[1 + c*x]) + (b*ArcCosh[c*x])/(2*c^4*d^2) + (x^2*(a + b*ArcCosh[c*x]))
/(2*c^2*d^2*(1 - c^2*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*c^4*d^2) + ((a + b
*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c^4*d^2) + (b*PolyLog[2, E^(2*
ArcCosh[c*x])])/(2*c^4*d^2)
```


Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx = \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{b\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2} - \frac{\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{c^2d}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \cosh^{-1}\right.}{c^4d^2}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d^2} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(}{1-}\right.}{}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}}{2c^4d^2\sqrt{1+cx}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d^2}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}}{2c^4d^2\sqrt{1+cx}} + \frac{b\cosh^{-1}(cx)}{2c^4d^2} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{(a+}{}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}}{2c^4d^2\sqrt{1+cx}} + \frac{b\cosh^{-1}(cx)}{2c^4d^2} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{(a+}{}$$

Mathematica [A] time = 0.620754, size = 209, normalized size = 1.17

$$\frac{4b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right) + 4b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right) - \frac{2a}{c^2x^2-1} + 2a\log\left(1-c^2x^2\right) - b\sqrt{\frac{cx-1}{cx+1}} + \frac{b\sqrt{\frac{cx-1}{cx+1}}}{1-cx} + \frac{bcx\sqrt{\frac{cx-1}{cx+1}}}{1-cx} - 2b}{4c^4d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (-(b*Sqrt[(-1 + c*x)/(1 + c*x)]) + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x)
 + (b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a)/(-1 + c^2*x^2) + (b
*ArcCosh[c*x])/(1 - c*x) + (b*ArcCosh[c*x])/(1 + c*x) - 2*b*ArcCosh[c*x]^2
+ 4*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 4*b*ArcCosh[c*x]*Log[1 + E^Arc
Cosh[c*x]] + 2*a*Log[1 - c^2*x^2] + 4*b*PolyLog[2, -E^ArcCosh[c*x]] + 4*b*P
olyLog[2, E^ArcCosh[c*x]])/(4*c^4*d^2)
```

Maple [A] time = 0.192, size = 309, normalized size = 1.7

$$-\frac{a}{4\,d^2c^4\,(cx-1)}+\frac{a\ln{(cx-1)}}{2\,d^2c^4}+\frac{a}{4\,d^2c^4\,(cx+1)}+\frac{a\ln{(cx+1)}}{2\,d^2c^4}-\frac{b\,(\operatorname{arccosh}{(cx)})^2}{2\,d^2c^4}-\frac{bx}{2\,c^3d^2\,(c^2x^2-1)}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/4/c^4*a/d^2/(c*x-1)+1/2/c^4*a/d^2*ln(c*x-1)+1/4/c^4*a/d^2/(c*x+1)+1/2/c^
4*a/d^2*ln(c*x+1)-1/2/c^4*b/d^2*arccosh(c*x)^2-1/2/c^3*b/d^2/(c^2*x^2-1)*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/2/c^2*b/d^2/(c^2*x^2-1)*x^2-1/2/c^4*b/d^2/(c^
2*x^2-1)*arccosh(c*x)-1/2/c^4*b/d^2/(c^2*x^2-1)+1/c^4*b/d^2*arccosh(c*x)*ln
(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/c^4*b/d^2*polylog(2,-c*x-(c*x-1)^(1/2
)*(c*x+1)^(1/2))+1/c^4*b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1
/2))+1/c^4*b/d^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\,b\left(\frac{\left(c^2x^2-1\right)\log{(cx+1)}^2+2\left(c^2x^2-1\right)\log{(cx+1)}\log{(cx-1)}+\left(c^2x^2-1\right)\log{(cx-1)}^2-4\left(\left(c^2x^2-1\right)\log{(cx+1}\right.\right.}{c^6d^2x^2-c^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b*(((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)*log(c*
x - 1) + (c^2*x^2 - 1)*log(c*x - 1)^2 - 4*((c^2*x^2 - 1)*log(c*x + 1) + (c^
2*x^2 - 1)*log(c*x - 1) - 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 2)/(c
^6*d^2*x^2 - c^4*d^2) - 8*integrate(1/2*((c^2*x^2 - 1)*log(c*x + 1) + (c^2*
x^2 - 1)*log(c*x - 1) - 1)/(c^8*d^2*x^5 - 2*c^6*d^2*x^3 + c^4*d^2*x + (c^7*
d^2*x^4 - 2*c^5*d^2*x^2 + c^3*d^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)))
, x)) - 1/2*a*(1/(c^6*d^2*x^2 - c^4*d^2) - log(c^2*x^2 - 1)/(c^4*d^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bx^3\operatorname{arcosh}{(cx)}+ax^3}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccosh(c*x) + a*x^3)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^3}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^3\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**3/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**3*acosh(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)x^3}{(c^2dx^2-d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^3/(c^2*d*x^2 - d)^2, x)
```

3.39 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=124

$$-\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2c^3d^2}+\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2c^3d^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx\right.}{c^3d^2}$$

```
[Out] -b/(2*c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*c
^2*d^2*(1 - c^2*x^2)) - ((a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^3
*d^2) - (b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c^3*d^2) + (b*PolyLog[2, E^ArcCo
sh[c*x]])/(2*c^3*d^2)
```

Rubi [A] time = 0.135397, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {5750, 74, 5694, 4182, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2c^3d^2}+\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2c^3d^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx\right.}{c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] -b/(2*c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*c
^2*d^2*(1 - c^2*x^2)) - ((a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^3
*d^2) - (b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c^3*d^2) + (b*PolyLog[2, E^ArcCo
sh[c*x]])/(2*c^3*d^2)
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
IntegerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{b\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx}{2c^2d}$$

$$= -\frac{b}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx, x, \cosh^{-1}\right.}{2c^3d^2}$$

$$= -\frac{b}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right.}{c^3d^2}$$

$$= -\frac{b}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right.}{c^3d^2}$$

$$= -\frac{b}{2c^3d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right.}{c^3d^2}$$

Mathematica [A] time = 0.739994, size = 206, normalized size = 1.66

$$\frac{-2b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)+2b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)-\frac{2acx}{c^2x^2-1}+a\log(1-cx)-a\log(cx+1)+\frac{bcx\sqrt{\frac{cx-1}{cx+1}}}{1-cx}+\frac{b\sqrt{\frac{cx-1}{cx+1}}}{1-cx}+}{4c^3d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (b*Sqrt[(-1 + c*x)/(1 + c*x)] + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) +
(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a*c*x)/(-1 + c^2*x^2) + (
b*ArcCosh[c*x])/(1 - c*x) - (b*ArcCosh[c*x])/(1 + c*x) + 2*b*ArcCosh[c*x]*L
og[1 - E^ArcCosh[c*x]] - 2*b*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + a*Log[1
 - c*x] - a*Log[1 + c*x] - 2*b*PolyLog[2, -E^ArcCosh[c*x]] + 2*b*PolyLog[2,
 E^ArcCosh[c*x]])/(4*c^3*d^2)
```

Maple [A] time = 0.099, size = 255, normalized size = 2.1

$$-\frac{a}{4\,c^3d^2\,(cx-1)}+\frac{a\ln(cx-1)}{4\,c^3d^2}-\frac{a}{4\,c^3d^2\,(cx+1)}-\frac{a\ln(cx+1)}{4\,c^3d^2}-\frac{b\text{arccosh}\,(cx)\,x}{2\,c^2d^2\left(c^2x^2-1\right)}-\frac{b}{2\,c^3d^2\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/4/c^3*a/d^2/(c*x-1)+1/4/c^3*a/d^2*ln(c*x-1)-1/4/c^3*a/d^2/(c*x+1)-1/4/c^
3*a/d^2*ln(c*x+1)-1/2/c^2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*x-1/2/c^3*b/d^2/(c
^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)-1/2/c^3*b/d^2*arccosh(c*x)*ln(1+c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))-1/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))/c^3/d^2+1/2/c^3*b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))
+1/2*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{64}\left(192\,c^3\int\frac{x^3\log\left(cx-1\right)}{8\left(c^6d^2x^4-2\,c^4d^2x^2+c^2d^2\right)}\,dx+8\,c^2\left(\frac{2\,x}{c^6d^2x^2-c^4d^2}+\frac{\log\left(cx+1\right)}{c^5d^2}-\frac{\log\left(cx-1\right)}{c^5d^2}\right)-64\,c^2\int\frac{x^2\,\mathrm{lo}}{8\left(c^6d^2x^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/64*(192*c^3*integrate(1/8*x^3*log(c*x - 1)/(c^6*d^2*x^4 - 2*c^4*d^2*x^2
+ c^2*d^2), x) + 8*c^2*(2*x/(c^6*d^2*x^2 - c^4*d^2) + log(c*x + 1)/(c^5*d^2
) - log(c*x - 1)/(c^5*d^2)) - 64*c^2*integrate(1/8*x^2*log(c*x - 1)/(c^6*d^
2*x^4 - 2*c^4*d^2*x^2 + c^2*d^2), x) + 3*(c*(2/(c^6*d^2*x - c^5*d^2) - log(
c*x + 1)/(c^5*d^2) + log(c*x - 1)/(c^5*d^2)) + 4*log(c*x - 1)/(c^6*d^2*x^2
- c^4*d^2))*c - 4*((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x +
 1)*log(c*x - 1) - 4*(2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*lo
g(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^5*d^2*x^2 - c^3*d^2)
 + 64*integrate(1/4*(2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*log
(c*x - 1))/(c^7*d^2*x^5 - 2*c^5*d^2*x^3 + c^3*d^2*x + (c^6*d^2*x^4 - 2*c^4*
d^2*x^2 + c^2*d^2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + 64*integrate(1/8*log(
c*x - 1)/(c^6*d^2*x^4 - 2*c^4*d^2*x^2 + c^2*d^2), x))*b - 1/4*a*(2*x/(c^4*d
^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3*d^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{arcosh}\left(cx\right)+ax^2}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^2}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**2*acosh(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)x^2}{(c^2dx^2-d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d)^2, x)
```

3.40 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=61

$$\frac{a+b\cosh^{-1}(cx)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{bx}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -(b*x)/(2*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*c^2
*d^2*(1 - c^2*x^2))
```

Rubi [A] time = 0.0521277, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5716, 39}

$$\frac{a+b\cosh^{-1}(cx)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{bx}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] -(b*x)/(2*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*c^2
*d^2*(1 - c^2*x^2))
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx = \frac{a+b\cosh^{-1}(cx)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{b\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2}$$
$$= -\frac{bx}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2c^2d^2\left(1-c^2x^2\right)}$$

Mathematica [A] time = 0.134365, size = 53, normalized size = 0.87

$$\frac{a+bcx\sqrt{cx-1}\sqrt{cx+1}+b\cosh^{-1}(cx)}{2c^2d^2-2c^4d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*ArcCosh[c*x])/(2*c^2*d^2 - 2*c^
4*d^2*x^2)
```

Maple [A] time = 0.013, size = 64, normalized size = 1.1

$$\frac{1}{c^2}\left(-\frac{a}{2\,d^2\left(c^2x^2-1\right)}+\frac{b}{d^2}\left(-\frac{\operatorname{arccosh}(cx)}{2\,c^2x^2-2}-\frac{cx}{2}\frac{1}{\sqrt{cx-1}}\frac{1}{\sqrt{cx+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] 1/c^2*(-1/2*a/d^2/(c^2*x^2-1)+b/d^2*(-1/2/(c^2*x^2-1)*arccosh(c*x)-1/2/(c*x
-1)^(1/2)/(c*x+1)^(1/2)*c*x))
```

Maxima [B] time = 1.27197, size = 223, normalized size = 3.66

$$-\frac{1}{4}\left(\frac{\left(\frac{\sqrt{c^2x^2-1}c^2d^2}{c^6d^4+\sqrt{c^6d^4}c^4d^2x}-\frac{\sqrt{c^2x^2-1}c^2d^2}{c^6d^4-\sqrt{c^6d^4}c^4d^2x}\right)c^5d^2}{\sqrt{c^6d^4}}+\frac{2\,\operatorname{arcosh}(cx)}{c^4d^2x^2-c^2d^2}\right)b-\frac{a}{2\left(c^4d^2x^2-c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((sqrt(c^2*x^2 - 1)*c^2*d^2/(c^6*d^4 + sqrt(c^6*d^4)*c^4*d^2*x) - sqrt
(c^2*x^2 - 1)*c^2*d^2/(c^6*d^4 - sqrt(c^6*d^4)*c^4*d^2*x))*c^5*d^2/sqrt(c^6
*d^4) + 2*arccosh(c*x)/(c^4*d^2*x^2 - c^2*d^2))*b - 1/2*a/(c^4*d^2*x^2 - c^
2*d^2)
```

Fricas [A] time = 1.76537, size = 136, normalized size = 2.23

$$-\frac{ac^2x^2 + \sqrt{c^2x^2-1}bcx + b\log\left(cx + \sqrt{c^2x^2-1}\right)}{2\left(c^4d^2x^2 - c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^2*x^2 + sqrt(c^2*x^2 - 1)*b*c*x + b*log(c*x + sqrt(c^2*x^2 - 1)))
/(c^4*d^2*x^2 - c^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x*acosh(c*x)/(
c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)x}{\left(c^2dx^2 - d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d)^2, x)
```

3.41 $\int \frac{a+b\cosh^{-1}(cx)}{(d-c^2dx^2)^2}\,dx$

Optimal. Leaf size=120

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2cd^2}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2cd^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{cd^2}$$

```
[Out] -b/(2*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*d^2
*(1 - c^2*x^2)) + ((a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c*d^2) +
(b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c*d^2) - (b*PolyLog[2, E^ArcCosh[c*x]])/
(2*c*d^2)
```

Rubi [A] time = 0.0946673, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5689, 74, 5694, 4182, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2cd^2}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2cd^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^2,x]
```

```
[Out] -b/(2*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*d^2
*(1 - c^2*x^2)) + ((a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c*d^2) +
(b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c*d^2) - (b*PolyLog[2, E^ArcCosh[c*x]])/
(2*c*d^2)
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{(bc)\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx}{2d}$$

$$= -\frac{b}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} - \frac{\text{Subst}\left(\int (a+bx)\,\text{csch}(x)\,dx, x, \cosh^{-1}(cx)\right)}{2cd^2}$$

$$= -\frac{b}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd^2} + \frac{b\,\text{S}}{}$$

$$= -\frac{b}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd^2} + \frac{b\,\text{S}}{}$$

$$= -\frac{b}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd^2} + \frac{b\text{L}}{}$$

Mathematica [A] time = 1.34477, size = 189, normalized size = 1.58

$$\frac{2b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right) - 2b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right) + \frac{ac^2x^2\log(cx+1)+\left(a-ac^2x^2\right)\log(1-cx)-2acx-a\log(cx+1)-2b\cosh^{-1}(cx)\left(\left(c^2x^2-1\right.\right.}{c^2x^2}}{4cd^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^2,x]
```

```
[Out] ((-2*a*c*x - 2*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 2*b*c*x*Sqrt[(-1 + c*x)/(1 +
c*x)] - 2*b*ArcCosh[c*x]*(c*x + (-1 + c^2*x^2)*Log[1 - E^ArcCosh[c*x]] + (1
 - c^2*x^2)*Log[1 + E^ArcCosh[c*x]]) + (a - a*c^2*x^2)*Log[1 - c*x] - a*Log
[1 + c*x] + a*c^2*x^2*Log[1 + c*x])/(-1 + c^2*x^2) + 2*b*PolyLog[2, -E^ArcC
osh[c*x]] - 2*b*PolyLog[2, E^ArcCosh[c*x]])/(4*c*d^2)
```

Maple [A] time = 0.062, size = 252, normalized size = 2.1

$$-\frac{a}{4\,cd^2\,(cx-1)} - \frac{a\ln(cx-1)}{4\,cd^2} - \frac{a}{4\,cd^2\,(cx+1)} + \frac{a\ln(cx+1)}{4\,cd^2} - \frac{b\text{arccosh}\,(cx)\,x}{2\,d^2\left(c^2x^2-1\right)} - \frac{b}{2\,cd^2\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1} + \frac{ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/4/c*a/d^2/(c*x-1)-1/4/c*a/d^2*ln(c*x-1)-1/4/c*a/d^2/(c*x+1)+1/4/c*a/d^2*
ln(c*x+1)-1/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*x-1/2/c*b/d^2/(c^2*x^2-1)*(c*x
+1)^(1/2)*(c*x-1)^(1/2)+1/2/c*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))+1/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2-1/2/c*b
/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/2*b*polylog(2,c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{64}\left(192\,c^{3}\int\frac{x^{3}\log\left(cx-1\right)}{8\left(c^{4}d^{2}x^{4}-2\,c^{2}d^{2}x^{2}+d^{2}\right)}\,dx-8\,c^{2}\left(\frac{2\,x}{c^{4}d^{2}x^{2}-c^{2}d^{2}}+\frac{\log\left(cx+1\right)}{c^{3}d^{2}}-\frac{\log\left(cx-1\right)}{c^{3}d^{2}}\right)-64\,c^{2}\int\frac{x^{2}\log\left(\right.}{8\left(c^{4}d^{2}x^{4}-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(192*c^3*integrate(1/8*x^3*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 +
 d^2), x) - 8*c^2*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - l
og(c*x - 1)/(c^3*d^2)) - 64*c^2*integrate(1/8*x^2*log(c*x - 1)/(c^4*d^2*x^4
 - 2*c^2*d^2*x^2 + d^2), x) + 3*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x + 1)/
(c^3*d^2) + log(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^2*d^2
))*c - 4*((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)*log(c
*x - 1) + 4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(c*x - 1
))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^3*d^2*x^2 - c*d^2) + 64*integ
rate(-1/4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(c*x - 1))
/(c^5*d^2*x^5 - 2*c^3*d^2*x^3 + c*d^2*x + (c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^
2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + 64*integrate(1/8*log(c*x - 1)/(c^4*d^
2*x^4 - 2*c^2*d^2*x^2 + d^2), x))*b - 1/4*a*(2*x/(c^2*d^2*x^2 - d^2) - log(
c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\left(cx\right)+a}{c^{4}d^{2}x^{4}-2\,c^{2}d^{2}x^{2}+d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{b\,\mathrm{acosh}\,(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*acosh(c*x)/(c**4
*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\operatorname{arcosh}\,(cx) + a}{\left(c^2dx^2 - d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(c^2*d*x^2 - d)^2, x)
```

3.42 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=116

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d^2}+\frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx\right.}{d^2}$$

```
[Out] -(b*c*x)/(2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*d^2
*(1 - c^2*x^2)) + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d^2
+ (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^2) - (b*PolyLog[2, E^(2*ArcCosh[
c*x])])/(2*d^2)
```

Rubi [A] time = 0.179596, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5754, 5721, 5461, 4182, 2279, 2391, 39}

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d^2}+\frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^2),x]
```



```
[Out] -(b*c*x)/(2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*d^2
*(1 - c^2*x^2)) + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d^2
+ (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^2) - (b*PolyLog[2, E^(2*ArcCosh[
c*x])])/(2*d^2)
```


Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
IntegerQ[p]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx = \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} + \frac{(bc)\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)}\,dx}{d}$$

$$= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} - \frac{\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\mathbf{sech}(x)\,dx, x, \cosh^{-1}(\right.}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} - \frac{2\,\text{Subst}\left(\int (a+bx)\mathbf{csch}(2x)\,dx, x, \cosh^{-1}(cx)\right)}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} + \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d^2} + \frac{b\,\text{S}}{}$$

$$= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} + \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d^2} + \frac{b\,\text{S}}{}$$

$$= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} + \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d^2} + \frac{b\text{L}}{}$$

Mathematica [A] time = 0.747899, size = 149, normalized size = 1.28

$$\frac{b\left(-\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right) + \text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right) + \frac{\cosh^{-1}(cx)}{1-c^2x^2} + \frac{cx\sqrt{\frac{cx-1}{cx+1}}}{1-cx} - 2\cosh^{-1}(cx)\log\left(1-e^{-2\cosh^{-1}(cx)}\right)\right.}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^2),x]
```

```
[Out] (a/(1 - c^2*x^2) + 2*a*Log[x] - a*Log[1 - c^2*x^2] + b*((c*x*Sqrt[(-1 + c*x
)/(1 + c*x)])/(1 - c*x) + ArcCosh[c*x]/(1 - c^2*x^2) - 2*ArcCosh[c*x]*Log[1
 - E^(-2*ArcCosh[c*x])] + 2*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - Pol
yLog[2, -E^(-2*ArcCosh[c*x])] + PolyLog[2, E^(-2*ArcCosh[c*x])]))/(2*d^2)
```

Maple [B] time = 0.094, size = 339, normalized size = 2.9

$$-\frac{a}{4\,d^2\,(cx-1)} - \frac{a\ln(cx-1)}{2\,d^2} + \frac{a\ln(cx)}{d^2} + \frac{a}{4\,d^2\,(cx+1)} - \frac{a\ln(cx+1)}{2\,d^2} - \frac{xbc}{2\,d^2\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1} + \frac{x^2bc^2}{2\,d^2\left(c^2x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/4*a/d^2/(c*x-1)-1/2*a/d^2*ln(c*x-1)+a/d^2*ln(c*x)+1/4*a/d^2/(c*x+1)-1/2*
a/d^2*ln(c*x+1)-1/2*b/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x+1/2*b
/d^2/(c^2*x^2-1)*c^2*x^2-1/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)-1/2*b/d^2/(c^2*
x^2-1)+b/d^2*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)+1/2*b*p
olylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-b/d^2*arccosh(c*x)*ln(1+
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))-b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d^2*poly
log(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{1}{c^2d^2x^2-d^2}+\frac{\log\left(cx+1\right)}{d^2}+\frac{\log\left(cx-1\right)}{d^2}-\frac{2\,\log\left(x\right)}{d^2}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{c^4d^2x^5-2\,c^2d^2x^3+d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(1/(c^2*d^2*x^2 - d^2) + log(c*x + 1)/d^2 + log(c*x - 1)/d^2 - 2*log
(x)/d^2) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^4*d^2*x^5
- 2*c^2*d^2*x^3 + d^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arcosh}\left(cx\right)+a}{c^4d^2x^5-2\,c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^4x^5-2c^2x^3+x}\,dx + \int \frac{b\,\mathrm{acosh}\,(cx)}{c^4x^5-2c^2x^3+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**5 - 2*c**2*x**3 + x), x) + Integral(b*acosh(c*x)/(c**4
*x**5 - 2*c**2*x**3 + x), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcosh}\,(cx) + a}{\left(c^2dx^2 - d\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x), x)
```

3.43 $\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=170

$$\frac{3bc\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2d^2}-\frac{3bc\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2d^2}+\frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)}+\frac{3c\tanh^{-1}\left(e\right.}{}$$

```
[Out] -(b*c)/(2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(d^2*x*(
1 - c^2*x^2)) + (3*c^2*x*(a + b*ArcCosh[c*x]))/(2*d^2*(1 - c^2*x^2)) + (b*c
*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 + (3*c*(a + b*ArcCosh[c*x])*ArcT
anh[E^ArcCosh[c*x]])/d^2 + (3*b*c*PolyLog[2, -E^ArcCosh[c*x]])/(2*d^2) - (3
*b*c*PolyLog[2, E^ArcCosh[c*x]])/(2*d^2)
```

Rubi [A] time = 0.184093, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5746, 104, 21, 92, 205, 5689, 74, 5694, 4182, 2279, 2391}

$$\frac{3bc\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2d^2}-\frac{3bc\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2d^2}+\frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)}+\frac{3c\tanh^{-1}\left(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^2),x]
```

```
[Out] -(b*c)/(2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(d^2*x*(
1 - c^2*x^2)) + (3*c^2*x*(a + b*ArcCosh[c*x]))/(2*d^2*(1 - c^2*x^2)) + (b*c
*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 + (3*c*(a + b*ArcCosh[c*x])*ArcT
anh[E^ArcCosh[c*x]])/d^2 + (3*b*c*PolyLog[2, -E^ArcCosh[c*x]])/(2*d^2) - (3
*b*c*PolyLog[2, E^ArcCosh[c*x]])/(2*d^2)
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
```

```
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \left(3c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx - \frac{(bc)\int \frac{1}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d^2}$$

$$= \frac{bc}{d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{b\int \frac{c+c^2x}{x\sqrt{-1+cx}(1+cx)^{3/2}}\,dx}{d^2} +$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} - \frac{(3c)\,\mathrm{Subst}\left(\int (a+bx\right.}{}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{3c\left(a+b\cosh^{-1}(cx)\right.}{d}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\right.}{d^2}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\right.}{d^2}$$

Mathematica [A] time = 0.744171, size = 283, normalized size = 1.66

$$\frac{6bc\,\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right) - 6bc\,\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right) - \frac{2ac^2x}{c^2x^2-1} - 3ac\log(1-cx) + 3ac\log(cx+1) - \frac{4a}{x} + \frac{4bc\sqrt{c^2x^2-1}}{\sqrt{cx}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^2),x]
```

```
[Out] ((-4*a)/x + b*c*Sqrt[(-1 + c*x)/(1 + c*x)] + (b*c*Sqrt[(-1 + c*x)/(1 + c*x)
])/(1 - c*x) + (b*c^2*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a*c^2*x)
/(-1 + c^2*x^2) - (4*b*ArcCosh[c*x])/x + (b*c*ArcCosh[c*x])/(1 - c*x) - (b*
c*ArcCosh[c*x])/(1 + c*x) + (4*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*
x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - 6*b*c*ArcCosh[c*x]*Log[1 - E^ArcCos
h[c*x]] + 6*b*c*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] - 3*a*c*Log[1 - c*x] +
 3*a*c*Log[1 + c*x] + 6*b*c*PolyLog[2, -E^ArcCosh[c*x]] - 6*b*c*PolyLog[2,
E^ArcCosh[c*x]])/(4*d^2)
```

Maple [A] time = 0.139, size = 259, normalized size = 1.5

$$-\frac{ca}{4\,d^2\,(cx-1)}-\frac{3\,ca\ln(cx-1)}{4\,d^2}-\frac{a}{d^2x}-\frac{ca}{4\,d^2\,(cx+1)}+\frac{3\,ca\ln(cx+1)}{4\,d^2}-\frac{3\,b\operatorname{arccosh}(cx)\,c^2x}{2\,d^2\left(c^2x^2-1\right)}-\frac{bc}{2\,d^2\left(c^2x^2-1\right)}\sqrt{cx-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/4*c*a/d^2/(c*x-1)-3/4*c*a/d^2*ln(c*x-1)-a/d^2/x-1/4*c*a/d^2/(c*x+1)+3/4*
c*a/d^2*ln(c*x+1)-3/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*c^2*x-1/2*c*b/d^2/(c^2
*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+b/d^2/x/(c^2*x^2-1)*arccosh(c*x)+2*c*b/
d^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*c*b/d^2*dilog(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))+3/2*c*b/d^2*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/
2*c*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(576*c^5*integrate(1/8*x^3*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 +
 d^2), x) - 24*c^4*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) -
log(c*x - 1)/(c^3*d^2)) - 192*c^4*integrate(1/8*x^2*log(c*x - 1)/(c^4*d^2*x
^4 - 2*c^2*d^2*x^2 + d^2), x) + 9*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x + 1
)/(c^3*d^2) + log(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^2*d
^2))*c^3 + 16*c^2*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x
 - 1)/(c*d^2)) + 192*c^2*integrate(1/8*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^
2*x^2 + d^2), x) - 4*(3*(c^3*x^3 - c*x)*log(c*x + 1)^2 + 6*(c^3*x^3 - c*x)*
log(c*x + 1)*log(c*x - 1) + 4*(6*c^2*x^2 - 3*(c^3*x^3 - c*x)*log(c*x + 1) +
 3*(c^3*x^3 - c*x)*log(c*x - 1) - 4)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
)/(c^2*d^2*x^3 - d^2*x) + 64*integrate(-1/4*(6*c^3*x^2 - 3*(c^4*x^3 - c^2*x
)*log(c*x + 1) + 3*(c^4*x^3 - c^2*x)*log(c*x - 1) - 4*c)/(c^5*d^2*x^6 - 2*c
^3*d^2*x^4 + c*d^2*x^2 + (c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)*sqrt(c*x + 1
)*sqrt(c*x - 1)), x))*b - 1/4*a*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) -
3*c*log(c*x + 1)/d^2 + 3*c*log(c*x - 1)/d^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{c^4d^2x^6-2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^4x^6-2c^2x^4+x^2}\,dx+\int \frac{b\,\text{acosh}\,(cx)}{c^4x^6-2c^2x^4+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**6 - 2*c**2*x**4 + x**2), x) + Integral(b*acosh(c*x)/(c
**4*x**6 - 2*c**2*x**4 + x**2), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^2), x)
```

3.44 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=152

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{d^2}-\frac{bc^2\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{d^2}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2\tanh^{-1}\left(\right.}{}$$

```
[Out] -(b*c)/(2*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*(a + b*ArcCosh[c*x]))/
(d^2*(1 - c^2*x^2)) - (a + b*ArcCosh[c*x])/(2*d^2*x^2*(1 - c^2*x^2)) + (4*c
^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d^2 + (b*c^2*PolyLog[2
, -E^(2*ArcCosh[c*x])])/d^2 - (b*c^2*PolyLog[2, E^(2*ArcCosh[c*x])])/d^2
```

Rubi [A] time = 0.261622, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5746, 103, 12, 39, 5754, 5721, 5461, 4182, 2279, 2391}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{d^2}-\frac{bc^2\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{d^2}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2\tanh^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```

```
[Out] -(b*c)/(2*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*(a + b*ArcCosh[c*x]))/
(d^2*(1 - c^2*x^2)) - (a + b*ArcCosh[c*x])/(2*d^2*x^2*(1 - c^2*x^2)) + (4*c
^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d^2 + (b*c^2*PolyLog[2
, -E^(2*ArcCosh[c*x])])/d^2 - (b*c^2*PolyLog[2, E^(2*ArcCosh[c*x])])/d^2
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
IntegerQ[p]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\left(2c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx-\frac{(bc)\int \frac{1}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{(bc)\int \frac{2c^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,d}{2d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^3x}{d^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{\left(4c^2\right)\text{Subst}\left(\int(a+bx)\right.}{}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)\text{t}}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)\text{t}}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)\text{t}}{d^2}$$

Mathematica [B] time = 0.561113, size = 319, normalized size = 2.1

$$\frac{-2bc^2x^2\left(c^2x^2-1\right)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+2bc^2x^2\left(c^2x^2-1\right)\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)-2ac^2x^2+4ac^4x^4\log(x)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```

```
[Out] (a - 2*a*c^2*x^2 - b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - b*c^2*x^2*Sqrt[(-1 +
c*x)/(1 + c*x)] + b*ArcCosh[c*x] - 2*b*c^2*x^2*ArcCosh[c*x] + 4*b*c^2*x^2*A
rcCosh[c*x]*Log[1 - E^(-2*ArcCosh[c*x])] - 4*b*c^4*x^4*ArcCosh[c*x]*Log[1 -
 E^(-2*ArcCosh[c*x])] - 4*b*c^2*x^2*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x]
)] + 4*b*c^4*x^4*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - 4*a*c^2*x^2*Lo
g[x] + 4*a*c^4*x^4*Log[x] + 2*a*c^2*x^2*Log[1 - c^2*x^2] - 2*a*c^4*x^4*Log[
1 - c^2*x^2] - 2*b*c^2*x^2*(-1 + c^2*x^2)*PolyLog[2, -E^(-2*ArcCosh[c*x])]
+ 2*b*c^2*x^2*(-1 + c^2*x^2)*PolyLog[2, E^(-2*ArcCosh[c*x])])/(2*d^2*x^2*(-
1 + c^2*x^2))
```

Maple [A] time = 0.112, size = 371, normalized size = 2.4

$$-\frac{c^2a}{4\,d^2\,(cx-1)}-\frac{c^2a\ln(cx-1)}{d^2}-\frac{a}{2\,d^2x^2}+2\,\frac{c^2a\ln(cx)}{d^2}+\frac{c^2a}{4\,d^2\,(cx+1)}-\frac{c^2a\ln(cx+1)}{d^2}-\frac{c^2b\text{arccosh}\,(cx)}{d^2\left(c^2x^2-1\right)}-\frac{bc}{2\,d^2x\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/4*c^2*a/d^2/(c*x-1)-c^2*a/d^2*ln(c*x-1)-1/2*a/d^2/x^2+2*c^2*a/d^2*ln(c*x
)+1/4*c^2*a/d^2/(c*x+1)-c^2*a/d^2*ln(c*x+1)-c^2*b/d^2/(c^2*x^2-1)*arccosh(c
*x)-1/2*c*b/d^2/x/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+1/2*b/d^2/x^2/(c^
2*x^2-1)*arccosh(c*x)+2*c^2*b/d^2*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))^2+1)+b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-2*c
^2*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*c^2*b/d^2*pol
ylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*c^2*b/d^2*arccosh(c*x)*ln(1-c*x-
(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*c^2*b/d^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{2\,c^2\log\left(cx+1\right)}{d^2}+\frac{2\,c^2\log\left(cx-1\right)}{d^2}-\frac{4\,c^2\log\left(x\right)}{d^2}+\frac{2\,c^2x^2-1}{c^2d^2x^4-d^2x^2}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{c^4d^2x^7-2\,c^2d^2x^5+d^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*c^2*log(c*x + 1)/d^2 + 2*c^2*log(c*x - 1)/d^2 - 4*c^2*log(x)/d^2
+ (2*c^2*x^2 - 1)/(c^2*d^2*x^4 - d^2*x^2)) + b*integrate(log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\left(cx\right)+a}{c^4d^2x^7-2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{c^4x^7-2c^2x^5+x^3}\,dx+\int\frac{b\,\text{acosh}\left(cx\right)}{c^4x^7-2c^2x^5+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**7 - 2*c**2*x**5 + x**3), x) + Integral(b*acosh(c*x)/(c
**4*x**7 - 2*c**2*x**5 + x**3), x))/d**2
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(c^2dx^2 - d\right)^2 x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^3), x)
```

3.45 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=248

$$\frac{5bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2d^2}-\frac{5bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2d^2}+\frac{5c^4x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^2x\left(1-c^2x^2\right)}-\frac{a+b}{3d^2x}$$

```
[Out] -(b*c^3)/(3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c)/(6*d^2*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(3*d^2*x^3*(1 - c^2*x^2)) - (5*c
^2*(a + b*ArcCosh[c*x]))/(3*d^2*x*(1 - c^2*x^2)) + (5*c^4*x*(a + b*ArcCosh[
c*x]))/(2*d^2*(1 - c^2*x^2)) + (13*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]])/(6*d^2) + (5*c^3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d^2 + (5
*b*c^3*PolyLog[2, -E^ArcCosh[c*x]])/(2*d^2) - (5*b*c^3*PolyLog[2, E^ArcCosh
[c*x]])/(2*d^2)
```

Rubi [A] time = 0.290807, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {5746, 103, 12, 104, 21, 92, 205, 5689, 74, 5694, 4182, 2279, 2391}

$$\frac{5bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2d^2}-\frac{5bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2d^2}+\frac{5c^4x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^2x\left(1-c^2x^2\right)}-\frac{a+b}{3d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```


```
[Out] -(b*c^3)/(3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c)/(6*d^2*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(3*d^2*x^3*(1 - c^2*x^2)) - (5*c
^2*(a + b*ArcCosh[c*x]))/(3*d^2*x*(1 - c^2*x^2)) + (5*c^4*x*(a + b*ArcCosh[
c*x]))/(2*d^2*(1 - c^2*x^2)) + (13*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]])/(6*d^2) + (5*c^3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d^2 + (5
*b*c^3*PolyLog[2, -E^ArcCosh[c*x]])/(2*d^2) - (5*b*c^3*PolyLog[2, E^ArcCosh
[c*x]])/(2*d^2)
```


Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}+\frac{1}{3}\left(5c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx-\frac{(bc)\int \frac{1}{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{3d^2}$$

$$= -\frac{bc}{6d^2x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^2x\left(1-c^2x^2\right)}+\left(5c^4\right)\int \frac{a+b\cosh^{-1}}{\left(d-c^2dx\right.}$$

$$= \frac{5bc^3}{3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^2x\left(1-c^2x^2\right)}$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right.}{3d^2x\left(1-c^2x^2\right)}$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right.}{3d^2x\left(1-c^2x^2\right)}$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right.}{3d^2x\left(1-c^2x^2\right)}$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right.}{3d^2x\left(1-c^2x^2\right)}$$

Mathematica [A] time = 1.66823, size = 377, normalized size = 1.52

$$-\frac{-30bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)+30bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)+\frac{6ac^4x}{c^2x^2-1}+\frac{24ac^2}{x}+15ac^3\log(1-cx)-15ac^3\log(cx+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```

```
[Out] -((4*a)/x^3 + (24*a*c^2)/x - 3*b*c^3*Sqrt[(-1 + c*x)/(1 + c*x)] + (3*b*c^3*
Sqrt[(-1 + c*x)/(1 + c*x)])/(-1 + c*x) + (3*b*c^4*x*Sqrt[(-1 + c*x)/(1 + c*
```

```
x)])/(-1 + c*x) - (2*b*c^3)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c)/(x^2*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*a*c^4*x)/(-1 + c^2*x^2) + (4*b*ArcCosh[c*
x])/x^3 + (24*b*c^2*ArcCosh[c*x])/x + (3*b*c^3*ArcCosh[c*x])/(-1 + c*x) + (
3*b*c^3*ArcCosh[c*x])/(1 + c*x) - (26*b*c^3*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[
-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + 30*b*c^3*ArcCosh[c*x]*Log[
1 - E^ArcCosh[c*x]] - 30*b*c^3*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + 15*a*
c^3*Log[1 - c*x] - 15*a*c^3*Log[1 + c*x] - 30*b*c^3*PolyLog[2, -E^ArcCosh[c
*x]] + 30*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/(12*d^2)
```

Maple [A] time = 0.188, size = 352, normalized size = 1.4

$$-\frac{c^3 a}{4 d^2 (cx-1)} - \frac{5 c^3 a \ln(cx-1)}{4 d^2} - \frac{a}{3 d^2 x^3} - 2\frac{c^2 a}{d^2 x} - \frac{c^3 a}{4 d^2 (cx+1)} + \frac{5 c^3 a \ln(cx+1)}{4 d^2} - \frac{5 c^4 b \operatorname{arccosh}(cx)\, x}{2 d^2 \left(c^2x^2-1\right)} - \frac{bc^3}{3 d^2 \left(c^2x^2 - \cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/4*c^3*a/d^2/(c*x-1)-5/4*c^3*a/d^2*ln(c*x-1)-1/3*a/d^2/x^3-2*c^2*a/d^2/x-
1/4*c^3*a/d^2/(c*x+1)+5/4*c^3*a/d^2*ln(c*x+1)-5/2*c^4*b/d^2/(c^2*x^2-1)*arc
cosh(c*x)*x-1/3*c^3*b/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+5/3*c^2*b
/d^2/x/(c^2*x^2-1)*arccosh(c*x)-1/6*c*b/d^2/x^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(
c*x-1)^(1/2)+1/3*b/d^2/x^3/(c^2*x^2-1)*arccosh(c*x)+13/3*c^3*b/d^2*arctan(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/2*c^3*b/d^2*dilog(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))+5/2*c^3*b/d^2*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/2*c^3*b/
d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(15*c^3*log(c*x + 1)/d^2 - 15*c^3*log(c*x - 1)/d^2 - 2*(15*c^4*x^4 - 1
0*c^2*x^2 - 2)/(c^2*d^2*x^5 - d^2*x^3))*a + 1/192*(8640*c^7*integrate(1/24*
x^5*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x) - 120*c^6*(2*x
/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3*d^2))
```

```
 - 2880*c^6*integrate(1/24*x^4*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 +
d^2*x^2), x) + 45*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x + 1)/(c^3*d^2) + lo
g(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^2*d^2))*c^5 + 80*c^
4*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d^2)) +
 2880*c^4*integrate(1/24*x^2*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^
2*x^2), x) + 16*c^2*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) - 3*c*log(c*x
+ 1)/d^2 + 3*c*log(c*x - 1)/d^2) - 4*(15*(c^5*x^5 - c^3*x^3)*log(c*x + 1)^2
 + 30*(c^5*x^5 - c^3*x^3)*log(c*x + 1)*log(c*x - 1) + 4*(30*c^4*x^4 - 20*c^
2*x^2 - 15*(c^5*x^5 - c^3*x^3)*log(c*x + 1) + 15*(c^5*x^5 - c^3*x^3)*log(c*
x - 1) - 4)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*d^2*x^5 - d^2*x^3)
 + 192*integrate(-1/12*(30*c^5*x^4 - 20*c^3*x^2 - 15*(c^6*x^5 - c^4*x^3)*lo
g(c*x + 1) + 15*(c^6*x^5 - c^4*x^3)*log(c*x - 1) - 4*c)/(c^5*d^2*x^8 - 2*c^
3*d^2*x^6 + c*d^2*x^4 + (c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3)*sqrt(c*x +
1)*sqrt(c*x - 1)), x))*b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx) + a}{c^4d^2x^8 - 2\,c^2d^2x^6 + d^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^4x^8-2c^2x^6+x^4}\,dx + \int \frac{b\,\text{acosh}\,(cx)}{c^4x^8-2c^2x^6+x^4}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**8 - 2*c**2*x**6 + x**4), x) + Integral(b*acosh(c*x)/(c
**4*x**8 - 2*c**2*x**6 + x**4), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(c^2 d x^2 - d\right)^2 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^4), x)
```

3.46 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=249

$$\frac{3b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8c^5d^3}-\frac{3b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8c^5d^3}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8c^4d^3\left(1-c^2x^2\right)}+\frac{3\tanh^{-1}\left(\right.}{}$$

```
[Out] (b*x^3)/(12*c^2*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(4*c^5*d^3*Sqrt[-
1 + c*x]*(1 + c*x)^(3/2)) - (b*(-1 + c*x)^(3/2))/(12*c^5*d^3*(1 + c*x)^(3/2
)) + (3*b)/(8*c^5*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^3*(a + b*ArcCosh[c
*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) - (3*x*(a + b*ArcCosh[c*x]))/(8*c^4*d^3*(
1 - c^2*x^2)) + (3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*c^5*d^3
) + (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c^5*d^3) - (3*b*PolyLog[2, E^ArcCo
sh[c*x]])/(8*c^5*d^3)
```

Rubi [A] time = 0.23922, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5750, 94, 89, 21, 37, 74, 5694, 4182, 2279, 2391}

$$\frac{3b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8c^5d^3}-\frac{3b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8c^5d^3}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8c^4d^3\left(1-c^2x^2\right)}+\frac{3\tanh^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```


```
[Out] (b*x^3)/(12*c^2*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(4*c^5*d^3*Sqrt[-
1 + c*x]*(1 + c*x)^(3/2)) - (b*(-1 + c*x)^(3/2))/(12*c^5*d^3*(1 + c*x)^(3/2
)) + (3*b)/(8*c^5*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^3*(a + b*ArcCosh[c
*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) - (3*x*(a + b*ArcCosh[c*x]))/(8*c^4*d^3*(
1 - c^2*x^2)) + (3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*c^5*d^3
) + (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c^5*d^3) - (3*b*PolyLog[2, E^ArcCo
sh[c*x]])/(8*c^5*d^3)
```


Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
```

```
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{x^3 \left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{b\int \frac{x^3}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4cd^3} - \frac{3\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8c^4d^3\left(1-c^2x^2\right)} - \frac{(3b)\int \frac{}{(-1}$$

$$= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^5d^3\sqrt{-1+cx}(1+cx)^{3/2}} + \frac{3b}{8c^5d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x^3(}{4}$$

$$= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^5d^3\sqrt{-1+cx}(1+cx)^{3/2}} + \frac{3b}{8c^5d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x^3(}{4}$$

$$= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^5d^3\sqrt{-1+cx}(1+cx)^{3/2}} - \frac{b(-1+cx)^{3/2}}{12c^5d^3(1+cx)^{3/2}} + \frac{}{8c^5d^3\sqrt{-1}}$$

$$= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^5d^3\sqrt{-1+cx}(1+cx)^{3/2}} - \frac{b(-1+cx)^{3/2}}{12c^5d^3(1+cx)^{3/2}} + \frac{}{8c^5d^3\sqrt{-1}}$$

Mathematica [A] time = 1.81093, size = 287, normalized size = 1.15

$$\frac{18b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right) - 18b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right) + \frac{30acx}{c^2x^2-1} + \frac{12acx}{\left(c^2x^2-1\right)^2} - 9a\log(1-cx) + 9a\log(cx+1) + \frac{b\sqrt{cx-1}(}{(cx+1}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (-((b*(-2 + c*x)*Sqrt[1 + c*x])/(-1 + c*x)^(3/2)) + (b*Sqrt[-1 + c*x]*(2 +
c*x))/(1 + c*x)^(3/2) + (12*a*c*x)/(-1 + c^2*x^2)^2 + (30*a*c*x)/(-1 + c^2*
x^2) + (3*b*ArcCosh[c*x])/(-1 + c*x)^2 - (3*b*ArcCosh[c*x])/(1 + c*x)^2 - 1
5*b*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)) - 15*b*(Sqrt
[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)) + (9*b*ArcCosh[c*x]*(ArcCo
sh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]))/2 - (9*b*ArcCosh[c*x]*(ArcCosh[c*x] -
 4*Log[1 + E^ArcCosh[c*x]]))/2 - 9*a*Log[1 - c*x] + 9*a*Log[1 + c*x] + 18*b
*PolyLog[2, -E^ArcCosh[c*x]] - 18*b*PolyLog[2, E^ArcCosh[c*x]])/(48*c^5*d^3
)
```

Maple [A] time = 0.425, size = 383, normalized size = 1.5

$$\frac{a}{16\,c^5d^3\,(cx-1)^2}+\frac{5\,a}{16\,c^5d^3\,(cx-1)}-\frac{3\,a\ln(cx-1)}{16\,c^5d^3}-\frac{a}{16\,c^5d^3\,(cx+1)^2}+\frac{5\,a}{16\,c^5d^3\,(cx+1)}+\frac{3\,a\ln(cx+1)}{16\,c^5d^3}+\frac{5\,b\mathrm{arc}}{8\,c^2d^3\,(c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/16/c^5*a/d^3/(c*x-1)^2+5/16/c^5*a/d^3/(c*x-1)-3/16/c^5*a/d^3*ln(c*x-1)-1/
16/c^5*a/d^3/(c*x+1)^2+5/16/c^5*a/d^3/(c*x+1)+3/16/c^5*a/d^3*ln(c*x+1)+5/8/
c^2*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x^3+5/8/c^3*b/d^3/(c^4*x^4-2*c
^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-3/8/c^4*b/d^3/(c^4*x^4-2*c^2*x^2+
1)*arccosh(c*x)*x-13/24/c^5*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-
1)^(1/2)+3/8/c^5*b/d^3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3
/8*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^3-3/8/c^5*b/d^3*arcc
osh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-3/8*b*polylog(2,c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))/c^5/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(18432*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^10*d^3*x^6 - 3*c^8*d^3
*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 80*c^4*(2*(5*c^2*x^3 - 3*x)/(c^12*d^3
*x^4 - 2*c^10*d^3*x^2 + c^8*d^3) + 3*log(c*x + 1)/(c^9*d^3) - 3*log(c*x - 1
)/(c^9*d^3)) - 6144*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^10*d^3*x^6 - 3*c
^8*d^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 18*(c*(2*(5*c^2*x^2 + 3*c*x - 6
)/(c^12*d^3*x^3 - c^11*d^3*x^2 - c^10*d^3*x + c^9*d^3) - 5*log(c*x + 1)/(c^
9*d^3) + 5*log(c*x - 1)/(c^9*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^12*
d^3*x^4 - 2*c^10*d^3*x^2 + c^8*d^3))*c^3 - 48*c^2*(2*(c^2*x^3 + x)/(c^10*d^
3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3) - log(c*x + 1)/(c^7*d^3) + log(c*x - 1)/(c
^7*d^3)) + 12288*c^2*integrate(1/32*x^2*log(c*x - 1)/(c^10*d^3*x^6 - 3*c^8*
d^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 9*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c
^10*d^3*x^3 - c^9*d^3*x^2 - c^8*d^3*x + c^7*d^3) - 3*log(c*x + 1)/(c^7*d^3)
 + 3*log(c*x - 1)/(c^7*d^3)) - 16*log(c*x - 1)/(c^10*d^3*x^4 - 2*c^8*d^3*x^
```

```
2 + c^6*d^3))*c - 32*(3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 6*(c^4*x
^4 - 2*c^2*x^2 + 1)*log(c*x + 1)*log(c*x - 1) - 4*(10*c^3*x^3 - 6*c*x + 3*(
c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x
 - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^9*d^3*x^4 - 2*c^7*d^3*x^2
 + c^5*d^3) + 2048*integrate(1/16*(10*c^3*x^3 - 6*c*x + 3*(c^4*x^4 - 2*c^2*
x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^11*d^3
*x^7 - 3*c^9*d^3*x^5 + 3*c^7*d^3*x^3 - c^5*d^3*x + (c^10*d^3*x^6 - 3*c^8*d^
3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - 6144*in
tegrate(1/32*log(c*x - 1)/(c^10*d^3*x^6 - 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 - c
^4*d^3), x))*b + 1/16*a*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 +
 c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5*d^3))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{bx^4 \operatorname{arcosh}(cx) + ax^4}{c^6d^3x^6 - 3\,c^4d^3x^4 + 3\,c^2d^3x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 - d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax^4}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{bx^4 \operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x**4/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integra
l(b*x**4*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(b \operatorname{arcosh}(cx) + a)x^4}{\left(c^2dx^2 - d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x^4/(c^2*d*x^2 - d)^3, x)
```

3.47 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=136

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{b\sqrt{cx-1}}{4c^4d^3\sqrt{cx+1}}+\frac{b}{4c^4d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\cosh^{-1}(cx)}{4c^4d^3}+\frac{bx^3}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

```
[Out] (b*x^3)/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(4*c^4*d^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (b*Sqrt[-1 + c*x])/(4*c^4*d^3*Sqrt[1 + c*x]) - (b*A
rcCosh[c*x])/(4*c^4*d^3) + (x^4*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^
2)
```

Rubi [A] time = 0.104914, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5722, 98, 21, 89, 12, 78, 52}

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{b\sqrt{cx-1}}{4c^4d^3\sqrt{cx+1}}+\frac{b}{4c^4d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\cosh^{-1}(cx)}{4c^4d^3}+\frac{bx^3}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```


```
[Out] (b*x^3)/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(4*c^4*d^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (b*Sqrt[-1 + c*x])/(4*c^4*d^3*Sqrt[1 + c*x]) - (b*A
rcCosh[c*x])/(4*c^4*d^3) + (x^4*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^
2)
```


Rule 5722

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d + e,
0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1] && IntegerQ[p]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{(bc)\int \frac{x^4}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{b\int \frac{x^2(-3-3cx)}{(-1+cx)^{3/2}(1+cx)^{5/2}}\,dx}{12cd^3}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{b\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{4cd^3}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^4d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{b\int \frac{}{\sqrt{-1+}}}{4}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^4d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{b\int \frac{}{\sqrt{-1+}}}{4}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^4d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{b\sqrt{-1+cx}}{4c^4d^3\sqrt{1+cx}} + \frac{x^4\left(a+b\cosh^{-1}\right.}{4d^3\left(1-c^2x^2\right.}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^4d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{b\sqrt{-1+cx}}{4c^4d^3\sqrt{1+cx}} - \frac{b\cosh^{-1}(cx)}{4c^4d^3} +$$

Mathematica [A] time = 0.233996, size = 83, normalized size = 0.61

$$\frac{a\left(6c^2x^2-3\right)+bcx\sqrt{cx-1}\sqrt{cx+1}\left(4c^2x^2-3\right)+3b\left(2c^2x^2-1\right)\cosh^{-1}(cx)}{12c^4d^3\left(c^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-3 + 4*c^2*x^2) + a*(-3 + 6*c^2*x^2) +
 3*b*(-1 + 2*c^2*x^2)*ArcCosh[c*x])/(12*c^4*d^3*(-1 + c^2*x^2)^2)
```

Maple [A] time = 0.023, size = 136, normalized size = 1.

$$\frac{1}{c^4}\left(-\frac{a}{d^3}\left(-\frac{1}{16\,(cx-1)^2}-\frac{3}{16\,cx-16}-\frac{1}{16\,(cx+1)^2}+\frac{3}{16\,cx+16}\right)-\frac{b}{d^3}\left(-\frac{\operatorname{arccosh}(cx)}{16\,(cx-1)^2}-\frac{3\operatorname{arccosh}(cx)}{16\,cx-16}-\frac{\operatorname{arccosh}(cx)}{16\,(cx+1)^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/c^4*(-a/d^3*(-1/16/(c*x-1)^2-3/16/(c*x-1)-1/16/(c*x+1)^2+3/16/(c*x+1))-b/
d^3*(-1/16*arccosh(c*x)/(c*x-1)^2-3/16*arccosh(c*x)/(c*x-1)-1/16*arccosh(c*
x)/(c*x+1)^2+3/16*arccosh(c*x)/(c*x+1)-1/12*c*x*(4*c^2*x^2-3)/(c*x+1)^(3/2)
/(c*x-1)^(3/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}b\left(\frac{4\,c^2x^2+4\left(2\,c^2x^2-1\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)-3}{c^8d^3x^4-2\,c^6d^3x^2+c^4d^3}+16\int\frac{2\,c^2}{4\left(c^{10}d^3x^7-3\,c^8d^3x^5+3\,c^6d^3x^3-c^4d^3x+\left(c^9d^3x^6\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b*((4*c^2*x^2 + 4*(2*c^2*x^2 - 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
)) - 3)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 16*integrate(1/4*(2*c^2*x
^2 - 1)/(c^10*d^3*x^7 - 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 - c^4*d^3*x + (c^9*d^
3*x^6 - 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 - c^3*d^3)*e^(1/2*log(c*x + 1) + 1/2*
log(c*x - 1))), x)) + 1/4*(2*c^2*x^2 - 1)*a/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 +
c^4*d^3)
```

Fricas [A] time = 1.9191, size = 209, normalized size = 1.54

$$\frac{3\,ac^4x^4+3\left(2\,bc^2x^2-b\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(4\,bc^3x^3-3\,bcx\right)\sqrt{c^2x^2-1}}{12\left(c^8d^3x^4-2\,c^6d^3x^2+c^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 + 3*(2*b*c^2*x^2 - b)*log(c*x + sqrt(c^2*x^2 - 1)) + (4*b
*c^3*x^3 - 3*b*c*x)*sqrt(c^2*x^2 - 1))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d
^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax^3}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{bx^3\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x**3/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integra
l(b*x**3*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(b\operatorname{arcosh}(cx)+a)x^3}{\left(c^2dx^2-d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x^3/(c^2*d*x^2 - d)^3, x)
```

3.48 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=186

$$-\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8c^3d^3}+\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8c^3d^3}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{8c^2d^3\left(1-c^2x^2\right)}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{\tanh^{-1}\left(e^{\cosh\right.}}{}$$

```
[Out] b/(12*c^3*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(8*c^3*d^3*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) -
(x*(a + b*ArcCosh[c*x]))/(8*c^2*d^3*(1 - c^2*x^2)) - ((a + b*ArcCosh[c*x])*
ArcTanh[E^ArcCosh[c*x]])/(4*c^3*d^3) - (b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c
^3*d^3) + (b*PolyLog[2, E^ArcCosh[c*x]])/(8*c^3*d^3)
```

Rubi [A] time = 0.179358, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5750, 74, 5689, 5694, 4182, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8c^3d^3}+\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8c^3d^3}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{8c^2d^3\left(1-c^2x^2\right)}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{\tanh^{-1}\left(e^{\cosh\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```


```
[Out] b/(12*c^3*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(8*c^3*d^3*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) -
(x*(a + b*ArcCosh[c*x]))/(8*c^2*d^3*(1 - c^2*x^2)) - ((a + b*ArcCosh[c*x])*
ArcTanh[E^ArcCosh[c*x]])/(4*c^3*d^3) - (b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c
^3*d^3) + (b*PolyLog[2, E^ArcCosh[c*x]])/(8*c^3*d^3)
```


Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
```

```
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{b\int \frac{x}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4cd^3} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{8c^2d^3\left(1-c^2x^2\right)} - \frac{b\int \frac{}{(-1+cx)}}{8}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{x\left(a+\right.}{8c^2d}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{x\left(a+\right.}{8c^2d}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{x\left(a+\right.}{8c^2d}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{x\left(a+\right.}{8c^2d}$$

Mathematica [A] time = 1.73316, size = 287, normalized size = 1.54

$$\frac{-6b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)+6b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)+\frac{6acx}{c^2x^2-1}+\frac{12acx}{\left(c^2x^2-1\right)^2}+3a\log(1-cx)-3a\log(cx+1)+\frac{b\sqrt{cx-}}{(cx+}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (-((b*(-2 + c*x)*Sqrt[1 + c*x])/(-1 + c*x)^(3/2)) + (b*Sqrt[-1 + c*x]*(2 +
c*x))/(1 + c*x)^(3/2) + (12*a*c*x)/(-1 + c^2*x^2)^2 + (6*a*c*x)/(-1 + c^2*x
^2) + (3*b*ArcCosh[c*x])/(-1 + c*x)^2 - (3*b*ArcCosh[c*x])/(1 + c*x)^2 - 3*
b*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)) - 3*b*(Sqrt[(-
1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)) - (3*b*ArcCosh[c*x]*(ArcCosh[
c*x] - 4*Log[1 - E^ArcCosh[c*x]]))/2 + (3*b*ArcCosh[c*x]*(ArcCosh[c*x] - 4*
Log[1 + E^ArcCosh[c*x]]))/2 + 3*a*Log[1 - c*x] - 3*a*Log[1 + c*x] - 6*b*Pol
```

```
yLog[2, -E^ArcCosh[c*x]] + 6*b*PolyLog[2, E^ArcCosh[c*x]])/(48*c^3*d^3)
```

Maple [A] time = 0.153, size = 380, normalized size = 2.

$$\frac{a}{16\,c^3d^3\,(cx-1)^2}+\frac{a}{16\,c^3d^3\,(cx-1)}+\frac{a\ln(cx-1)}{16\,c^3d^3}-\frac{a}{16\,c^3d^3\,(cx+1)^2}+\frac{a}{16\,c^3d^3\,(cx+1)}-\frac{a\ln(cx+1)}{16\,c^3d^3}+\frac{b\operatorname{arccosh}(c}{8\,d^3\left(c^4x^4-2\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/16/c^3*a/d^3/(c*x-1)^2+1/16/c^3*a/d^3/(c*x-1)+1/16/c^3*a/d^3*ln(c*x-1)-1/
16/c^3*a/d^3/(c*x+1)^2+1/16/c^3*a/d^3/(c*x+1)-1/16/c^3*a/d^3*ln(c*x+1)+1/8*
b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x^3+1/8/c*b/d^3/(c^4*x^4-2*c^2*x^2
+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/8/c^2*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arc
cosh(c*x)*x-1/24/c^3*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2
)-1/8/c^3*b/d^3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/8*b*po
lylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^3+1/8/c^3*b/d^3*arccosh(c*x
)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/8*b*polylog(2,c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))/c^3/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2048*(6144*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*d^3*
x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x) - 16*c^4*(2*(5*c^2*x^3 - 3*x)/(c^10*d^3*
x^4 - 2*c^8*d^3*x^2 + c^6*d^3) + 3*log(c*x + 1)/(c^7*d^3) - 3*log(c*x - 1)/
(c^7*d^3)) - 2048*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*
d^3*x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x) + 6*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c
^10*d^3*x^3 - c^9*d^3*x^2 - c^8*d^3*x + c^7*d^3) - 5*log(c*x + 1)/(c^7*d^3)
 + 5*log(c*x - 1)/(c^7*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^10*d^3*x^
4 - 2*c^8*d^3*x^2 + c^6*d^3))*c^3 - 16*c^2*(2*(c^2*x^3 + x)/(c^8*d^3*x^4 -
2*c^6*d^3*x^2 + c^4*d^3) - log(c*x + 1)/(c^5*d^3) + log(c*x - 1)/(c^5*d^3))
 + 4096*c^2*integrate(1/32*x^2*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*d^3*x^4 +
```

```
3*c^4*d^3*x^2 - c^2*d^3), x) + 3*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^8*d^3*x^3
 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 3*log(c*x + 1)/(c^5*d^3) + 3*log(c*
x - 1)/(c^5*d^3)) - 16*log(c*x - 1)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)
)*c - 32*((c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 2*(c^4*x^4 - 2*c^2*x^2
 + 1)*log(c*x + 1)*log(c*x - 1) + 4*(2*c^3*x^3 + 2*c*x - (c^4*x^4 - 2*c^2*x
^2 + 1)*log(c*x + 1) + (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1)))/(c^7*d^3*x^4 - 2*c^5*d^3*x^2 + c^3*d^3) + 2048*
integrate(-1/16*(2*c^3*x^3 + 2*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)
 + (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^9*d^3*x^7 - 3*c^7*d^3*x^5 + 3
*c^5*d^3*x^3 - c^3*d^3*x + (c^8*d^3*x^6 - 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 - c
^2*d^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - 2048*integrate(1/32*log(c*x - 1)
/(c^8*d^3*x^6 - 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x))*b + 1/16*a*(2
*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*
d^3) + log(c*x - 1)/(c^3*d^3))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{bx^2\operatorname{arcosh}(cx) + ax^2}{c^6d^3x^6 - 3\,c^4d^3x^4 + 3\,c^2d^3x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2*arccosh(c*x) + a*x^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 - d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{ax^2}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{bx^2\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integra
l(b*x**2*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(b\operatorname{arcosh}(cx)+a)x^2}{\left(c^2dx^2-d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d)^3, x)
```

3.49 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=91

$$\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{bx}{6cd^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

```
[Out] (b*x)/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (b*x)/(6*c*d^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(4*c^2*d^3*(1 - c^2*x^2)^2)
```

Rubi [A] time = 0.0582914, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5716, 40, 39}

$$\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{bx}{6cd^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (b*x)/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (b*x)/(6*c*d^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(4*c^2*d^3*(1 - c^2*x^2)^2)
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx &= \frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\int\frac{1}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4cd^3}\\ &= \frac{bx}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b\int\frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{6cd^3}\\ &= \frac{bx}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{bx}{6cd^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.20381, size = 64, normalized size = 0.7

$$\frac{3a+bcx\sqrt{cx-1}\sqrt{cx+1}\left(3-2c^2x^2\right)+3b\cosh^{-1}(cx)}{12c^2d^3\left(c^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (3*a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - 2*c^2*x^2) + 3*b*ArcCosh[c*x
])/(12*c^2*d^3*(-1 + c^2*x^2)^2)
```

Maple [A] time = 0.015, size = 86, normalized size = 1.

$$\frac{1}{c^2}\left(\frac{a}{4\,d^3\left(c^2x^2-1\right)^2}-\frac{b}{d^3}\left(-\frac{\operatorname{arccosh}(cx)}{4\left(c^2x^2-1\right)^2}+\frac{cx\left(2\,c^2x^2-3\right)}{12\,c^2x^2-12}\frac{1}{\sqrt{cx-1}}\frac{1}{\sqrt{cx+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/c^2*(1/4*a/d^3/(c^2*x^2-1)^2-b/d^3*(-1/4/(c^2*x^2-1)^2*arccosh(c*x)+1/12/
(c*x-1)^(1/2)/(c*x+1)^(1/2)*c*x*(2*c^2*x^2-3)/(c^2*x^2-1)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}b\left(\frac{4\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+1}{c^6d^3x^4-2c^4d^3x^2+c^2d^3}+16\int\frac{1}{4\left(c^8d^3x^7-3c^6d^3x^5+3c^4d^3x^3-c^2d^3x+\left(c^7d^3x^6-3c^5d^3x^4+3c^3d^3x^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b*((4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 1)/(c^6*d^3*x^4 - 2*c^4
*d^3*x^2 + c^2*d^3) + 16*integrate(1/4/(c^8*d^3*x^7 - 3*c^6*d^3*x^5 + 3*c^4
*d^3*x^3 - c^2*d^3*x + (c^7*d^3*x^6 - 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 - c*d^3
)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)) + 1/4*a/(c^6*d^3*x^4 - 2*c^
4*d^3*x^2 + c^2*d^3)
```

Fricas [A] time = 1.86284, size = 208, normalized size = 2.29

$$-\frac{3\,ac^4x^4-6\,ac^2x^2-3\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(2\,bc^3x^3-3\,bcx\right)\sqrt{c^2x^2-1}}{12\left(c^6d^3x^4-2\,c^4d^3x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*c^4*x^4 - 6*a*c^2*x^2 - 3*b*log(c*x + sqrt(c^2*x^2 - 1)) + (2*b*
c^3*x^3 - 3*b*c*x)*sqrt(c^2*x^2 - 1))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^
3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{ax}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int\frac{bx\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b
*x*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(b \operatorname{arcosh}(cx) + a)x}{(c^2dx^2 - d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d)^3, x)
```

3.50 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=180

$$\frac{3b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8cd^3}-\frac{3b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8cd^3}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{3\tanh^{-1}\left(\right.}{}$$

```
[Out] b/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (3*b)/(8*c*d^3*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) + (3*x
*(a + b*ArcCosh[c*x]))/(8*d^3*(1 - c^2*x^2)) + (3*(a + b*ArcCosh[c*x])*ArcT
anh[E^ArcCosh[c*x]])/(4*c*d^3) + (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c*d^3
) - (3*b*PolyLog[2, E^ArcCosh[c*x]])/(8*c*d^3)
```

Rubi [A] time = 0.134077, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5689, 74, 5694, 4182, 2279, 2391}

$$\frac{3b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8cd^3}-\frac{3b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8cd^3}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{3\tanh^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^3,x]
```


```
[Out] b/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (3*b)/(8*c*d^3*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) + (3*x
*(a + b*ArcCosh[c*x]))/(8*d^3*(1 - c^2*x^2)) + (3*(a + b*ArcCosh[c*x])*ArcT
anh[E^ArcCosh[c*x]])/(4*c*d^3) + (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c*d^3
) - (3*b*PolyLog[2, E^ArcCosh[c*x]])/(8*c*d^3)
```


Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
```

```
egerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{(bc)\int \frac{x}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3} + \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx}{4d}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)} + \frac{(3bc)\int \frac{}{(-1+cx)}}{8d}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a+b\cos\right.}{8d^3\left(1-c\right.}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a+b\cos\right.}{8d^3\left(1-c\right.}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a+b\cos\right.}{8d^3\left(1-c\right.}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a+b\cos\right.}{8d^3\left(1-c\right.}$$

Mathematica [A] time = 1.1607, size = 316, normalized size = 1.76

$$-\frac{3b\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(e^{\cosh^{-1}(cx)}+1\right)\right)-4\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)\right)}{2c} + \frac{3b\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(1-e^{\cosh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,e^{\cosh^{-1}\right.\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^3,x]
```

```
[Out] ((4*a*x)/(-1 + c^2*x^2)^2 - (6*a*x)/(-1 + c^2*x^2) + (b*(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(2 + c*x) - 3*ArcCosh[c*x]))/(3*c*(1 + c*x)^2) + (b*((2 - c*x)*S
qrt[-1 + c*x]*Sqrt[1 + c*x] + 3*ArcCosh[c*x]))/(3*c*(-1 + c*x)^2) + (3*b*(-
(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)))/c + (3*b*(Sqrt[(-
1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)))/c - (3*a*Log[1 - c*x])/c + (
3*a*Log[1 + c*x])/c - (3*b*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^ArcCos
h[c*x]]) - 4*PolyLog[2, -E^ArcCosh[c*x]]))/(2*c) + (3*b*(ArcCosh[c*x]*(ArcC
osh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]]))/(2*c
```

```
))/(16*d^3)
```

Maple [A] time = 0.106, size = 378, normalized size = 2.1

$$\frac{a}{16\,cd^3\,(cx-1)^2}-\frac{3\,a}{16\,cd^3\,(cx-1)}-\frac{3\,a\ln(cx-1)}{16\,cd^3}-\frac{a}{16\,cd^3\,(cx+1)^2}-\frac{3\,a}{16\,cd^3\,(cx+1)}+\frac{3\,a\ln(cx+1)}{16\,cd^3}-\frac{3\,c^2b\mathrm{arccosh}\,(}{8\,d^3\left(c^4x^4-2\,c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/16/c*a/d^3/(c*x-1)^2-3/16/c*a/d^3/(c*x-1)-3/16/c*a/d^3*ln(c*x-1)-1/16/c*a
/d^3/(c*x+1)^2-3/16/c*a/d^3/(c*x+1)+3/16/c*a/d^3*ln(c*x+1)-3/8*c^2*b/d^3/(c
^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x^3-3/8*c*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^2+5/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x+
11/24/c*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+3/8/c*b/d^3
*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/8*b*polylog(2,-c*x-(c
*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^3-3/8/c*b/d^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^
(1/2)*(c*x+1)^(1/2))-3/8*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(18432*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*
x^4 + 3*c^2*d^3*x^2 - d^3), x) - 48*c^4*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4 -
 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5*
d^3)) - 6144*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x
^4 + 3*c^2*d^3*x^2 - d^3), x) + 18*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c^8*d^3*x
^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^5*d^3) + 5*log(
c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d^3*x^4 - 2*c^6*
d^3*x^2 + c^4*d^3))*c^3 + 80*c^2*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d^3*
x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3)) + 12288*c
^2*integrate(1/32*x^2*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3
*x^2 - d^3), x) + 9*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^6*d^3*x^3 - c^5*d^3*x^
```

```
2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d
^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3))*c - 32*(3*(
c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 6*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c
*x + 1)*log(c*x - 1) + 4*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*
log(c*x + 1) + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3) + 2048*integrat
e(-1/16*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) + 3*
(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^7*d^3*x^7 - 3*c^5*d^3*x^5 + 3*c^
3*d^3*x^3 - c*d^3*x + (c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3)*s
qrt(c*x + 1)*sqrt(c*x - 1)), x) - 6144*integrate(1/32*log(c*x - 1)/(c^6*d^3
*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x))*b - 1/16*a*(2*(3*c^2*x^3 -
 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log(c*x + 1)/(c*d^3) + 3*log(
c*x - 1)/(c*d^3))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\,\text{arcosh}\,(cx)+a}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
 - d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{a}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int\frac{b\,\text{acosh}\,(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b*a
cosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\,\mathrm{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d)^3, x)
```

3.51 $\int \frac{a+b\cosh^{-1}(cx)}{x(d-c^2dx^2)^3}\,dx$

Optimal. Leaf size=171

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d^3}+\frac{a+b\cosh^{-1}(cx)}{2d^3(1-c^2x^2)}+\frac{a+b\cosh^{-1}(cx)}{4d^3(1-c^2x^2)^2}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(c\ldots\right.}}{\ldots}$$

```
[Out] (b*c*x)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (2*b*c*x)/(3*d^3*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(4*d^3*(1 - c^2*x^2)^2) + (a
 + b*ArcCosh[c*x])/(2*d^3*(1 - c^2*x^2)) + (2*(a + b*ArcCosh[c*x])*ArcTanh[
E^(2*ArcCosh[c*x])])/d^3 + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^3) - (b
*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*d^3)
```

Rubi [A] time = 0.260796, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5754, 5721, 5461, 4182, 2279, 2391, 39, 40}

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d^3}+\frac{a+b\cosh^{-1}(cx)}{2d^3(1-c^2x^2)}+\frac{a+b\cosh^{-1}(cx)}{4d^3(1-c^2x^2)^2}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(c\ldots\right.}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^3),x]
```


```
[Out] (b*c*x)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (2*b*c*x)/(3*d^3*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(4*d^3*(1 - c^2*x^2)^2) + (a
 + b*ArcCosh[c*x])/(2*d^3*(1 - c^2*x^2)) + (2*(a + b*ArcCosh[c*x])*ArcTanh[
E^(2*ArcCosh[c*x])])/d^3 + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^3) - (b
*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*d^3)
```


Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
```

```
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
IntegerQ[p]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
```

```
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx = \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} - \frac{(bc)\int \frac{1}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3} + \frac{\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx}{d}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)} + \frac{(bc)\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,d}{6d^3}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)} -$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)} -$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)} +$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)} +$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)} +$$

Mathematica [A] time = 1.42309, size = 210, normalized size = 1.23

$$-\frac{b\left(6\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-6\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)+\frac{6\cosh^{-1}(cx)}{c^2x^2-1}-\frac{3\cosh^{-1}(cx)}{\left(c^2x^2-1\right)^2}-\frac{cx\left(\frac{cx-1}{cx+1}\right)^{3/2}}{(cx-1)^3}+\frac{8cx\sqrt{\frac{cx-1}{cx+1}}}{cx-1}+12\,\text{co}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^3),x]
```

```
[Out] -((-3*a)/(-1 + c^2*x^2)^2 + (6*a)/(-1 + c^2*x^2) - 12*a*Log[x] + 6*a*Log[1
- c^2*x^2] + b*((8*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(-1 + c*x) - (c*x*((-1 +
```

```
 c*x)/(1 + c*x))^(3/2))/(-1 + c*x)^3 - (3*ArcCosh[c*x])/(-1 + c^2*x^2)^2 +
(6*ArcCosh[c*x])/(-1 + c^2*x^2) + 12*ArcCosh[c*x]*Log[1 - E^(-2*ArcCosh[c*x
])] - 12*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] + 6*PolyLog[2, -E^(-2*Ar
cCosh[c*x])] - 6*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(12*d^3)
```

Maple [B] time = 0.182, size = 508, normalized size = 3.

$$\frac{a}{16\,d^{3}\left(cx-1\right)^{2}}-\frac{5\,a}{16\,d^{3}\left(cx-1\right)}-\frac{a\ln\left(cx-1\right)}{2\,d^{3}}+\frac{a\ln\left(cx\right)}{d^{3}}+\frac{a}{16\,d^{3}\left(cx+1\right)^{2}}+\frac{5\,a}{16\,d^{3}\left(cx+1\right)}-\frac{a\ln\left(cx+1\right)}{2\,d^{3}}-\frac{2\,x^{3}}{3\,d^{3}\left(c^{4}x^{4}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/16*a/d^3/(c*x-1)^2-5/16*a/d^3/(c*x-1)-1/2*a/d^3*ln(c*x-1)+a/d^3*ln(c*x)+1
/16*a/d^3/(c*x+1)^2+5/16*a/d^3/(c*x+1)-1/2*a/d^3*ln(c*x+1)-2/3*b/d^3/(c^4*x
^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*x^3+2/3*b/d^3/(c^4*x^4-2*c^
2*x^2+1)*c^4*x^4-1/2*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c^2*x^2+3/4*b
/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x-4/3*b/d^3/(c^4*x
^4-2*c^2*x^2+1)*c^2*x^2+3/4*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)+2/3*b/
d^3/(c^4*x^4-2*c^2*x^2+1)-b/d^3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))-b/d^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d^3*arccosh(c*x
)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)+1/2*b*polylog(2,-(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2)/d^3-b/d^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))-b/d^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}\,a\left(\frac{2\,c^{2}x^{2}-3}{c^{4}d^{3}x^{4}-2\,c^{2}d^{3}x^{2}+d^{3}}+\frac{2\,\log\left(cx+1\right)}{d^{3}}+\frac{2\,\log\left(cx-1\right)}{d^{3}}-\frac{4\,\log\left(x\right)}{d^{3}}\right)-b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{c^{6}d^{3}x^{7}-3\,c^{4}d^{3}x^{5}+3\,c^{2}d^{3}x^{3}-d^{3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((2*c^2*x^2 - 3)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) + 2*log(c*x + 1
)/d^3 + 2*log(c*x - 1)/d^3 - 4*log(x)/d^3) - b*integrate(log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x),
```

```
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\,\text{arcosh}\,(cx)+a}{c^6d^3x^7-3\,c^4d^3x^5+3\,c^2d^3x^3-d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3
 - d^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{a}{c^6x^7-3c^4x^5+3c^2x^3-x}\,dx+\int\frac{b\,\text{acosh}\,(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x) + Integral(b*a
cosh(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{b\,\text{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x), x)
```

3.52 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)^3}\,dx$

Optimal. Leaf size=230

$$\frac{15bc\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8d^3}-\frac{15bc\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8d^3}+\frac{15c^2x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{a}{d}$$

```
[Out] (b*c)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (7*b*c)/(8*d^3*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(d^3*x*(1 - c^2*x^2)^2) + (5*c^2
*x*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) + (15*c^2*x*(a + b*ArcCosh
[c*x]))/(8*d^3*(1 - c^2*x^2)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d^3 + (15*c*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*d^3) + (15*b*c
*PolyLog[2, -E^ArcCosh[c*x]])/(8*d^3) - (15*b*c*PolyLog[2, E^ArcCosh[c*x]])
/(8*d^3)
```

Rubi [A] time = 0.244466, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5746, 104, 21, 92, 205, 5689, 74, 5694, 4182, 2279, 2391}

$$\frac{15bc\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8d^3}-\frac{15bc\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8d^3}+\frac{15c^2x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{a}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

```
[Out] (b*c)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (7*b*c)/(8*d^3*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(d^3*x*(1 - c^2*x^2)^2) + (5*c^2
*x*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) + (15*c^2*x*(a + b*ArcCosh
[c*x]))/(8*d^3*(1 - c^2*x^2)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d^3 + (15*c*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*d^3) + (15*b*c
*PolyLog[2, -E^ArcCosh[c*x]])/(8*d^3) - (15*b*c*PolyLog[2, E^ArcCosh[c*x]])
/(8*d^3)
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
```

```
egerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \left(5c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^3}$$

$$= -\frac{bc}{3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{b\int \frac{3c+3c^2x}{x(-1+cx)^{3/2}(1+c}}{3d^3}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{15c^2x\left(a+b\cosh\right.}{8d^3\left(1-c^2x\right.}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c^2x\left(a+b\cosh^{-1}(\right.}{4d^3\left(1-c^2x^2\right)^2}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c^2x\left(a+b\cosh^{-1}(\right.}{4d^3\left(1-c^2x^2\right)^2}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c^2x\left(a+b\cosh^{-1}(\right.}{4d^3\left(1-c^2x^2\right)^2}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c^2x\left(a+b\cosh^{-1}(\right.}{4d^3\left(1-c^2x^2\right)^2}$$

Mathematica [A] time = 1.85133, size = 362, normalized size = 1.57

$$\frac{-45bc\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(e^{\cosh^{-1}(cx)}+1\right)\right)-4\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)\right)+45bc\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

```
[Out] ((-96*a)/x + (24*a*c^2*x)/(-1 + c^2*x^2)^2 - (84*a*c^2*x)/(-1 + c^2*x^2) -
(2*b*c*((-2 + c*x)*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*ArcCosh[c*x]))/(-1 + c*
x)^2 + (2*b*c*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c*x) - 3*ArcCosh[c*x]))/(1
 + c*x)^2 - (96*b*ArcCosh[c*x])/x + 42*b*c*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)])
 + ArcCosh[c*x]/(1 - c*x)) + 42*b*c*(Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c
*x]/(1 + c*x)) + (96*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sq
```

```
rt[-1 + c*x]*Sqrt[1 + c*x]) - 90*a*c*Log[1 - c*x] + 90*a*c*Log[1 + c*x] - 4
5*b*c*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^ArcCosh[c*x]]) - 4*PolyLog[
2, -E^ArcCosh[c*x]]) + 45*b*c*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 - E^Arc
Cosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]]))/(96*d^3)
```

Maple [A] time = 0.18, size = 392, normalized size = 1.7

$$\frac{ca}{16\,d^3\,(cx-1)^2}-\frac{7\,ca}{16\,d^3\,(cx-1)}-\frac{15\,ca\ln(cx-1)}{16\,d^3}-\frac{a}{d^3x}-\frac{ca}{16\,d^3\,(cx+1)^2}-\frac{7\,ca}{16\,d^3\,(cx+1)}+\frac{15\,ca\ln(cx+1)}{16\,d^3}-\frac{15\,b\mathrm{arcc}}{8\,d^3\,(c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/16*c*a/d^3/(c*x-1)^2-7/16*c*a/d^3/(c*x-1)-15/16*c*a/d^3*ln(c*x-1)-a/d^3/x
-1/16*c*a/d^3/(c*x+1)^2-7/16*c*a/d^3/(c*x+1)+15/16*c*a/d^3*ln(c*x+1)-15/8*b
/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c^4*x^3-7/8*b/d^3/(c^4*x^4-2*c^2*x^
2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*x^2+25/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*a
rccosh(c*x)*c^2*x+23/24*c*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)
^(1/2)-b/d^3/x/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)+2*c*b/d^3*arctan(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))+15/8*c*b/d^3*dilog(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)+15/8*c*b/d^3*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+15/8*c*b/d^3*arccos
h(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(92160*c^7*integrate(1/32*x^5*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*
x^4 + 3*c^2*d^3*x^2 - d^3), x) - 240*c^6*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4
- 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5
*d^3)) - 30720*c^6*integrate(1/32*x^4*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3
*x^4 + 3*c^2*d^3*x^2 - d^3), x) + 90*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c^8*d^3
*x^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^5*d^3) + 5*lo
g(c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d^3*x^4 - 2*c^
```

```
6*d^3*x^2 + c^4*d^3))*c^5 + 400*c^4*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d
^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3)) + 6144
0*c^4*integrate(1/32*x^2*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 - d^3), x) + 45*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^6*d^3*x^3 - c^5*d^
3*x^2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c
^3*d^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3))*c^3 + 1
28*c^2*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log(c*x
 + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) - 30720*c^2*integrate(1/32*log(c*x
- 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x) - 32*(15*(c^5*
x^5 - 2*c^3*x^3 + c*x)*log(c*x + 1)^2 + 30*(c^5*x^5 - 2*c^3*x^3 + c*x)*log(
c*x + 1)*log(c*x - 1) + 4*(30*c^4*x^4 - 50*c^2*x^2 - 15*(c^5*x^5 - 2*c^3*x^
3 + c*x)*log(c*x + 1) + 15*(c^5*x^5 - 2*c^3*x^3 + c*x)*log(c*x - 1) + 16)*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x
) + 2048*integrate(-1/16*(30*c^5*x^4 - 50*c^3*x^2 - 15*(c^6*x^5 - 2*c^4*x^3
 + c^2*x)*log(c*x + 1) + 15*(c^6*x^5 - 2*c^4*x^3 + c^2*x)*log(c*x - 1) + 16
*c)/(c^7*d^3*x^8 - 3*c^5*d^3*x^6 + 3*c^3*d^3*x^4 - c*d^3*x^2 + (c^6*d^3*x^7
 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x)*sqrt(c*x + 1)*sqrt(c*x - 1)), x))
*b - 1/16*a*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 +
 d^3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(c*x - 1)/d^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\,\text{arcosh}\,(cx)+a}{c^6d^3x^8-3\,c^4d^3x^6+3\,c^2d^3x^4-d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
 - d^3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{a}{c^6x^8-3c^4x^6+3c^2x^4-x^2}\,dx+\int\frac{b\,\text{acosh}\,(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x) + Integral(
b*acosh(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b \operatorname{arcosh}(cx) + a}{\left(c^2 d x^2 - d\right)^3 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^2), x)
```

3.53 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=250

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d^3}-\frac{3bc^2\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d^3}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d^3\left(1-c^2x^2\right)}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{a}{2d}$$

```
[Out] (b*c)/(2*d^3*x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (5*b*c^3*x)/(12*d^3*(-1
+ c*x)^(3/2)*(1 + c*x)^(3/2)) - (2*b*c^3*x)/(3*d^3*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (3*c^2*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) - (a + b*ArcCo
sh[c*x])/(2*d^3*x^2*(1 - c^2*x^2)^2) + (3*c^2*(a + b*ArcCosh[c*x]))/(2*d^3*
(1 - c^2*x^2)) + (6*c^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d
^3 + (3*b*c^2*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^3) - (3*b*c^2*PolyLog[2
, E^(2*ArcCosh[c*x])])/(2*d^3)
```

Rubi [A] time = 0.366852, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5746, 103, 12, 40, 39, 5754, 5721, 5461, 4182, 2279, 2391}

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d^3}-\frac{3bc^2\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d^3}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d^3\left(1-c^2x^2\right)}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{a}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^3),x]
```


```
[Out] (b*c)/(2*d^3*x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (5*b*c^3*x)/(12*d^3*(-1
+ c*x)^(3/2)*(1 + c*x)^(3/2)) - (2*b*c^3*x)/(3*d^3*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (3*c^2*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) - (a + b*ArcCo
sh[c*x])/(2*d^3*x^2*(1 - c^2*x^2)^2) + (3*c^2*(a + b*ArcCosh[c*x]))/(2*d^3*
(1 - c^2*x^2)) + (6*c^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d
^3 + (3*b*c^2*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^3) - (3*b*c^2*PolyLog[2
, E^(2*ArcCosh[c*x])])/(2*d^3)
```


Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
```

```
IntegerQ[p]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2} + \left(3c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{2d^3}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{a+b\cosh^{-1}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2} + \frac{(bc)\int \frac{4c^2}{(-1+cx)^{5/2}(1+cx}}{2d^3}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{bc^3x}{4d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{a+b\cosh}{2d^3x^2\left(1-\right.}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bc^3x}{d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c^2\left(a+b\,\mathrm{co}\right.}{4d^3\left(1-\right.}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bc^3x}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c^2\left(a+b\right.}{4d^3\left(1-\right.}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bc^3x}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c^2\left(a+b\right.}{4d^3\left(1-\right.}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bc^3x}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c^2\left(a+b\right.}{4d^3\left(1-\right.}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bc^3x}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c^2\left(a+b\right.}{4d^3\left(1-\right.}$$

Mathematica [A] time = 2.90529, size = 273, normalized size = 1.09

$$-\frac{bc^2\left(18\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-18\mathrm{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)+\frac{12\cosh^{-1}(cx)}{c^2x^2-1}-\frac{3\cosh^{-1}(cx)}{\left(c^2x^2-1\right)^2}+\frac{6\cosh^{-1}(cx)}{c^2x^2}+\frac{14cx}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^3),x]
```

```
[Out] -((6*a)/x^2 - (3*a*c^2)/(-1 + c^2*x^2)^2 + (12*a*c^2)/(-1 + c^2*x^2) - 36*a
*c^2*Log[x] + 18*a*c^2*Log[1 - c^2*x^2] + b*c^2*(-((c*x)/(((-1 + c*x)/(1 +
c*x))^(3/2)*(1 + c*x)^3)) + (14*c*x)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

```
 - (6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x) + (6*ArcCosh[c*x])/(c^2*x
^2) - (3*ArcCosh[c*x])/(-1 + c^2*x^2)^2 + (12*ArcCosh[c*x])/(-1 + c^2*x^2)
+ 36*ArcCosh[c*x]*Log[1 - E^(-2*ArcCosh[c*x])] - 36*ArcCosh[c*x]*Log[1 + E^
(-2*ArcCosh[c*x])] + 18*PolyLog[2, -E^(-2*ArcCosh[c*x])] - 18*PolyLog[2, E^
(-2*ArcCosh[c*x])]))/(12*d^3)
```

Maple [B] time = 0.238, size = 641, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/16*c^2*a/d^3/(c*x-1)^2-9/16*c^2*a/d^3/(c*x-1)-3/2*c^2*a/d^3*ln(c*x-1)-1/2
*a/d^3/x^2+3*c^2*a/d^3*ln(c*x)+1/16*c^2*a/d^3/(c*x+1)^2+9/16*c^2*a/d^3/(c*x
+1)-3/2*c^2*a/d^3*ln(c*x+1)-2/3*c^5*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3+2/3*c^6*b/d^3/(c^4*x^4-2*c^2*x^2+1)*x^4-3/2*c^4*b/d^3/
(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x^2+1/4*c^3*b/d^3/(c^4*x^4-2*c^2*x^2+1)*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-4/3*c^4*b/d^3/(c^4*x^4-2*c^2*x^2+1)*x^2+9/4*c
^2*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)+1/2*c*b/d^3/(c^4*x^4-2*c^2*x^2+
1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)+2/3*c^2*b/d^3/(c^4*x^4-2*c^2*x^2+1)-1/2*b/
d^3/(c^4*x^4-2*c^2*x^2+1)/x^2*arccosh(c*x)-3*c^2*b/d^3*arccosh(c*x)*ln(1+c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-3*c^2*b/d^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*
x+1)^(1/2))+3*c^2*b/d^3*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2
+1)+3/2*b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3-3*c^2*b/d
^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-3*c^2*b/d^3*polylog(2
,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}a\left(\frac{6c^4x^4-9c^2x^2+2}{c^4d^3x^6-2c^2d^3x^4+d^3x^2}+\frac{6c^2\log(cx+1)}{d^3}+\frac{6c^2\log(cx-1)}{d^3}-\frac{12c^2\log(x)}{d^3}\right)-b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx}\right.}{c^6d^3x^9-3c^4d^3x^7+3c^2d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*c^4*x^4 - 9*c^2*x^2 + 2)/(c^4*d^3*x^6 - 2*c^2*d^3*x^4 + d^3*x^2)
 + 6*c^2*log(c*x + 1)/d^3 + 6*c^2*log(c*x - 1)/d^3 - 12*c^2*log(x)/d^3) - b
```

```
*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^6*d^3*x^9 - 3*c^4*d^3*
x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\,\text{arcosh}\,(cx)+a}{c^6d^3x^9-3\,c^4d^3x^7+3\,c^2d^3x^5-d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5
 - d^3*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{a}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx+\int\frac{b\,\text{acosh}\,(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x) + Integral(
b*acosh(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{b\,\text{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^3), x)
```

3.54 $\int \frac{a+b\cosh^{-1}(cx)}{x^4(d-c^2dx^2)^3}\,dx$

Optimal. Leaf size=310

$$\frac{35bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8d^3}-\frac{35bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8d^3}+\frac{35c^4x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{35c^4x\left(a+b\cosh^{-1}(cx)\right)}{12d^3\left(1-c^2x^2\right)^2}$$

```
[Out] -(b*c^3)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (b*c)/(6*d^3*x^2*(-1 +
 c*x)^(3/2)*(1 + c*x)^(3/2)) - (29*b*c^3)/(24*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) - (a + b*ArcCosh[c*x])/(3*d^3*x^3*(1 - c^2*x^2)^2) - (7*c^2*(a + b*Arc
Cosh[c*x]))/(3*d^3*x*(1 - c^2*x^2)^2) + (35*c^4*x*(a + b*ArcCosh[c*x]))/(12
*d^3*(1 - c^2*x^2)^2) + (35*c^4*x*(a + b*ArcCosh[c*x]))/(8*d^3*(1 - c^2*x^2
)) + (19*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(6*d^3) + (35*c^3*(a +
 b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*d^3) + (35*b*c^3*PolyLog[2, -E
^ArcCosh[c*x]])/(8*d^3) - (35*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/(8*d^3)
```

Rubi [A] time = 0.387042, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {5746, 103, 12, 104, 21, 92, 205, 5689, 74, 5694, 4182, 2279, 2391}

$$\frac{35bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8d^3}-\frac{35bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8d^3}+\frac{35c^4x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{35c^4x\left(a+b\cosh^{-1}(cx)\right)}{12d^3\left(1-c^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^3),x]
```


```
[Out] -(b*c^3)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (b*c)/(6*d^3*x^2*(-1 +
 c*x)^(3/2)*(1 + c*x)^(3/2)) - (29*b*c^3)/(24*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) - (a + b*ArcCosh[c*x])/(3*d^3*x^3*(1 - c^2*x^2)^2) - (7*c^2*(a + b*Arc
Cosh[c*x]))/(3*d^3*x*(1 - c^2*x^2)^2) + (35*c^4*x*(a + b*ArcCosh[c*x]))/(12
*d^3*(1 - c^2*x^2)^2) + (35*c^4*x*(a + b*ArcCosh[c*x]))/(8*d^3*(1 - c^2*x^2
)) + (19*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(6*d^3) + (35*c^3*(a +
 b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*d^3) + (35*b*c^3*PolyLog[2, -E
^ArcCosh[c*x]])/(8*d^3) - (35*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/(8*d^3)
```


Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{1}{3}\left(7c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx+\frac{(bc)\int \frac{1}{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{3d^3}$$

$$= \frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{7c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^3x\left(1-c^2x^2\right)^2}+\frac{1}{3}\left(35c^4\right)\int \frac{a+b}{\left(d-\right.}$$

$$= -\frac{7bc^3}{9d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{7c^2\left(a+b\cos\right.}{3d^3x\left(1-\right.}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{7c^2\left(a+b\,\mathrm{co}\right.}{3d^3x\left(1-\right.}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{49bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\,\mathrm{c}}{3d^3x^3\left(\right.}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\,\mathrm{c}}{3d^3x^3\left(\right.}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\,\mathrm{c}}{3d^3x^3\left(\right.}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\,\mathrm{c}}{3d^3x^3\left(\right.}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\,\mathrm{c}}{3d^3x^3\left(\right.}$$

Mathematica [A] time = 1.96871, size = 471, normalized size = 1.52

$$-\frac{105}{2}bc^3\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(e^{\cosh^{-1}(cx)}+1\right)\right)-4\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)\right)+\frac{105}{2}bc^3\left(\cosh^{-1}(cx)\left(\cosh^{-1}(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^3),x]
```

```
[Out] ((-16*a)/x^3 - (144*a*c^2)/x + (12*a*c^4*x)/(-1 + c^2*x^2)^2 - (66*a*c^4*x)
/(-1 + c^2*x^2) - (b*c^3*((-2 + c*x)*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*ArcCo
sh[c*x]))/(-1 + c*x)^2 + (b*c^3*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c*x) - 3
*ArcCosh[c*x]))/(1 + c*x)^2 + 33*b*c^3*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + A
rcCosh[c*x]/(1 - c*x)) + 33*b*c^3*(Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x
]/(1 + c*x)) + 144*b*c^2*(-(ArcCosh[c*x]/x) + (c*Sqrt[-1 + c^2*x^2]*ArcTan[
Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (8*b*(-2*ArcCosh[c*x
] + (c*x*(-1 + c^2*x^2 + c^2*x^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^
2]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/x^3 - 105*a*c^3*Log[1 - c*x] + 105*a
*c^3*Log[1 + c*x] - (105*b*c^3*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^Ar
cCosh[c*x]]) - 4*PolyLog[2, -E^ArcCosh[c*x]]))/2 + (105*b*c^3*(ArcCosh[c*x]
*(ArcCosh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]])
)/2)/(48*d^3)
```

Maple [A] time = 0.237, size = 504, normalized size = 1.6

$$\frac{c^3a}{16d^3(cx-1)^2}-\frac{11c^3a}{16d^3(cx-1)}-\frac{35c^3a\ln(cx-1)}{16d^3}-\frac{a}{3d^3x^3}-3\frac{c^2a}{d^3x}-\frac{c^3a}{16d^3(cx+1)^2}-\frac{11c^3a}{16d^3(cx+1)}+\frac{35c^3a\ln(cx+}{16d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/16*c^3*a/d^3/(c*x-1)^2-11/16*c^3*a/d^3/(c*x-1)-35/16*c^3*a/d^3*ln(c*x-1)-
1/3*a/d^3/x^3-3*c^2*a/d^3/x-1/16*c^3*a/d^3/(c*x+1)^2-11/16*c^3*a/d^3/(c*x+1
)+35/16*c^3*a/d^3*ln(c*x+1)-35/8*c^6*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*
x)*x^3-29/24*c^5*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
2+175/24*c^4*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x+9/8*c^3*b/d^3/(c^4*
x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)-7/3*c^2*b/d^3/x/(c^4*x^4-2*c^2
*x^2+1)*arccosh(c*x)+1/6*c*b/d^3/(c^4*x^4-2*c^2*x^2+1)/x^2*(c*x+1)^(1/2)*(c
```

```
*x-1)^(1/2)-1/3*b/d^3/(c^4*x^4-2*c^2*x^2+1)/x^3*arccosh(c*x)+19/3*c^3*b/d^3
*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+35/8*c^3*b/d^3*dilog(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))+35/8*c^3*b/d^3*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
+35/8*c^3*b/d^3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/6144*(1935360*c^9*integrate(1/96*x^7*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*d^
3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) - 1680*c^8*(2*(5*c^2*x^3 - 3*x)/(c^8*d
^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x -
1)/(c^5*d^3)) - 645120*c^8*integrate(1/96*x^6*log(c*x - 1)/(c^6*d^3*x^8 - 3
*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) + 630*(c*(2*(5*c^2*x^2 + 3*c*x
- 6)/(c^8*d^3*x^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^
5*d^3) + 5*log(c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d
^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3))*c^7 + 2800*c^6*(2*(c^2*x^3 + x)/(c^6*d^3
*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^
3*d^3)) + 1290240*c^6*integrate(1/96*x^4*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*
d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) + 315*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/
(c^6*d^3*x^3 - c^5*d^3*x^2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3
) + 3*log(c*x - 1)/(c^3*d^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^
2 + c^2*d^3))*c^5 + 896*c^4*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x
^2 + d^3) - 3*log(c*x + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) - 645120*c^4*i
ntegrate(1/96*x^2*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
 - d^3*x^2), x) + 128*c^2*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)/(c^4*d^3*x^5 - 2
*c^2*d^3*x^3 + d^3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(c*x - 1)/d^3) - 32
*(105*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x + 1)^2 + 210*(c^7*x^7 - 2*c^5
*x^5 + c^3*x^3)*log(c*x + 1)*log(c*x - 1) + 4*(210*c^6*x^6 - 350*c^4*x^4 +
112*c^2*x^2 - 105*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x + 1) + 105*(c^7*x
^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x - 1) + 16)*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1)))/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3) + 6144*integrate(-1/48*(2
10*c^7*x^6 - 350*c^5*x^4 + 112*c^3*x^2 - 105*(c^8*x^7 - 2*c^6*x^5 + c^4*x^3
)*log(c*x + 1) + 105*(c^8*x^7 - 2*c^6*x^5 + c^4*x^3)*log(c*x - 1) + 16*c)/(
c^7*d^3*x^10 - 3*c^5*d^3*x^8 + 3*c^3*d^3*x^6 - c*d^3*x^4 + (c^6*d^3*x^9 - 3
*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x))*b
 + 1/48*a*(105*c^3*log(c*x + 1)/d^3 - 105*c^3*log(c*x - 1)/d^3 - 2*(105*c^6
*x^6 - 175*c^4*x^4 + 56*c^2*x^2 + 8)/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3
```

```
))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\operatorname{arcosh}(cx)+a}{c^6d^3x^{10}-3\,c^4d^3x^8+3\,c^2d^3x^6-d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^
6 - d^3*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\operatorname{arcosh}(cx)+a}{\left(c^2dx^2-d\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^4), x)
```

3.55 $\int \frac{\cosh^{-1}(ax)}{c-a^2cx^2} dx$

Optimal. Leaf size=53

$$\frac{\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

```
[Out] (2*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c) + PolyLog[2, -E^ArcCosh[a*x]
]/(a*c) - PolyLog[2, E^ArcCosh[a*x]]/(a*c)
```

Rubi [A] time = 0.0519342, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5694, 4182, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] (2*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c) + PolyLog[2, -E^ArcCosh[a*x]
]/(a*c) - PolyLog[2, E^ArcCosh[a*x]]/(a*c)
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{c-a^2cx^2}\,dx = -\frac{\text{Subst}\left(\int x\,\text{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac}$$
$$= \frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\text{Subst}\left(\int \log(1-e^x)\,dx, x, \cosh^{-1}(ax)\right)}{ac} - \frac{\text{Subst}\left(\int \log(1+e\right.}{}$$
$$= \frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\right.}}{ac}$$
$$= \frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

Mathematica [A] time = 0.0466481, size = 77, normalized size = 1.45

$$\frac{\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\cosh^{-1}(ax)\log\left(1-e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\cosh^{-1}(ax)\log\left(e^{\cosh^{-1}(ax)}+1\right.}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] -((ArcCosh[a*x]*Log[1 - E^ArcCosh[a*x]])/(a*c)) + (ArcCosh[a*x]*Log[1 + E^A
rcCosh[a*x]])/(a*c) + PolyLog[2, -E^ArcCosh[a*x]]/(a*c) - PolyLog[2, E^ArcC
osh[a*x]]/(a*c)
```

Maple [C] time = 0.017, size = 321, normalized size = 6.1

$$\frac{\text{Artanh}(ax)\operatorname{arccosh}(ax)}{ac} + \frac{2i\text{Artanh}(ax)}{ac(ax-1)(ax+1)}\sqrt{\frac{1}{2}+\frac{ax}{2}}\sqrt{-a^2x^2+1}\sqrt{-\frac{1}{2}+\frac{ax}{2}}\ln\left(1+i(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{2}{ac(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c),x)
```

```
[Out] 1/a/c*arctanh(a*x)*arccosh(a*x)+2*I/a/c*(1/2+1/2*a*x)^(1/2)*(-a^2*x^2+1)^(1
/2)*(-1/2+1/2*a*x)^(1/2)/(a*x-1)/(a*x+1)*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*
x^2+1)^(1/2))-2*I/a/c*(1/2+1/2*a*x)^(1/2)*(-a^2*x^2+1)^(1/2)*(-1/2+1/2*a*x)
^(1/2)/(a*x-1)/(a*x+1)*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+2*I/
a/c*(1/2+1/2*a*x)^(1/2)*(-a^2*x^2+1)^(1/2)*(-1/2+1/2*a*x)^(1/2)/(a*x-1)/(a*
x+1)*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-2*I/a/c*(1/2+1/2*a*x)^(1/2)*(-a^
2*x^2+1)^(1/2)*(-1/2+1/2*a*x)^(1/2)/(a*x-1)/(a*x+1)*dilog(1-I*(a*x+1)/(-a^2
*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\left(\log\left(ax+1\right)-\log\left(ax-1\right)\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)-\log\left(ax+1\right)^{2}-2\,\log\left(ax+1\right)\log\left(ax-1\right)+\log\left(ax-1\right)^{2}}{8\,ac}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/8*(4*(log(a*x + 1) - log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))
 - log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)/(a*c) + 1
/2*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/(a*c) + integr
ate(1/2*(log(a*x + 1) - log(a*x - 1))/(a^3*c*x^3 - a*c*x + (a^2*c*x^2 - c)*
sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{arcosh}\left(ax\right)}{a^{2}cx^{2}-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\operatorname{acosh}(ax)}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(acosh(a*x)/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arcosh}(ax)}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)/(a^2*c*x^2 - c), x)
```

3.56 $\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^2} \, dx$

Optimal. Leaf size=109

$$\frac{\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{2ac^2}+\frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)}-\frac{1}{2ac^2\sqrt{ax-1}\sqrt{ax+1}}+\frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cos\cdots}\right.}{ac^2}$$

```
[Out] -1/(2*a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x])/(2*c^2*(1 - a^
2*x^2)) + (ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) + PolyLog[2, -E^Ar
cCosh[a*x]]/(2*a*c^2) - PolyLog[2, E^ArcCosh[a*x]]/(2*a*c^2)
```

Rubi [A] time = 0.0804365, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5689, 74, 5694, 4182, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{2ac^2}+\frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)}-\frac{1}{2ac^2\sqrt{ax-1}\sqrt{ax+1}}+\frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cos\cdots}\right.}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/(2*a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x])/(2*c^2*(1 - a^
2*x^2)) + (ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) + PolyLog[2, -E^Ar
cCosh[a*x]]/(2*a*c^2) - PolyLog[2, E^ArcCosh[a*x]]/(2*a*c^2)
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c-a^2cx^2\right)^2}\,dx = \frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)} + \frac{a\int \frac{x}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{2c^2} + \frac{\int \frac{\cosh^{-1}(ax)}{c-a^2cx^2}\,dx}{2c}$$

$$= -\frac{1}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)} - \frac{\text{Subst}\left(\int x\,\text{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{2ac^2}$$

$$= -\frac{1}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\text{Subst}\left(\int \log\left(1-e^{x}\right.\right.}{2a}$$

$$= -\frac{1}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx\right.}{2ac^2}$$

$$= -\frac{1}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{2ac^2} -$$

Mathematica [A] time = 0.83427, size = 120, normalized size = 1.1

$$\frac{2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right) - 2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right) - \frac{2\left(\cosh^{-1}(ax)\left(\left(a^2x^2-1\right)\log\left(1-e^{\cosh^{-1}(ax)}\right)+\left(1-a^2x^2\right)\log\left(e^{\cosh^{-1}(ax)}+1\right)+ax\right)+\sqrt{\frac{a}{a}}\right.}{a^2x^2-1}}{4ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] ((-2*(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + ArcCosh[a*x]*(a*x + (-1 + a^2*
x^2)*Log[1 - E^ArcCosh[a*x]] + (1 - a^2*x^2)*Log[1 + E^ArcCosh[a*x]])))/(-1
 + a^2*x^2) + 2*PolyLog[2, -E^ArcCosh[a*x]] - 2*PolyLog[2, E^ArcCosh[a*x]])
/(4*a*c^2)
```

Maple [A] time = 0.078, size = 184, normalized size = 1.7

$$-\frac{x\text{arccosh}\,(ax)}{\left(2\,a^2x^2-2\right)c^2} - \frac{1}{2\,a\left(a^2x^2-1\right)c^2}\sqrt{ax-1}\sqrt{ax+1} + \frac{\text{arccosh}\,(ax)}{2\,ac^2}\ln\left(1+ax+\sqrt{ax-1}\sqrt{ax+1}\right) + \frac{1}{2\,ac^2}\text{polylog}\left(2,-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/(a^2*x^2-1)/c^2*x*arccosh(a*x)-1/2/a/(a^2*x^2-1)/c^2*(a*x-1)^(1/2)*(a*
x+1)^(1/2)+1/2/a/c^2*arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+1/2
*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-1/2/a/c^2*arccosh(a*x)*l
n(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-1/2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1
)^(1/2))/a/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(a^2x^2-1\right)\log\left(ax+1\right)^2+2\left(a^2x^2-1\right)\log\left(ax+1\right)\log\left(ax-1\right)-\left(a^2x^2-1\right)\log\left(ax-1\right)^2+4\,ax+4\left(2\,ax-\left(a^2x^2-1\right)\right.}{16\left(a^3c^2x^2-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 + 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
- 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 + 4*a*x + 4*(2*a*x - (a^2*x^2 - 1)*log(
a*x + 1) + (a^2*x^2 - 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1
)) - 2*(a^2*x^2 - 1)*log(a*x - 1))/(a^3*c^2*x^2 - a*c^2) + 1/4*(log(a*x - 1
)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/(a*c^2) - 1/8*log(a*x + 1)/(a
*c^2) + integrate(-1/4*(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*
log(a*x - 1))/(a^5*c^2*x^5 - 2*a^3*c^2*x^3 + a*c^2*x + (a^4*c^2*x^4 - 2*a^2
*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{arcosh}\left(ax\right)}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{acosh}(ax)}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(acosh(a*x)/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/(a^2*c*x^2 - c)^2, x)
```

3.57 $\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=164

$$\frac{3\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{3\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{8ac^3}+\frac{3x\cosh^{-1}(ax)}{8c^3\left(1-a^2x^2\right)}+\frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2}-\frac{3}{8ac^3\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{12\ldots}$$

```
[Out] 1/(12*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)) - 3/(8*a*c^3*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (x*ArcCosh[a*x])/(4*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x
])/(8*c^3*(1 - a^2*x^2)) + (3*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(4*a*c^
3) + (3*PolyLog[2, -E^ArcCosh[a*x]])/(8*a*c^3) - (3*PolyLog[2, E^ArcCosh[a*
x]])/(8*a*c^3)
```

Rubi [A] time = 0.112888, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5689, 74, 5694, 4182, 2279, 2391}

$$\frac{3\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{3\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{8ac^3}+\frac{3x\cosh^{-1}(ax)}{8c^3\left(1-a^2x^2\right)}+\frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2}-\frac{3}{8ac^3\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{12\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)) - 3/(8*a*c^3*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (x*ArcCosh[a*x])/(4*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x
])/(8*c^3*(1 - a^2*x^2)) + (3*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(4*a*c^
3) + (3*PolyLog[2, -E^ArcCosh[a*x]])/(8*a*c^3) - (3*PolyLog[2, E^ArcCosh[a*
x]])/(8*a*c^3)
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
```

egerQ[p]

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c-a^2cx^2\right)^3}\,dx = \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} - \frac{a\int \frac{x}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{4c^3} + \frac{3\int \frac{\cosh^{-1}(ax)}{\left(c-a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)}{8c^3\left(1-a^2x^2\right)} + \frac{(3a)\int \frac{x}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{8c^3} + \frac{3}{}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)}{8c^3\left(1-a^2x^2\right)} - \frac{3\,\mathrm{Sub}}{}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)}{8c^3\left(1-a^2x^2\right)} + \frac{3\cos}{}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)}{8c^3\left(1-a^2x^2\right)} + \frac{3\cos}{}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)}{8c^3\left(1-a^2x^2\right)} + \frac{3\cos}{}$$

Mathematica [A] time = 2.34331, size = 223, normalized size = 1.36

$$36\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right) - 36\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right) - \frac{2\sqrt{ax+1}(ax-2)}{(ax-1)^{3/2}} + \frac{2\sqrt{ax-1}(ax+2)}{(ax+1)^{3/2}} + \frac{6\cosh^{-1}(ax)}{(ax-1)^2} - \frac{6\cosh^{-1}(ax)}{(ax+1)^2} + 18\Big(\cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] ((-2*(-2 + a*x)*Sqrt[1 + a*x])/(-1 + a*x)^(3/2) + (2*Sqrt[-1 + a*x]*(2 + a*
x))/(1 + a*x)^(3/2) + (6*ArcCosh[a*x])/(-1 + a*x)^2 - (6*ArcCosh[a*x])/(1 +
 a*x)^2 + 18*(-(1/Sqrt[(-1 + a*x)/(1 + a*x)]) + ArcCosh[a*x]/(1 - a*x)) + 1
8*(Sqrt[(-1 + a*x)/(1 + a*x)] - ArcCosh[a*x]/(1 + a*x)) + 9*ArcCosh[a*x]*(A
rcCosh[a*x] - 4*Log[1 - E^ArcCosh[a*x]]) - 9*ArcCosh[a*x]*(ArcCosh[a*x] - 4
*Log[1 + E^ArcCosh[a*x]]) + 36*PolyLog[2, -E^ArcCosh[a*x]] - 36*PolyLog[2,
E^ArcCosh[a*x]])/(96*a*c^3)
```

Maple [A] time = 0.088, size = 276, normalized size = 1.7

$$-\frac{3a^2x^3\mathrm{arccosh}\,(ax)}{(8x^4a^4-16a^2x^2+8)c^3}-\frac{3ax^2}{(8x^4a^4-16a^2x^2+8)c^3}\sqrt{ax-1}\sqrt{ax+1}+\frac{5x\mathrm{arccosh}\,(ax)}{(8x^4a^4-16a^2x^2+8)c^3}+\frac{11}{24a(x^4a^4-2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -3/8*a^2/(a^4*x^4-2*a^2*x^2+1)/c^3*x^3*arccosh(a*x)-3/8*a/(a^4*x^4-2*a^2*x^
2+1)/c^3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2+5/8/(a^4*x^4-2*a^2*x^2+1)/c^3*x*ar
ccosh(a*x)+11/24/a/(a^4*x^4-2*a^2*x^2+1)/c^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+3/
8/a/c^3*arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/8*polylog(2,-a
*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-3/8/a/c^3*arccosh(a*x)*ln(1-a*x-(a*x-
1)^(1/2)*(a*x+1)^(1/2))-3/8*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{10a^3x^3+3(a^4x^4-2a^2x^2+1)\log(ax+1)^2+6(a^4x^4-2a^2x^2+1)\log(ax+1)\log(ax-1)-3(a^4x^4-2a^2x^2+1)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(10*a^3*x^3 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4
 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*l
og(a*x - 1)^2 - 14*a*x + 4*(6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x + 1) + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a
*x + 1)*sqrt(a*x - 1)) - 7*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))/(a^5*c^3
*x^4 - 2*a^3*c^3*x^2 + a*c^3) + 3/16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dil
og(-1/2*a*x + 1/2))/(a*c^3) - 7/64*log(a*x + 1)/(a*c^3) + integrate(-1/16*(
6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*(a^4*x^4
- 2*a^2*x^2 + 1)*log(a*x - 1))/(a^7*c^3*x^7 - 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3
 - a*c^3*x + (a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(a*x +
 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{arcosh}(ax)}{a^6c^3x^6 - 3\,a^4c^3x^4 + 3\,a^2c^3x^2 - c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\operatorname{acosh}(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(acosh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arcosh}(ax)}{\left(a^2cx^2 - c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)/(a^2*c*x^2 - c)^3, x)
```

3.58 $\int x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=278

$$\frac{1}{6}x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{24c^2}-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16c^4}-\frac{\sqrt{d-c^2dx^2}\left(a\right.}{32bc^5\sqrt{c}}$$

```
[Out] (b*x^2*Sqrt[d - c^2*d*x^2])/(32*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^4*
Sqrt[d - c^2*d*x^2])/(96*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^6*Sqrt[d
- c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(16*c^4) - (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
))/(24*c^2) + (x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.782305, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5743, 5759, 5676, 30}

$$\frac{1}{6}x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{24c^2}-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16c^4}-\frac{\sqrt{d-c^2dx^2}\left(a\right.}{32bc^5\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (b*x^2*Sqrt[d - c^2*d*x^2])/(32*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^4*
Sqrt[d - c^2*d*x^2])/(96*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^6*Sqrt[d
- c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(16*c^4) - (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
))/(24*c^2) + (x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\sqrt{d-c^2dx^2}\int x^4\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{1}{6}x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\int\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{6\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(bc\sqrt{d\right.}}{6\sqrt{-}}$$

$$=-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}}-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{24c^2}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}\left(a\right.$$

$$=\frac{bx^4\sqrt{d-c^2dx^2}}{96c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}}-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16c^4}$$

$$=\frac{bx^2\sqrt{d-c^2dx^2}}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx^4\sqrt{d-c^2dx^2}}{96c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}}-\frac{x\sqrt{d-}}{}$$

Mathematica [A] time = 1.14238, size = 198, normalized size = 0.71

$$\frac{48acx\left(8c^4x^4-2c^2x^2-3\right)\sqrt{d-c^2dx^2}-144a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+\frac{b\sqrt{d-c^2dx^2}\left(-72\cosh^{-1}(cx)^2+18\cosh\left(2\cosh^{-1}(cx)\right)-9\cosh\left(4\cos\right.\right.}{}}{2304c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (48*a*c*x*Sqrt[d - c^2*d*x^2]*(-3 - 2*c^2*x^2 + 8*c^4*x^4) - 144*a*Sqrt[d]*
ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*Sqrt[d - c^
2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c
*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] +
3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x)))/(2304*c^5)
```

Maple [A] time = 0.514, size = 449, normalized size = 1.6

$$-\frac{x^3a}{6c^2d}\left(-c^2dx^2+d\right)^{\frac{3}{2}}-\frac{ax}{8dc^4}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+\frac{ax}{16c^4}\sqrt{-c^2dx^2+d}+\frac{ad}{16c^4}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}-\frac{b\,(\mathrm{arcco}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/6*a*x^3*(-c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^4*x*(-c^2*d*x^2+d)^(3/2)/d+1/
16*a/c^4*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^4*d/(c^2*d)^(1/2)*arctan((c^2*d)^(
1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)/c^5*arccosh(c*x)^2-1/36*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)*
c/(c*x-1)^(1/2)*x^6+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)*c^2/(c*x-1)*arccos
h(c*x)*x^7-5/24*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*x^5-1
/48*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x^3+1/16*b*(-
d*(c^2*x^2-1))^(1/2)/(c*x+1)/c^4/(c*x-1)*arccosh(c*x)*x-25/2304*b*(-d*(c^2*
x^2-1))^(1/2)/(c*x+1)^(1/2)/c^5/(c*x-1)^(1/2)+1/96*b*(-d*(c^2*x^2-1))^(1/2)
/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^4+1/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1
/2)/c^3/(c*x-1)^(1/2)*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 \operatorname{arcosh}(cx) + ax^4\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2+d}(b\operatorname{arcosh}\left(cx\right)+a)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*x^4, x)
```

3.59 $\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=201

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^2}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^4\sqrt{d-c^2}}{16\sqrt{cx-1}\sqrt{c}}$$

```
[Out] (b*x^2*Sqrt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^4*
Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(8*c^2) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/4 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.57894, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5743, 5759, 5676, 30}

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^2}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^4\sqrt{d-c^2}}{16\sqrt{cx-1}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (b*x^2*Sqrt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^4*
Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(8*c^2) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/4 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{\sqrt{d-c^2dx^2}\int x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(bc\sqrt{\right.}{4\sqrt{}}$$

$$= -\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^2} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a\right.$$

$$= \frac{bx^2\sqrt{d-c^2dx^2}}{16c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{8c^2}$$

Mathematica [A] time = 1.03536, size = 151, normalized size = 0.75

$$-\frac{-16acx\left(2c^2x^2-1\right)\sqrt{d-c^2dx^2}+16a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+\frac{b\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx)\sinh\left(4\cosh\right.\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}}{128c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(-16*a*c*x*(-1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2] + 16*a*Sqrt[d]*ArcTan[(c*x
*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*Sqrt[d - c^2*d*x^2]*(8
*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]
]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/(128*c^3)
```

Maple [B] time = 0.269, size = 346, normalized size = 1.7

$$-\frac{ax}{4\,c^2d}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+\frac{ax}{8\,c^2}\sqrt{-c^2dx^2+d}+\frac{ad}{8\,c^2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}+\frac{bc^2\mathrm{arccosh}\,(cx)\,x^5}{(4\,cx+4)\,(cx-1)}\sqrt{-d\left(c^2x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a/c^2*x*(-c^2*d*x^2+d)^(1/2)+1/8*a/
c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/4*b*(-d*
```

```
(c^2*x^2-1))^(1/2)/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5-3/8*b*(-d*(c^2*x^2-
1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/8*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x+1)/c^2/(c*x-1)*arccosh(c*x)*x-1/128*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2
)/c^3/(c*x-1)^(1/2)-1/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)/c^3*arccosh(c*x)^2-1/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)*c/(c*x-1)
^(1/2)*x^4+1/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(bx^2\operatorname{arcosh}(cx)+ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2 + d}(b \operatorname{arcosh}(cx) + a)x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*x^2, x)
```

3.60 $\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=124

$$\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -(b*c*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.204154, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5713, 5683, 5676, 30}

$$\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*c*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
```

```
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int \sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(bc\sqrt{d-c^2\right.}{2\sqrt{-1+c}}$$

$$= -\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{4bc\sqrt{-1+cx}\sqrt{}}$$

Mathematica [A] time = 0.568755, size = 144, normalized size = 1.16

$$\frac{1}{8}\left(4ax\sqrt{d-c^2dx^2} - \frac{4a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(c^2x^2-1)}\right)}{c} - \frac{b\sqrt{d-c^2dx^2}\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)-2\cosh^{-1}(cx)\sinh\right.}{c\sqrt{\frac{cx-1}{cx+1}}(cx+1)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (4*a*x*Sqrt[d - c^2*d*x^2] - (4*a*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/
(Sqrt[d]*(-1 + c^2*x^2))])/c - (b*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2 + C
```

```
osh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(c*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)))/8
```

Maple [B] time = 0.161, size = 239, normalized size = 1.9

$$\frac{ax}{2}\sqrt{-c^2dx^2+d}+\frac{ad}{2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}-\frac{b(\operatorname{arccosh}(cx))^2}{4c}\sqrt{-d\left(c^2x^2-1\right)}\frac{1}{\sqrt{cx-1}}\frac{1}{\sqrt{cx+1}}+\frac{bc^2\operatorname{arcc}}{(2cx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(
-c^2*d*x^2+d)^(1/2))-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)/c*arccosh(c*x)^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*c^2*arccos
h(c*x)*x^3-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x^2-1
/2*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*x+1/8*b*(-d*(c^2*x
^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-d(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.61 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=118

$$\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x) + (c*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^
2*d*x^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.367616, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5738, 29, 5676}

$$\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x) + (c*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^
2*d*x^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
```

```
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{1}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d-c^2dx^2}\right)\int$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc\sqrt{d-c}}{\sqrt{-1+}}$$

Mathematica [A] time = 0.432655, size = 137, normalized size = 1.16

$$-\frac{a\sqrt{d-c^2dx^2}}{x}+ac\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+\frac{1}{2}bc\sqrt{d-c^2dx^2}\left(\frac{2\log(cx)+\cosh^{-1}(cx)^2}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}-\frac{2\cosh^{-1}(cx)}{cx}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/x) + a*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])
/(Sqrt[d]*(-1 + c^2*x^2))] + (b*c*Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x])/(c
*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))
)/2
```

Maple [B] time = 0.241, size = 286, normalized size = 2.4

$$-\frac{a}{dx}\left(-c^2dx^2+d\right)^{\frac{3}{2}}-ac^2x\sqrt{-c^2dx^2+d}-ac^2d\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}+\frac{b\left(\operatorname{arccosh}(cx)\right)^2c}{2}\sqrt{-d\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(3/2)-a*c^2*x*(-c^2*d*x^2+d)^(1/2)-a*c^2*d/(c^2*d)^(1
/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c-b*(-d*(c^2*x^2-1))^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c
*x)/(c*x+1)/(c*x-1)*x*c^2+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+1)/(c*
x-1)/x+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2+1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\operatorname{arcosh}(cx)+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^{2}dx^{2}+d}(b\operatorname{arcosh}\left(cx\right)+a)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^2, x)
```

3.62 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=119

$$-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc^3\log(x)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2
*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(3*d*x^3) - (b*c^3*Sqrt[d - c^2*d*x^2]*
Log[x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.285478, antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5798, 5724, 14}

$$-\frac{(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc^3\log(x)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((1 - c*x
)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^3) - (b*c^3*Sqrt
[d - c^2*d*x^2]*Log[x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
```

```
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{-1+c^2x^2}{x^3}\,dx}{3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \left(-\frac{1}{x^3}+\frac{c^2}{x}\right)}{3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{bc^3\sqrt{}}{3\sqrt{}}$$

Mathematica [A] time = 0.121119, size = 88, normalized size = 0.74

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{1}{3}bc\left(c^2\log(x)+\frac{1}{2x^2}\right)\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]
))/(3*x^3) - (b*c*(1/(2*x^2) + c^2*Log[x]))/3))/(Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Maple [B] time = 0.286, size = 1017, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x)
```

```
[Out] -1/3*a/d/x^3*(-c^2*d*x^2+d)^(3/2)+2/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)*arccosh(c*x)*c^3-b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*
x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+b*(-d*(c^2*x^2-1))^
(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-1/6*b*(-
d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3*c^6+1/6*b*(-d*(c^2*x^2-1))
^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8+b*(-d*(c^2*x^2-1))^(
1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c
^5-3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*a
rccosh(c*x)*c^6+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x*c^4-
1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*c^5-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x
+1)/(c*x-1)*c^6-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3+10/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4
*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+1/2*b*(-d*(c^2*x^2-1))
^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+1/6*b*(-d*(c
^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-5/3*b*(-d*(c
^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2
-1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*
x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^3/(c*x+
1)/(c*x-1)*arccosh(c*x)-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.26496, size = 986, normalized size = 8.29

$$\left[\frac{2\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(bc^5x^5-bc^3x^3\right)\sqrt{-d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\left(x^4\right.}{c^2x^4-x^2}\right.}{6\left(c^2x^5-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/6*(2*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 - 1)) + (b*c^5*x^5 - b*c^3*x^3)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2
- d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c
^2*x^4 - x^2)) + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) +
 2*(a*c^4*x^4 - 2*a*c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3), -1/
6*(2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x
^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 2*(b*c^4*x^4
 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - sqr
t(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) - 2*(a*c^4*x^4 - 2*a*
c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^4, x)
```

3.63 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=199

$$-\frac{2c^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{15dx^3}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{5dx^5}+\frac{bc^3\sqrt{d-c^2dx^2}}{30x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}}{20x^4\sqrt{cx-1}\sqrt{cx-}}$$

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(20*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*S
qrt[d - c^2*d*x^2])/(30*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2
)^(3/2)*(a + b*ArcCosh[c*x]))/(5*d*x^5) - (2*c^2*(d - c^2*d*x^2)^(3/2)*(a +
 b*ArcCosh[c*x]))/(15*d*x^3) - (2*b*c^5*Sqrt[d - c^2*d*x^2]*Log[x])/(15*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.345629, antiderivative size = 226, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 97, 12, 103, 95, 5733, 14}

$$\frac{2c^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15x}+\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15x^3}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5}+\frac{bc^3\sqrt{d-c^2}}{30x^2\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(20*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*S
qrt[d - c^2*d*x^2])/(30*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/(5*x^5) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x]))/(15*x^3) + (2*c^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*x
) - (2*b*c^5*Sqrt[d - c^2*d*x^2]*Log[x])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15x^3} + \frac{2c^4\sqrt{d-}}{}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15x^3} + \frac{2c^4\sqrt{d-}}{}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15x^3} + \frac{2c^4\sqrt{d-}}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{30x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{5x^5}$$

Mathematica [A] time = 0.200989, size = 128, normalized size = 0.64

$$\frac{\sqrt{d-c^2dx^2}\left(8c^2x^2(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+12(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)-bcx\left(-2c^2x^2+\right.\right.}{60x^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(12*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*
x]) + 8*c^2*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) - b*c
*x*(3 - 2*c^2*x^2 + 8*c^4*x^4*Log[x])))/(60*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Maple [B] time = 0.335, size = 1741, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x)
```

```
[Out] 12/5*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x/(c*x+1)
/(c*x-1)*arccosh(c*x)*c^4-1/5*a/d/x^5*(-c^2*d*x^2+d)^(3/2)+2*b*(-d*(c^2*x^2
-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1
/2)*arccosh(c*x)*c^7-2*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^
2*x^2+9)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11-27/5*b*(-d*(c^2*
x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^3/(c*x+1)/(c*x-1)*arcco
sh(c*x)*c^2+2*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*
x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12-5/3*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*
x^6-5*c^4*x^4-15*c^2*x^2+9)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10-17/3*b*(-
d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^3/(c*x+1)/(c*x-1
)*arccosh(c*x)*c^8+98/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*
c^2*x^2+9)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+2/15*b*(-d*(c^2*x^2-1))^(1/2)
/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^7*c^12+2/3*b*(-d*(c^2*x^2-1))^(1/2)/
(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh
(c*x)*c^9-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/
(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+4/15*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)*arccosh(c*x)*c^5-2/15*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c^5-9/20*b*(-d*(c^
2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^4/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*c-6/5*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)
/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5+1/2*b*(-d*(c^2*x^2-1))^(1/2)/
(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9-11/
12*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^2/(c*x+1)
^(1/2)/(c*x-1)^(1/2)*c^7+21/20*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x
^4-15*c^2*x^2+9)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-2/15*b*(-d*(c^2*x^2-1)
)^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^9/(c*x+1)/(c*x-1)*c^14+4/15*b
*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^7/(c*x+1)/(c*
x-1)*c^12+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*
x^5/(c*x+1)/(c*x-1)*c^10-3/5*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4
-15*c^2*x^2+9)*x^3/(c*x+1)/(c*x-1)*c^8+3/10*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^
6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x/(c*x+1)/(c*x-1)*c^6+9/5*b*(-d*(c^2*x^2-1))^
(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)-
2/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^5*c^10-
3/10*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^3*c^8+3
/10*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x*c^6-2/15
*a*c^2/d/x^3*(-c^2*d*x^2+d)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28829, size = 1158, normalized size = 5.82

$$\left[\frac{4\left(2\,bc^{6}x^{6}-bc^{4}x^{4}-4\,bc^{2}x^{2}+3\,b\right)\sqrt{-c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)+4\left(bc^{7}x^{7}-bc^{5}x^{5}\right)\sqrt{-d}\log\left(\frac{c^{2}dx^{6}+c^{2}dx^{2}-dx^{4}+\sqrt{-c^{2}d}}{c^{2}x}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x, algorithm="fricas"
)
```

```
[Out] [1/60*(4*(2*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 3*b)*sqrt(-c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1)) + 4*(b*c^7*x^7 - b*c^5*x^5)*sqrt(-d)*log((c^2
*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 -
1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 -
3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 4*(2*a*c^6*x^6 - a*c^4*x^
4 - 4*a*c^2*x^2 + 3*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5), -1/60*(8*(b*c
^7*x^7 - b*c^5*x^5)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(
x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 4*(2*b*c^6*x^6 - b*c^
4*x^4 - 4*b*c^2*x^2 + 3*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)
) - (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sq
rt(c^2*x^2 - 1) - 4*(2*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + 3*a)*sqrt(-c^2
*d*x^2 + d))/(c^2*x^7 - x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^6, x)
```

3.64 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=279

$$-\frac{8c^4\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{105dx^3}-\frac{4c^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{35dx^5}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{7dx^7}+\frac{}{10}$$

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(42*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*S
qrt[d - c^2*d*x^2])/(140*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^5*Sqrt[
d - c^2*d*x^2])/(105*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(
3/2)*(a + b*ArcCosh[c*x]))/(7*d*x^7) - (4*c^2*(d - c^2*d*x^2)^(3/2)*(a + b*
ArcCosh[c*x]))/(35*d*x^5) - (8*c^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x
]))/(105*d*x^3) - (8*b*c^7*Sqrt[d - c^2*d*x^2]*Log[x])/(105*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])
```

Rubi [A] time = 0.377553, antiderivative size = 303, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 97, 12, 103, 95, 5733, 14}

$$\frac{8c^6\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105x}+\frac{4c^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105x^3}+\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5}-\frac{\sqrt{d-c^2dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(42*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*S
qrt[d - c^2*d*x^2])/(140*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^5*Sqrt[
d - c^2*d*x^2])/(105*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(7*x^7) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x]))/(35*x^5) + (4*c^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(105*x^3
) + (8*c^6*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(105*x) - (8*b*c^7*Sqr
t[d - c^2*d*x^2]*Log[x])/(105*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx=\frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7}+\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5}+\frac{4c^4\sqrt{d-}}{}$$

$$=-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7}+\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5}+\frac{4c^4\sqrt{d-}}{}$$

$$=-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7}+\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5}+\frac{4c^4\sqrt{d-}}{}$$

$$=-\frac{bc\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3\sqrt{d-c^2dx^2}}{140x^4\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bc^5\sqrt{d-c^2dx^2}}{105x^2\sqrt{-1+cx}\sqrt{1+cx}}-$$

Mathematica [A] time = 0.257163, size = 146, normalized size = 0.52

$$\frac{\sqrt{d-c^2dx^2}\left(16c^2x^2(cx-1)^{3/2}(cx+1)^{3/2}\left(2c^2x^2+3\right)\left(a+b\cosh^{-1}(cx)\right)+60(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)-b}{420x^7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(60*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*
x]) + 16*c^2*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(3 + 2*c^2*x^2)*(a + b*Ar
cCosh[c*x]) - b*c*x*(10 - 3*c^2*x^2 - 8*c^4*x^4 + 32*c^6*x^6*Log[x])))/(420
*x^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.363, size = 2534, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x)
```

```
[Out] -8/105*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2+1)*c^7-73/20*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-
105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+16/
105*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^7+1
28/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2
*x^2+225)*x^11*c^18-64/3*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-
21*c^4*x^4-315*c^2*x^2+225)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^
15+8*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x
^2+225)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^13+8/5*b*(-d*(c^2*x^
2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^4/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11+24*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^
8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/
2)*arccosh(c*x)*c^9+3057/35*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x
^6-21*c^4*x^4-315*c^2*x^2+225)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-594/35*b*
(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)
/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+342/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8
*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^3/(c*x+1)/(c*x-1)*arccosh(c*
x)*c^4-585/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-3
15*c^2*x^2+225)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2+64/3*b*(-d*(c^2*x^2-1)
)^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^9/(c*x+1)/(c
*x-1)*arccosh(c*x)*c^16-56/3*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*
x^6-21*c^4*x^4-315*c^2*x^2+225)*x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^14-4/15*
b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+22
5)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12-351/5*b*(-d*(c^2*x^2-1))^(1/2)/(28
0*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^3/(c*x+1)/(c*x-1)*arcco
sh(c*x)*c^10+71/28*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4
*x^4-315*c^2*x^2+225)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+255/28*b*(-d*(c^2
*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^4/(c*
x+1)^(1/2)/(c*x-1)^(1/2)*c^3-75/14*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-10
5*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-469
/60*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^
2+225)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9-120/7*b*(-d*(c^2*x^2-1))^(1/2)/(
280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/(c*x+1)^(1/2)/(c*x-1)^(
1/2)*arccosh(c*x)*c^7+16/3*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^
6-21*c^4*x^4-315*c^2*x^2+225)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^13+20/7*b*(
-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*
x/(c*x+1)/(c*x-1)*c^8+225/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x
^6-21*c^4*x^4-315*c^2*x^2+225)/x^7/(c*x+1)/(c*x-1)*arccosh(c*x)-128/105*b*(
-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*
x^13/(c*x+1)/(c*x-1)*c^20+16/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*
```

```
c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^11/(c*x+1)/(c*x-1)*c^18+40/21*b*(-d*(
c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^9/
(c*x+1)/(c*x-1)*c^16+214/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*
x^6-21*c^4*x^4-315*c^2*x^2+225)*x^7/(c*x+1)/(c*x-1)*c^14-152/105*b*(-d*(c^2
*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^5/(c*
x+1)/(c*x-1)*c^12-30/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21
*c^4*x^4-315*c^2*x^2+225)*x^3/(c*x+1)/(c*x-1)*c^10-1/7*a/d/x^7*(-c^2*d*x^2+
d)^(3/2)+16/15*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4
-315*c^2*x^2+225)*x^9*c^16+20/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c
^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x*c^8-10/7*b*(-d*(c^2*x^2-1))^(1/2)/(280
*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^3*c^10-302/105*b*(-d*(c^
2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^5*c^
12-88/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*
c^2*x^2+225)*x^7*c^14-4/35*a*c^2/d/x^5*(-c^2*d*x^2+d)^(3/2)-8/105*a*c^4/d/x
^3*(-c^2*d*x^2+d)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.26926, size = 1320, normalized size = 4.73

$$\left[\frac{4\left(8\,bc^8x^8-4\,bc^6x^6-bc^4x^4-18\,bc^2x^2+15\,b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+16\left(bc^9x^9-bc^7x^7\right)\sqrt{-d}\log\left(\frac{c^2dx^6+c^2}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x, algorithm="fricas"
)
```

```
[Out] [1/420*(4*(8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c^4*x^4 - 18*b*c^2*x^2 + 15*b)*sqr
t(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 16*(b*c^9*x^9 - b*c^7*x^7)
*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^
2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (8*b*c^5*x^5 - (8*b*c
^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^3 - 10*b*c*x)*sqrt(-c^2*d*x^2 + d)*s
qrt(c^2*x^2 - 1) + 4*(8*a*c^8*x^8 - 4*a*c^6*x^6 - a*c^4*x^4 - 18*a*c^2*x^2
+ 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7), -1/420*(32*(b*c^9*x^9 - b*c^
7*x^7)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt
(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 4*(8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c
^4*x^4 - 18*b*c^2*x^2 + 15*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
 1)) - (8*b*c^5*x^5 - (8*b*c^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^3 - 10*b
*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(8*a*c^8*x^8 - 4*a*c^6*x^6
 - a*c^4*x^4 - 18*a*c^2*x^2 + 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2 + d}(b\,\mathrm{arcosh}\,(cx) + a)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^8, x)
```

3.65 $\int x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=272

$$-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^6d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^6d}-\frac{bcx^7\sqrt{}}{49\sqrt{cx-}}$$

```
[Out] (8*b*x*Sqrt[d - c^2*d*x^2])/(105*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*x
^3*Sqrt[d - c^2*d*x^2])/(315*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^5*Sqr
t[d - c^2*d*x^2])/(175*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^7*Sqrt[d -
c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)*(a +
 b*ArcCosh[c*x]))/(3*c^6*d) + (2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])
)/(5*c^6*d^2) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^6*d^3)
```

Rubi [A] time = 0.35236, antiderivative size = 302, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 100, 12, 74, 5733}

$$-\frac{x^4(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{4x^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4}-\frac{8(1-cx)(cx+}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*x*Sqrt[d - c^2*d*x^2])/(105*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*x
^3*Sqrt[d - c^2*d*x^2])/(315*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^5*Sqr
t[d - c^2*d*x^2])/(175*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^7*Sqrt[d -
c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*(1 - c*x)*(1 + c*x)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(105*c^6) - (4*x^2*(1 - c*x)*(1 + c*x
)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*c^4) - (x^4*(1 - c*x)*(1 +
c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rubi steps

$$\int x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\sqrt{d-c^2dx^2}\int x^5\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{8(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105c^6}-\frac{4x^2(1-cx)(1+cx)\sqrt{d-c}}{35c}$$

$$=-\frac{8(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105c^6}-\frac{4x^2(1-cx)(1+cx)\sqrt{d-c}}{35c}$$

$$=\frac{8bx\sqrt{d-c^2dx^2}}{105c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4bx^3\sqrt{d-c^2dx^2}}{315c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx^5\sqrt{d-c^2dx^2}}{175c\sqrt{-1+cx}\sqrt{1+cx}}-$$

Mathematica [A] time = 0.341505, size = 152, normalized size = 0.56

$$\frac{\sqrt{d-c^2dx^2}\left(15c^3x^4(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{4(cx-1)^{3/2}(cx+1)^{3/2}\left(3c^2x^2+2\right)\left(a+b\cosh^{-1}(cx)\right)}{c}+b\left(-\frac{15}{7}c^6x^7+\frac{3c^4x^5}{5}\right.\right.}{105c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (15*c^6*x^7)
/7) + 15*c^3*x^4*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + (4
*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(2 + 3*c^2*x^2)*(a + b*ArcCosh[c*x]))/c))
/(105*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.421, size = 988, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(-1/7*x^4*(-c^2*d*x^2+d)^(3/2)/c^2/d+4/7/c^2*(-1/5*x^2*(-c^2*d*x^2+d)^(3/
2)/c^2/d-2/15/d/c^4*(-c^2*d*x^2+d)^(3/2)))+b*(1/6272*(-d*(c^2*x^2-1))^(1/2)
*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4
```

```
-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))/(c
*x+1)/c^6/(c*x-1)+3/3200*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))/(c*x+1)/
c^6/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccos
h(c*x))/(c*x+1)/c^6/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/(c*x+1)/c^6/(c*x-1)-5/128*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))/(c*x+1)/c^6/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1
)*(1+3*arccosh(c*x))/(c*x+1)/c^6/(c*x-1)+3/3200*(-d*(c^2*x^2-1))^(1/2)*(-16
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5
*arccosh(c*x))/(c*x+1)/c^6/(c*x-1)+1/6272*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))/(c*x+1)/c^6/(
c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87733, size = 448, normalized size = 1.65

$$\frac{105\left(15\,bc^8x^8-18\,bc^6x^6-bc^4x^4-4\,bc^2x^2+8\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(225\,bc^7x^7-63\,bc^5x^5-140\,bc^3x^3-\right.}{11025\left(c^8x^2-c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/11025*(105*(15*b*c^8*x^8 - 18*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 8*b)*
sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (225*b*c^7*x^7 - 63*b*c
^5*x^5 - 140*b*c^3*x^3 - 840*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
+ 105*(15*a*c^8*x^8 - 18*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + 8*a)*sqrt(-c
^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.66 $\int x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=195

$$\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^4d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^4d}-\frac{bcx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx^3\sqrt{d-c^2dx^2}}{45c\sqrt{cx-1}\sqrt{cx+1}}+\frac{2l}{15c^3}$$

```
[Out] (2*b*x*Sqrt[d - c^2*d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^3*
Sqrt[d - c^2*d*x^2])/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^5*Sqrt[d
- c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)*(a
 + b*ArcCosh[c*x]))/(3*c^4*d) + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])
)/(5*c^4*d^2)
```

Rubi [A] time = 0.323137, antiderivative size = 214, normalized size of antiderivative = 1.1, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 100, 12, 74, 5733}

$$-\frac{x^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}-\frac{2(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4}-\frac{bcx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*x*Sqrt[d - c^2*d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^3*
Sqrt[d - c^2*d*x^2])/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^5*Sqrt[d
- c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*(1 - c*x)*(1 + c*x)*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*c^4) - (x^2*(1 - c*x)*(1 + c*x)
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rubi steps

$$\int x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\sqrt{d-c^2dx^2}\int x^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4}-\frac{x^2(1-cx)(1+cx)\sqrt{d-c^2dx}}{5c^2}$$

$$=-\frac{2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4}-\frac{x^2(1-cx)(1+cx)\sqrt{d-c^2dx}}{5c^2}$$

$$=\frac{2bx\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx^3\sqrt{d-c^2dx^2}}{45c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2(1-}{}$$

Mathematica [A] time = 0.179567, size = 128, normalized size = 0.66

$$\frac{\sqrt{d-c^2dx^2}\left(3c^2x^2(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+2(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{15}bcx\left(-9c^4x^4+\right.\right.}{15c^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*((b*c*x*(30 + 5*c^2*x^2 - 9*c^4*x^4))/15 + 2*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + 3*c^2*x^2*(-1 + c*x)^(3/2)*(
1 + c*x)^(3/2)*(a + b*ArcCosh[c*x])))/(15*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.353, size = 640, normalized size = 3.3

$$a\left(-\frac{x^2}{5\,c^2d}\left(-c^2dx^2+d\right)^{\frac{3}{2}}-\frac{2}{15\,dc^4}\left(-c^2dx^2+d\right)^{\frac{3}{2}}\right)+b\left(\frac{-1+5\,\mathrm{arccosh}\,(cx)}{(800\,cx+800)\,c^4\,(cx-1)}\sqrt{-d\left(c^2x^2-1\right)}\left(16\,c^6x^6-28\,c^4x^4+16\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(-1/5*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(-c^2*d*x^2+d)^(3/2))+b*(
1/800*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/288*(
```

```
-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^4/
(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^
2-1)*(-1+arccosh(c*x))/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^4/(c*x-
1)+1/288*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c
^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c
*x+1)/c^4/(c*x-1)+1/800*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))/(c*x+1)/c
^4/(c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83329, size = 373, normalized size = 1.91

$$\frac{15\left(3\,bc^{6}x^{6}-4\,bc^{4}x^{4}-bc^{2}x^{2}+2\,b\right)\sqrt{-c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-\left(9\,bc^{5}x^{5}-5\,bc^{3}x^{3}-30\,bcx\right)\sqrt{-c^{2}dx^{2}+d}\sqrt{c^{2}x^{2}}}{225\left(c^{6}x^{2}-c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/225*(15*(3*b*c^6*x^6 - 4*b*c^4*x^4 - b*c^2*x^2 + 2*b)*sqrt(-c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^5*x^5 - 5*b*c^3*x^3 - 30*b*c*x)*sqr
t(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(3*a*c^6*x^6 - 4*a*c^4*x^4 - a*c^2
*x^2 + 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.67 $\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=118

$$-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}-\frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^3*Sqr
t[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)
*(a + b*ArcCosh[c*x]))/(3*c^2*d)
```

Rubi [A] time = 0.211942, antiderivative size = 126, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {5798, 5718}

$$-\frac{(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2}-\frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^3*Sqr
t[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((1 - c*x)*(1 + c*x)*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
```

```
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2} - \frac{\left(b\sqrt{d-c^2dx^2}\right)\int\left(-1+c^2x^2\right)}{3c\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{c}\right.}{3c^2}$$

Mathematica [A] time = 0.128441, size = 98, normalized size = 0.83

$$\frac{\sqrt{d-c^2dx^2}\left(3a\left(c^2x^2-1\right)^2+bcx\sqrt{cx-1}\sqrt{cx+1}\left(3-c^2x^2\right)+3b\left(c^2x^2-1\right)^2\cosh^{-1}(cx)\right)}{9c^2\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - c^2*x^2) + 3*
a*(-1 + c^2*x^2)^2 + 3*b*(-1 + c^2*x^2)^2*ArcCosh[c*x]))/(9*c^2*(-1 + c^2*x
^2))
```

Maple [B] time = 0.244, size = 356, normalized size = 3.

$$-\frac{a}{3c^2d}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+b\left(\frac{-1+3\,\mathrm{arccosh}\,(cx)}{(72\,cx+72)\,c^2\,(cx-1)}\sqrt{-d\left(c^2x^2-1\right)}\left(4\,c^4x^4-5\,c^2x^2+4\,\sqrt{cx+1}\sqrt{cx-1}x^3c^3-3\,\sqrt{cx+1}\sqrt{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a/c^2/d*(-c^2*d*x^2+d)^(3/2)+b*(1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4
-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/
2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/(c*x+1)/c^
2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*
x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)+1/72*(-d*(c^2*x^2-1))^(1/2)*(-4
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c*x+1)/c^2/(c*x-1))
```

Maxima [A] time = 1.13872, size = 109, normalized size = 0.92

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}b\,\text{arcosh}\,(cx)}{3\,c^2d}-\frac{\left(c^2\sqrt{-d}dx^3-3\,\sqrt{-d}dx\right)b}{9\,cd}-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}a}{3\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-c^2*d*x^2 + d)^(3/2)*b*arccosh(c*x)/(c^2*d) - 1/9*(c^2*sqrt(-d)*d*x^
3 - 3*sqrt(-d)*d*x)*b/(c*d) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

Fricas [A] time = 1.77673, size = 301, normalized size = 2.55

$$\frac{3\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^3x^3-3\,bcx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}+3\left(ac^4x^4-2\,ac^2x^2\right.}{9\left(c^4x^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 - 1)) - (b*c^3*x^3 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
+ 3*(a*c^4*x^4 - 2*a*c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\text{acosh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.68 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=213

$$\frac{ib\sqrt{d-c^2dx^2}\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{ib\sqrt{d-c^2dx^2}\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{2\sqrt{}}{}$$

```
[Out] -((b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]) - (2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b*Sqrt[d - c^2
*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
I*b*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rubi [A] time = 0.524453, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5743, 5761, 4180, 2279, 2391, 8}

$$\frac{ib\sqrt{d-c^2dx^2}\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{ib\sqrt{d-c^2dx^2}\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -((b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]) - (2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b*Sqrt[d - c^2
*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
I*b*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(bc\sqrt{d-c^2dx\right.}{\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int(a\right.}{\sqrt{-1}}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{\sqrt{-1+c}}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{\sqrt{-1+c}}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{\sqrt{-1+c}}$$

Mathematica [A] time = 0.834212, size = 233, normalized size = 1.09

$$\frac{b\sqrt{d-c^2dx^2}\left(i\mathrm{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right) - i\mathrm{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right) - cx + cx\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx) + \sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx)\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] a*Sqrt[d - c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d
 - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)
]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*
x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I
*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.247, size = 394, normalized size = 1.9

$$-\sqrt{d}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{-c^2dx^2+d}\right)\right)a+a\sqrt{-c^2dx^2+d}+\frac{bx^2\mathrm{arccosh}\,(cx)\,c^2}{(cx+1)\,(cx-1)}\sqrt{-d\left(c^2x^2-1\right)}-xbc\sqrt{-d\left(c^2x^2-1\right)}\frac{1}{\sqrt{cx-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] -d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)*a+a*(-c^2*d*x^2+d)^(1/2
)+b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2*c^2-b*(-d*(c^2*
x^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x*c-b*(-d*(c^2*x^2-1))^(1/2)/(c*x
+1)/(c*x-1)*arccosh(c*x)+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^
2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*
dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}{(cx)})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x, x)
```

3.69 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=235

$$-\frac{ibc^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{ibc^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+$$

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*x^2) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/2
)*b*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + ((I/2)*b*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[
c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.52011, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5738, 30, 5761, 4180, 2279, 2391}

$$-\frac{ibc^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{ibc^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^3,x]
```


```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*x^2) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/2
)*b*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + ((I/2)*b*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[
c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d-c^2dx^2}\right)\int \frac{a}{x}}{2\sqrt{-1+cx}}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{\left(c^2\sqrt{d-c^2dx^2}\right)\text{Subst}}{2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{\sqrt{-1+}}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{\sqrt{-1+}}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{\sqrt{-1+}}$$

Mathematica [A] time = 1.02351, size = 307, normalized size = 1.31

$$\frac{1}{2}\left(\frac{bd(cx+1)\left(ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx)\,\mathrm{lo}}{x^2\sqrt{d-c^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-((a*Sqrt[d - c^2*d*x^2])/x^2) - a*c^2*Sqrt[d]*Log[x] + a*c^2*Sqrt[d]*Log[
d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1 +
 c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c
*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1
+ c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/
(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 +
```

```
c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

Maple [A] time = 0.266, size = 438, normalized size = 1.9

$$-\frac{a}{2\,dx^2}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+\frac{ac^2}{2}\sqrt{d}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{-c^2dx^2+d}\right)\right)-\frac{ac^2}{2}\sqrt{-c^2dx^2+d}-\frac{b\mathrm{arccosh}\,(cx)\,c^2}{(2\,cx+2)\,(cx-1)}\sqrt{-d\left(c^2x^2-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(3/2)+1/2*a*d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x
^2+d)^(1/2))/x)*c^2-1/2*a*(-c^2*d*x^2+d)^(1/2)*c^2-1/2*b*(-d*(c^2*x^2-1))^(
1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/x/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)/x^2/(c*x+1)/(c*x-1)*ar
ccosh(c*x)-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arcco
sh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^2-
1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2)))*c^2-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^2
-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^3, x)
```

3.70 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=315

$$-\frac{ibc^4\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{ibc^4\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{8x^2}$$

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(12*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*S
qrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(4*x^4) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x]))/(8*x^2) + (c^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^Arc
Cosh[c*x]])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/8)*b*c^4*Sqrt[d - c^2*d*
x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((I/
8)*b*c^4*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x])
```

Rubi [A] time = 0.743373, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5738, 30, 5748, 5761, 4180, 2279, 2391}

$$-\frac{ibc^4\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{ibc^4\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^5,x]
```


```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(12*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*S
qrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(4*x^4) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x]))/(8*x^2) + (c^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^Arc
Cosh[c*x]])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/8)*b*c^4*Sqrt[d - c^2*d*
x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((I/
8)*b*c^4*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x])
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
```

```
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{1}{x^4}\,dx}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d-c^2dx^2}\right)\int}{4\sqrt{-1+c}}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+\right.}{8x^2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{4x^4}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{4x^4}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{4x^4}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{4x^4}$$

Mathematica [A] time = 1.0241, size = 290, normalized size = 0.92

$$\frac{1}{24}\left(\frac{b\sqrt{d-c^2dx^2}\left(-3ic^4x^4\left(\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)+3c^3x^3+3c^2x^2\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh\right.}{x^4\sqrt{\frac{cx-1}{cx+1}}(c}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] ((3*a*(-2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^4 - 3*a*c^4*Sqrt[d]*Log[x] + 3*
a*c^4*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^2]
*(-2*c*x + 3*c^3*x^3 - 6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]
+ 3*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - (3*I)*c^4*x
^4*ArcCosh[c*x]*(Log[1 - I/E^ArcCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]) - (
3*I)*c^4*x^4*(PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]
])))/(x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/24
```

Maple [A] time = 0.327, size = 541, normalized size = 1.7

$$-\frac{a}{4\,dx^4}\left(-c^2dx^2+d\right)^{\frac{3}{2}}-\frac{ac^2}{8\,dx^2}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+\frac{ac^4}{8}\sqrt{d}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{-c^2dx^2+d}\right)\right)-\frac{ac^4}{8}\sqrt{-c^2dx^2+d}+\frac{b\text{arccosh}}{(8\,cx+8)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x)
```

```
[Out] -1/4*a/d/x^4*(-c^2*d*x^2+d)^(3/2)-1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(3/2)+1/8*
a*c^4*d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-1/8*a*c^4*(-c^2*d*
x^2+d)^(1/2)+1/8*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+
1/8*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/x/(c*x-1)^(1/2)*c^3-3/8*b*(-d*(c
^2*x^2-1))^(1/2)/(c*x+1)/x^2/(c*x-1)*arccosh(c*x)*c^2-1/12*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x+1)^(1/2)/x^3/(c*x-1)^(1/2)*c+1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x+1)/x^4/(c*x-1)*arccosh(c*x)-1/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4+1/
8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4-1/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4+1
/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\text{acosh}\,(cx))}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^5, x)
```

3.71 $\int x^4 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=360

$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{16}dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{64c^2}-\frac{3d}{}$$

```
[Out] (3*b*d*x^2*Sqrt[d - c^2*d*x^2])/(256*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*d*x^4*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*x
^6*Sqrt[d - c^2*d*x^2])/(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^8*Sq
rt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*d*x*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^4) - (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/(64*c^2) + (d*x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/
16 + (x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/8 - (3*d*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])^2)/(256*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.06425, antiderivative size = 372, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5745, 5743, 5759, 5676, 30, 14}

$$\frac{1}{16}dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}dx^5(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{64c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d*x^2*Sqrt[d - c^2*d*x^2])/(256*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*d*x^4*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*x
^6*Sqrt[d - c^2*d*x^2])/(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^8*Sq
rt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*d*x*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^4) - (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/(64*c^2) + (d*x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/
16 + (d*x^5*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8
 - (3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(256*b*c^5*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
```

```
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int x^4\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^4(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{8}dx^5(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{\left(3d\sqrt{d-c^2dx^2}\right)\int x^4$$

$$= \frac{1}{16}dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}dx^5(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a$$

$$= -\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^8\sqrt{d-c^2dx^2}}{64\sqrt{-1+cx}\sqrt{1+cx}}-\frac{dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{64c^2}$$

$$= \frac{bdx^4\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^8\sqrt{d-c^2dx^2}}{64\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3d}{}$$

$$= \frac{3bdx^2\sqrt{d-c^2dx^2}}{256c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bdx^4\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{-1+cx}\sqrt{1+cx}}+$$

Mathematica [A] time = 4.51278, size = 337, normalized size = 0.94

$$d\left(-576acx\left(16c^6x^6-24c^4x^4+2c^2x^2+3\right)\sqrt{d-c^2dx^2}-1728a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+\frac{32b\sqrt{d-c^2dx^2}\left(-72\cosh^{-1}(cx)^2+18\cosh(\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(-576*a*c*x*Sqrt[d - c^2*d*x^2]*(3 + 2*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6
) - 1728*a*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2)
)] + (32*b*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]
] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*S
inh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqr
t[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*Sqrt[d - c^2*d*x^2]*(1440*ArcCosh[c
*x]^2 - 576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cosh[6*Arc
Cosh[c*x]] + 9*Cosh[8*ArcCosh[c*x]] - 24*ArcCosh[c*x]*(-48*Sinh[2*ArcCosh[c
*x]] + 24*Sinh[4*ArcCosh[c*x]] + 16*Sinh[6*ArcCosh[c*x]] + 3*Sinh[8*ArcCosh
[c*x]])))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(73728*c^5)
```

Maple [A] time = 0.353, size = 561, normalized size = 1.6

$$-\frac{x^3 a}{8\,c^2 d}\left(-c^2 d x^2+d\right)^{\frac{5}{2}}-\frac{ax}{16\,dc^4}\left(-c^2 d x^2+d\right)^{\frac{5}{2}}+\frac{ax}{64\,c^4}\left(-c^2 d x^2+d\right)^{\frac{3}{2}}+\frac{3\,adx}{128\,c^4}\sqrt{-c^2 d x^2+d}+\frac{3\,ad^2}{128\,c^4}\arctan\left(x\sqrt{c^2 d}\frac{}{\sqrt{-c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/8*a*x^3*(-c^2*d*x^2+d)^(5/2)/c^2/d-1/16*a/c^4*x*(-c^2*d*x^2+d)^(5/2)/d+1
/64*a/c^4*x*(-c^2*d*x^2+d)^(3/2)+3/128*a/c^4*d*x*(-c^2*d*x^2+d)^(1/2)+3/128
*a/c^4*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/8*b
*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^9+5/16*b*(-d*(
c^2*x^2-1))^(1/2)*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^7-13/64*b*(-d*(c^2*x
^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^5-1/128*b*(-d*(c^2*x^2-1))^(1
/2)*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x^3+3/128*b*(-d*(c^2*x^2-1))^(1/2)*d
/(c*x+1)/c^4/(c*x-1)*arccosh(c*x)*x-15/8192*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x
+1)^(1/2)/c^5/(c*x-1)^(1/2)-3/256*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)/c^5*arccosh(c*x)^2*d+1/64*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1
/2)*c^3/(c*x-1)^(1/2)*x^8-1/32*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c/(
c*x-1)^(1/2)*x^6+1/256*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c/(c*x-1)^(
1/2)*x^4+3/256*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)*x
^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(ac^2dx^6 - adx^4 + \left(bc^2dx^6 - bdx^4\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^6 - a*d*x^4 + (b*c^2*d*x^6 - b*d*x^4)*arccosh(c*x))*sq
rt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2 + d\right)^{\frac{3}{2}}(b\,\text{arcosh}\left(cx\right) + a)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*x^4, x)
```

3.72 $\int x^2 \left(d-c^2 d x^2\right)^{3/2} \left(a+b \cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=281

$$\frac{1}{6} x^3 \left(d-c^2 d x^2\right)^{3/2} \left(a+b \cosh^{-1}(cx)\right)+\frac{1}{8} d x^3 \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right)-\frac{d x \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right)}{16 c^2}-\frac{d \sqrt{d}}{}$$

```
[Out] (b*d*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c*
d*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^6
*Sqrt[d - c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*x*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c^2) + (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/8 + (x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/6 - (d*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])
```

Rubi [A] time = 0.840716, antiderivative size = 293, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5745, 5743, 5759, 5676, 30, 14}

$$\frac{1}{8} d x^3 \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right)+\frac{1}{6} d x^3 (1-cx)(cx+1) \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right)-\frac{d x \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}\right.}{16 c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c*
d*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^6
*Sqrt[d - c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*x*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c^2) + (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/8 + (d*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x]))/6 - (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c^3*S
qrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
```

```
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{1}{6}dx^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^2\cdots}{\cdots}$$

$$=\frac{1}{8}dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}dx^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+\cdots\right.$$

$$=-\frac{7bcdx^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}}-\frac{dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{\cdots}\right.}{16c^2}$$

$$=\frac{bdx^2\sqrt{d-c^2dx^2}}{32c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{7bcdx^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}}-\frac{dx\cdots}{\cdots}$$

Mathematica [A] time = 1.84984, size = 270, normalized size = 0.96

$$\frac{d\left(-48acx\left(8c^4x^4-14c^2x^2+3\right)\sqrt{d-c^2dx^2}-144a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-\frac{18b\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh\cdots\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(-48*a*c*x*Sqrt[d - c^2*d*x^2]*(3 - 14*c^2*x^2 + 8*c^4*x^4) - 144*a*Sqrt
[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - (18*b*Sqrt
[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*S
inh[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*Sqrt[d -
c^2*d*x^2]*(72*ArcCosh[c*x]^2 - 18*Cosh[2*ArcCosh[c*x]] + 9*Cosh[4*ArcCosh[
c*x]] + 2*Cosh[6*ArcCosh[c*x]] - 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] +
 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x))))/(2304*c^3)
```

Maple [A] time = 0.289, size = 456, normalized size = 1.6

$$-\frac{ax}{6\,c^2d}\left(-c^2dx^2+d\right)^{\frac{5}{2}}+\frac{ax}{24\,c^2}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+\frac{adx}{16\,c^2}\sqrt{-c^2dx^2+d}+\frac{ad^2}{16\,c^2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}-\frac{bdc^4\mathrm{arc}}{(6\,cx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/6*a*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a/c^2*x*(-c^2*d*x^2+d)^(3/2)+1/16*
a/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^
(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^4/(c
*x-1)*arccosh(c*x)*x^7+11/24*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^2/(c*x-1)
*arccosh(c*x)*x^5-17/48*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(
c*x)*x^3+1/16*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x
+7/2304*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)-1/32*b*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*d+1/36
*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-7/96*b*(-d*
(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+1/32*b*(-d*(c^2*x^2-
1))^(1/2)*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(ac^2dx^4 - adx^2 + \left(bc^2dx^4 - bdx^2\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^4 - a*d*x^2 + (b*c^2*d*x^4 - b*d*x^2)*arccosh(c*x))*sq
rt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2 + d\right)^{\frac{3}{2}}(b\,\text{arcosh}\left(cx\right) + a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*x^2, x)
```

3.73 $\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=200

$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3dx^4}{16\sqrt{cx}}$$

```
[Out] (-5*b*c*d*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c
^3*d*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*x*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (x*(d - c^2*d*x^2)^(3/2)*(a + b
*ArcCosh[c*x]))/4 - (3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.321291, antiderivative size = 212, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5713, 5685, 5683, 5676, 30, 14}

$$\frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dx(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{16bc\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5*b*c*d*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c
^3*d*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*x*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (d*x*(1 - c*x)*(1 + c*x)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (3*d*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x])^2)/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}dx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{\left(3d\sqrt{d-c^2dx^2}\right)\int\sqrt{-1+}}{4\sqrt{-1}}$$

$$= \frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cos\right.$$

$$= -\frac{5bcdx^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.$$

Mathematica [A] time = 1.19486, size = 235, normalized size = 1.18

$$-\frac{3ad^{3/2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(c^2x^2-1)}\right)}{8c}-\frac{1}{8}adx\left(2c^2x^2-5\right)\sqrt{d-c^2dx^2}-\frac{bd\sqrt{d-c^2dx^2}\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)-2\cos\right.}{8c\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(a*d*x*(-5 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/8 - (3*a*d^(3/2)*ArcTan[(c*x*
Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/(8*c) - (b*d*Sqrt[d - c^2*d
*x^2]*(2*ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcC
osh[c*x]]))/(8*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*d*Sqrt[d - c^2*
d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*Arc
Cosh[c*x]]))/(128*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] time = 0.155, size = 344, normalized size = 1.7

$$\frac{ax}{4}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+\frac{3\,adx}{8}\sqrt{-c^2dx^2+d}+\frac{3\,ad^2}{8}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}-\frac{5\,bdcx^2}{16}\sqrt{-d\left(c^2x^2-1\right)}\frac{1}{\sqrt{cx-1}}\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/4*a*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*d^2/(c^2*
d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-5/16*b*(-d*(c^2*x^2-1
))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x^2+1/16*b*(-d*(c^2*x^2-1))^(1/2)*
d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3*x^4-1/4*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1
)/(c*x-1)*c^4*arccosh(c*x)*x^5+7/8*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-
1)*c^2*arccosh(c*x)*x^3-5/8*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arcc
osh(c*x)*x+17/128*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c-
3/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2*
d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}\left(a + b\operatorname{acosh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)

[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")

[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a), x)
```

3.74 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=197

$$-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{bc^3d}{4\sqrt{c}}$$

```
[Out] (b*c^3*d*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2
*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - ((d - c^2*d*x^2)^(3/2)*(
a + b*ArcCosh[c*x]))/x + (3*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)
/(4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d*Sqrt[d - c^2*d*x^2]*Log[x])/(S
qrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.536116, antiderivative size = 209, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 5740, 5683, 5676, 30, 14}

$$-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4b\sqrt{cx-1}\sqrt{cx+1}}-\frac{d(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (b*c^3*d*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2
*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (d*(1 - c*x)*(1 + c*x)*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x + (3*c*d*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])^2)/(4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d*Sqrt[d - c
^2*d*x^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} - \frac{\left(bcd\sqrt{d-c^2dx^2}\right)\int \frac{-1+c^2x}{x}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{c}\right.}{x}$$

$$= \frac{bc^3dx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d(1-cx)(1+cx}{}$$

Mathematica [A] time = 1.01072, size = 223, normalized size = 1.13

$$\frac{1}{8}\left(12acd^{3/2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - \frac{4ad\left(c^2x^2+2\right)\sqrt{d-c^2dx^2}}{x} + 4bcd\sqrt{d-c^2dx^2}\left(\frac{2\log(cx)+\cosh^{-1}(cx)^2}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)} - \frac{2\cosh^{-1}}{cx}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] ((-4*a*d*(2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/x + 12*a*c*d^(3/2)*ArcTan[(c*x*
Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 4*b*c*d*Sqrt[d - c^2*d*x^2
]*((-2*ArcCosh[c*x])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x))) + (b*c*d*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2 + Co
sh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)))/8
```

Maple [B] time = 0.204, size = 427, normalized size = 2.2

$$-\frac{a}{dx}\left(-c^2dx^2+d\right)^{\frac{5}{2}} - ac^2x\left(-c^2dx^2+d\right)^{\frac{3}{2}} - \frac{3\,dac^2x}{2}\sqrt{-c^2dx^2+d} - \frac{3\,ac^2d^2}{2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}} + \frac{3\,b\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(5/2)-a*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a*c^2*d*x*(-c^
2*d*x^2+d)^(1/2)-3/2*a*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d
*x^2+d)^(1/2))+3/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arc
cosh(c*x)^2*c*d-1/2*b*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)*arccosh(
c*x)*x^3+1/4*b*(-d*(c^2*x^2-1))^(1/2)*c^3*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2
-b*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)-1/2*
b*(-d*(c^2*x^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x-1/8*b*(-d*(c^
2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)*ar
ccosh(c*x)*d/(c*x+1)/(c*x-1)/x+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\text{arcosh}\,(cx)\right)\sqrt{-c^2dx^2 + d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\text{acosh}\,(cx)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)/x^2, x)
```

3.75 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=203

$$-\frac{c^3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{bcd\sqrt{d}}{6x^2\sqrt{cx-}}$$

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*d*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x - ((d - c^2*d*x^2)^(3/2)*(a + b
*ArcCosh[c*x]))/(3*x^3) - (c^3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2
)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c^3*d*Sqrt[d - c^2*d*x^2]*Log[x
])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.633683, antiderivative size = 215, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 5740, 5738, 29, 5676, 14}

$$-\frac{c^3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*d*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x - (d*(1 - c*x)*(1 + c*x)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^3) - (c^3*d*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c^3*d*Sqrt[
d - c^2*d*x^2]*Log[x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{\left(bcd\sqrt{d-c^2dx^2}\right)\int \frac{-1+c^2x^2}{x^3}}{3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} - \frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{3x^3}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} - \frac{d(1-cx)(1+cx}{}$$

Mathematica [A] time = 0.762843, size = 259, normalized size = 1.28

$$\frac{d^2\left(-2a\sqrt{\frac{cx-1}{cx+1}}\left(4c^4x^4-5c^2x^2+1\right)+8bc^3x^3(cx-1)\log(cx)+bcx(cx-1)\right)-6ac^3d^{3/2}x^3\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx}}{\sqrt{d}(c^2x^2-}\right.}{6x^3\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (-2*b*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 - 5*c^2*x^2 + 4*c^4*x^4)*ArcCosh[c*
x] + 3*b*c^3*d^2*x^3*(-1 + c*x)*ArcCosh[c*x]^2 - 6*a*c^3*d^(3/2)*x^3*Sqrt[(
-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(
Sqrt[d]*(-1 + c^2*x^2))] + d^2*(b*c*x*(-1 + c*x) - 2*a*Sqrt[(-1 + c*x)/(1 +
 c*x)]*(1 - 5*c^2*x^2 + 4*c^4*x^4) + 8*b*c^3*x^3*(-1 + c*x)*Log[c*x]))/(6*x
^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.227, size = 1181, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] -1/3*a/d/x^3*(-c^2*d*x^2+d)^(5/2)+2/3*a*c^2/d/x*(-c^2*d*x^2+d)^(5/2)+2/3*a*
c^4*x*(-c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+a*c^4*d^2/(c^2*d)
^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(c^2*x^2-1))^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c^3*d+8/3*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^3*d-32*b*(-d*(c^2*x^2-1
))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh
(c*x)*c^7+32*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1
)/(c*x-1)*arccosh(c*x)*c^8-8/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)*x^3*c^6+8/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5
/(c*x+1)/(c*x-1)*c^8+12*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)
*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-52*b*(-d*(c^2*x^2-1))^(1/
2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+2/3*b*(-
d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x*c^4-4*b*(-d*(c^2*x^2-1))^
(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-10/3*b
*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6-
4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*arccosh(c*x)*c^3+73/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+3/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(
24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+2/3*b*(-d*(c^2*x^2-
1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-14/3*b*(-d*(c^2*
x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2
-1/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^3
/(c*x+1)/(c*x-1)*arccosh(c*x)-4/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c^3*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)/x^4, x)
```

3.76 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5dx^5}+\frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^5d\log(x)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(20*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3
*d*Sqrt[d - c^2*d*x^2])/(5*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*
x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(5*d*x^5) + (b*c^5*d*Sqrt[d - c^2*d*x^2]*L
og[x])/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.354473, antiderivative size = 179, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5724, 266, 43}

$$-\frac{d(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5}+\frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^5d\log(x)\sqrt{d-c^2}}{5\sqrt{cx-1}\sqrt{cx+}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(20*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3
*d*Sqrt[d - c^2*d*x^2])/(5*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(1 - c*x)
^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*x^5) + (b*c^5*d
*Sqrt[d - c^2*d*x^2]*Log[x])/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
```

```
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd\sqrt{d-c^2dx^2}\right)\int \frac{(-1+c}{x^5}}{5\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\right.}{10\sqrt{-1+cx}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\right.}{10\sqrt{-1+}}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2}}{5x^5}$$

Mathematica [A] time = 0.0765298, size = 94, normalized size = 0.57

$$-\frac{d\sqrt{d-c^2dx^2}\left(4(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+bcx\left(-4c^2x^2-4c^4x^4\log(x)+1\right)\right)}{20x^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(4*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[
c*x]) + b*c*x*(1 - 4*c^2*x^2 - 4*c^4*x^4*Log[x])))/(20*x^5*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Maple [B] time = 0.256, size = 2171, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x)
```

```
[Out] 2*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*
x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-b*(-d*(c^2*x^2-1))^(1/2)*d
/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1
/2)*arccosh(c*x)*c^7+b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^
4*x^4-5*c^2*x^2+1)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^13-2*b*(-
d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^6/(c
*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11-b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c
^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*
c^14+5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^
2+1)*x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12-11*b*(-d*(c^2*x^2-1))^(1/2)*d/(5
*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x
)*c^10+14*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2
*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-56/5*b*(-d*(c^2*x^2-1))^(1/2)*
d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c
*x)*c^6+28/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*
c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-8/5*b*(-d*(c^2*x^2-1))^(1/2)*
d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh
(c*x)*c^2+3/10*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-
5*c^2*x^2+1)*x^3*c^8-1/5*a/d/x^5*(-c^2*d*x^2+d)^(5/2)-2/5*b*(-d*(c^2*x^2-1)
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^5*d-3/4*b*(-d*(c^2*x^2-1
```

```
))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1
)*c^10+7/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c
^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^8-1/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^
8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^6+1/5*b*(-d*(c^2*x
^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x^5/(c*x+1)/(c
*x-1)*arccosh(c*x)-1/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*
c^4*x^4-5*c^2*x^2+1)*x^9/(c*x+1)/(c*x-1)*c^14+13/20*b*(-d*(c^2*x^2-1))^(1/2
)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^7/(c*x+1)/(c*x-1)*c^12+
1/5*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2+1)*c^5*d-3/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*
c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-9/4*b*(-d*(
c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^4/(c*x+
1)^(1/2)/(c*x-1)^(1/2)*c^9+5/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6
*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+9/20*b*(-d
*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x^2/(c*
x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*
c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+1/5*b*(-d
*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/(c*x+1)
^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5+b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8
-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^11-1/
20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)
*x*c^6+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^
2*x^2+1)*x^7*c^12-9/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*
c^4*x^4-5*c^2*x^2+1)*x^5*c^10
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69814, size = 1207, normalized size = 7.27

$$\left[-\frac{4\left(bc^6dx^6-3\,bc^4dx^4+3\,bc^2dx^2-bd\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-2\left(bc^7dx^7-bc^5dx^5\right)\sqrt{-d}\log\left(\frac{c^2dx^6+c^2dx^2-dx}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="fricas"
)
```

```
[Out] [-1/20*(4*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*x
^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^7*d*x^7 - b*c^5*d*x^5)*sqrt(-
d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 -
 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) - (4*b*c^3*d*x^3 - (4*b*c^3 -
b*c)*d*x^5 - b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 4*(a*c^6*d*x
^6 - 3*a*c^4*d*x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 -
x^5), 1/20*(4*(b*c^7*d*x^7 - b*c^5*d*x^5)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 +
d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) -
 4*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1)) + (4*b*c^3*d*x^3 - (4*b*c^3 - b*c)*d*x^5 - b*
c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(a*c^6*d*x^6 - 3*a*c^4*d*
x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)/x^6, x)
```

3.77 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=247

$$-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{35dx^5}-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{7dx^7}-\frac{bc^5d\sqrt{d-c^2dx^2}}{70x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bc^3d\sqrt{d-c^2dx^2}}{35x^4\sqrt{cx-1}\sqrt{cx}}$$

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(42*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c
^3*d*Sqrt[d - c^2*d*x^2])/(35*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d*
Sqrt[d - c^2*d*x^2])/(70*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^
2)^(5/2)*(a + b*ArcCosh[c*x]))/(7*d*x^7) - (2*c^2*(d - c^2*d*x^2)^(5/2)*(a
+ b*ArcCosh[c*x]))/(35*d*x^5) + (2*b*c^7*d*Sqrt[d - c^2*d*x^2]*Log[x])/(35*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.445279, antiderivative size = 322, normalized size of antiderivative = 1.3, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 97, 12, 103, 95, 5733, 446, 76}

$$-\frac{2c^6d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x}-\frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^3}+\frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5}-\frac{d(1-}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(42*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c
^3*d*Sqrt[d - c^2*d*x^2])/(35*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d*
Sqrt[d - c^2*d*x^2])/(70*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*c^2*d*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*x^5) - (c^4*d*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/(35*x^3) - (2*c^6*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/(35*x) - (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x]))/(7*x^7) + (2*b*c^7*d*Sqrt[d - c^2*d*x^2]*Log[x])/(35*Sqrt[-1 + c*x]
*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
```

```
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^3} - \frac{2c}{}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^3} - \frac{2c}{}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^3} - \frac{2c}{}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^3} - \frac{2c}{}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^3d\sqrt{d-c^2dx^2}}{35x^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d\sqrt{d-c^2dx^2}}{70x^2\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 0.137776, size = 136, normalized size = 0.55

$$-\frac{d\sqrt{d-c^2dx^2}\left(12c^2x^2(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+30(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+bcx\left(3c^4x^4\right.\right.}{210x^7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(30*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh
[c*x]) + 12*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x]) +
 b*c*x*(5 - 12*c^2*x^2 + 3*c^4*x^4 - 12*c^6*x^6*Log[x])))/(210*x^7*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.311, size = 3144, normalized size = 12.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x)
```

```
[Out] -2/35*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^
4*x^4-105*c^2*x^2+25)*x^11*c^18+2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-
35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^10/(c*x+1)^(1/2)/(c*x-1
)^(1/2)*arccosh(c*x)*c^17-2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8
*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)
*arccosh(c*x)*c^15-4*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70
*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccos
h(c*x)*c^13+44/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6
*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*
x)*c^11-6*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+15
4*c^4*x^4-105*c^2*x^2+25)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-
170/7*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^
4*x^4-105*c^2*x^2+25)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-2*b*(-d*(c^2*x^2
-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25
)*x^11/(c*x+1)/(c*x-1)*arccosh(c*x)*c^18+3*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c
^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^9/(c*x+1)/(c*x
-1)*arccosh(c*x)*c^16+12*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^
8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c
^14-164/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+15
4*c^4*x^4-105*c^2*x^2+25)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12+52/5*b*(-d*
(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^
2*x^2+25)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10+1966/35*b*(-d*(c^2*x^2-1))^
(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x/(
c*x+1)/(c*x-1)*arccosh(c*x)*c^8-3272/35*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10
*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x/(c*x+1)/(c*x-1)*a
```

rccosh(c*x)*c^6+472/7*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-1/7*a/d/x^7*(-c^2*d*x^2+d)^(5/2)+2/35*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^13/(c*x+1)/(c*x-1)*c^20-9/35*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^11/(c*x+1)/(c*x-1)*c^18-1/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^9/(c*x+1)/(c*x-1)*c^16+142/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^7/(c*x+1)/(c*x-1)*c^14-72/35*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^5/(c*x+1)/(c*x-1)*c^12+25/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^3/(c*x+1)/(c*x-1)*c^10-5/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x/(c*x+1)/(c*x-1)*c^8+25/7*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^7/(c*x+1)/(c*x-1)*arccosh(c*x)-1/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^15+5/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^13-11/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^11-161/30*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9-421/42*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+55/14*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-25/42*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^9*c^16+26/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^7*c^14-116/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^5*c^12-2/35*a*c^2/d/x^5*(-c^2*d*x^2+d)^(5/2)+10/7*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+359/30*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+2/35*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c^7*d-4/35*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^7*d+20/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^3*c^10-5/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x*c^8

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.77759, size = 1400, normalized size = 5.67

$$\left[-\frac{6\left(2\,bc^8dx^8 - bc^6dx^6 - 9\,bc^4dx^4 + 13\,bc^2dx^2 - 5\,bd\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right) - 6\left(bc^9dx^9 - bc^7dx^7\right)\sqrt{-d}\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="fricas"
)
```

```
[Out] [-1/210*(6*(2*b*c^8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 -
5*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 6*(b*c^9*d*x^9 -
 b*c^7*d*x^7)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (3*b*c^5
*d*x^5 - (3*b*c^5 - 12*b*c^3 + 5*b*c)*d*x^7 - 12*b*c^3*d*x^3 + 5*b*c*d*x)*s
qrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 6*(2*a*c^8*d*x^8 - a*c^6*d*x^6 - 9*
a*c^4*d*x^4 + 13*a*c^2*d*x^2 - 5*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)
, 1/210*(12*(b*c^9*d*x^9 - b*c^7*d*x^7)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)
*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 6
*(2*b*c^8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 - 5*b*d)*sqr
t(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (3*b*c^5*d*x^5 - (3*b*c^5
- 12*b*c^3 + 5*b*c)*d*x^7 - 12*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(-c^2*d*x^2 + d
)*sqrt(c^2*x^2 - 1) - 6*(2*a*c^8*d*x^8 - a*c^6*d*x^6 - 9*a*c^4*d*x^4 + 13*a
*c^2*d*x^2 - 5*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx) + a)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)/x^8, x)
```

3.78 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx$

Optimal. Leaf size=328

$$-\frac{8c^4\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{315dx^5}-\frac{4c^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{63dx^7}-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{9dx^9}-\frac{2b}{315}$$

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(72*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c
^3*d*Sqrt[d - c^2*d*x^2])/(189*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d
*Sqrt[d - c^2*d*x^2])/(420*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^7*d*S
qrt[d - c^2*d*x^2])/(315*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^
2)^(5/2)*(a + b*ArcCosh[c*x]))/(9*d*x^9) - (4*c^2*(d - c^2*d*x^2)^(5/2)*(a
+ b*ArcCosh[c*x]))/(63*d*x^7) - (8*c^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh
[c*x]))/(315*d*x^5) + (8*b*c^9*d*Sqrt[d - c^2*d*x^2]*Log[x])/(315*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.51281, antiderivative size = 401, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 97, 12, 103, 95, 5733, 1251, 893}

$$-\frac{8c^8d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{315x}-\frac{4c^6d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{315x^3}-\frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105x^5}+\frac{c^2d\sqrt{d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(72*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c
^3*d*Sqrt[d - c^2*d*x^2])/(189*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d
*Sqrt[d - c^2*d*x^2])/(420*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^7*d*S
qrt[d - c^2*d*x^2])/(315*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*d*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(21*x^7) - (c^4*d*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(105*x^5) - (4*c^6*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x]))/(315*x^3) - (8*c^8*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(315
*x) - (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*x
^9) + (8*b*c^9*d*Sqrt[d - c^2*d*x^2]*Log[x])/(315*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
```

```
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105x^5} - \frac{4c^6d}{}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105x^5} - \frac{4c^6d}{}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105x^5} - \frac{4c^6d}{}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105x^5} - \frac{4c^6d}{}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{189x^6\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d\sqrt{d-c^2dx^2}}{420x^4\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.312996, size = 154, normalized size = 0.47

$$-\frac{d\sqrt{d-c^2dx^2}\left(96c^2x^2(cx-1)^{5/2}\left(2c^2x^2+5\right)(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+840(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{7560x^9\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(840*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCos
h[c*x]) + 96*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(5 + 2*c^2*x^2)*(a +
b*ArcCosh[c*x]) + b*c*x*(105 - 200*c^2*x^2 + 18*c^4*x^4 + 48*c^6*x^6 - 192*
c^8*x^8*Log[x])))/(7560*x^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.387, size = 4259, normalized size = 13.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x)
```

```
[Out] -1/9*a/d/x^9*(-c^2*d*x^2+d)^(5/2)+104/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^1
2*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+122
5)*x^11/(c*x+1)/(c*x-1)*arccosh(c*x)*c^20-7700/9*b*(-d*(c^2*x^2-1))^(1/2)*d
/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^
2*x^2+1225)/x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-212/15*b*(-d*(c^2*x^2-1))^
(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-
4725*c^2*x^2+1225)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^18+3151/15*b*(-d*(c^2
*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210
*c^4*x^4-4725*c^2*x^2+1225)*x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^16-60632/105
*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c
^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^1
4+59884/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8
*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^3/(c*x+1)/(c*x-1)*arcco
sh(c*x)*c^12-43264/63*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^
10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x/(c*x+1)/(c*x-
1)*arccosh(c*x)*c^10+113594/63*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-94
5*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/x/(c*x
+1)/(c*x-1)*arccosh(c*x)*c^8-174520/63*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12
*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225
```

)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+19540/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(
840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*
x^2+1225)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+1104/7*b*(-d*(c^2*x^2-1))^(1
/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-47
25*c^2*x^2+1225)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^13-120*b*(-
d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^
6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c
*x)*c^11+64/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c
^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^12/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*arccosh(c*x)*c^21-24*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-94
5*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^10/(
c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^19+24/5*b*(-d*(c^2*x^2-1))^(1/2)*
d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c
^2*x^2+1225)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^17-208/3*b*(-d*
(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+
6210*c^4*x^4-4725*c^2*x^2+1225)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x
)*c^15-40/63*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^
8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^7*c^16-35/9*b*(-d*(c^2
*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210
*c^4*x^4-4725*c^2*x^2+1225)*x/(c*x+1)/(c*x-1)*c^10-16/3*b*(-d*(c^2*x^2-1))^
(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-
4725*c^2*x^2+1225)*x^10/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^19+4*b*(-d*(c^2*x^2-1
))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x
^4-4725*c^2*x^2+1225)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^17+280/9*b*(-d*(c^2
*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210
*c^4*x^4-4725*c^2*x^2+1225)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9+41
89/180*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-
2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2
)*c^15-1187/60*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*
c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^4/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*c^13-829/56*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^
10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^2/(c*x+1)^(1/
2)/(c*x-1)^(1/2)*c^11+25915/126*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-9
45*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/x^2/(
c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7-1285/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*
x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)
/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+21175/216*b*(-d*(c^2*x^2-1))^(1/2)*d/(
840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*
x^2+1225)/x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1225/72*b*(-d*(c^2*x^2-1))^(1
/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-47
25*c^2*x^2+1225)/x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+1225/9*b*(-d*(c^2*x^2-1)
)^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^
4-4725*c^2*x^2+1225)/x^9/(c*x+1)/(c*x-1)*arccosh(c*x)+128/315*b*(-d*(c^2*x^
2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^
4*x^4-4725*c^2*x^2+1225)*x^17/(c*x+1)/(c*x-1)*c^26-16/315*b*(-d*(c^2*x^2-1)

```
)^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^
4-4725*c^2*x^2+1225)*x^15/(c*x+1)/(c*x-1)*c^24-344/189*b*(-d*(c^2*x^2-1))^(
1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4
725*c^2*x^2+1225)*x^13/(c*x+1)/(c*x-1)*c^22-922/945*b*(-d*(c^2*x^2-1))^(1/2
)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725
*c^2*x^2+1225)*x^11/(c*x+1)/(c*x-1)*c^20+2906/945*b*(-d*(c^2*x^2-1))^(1/2)*
d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c
^2*x^2+1225)*x^9/(c*x+1)/(c*x-1)*c^18+2069/189*b*(-d*(c^2*x^2-1))^(1/2)*d/(
840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*
x^2+1225)*x^7/(c*x+1)/(c*x-1)*c^16-4639/189*b*(-d*(c^2*x^2-1))^(1/2)*d/(840
*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2
+1225)*x^5/(c*x+1)/(c*x-1)*c^14+455/27*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12
*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225
)*x^3/(c*x+1)/(c*x-1)*c^12-64/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-9
45*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^13/
(c*x+1)/(c*x-1)*arccosh(c*x)*c^22-4/63*a*c^2/d/x^7*(-c^2*d*x^2+d)^(5/2)-300
55/504*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-
2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^
9+8/315*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))^2+1)*c^9*d-16/315*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^9*d-8/315*a*c^4/d/x^5*(-c^2*d*x^2+d)^(5/2
)-2189/189*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*
x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^5*c^14+350/27*b*(-d*(c^2
*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210
*c^4*x^4-4725*c^2*x^2+1225)*x^3*c^12-35/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c
^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1
225)*x*c^10-128/315*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10
+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^15*c^24-16/45*b
*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6
*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^13*c^22+1384/945*b*(-d*(c^2*x^2-1))^
(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-
4725*c^2*x^2+1225)*x^11*c^20+2306/945*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*
x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)
*x^9*c^18
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.9761, size = 1624, normalized size = 4.95

$$\left[-\frac{24\left(8\,bc^{10}dx^{10}-4\,bc^{8}dx^{8}-bc^{6}dx^{6}-53\,bc^{4}dx^{4}+85\,bc^{2}dx^{2}-35\,bd\right)\sqrt{-c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-96\left(bc^{11}dx^{11}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="fricas
")
```

```
[Out] [-1/7560*(24*(8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*d*x^
4 + 85*b*c^2*d*x^2 - 35*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
1)) - 96*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2
- d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c
^2*x^4 - x^2)) + (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c^5 -
200*b*c^3 + 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1) + 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^6*d*x^6
 - 53*a*c^4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^1
1 - x^9), 1/7560*(192*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(d)*arctan(sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x
^2 + d)) - 24*(8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*d*x
^4 + 85*b*c^2*d*x^2 - 35*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
 1)) - (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c^5 - 200*b*c^3
+ 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^6*d*x^6 - 53*a*c^
4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**10,x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)/x^10, x)
```

3.79 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx$

Optimal. Leaf size=409

$$-\frac{16c^6\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{1155dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{231dx^7}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{33dx^9}-$$

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(110*x^10*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c
^3*d*Sqrt[d - c^2*d*x^2])/(66*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d*
Sqrt[d - c^2*d*x^2])/(1386*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^7*d*Sqr
t[d - c^2*d*x^2])/(770*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c^9*d*Sqrt[
d - c^2*d*x^2])/(1155*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^
(5/2)*(a + b*ArcCosh[c*x]))/(11*d*x^11) - (2*c^2*(d - c^2*d*x^2)^(5/2)*(a +
 b*ArcCosh[c*x]))/(33*d*x^9) - (8*c^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[
c*x]))/(231*d*x^7) - (16*c^6*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(1
155*d*x^5) + (16*b*c^11*d*Sqrt[d - c^2*d*x^2]*Log[x])/(1155*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])
```

Rubi [A] time = 0.606418, antiderivative size = 480, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 97, 12, 103, 95, 5733, 1799, 1620}

$$-\frac{16c^{10}d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{1155x}-\frac{8c^8d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{1155x^3}-\frac{2c^6d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{385x^5}-\frac{c^4d}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(110*x^10*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c
^3*d*Sqrt[d - c^2*d*x^2])/(66*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d*
Sqrt[d - c^2*d*x^2])/(1386*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^7*d*Sqr
t[d - c^2*d*x^2])/(770*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c^9*d*Sqrt[
d - c^2*d*x^2])/(1155*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*d*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(33*x^9) - (c^4*d*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x]))/(231*x^7) - (2*c^6*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*
x]))/(385*x^5) - (8*c^8*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(1155*x
^3) - (16*c^10*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(1155*x) - (d*(1
 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(11*x^11) + (16
```

```
*b*c^11*d*Sqrt[d - c^2*d*x^2]*Log[x])/(1155*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} - \frac{2c^6d\cdots}{\cdots}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} - \frac{2c^6d\cdots}{\cdots}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} - \frac{2c^6d\cdots}{\cdots}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} - \frac{2c^6d\cdots}{\cdots}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d\sqrt{d-c^2dx^2}}{66x^8\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d\sqrt{d-c^2dx^2}}{1386x^6\sqrt{-1+cx}\sqrt{1+c\cdots}}$$

Mathematica [A] time = 0.41739, size = 170, normalized size = 0.42

$$-\frac{d\sqrt{d-c^2dx^2}\left(12c^2x^2(cx-1)^{5/2}\left(8c^4x^4+20c^2x^2+35\right)(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+630(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\right.\right.}{6930x^{11}\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(630*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCos
h[c*x]) + 12*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(35 + 20*c^2*x^2 + 8*
c^4*x^4)*(a + b*ArcCosh[c*x]) + b*c*x*(63 - 105*c^2*x^2 + 5*c^4*x^4 + 9*c^6
*x^6 + 24*c^8*x^8 - 96*c^10*x^10*Log[x])))/(6930*x^11*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])
```

Maple [B] time = 0.493, size = 5518, normalized size = 13.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.07987, size = 1806, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="fricas
")
```

```
[Out] [-1/6930*(6*(16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*c^6*d*x
^6 - 145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*sqrt(-c^2*d*x^2 + d)*log(
c*x + sqrt(c^2*x^2 - 1)) - 48*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(-d)*log(
(c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^
4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (24*b*c^9*d*x^9 + 9*b*c^7*d*x^7 - (
24*b*c^9 + 9*b*c^7 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d*x^5 -
 105*b*c^3*d*x^3 + 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 6*(
16*a*c^12*d*x^12 - 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 - 145*a*c^
4*d*x^4 + 245*a*c^2*d*x^2 - 105*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11
), 1/6930*(96*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(d)*arctan(sqrt(-c^2*d*x^
2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d
)) - 6*(16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*c^6*d*x^6 -
145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 - 1)) - (24*b*c^9*d*x^9 + 9*b*c^7*d*x^7 - (24*b*c^9 + 9*b*c^7
 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d*x^5 - 105*b*c^3*d*x^3 +
 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 6*(16*a*c^12*d*x^12 -
 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 - 145*a*c^4*d*x^4 + 245*a*c^
2*d*x^2 - 105*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**12,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)/x^12, x)
```

3.80 $\int x^7 \left(d - c^2 d x^2\right)^{3/2} \left(a + b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=399

$$\frac{\left(d - c^2 d x^2\right)^{11/2} \left(a + b \cosh^{-1}(cx)\right)}{11 c^8 d^4} - \frac{\left(d - c^2 d x^2\right)^{9/2} \left(a + b \cosh^{-1}(cx)\right)}{3 c^8 d^3} + \frac{3 \left(d - c^2 d x^2\right)^{7/2} \left(a + b \cosh^{-1}(cx)\right)}{7 c^8 d^2} - \frac{\left(d - c^2 d x^2\right.}{}$$

```
[Out] (16*b*d*x*Sqrt[d - c^2*d*x^2])/(1155*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8
*b*d*x^3*Sqrt[d - c^2*d*x^2])/(3465*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*
b*d*x^5*Sqrt[d - c^2*d*x^2])/(1925*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d
*x^7*Sqrt[d - c^2*d*x^2])/(1617*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c*d*
x^9*Sqrt[d - c^2*d*x^2])/(297*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^11
*Sqrt[d - c^2*d*x^2])/(121*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)
^(5/2)*(a + b*ArcCosh[c*x]))/(5*c^8*d) + (3*(d - c^2*d*x^2)^(7/2)*(a + b*Ar
cCosh[c*x]))/(7*c^8*d^2) - ((d - c^2*d*x^2)^(9/2)*(a + b*ArcCosh[c*x]))/(3*
c^8*d^3) + ((d - c^2*d*x^2)^(11/2)*(a + b*ArcCosh[c*x]))/(11*c^8*d^4)
```

Rubi [A] time = 0.495963, antiderivative size = 460, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 100, 12, 74, 5733, 1810}

$$-\frac{d x^6 (1 - cx)^2 (cx + 1)^2 \sqrt{d - c^2 d x^2} \left(a + b \cosh^{-1}(cx)\right)}{11 c^2} - \frac{2 d x^4 (1 - cx)^2 (cx + 1)^2 \sqrt{d - c^2 d x^2} \left(a + b \cosh^{-1}(cx)\right)}{33 c^4} - \frac{8 d x^2 (1 -}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d*x*Sqrt[d - c^2*d*x^2])/(1155*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8
*b*d*x^3*Sqrt[d - c^2*d*x^2])/(3465*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*
b*d*x^5*Sqrt[d - c^2*d*x^2])/(1925*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d
*x^7*Sqrt[d - c^2*d*x^2])/(1617*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c*d*
x^9*Sqrt[d - c^2*d*x^2])/(297*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^11
*Sqrt[d - c^2*d*x^2])/(121*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (16*d*(1 - c*x)^
2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(1155*c^8) - (8*d*x
^2*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(231*c
^6) - (2*d*x^4*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x]))/(33*c^4) - (d*x^6*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/(11*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^7 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^7(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{16d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{1155c^8} - \frac{8dx^2(1-cx)^2(1+cx}{}$$

$$= -\frac{16d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{1155c^8} - \frac{8dx^2(1-cx)^2(1+cx}{}$$

$$= -\frac{16d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{1155c^8} - \frac{8dx^2(1-cx)^2(1+cx}{}$$

$$= \frac{16bdx\sqrt{d-c^2dx^2}}{1155c^7\sqrt{-1+cx}\sqrt{1+cx}} + \frac{8bdx^3\sqrt{d-c^2dx^2}}{3465c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bdx^5\sqrt{d-c^2dx^2}}{1925c^3\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A] time = 0.222922, size = 182, normalized size = 0.46

$$-\frac{d\sqrt{d-c^2dx^2}\left(105c^5x^6(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{2(cx-1)^{5/2}(cx+1)^{5/2}\left(35c^4x^4+20c^2x^2+8\right)\left(a+b\cosh^{-1}(cx)\right)}{c}-b\left(\frac{105c^{10}x}{11}\right.\right.}{1155c^7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(-(b*(16*x + (8*c^2*x^3)/3 + (6*c^4*x^5)/5 + (5*c^6
*x^7)/7 - (140*c^8*x^9)/9 + (105*c^10*x^11)/11)) + 105*c^5*x^6*(-1 + c*x)^(
5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x]) + (2*(-1 + c*x)^(5/2)*(1 + c*x)^(
5/2)*(8 + 20*c^2*x^2 + 35*c^4*x^4)*(a + b*ArcCosh[c*x]))/c))/(1155*c^7*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.497, size = 1846, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/11*x^6*(-c^2*d*x^2+d)^(5/2)/c^2/d+6/11/c^2*(-1/9*x^4*(-c^2*d*x^2+d)^(
5/2)/c^2/d+4/9/c^2*(-1/7*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(-c^2*d*
x^2+d)^(5/2))))+b*(-1/247808*(-d*(c^2*x^2-1))^(1/2)*(1+4096*c^8*x^8-2352*c^
6*x^6+620*c^4*x^4-61*c^2*x^2+1024*x^12*c^12-3328*x^10*c^10+220*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3*c^3-11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1024*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^11*c^11-2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+2816*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-1232*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5)
*(-1+11*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)-1/55296*(-d*(c^2*x^2-1))^(1/2)*
(256*x^10*c^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+688*c^6*
x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-280*c^4*x^4+432*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)
+1/100352*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25
*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*x*c+1)*(-1+7*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)+11/51200*(-d*(c^2*x^2-1
))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c
^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c-1)*(-1+5*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)+1/3072*(-d*(c^2*x^2-1))^(
1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)-7/1024*
(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arcc
osh(c*x))*d/(c*x+1)/c^8/(c*x-1)-7/1024*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)+1/30
72*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4
+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1
)/c^8/(c*x-1)+11/51200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/
c^8/(c*x-1)+1/100352*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)-1/55296*(-d*
(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*x^10*c^10+
576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^5*c^5+688*c^6*x^6+120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^
4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d/
(c*x+1)/c^8/(c*x-1)-1/247808*(-d*(c^2*x^2-1))^(1/2)*(-1024*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^11*c^11+1024*x^12*c^12+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c
^9-3328*x^10*c^10-2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+4096*c^8*x^8+123
2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-2352*c^6*x^6-220*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^3*c^3+620*c^4*x^4+11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-61*c^2*x^2+1
```

```
)*(1+11*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.30439, size = 670, normalized size = 1.68

$$-\frac{3465\left(105\,bc^{12}dx^{12} - 245\,bc^{10}dx^{10} + 145\,bc^{8}dx^{8} + bc^{6}dx^{6} + 2\,bc^{4}dx^{4} + 8\,bc^{2}dx^{2} - 16\,bd\right)\sqrt{-c^{2}dx^{2} + d}\,\log\left(cx + \sqrt{c^{2}x^{2} -}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] -1/4002075*(3465*(105*b*c^12*d*x^12 - 245*b*c^10*d*x^10 + 145*b*c^8*d*x^8 +
 b*c^6*d*x^6 + 2*b*c^4*d*x^4 + 8*b*c^2*d*x^2 - 16*b*d)*sqrt(-c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1)) - (33075*b*c^11*d*x^11 - 53900*b*c^9*d*x^9 +
2475*b*c^7*d*x^7 + 4158*b*c^5*d*x^5 + 9240*b*c^3*d*x^3 + 55440*b*c*d*x)*sqr
t(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 3465*(105*a*c^12*d*x^12 - 245*a*c^10*
d*x^10 + 145*a*c^8*d*x^8 + a*c^6*d*x^6 + 2*a*c^4*d*x^4 + 8*a*c^2*d*x^2 - 16
*a*d)*sqrt(-c^2*d*x^2 + d))/(c^10*x^2 - c^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.81 $\int x^5 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=321

$$-\frac{\left(d-c^2dx^2\right)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{9c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^6d}+\frac{bc^3dx^9\sqrt{d}}{81\sqrt{cx-1}}$$

```
[Out] (8*b*d*x*Sqrt[d - c^2*d*x^2])/(315*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b
*d*x^3*Sqrt[d - c^2*d*x^2])/(945*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*x
^5*Sqrt[d - c^2*d*x^2])/(525*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (10*b*c*d*x^
7*Sqrt[d - c^2*d*x^2])/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^9*Sq
rt[d - c^2*d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/
2)*(a + b*ArcCosh[c*x]))/(5*c^6*d) + (2*(d - c^2*d*x^2)^(7/2)*(a + b*ArcCos
h[c*x]))/(7*c^6*d^2) - ((d - c^2*d*x^2)^(9/2)*(a + b*ArcCosh[c*x]))/(9*c^6*
d^3)
```

Rubi [A] time = 0.437273, antiderivative size = 366, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 100, 12, 74, 5733, 1153}

$$-\frac{dx^4(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9c^2}-\frac{4dx^2(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4}-\frac{8d(1-cx}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d*x*Sqrt[d - c^2*d*x^2])/(315*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b
*d*x^3*Sqrt[d - c^2*d*x^2])/(945*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*x
^5*Sqrt[d - c^2*d*x^2])/(525*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (10*b*c*d*x^
7*Sqrt[d - c^2*d*x^2])/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^9*Sq
rt[d - c^2*d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*d*(1 - c*x)^2*(1
+ c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(315*c^6) - (4*d*x^2*(1
- c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(63*c^4) - (
d*x^4*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*
c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^5 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^5(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{8d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{315c^6} - \frac{4dx^2(1-cx)^2(1+cx)}{}$$

$$= -\frac{8d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{315c^6} - \frac{4dx^2(1-cx)^2(1+cx)}{}$$

$$= -\frac{8d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{315c^6} - \frac{4dx^2(1-cx)^2(1+cx)}{}$$

$$= \frac{8bdx\sqrt{d-c^2dx^2}}{315c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4bdx^3\sqrt{d-c^2dx^2}}{945c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bdx^5\sqrt{d-c^2dx^2}}{525c\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.175486, size = 164, normalized size = 0.51

$$-\frac{d\sqrt{d-c^2dx^2}\left(35c^3x^4(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{4(cx-1)^{5/2}(cx+1)^{5/2}\left(5c^2x^2+2\right)\left(a+b\cosh^{-1}(cx)\right)}{c}-b\left(\frac{35c^8x^9}{9}-\frac{50c^6x^7}{7}\right.\right.}{315c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(-(b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (50*c^6
*x^7)/7 + (35*c^8*x^9)/9)) + 35*c^3*x^4*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a
 + b*ArcCosh[c*x]) + (4*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(2 + 5*c^2*x^2)*(a
 + b*ArcCosh[c*x]))/c))/(315*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.379, size = 1376, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/9*x^4*(-c^2*d*x^2+d)^(5/2)/c^2/d+4/9/c^2*(-1/7*x^2*(-c^2*d*x^2+d)^(5/
2)/c^2/d-2/35/d/c^4*(-c^2*d*x^2+d)^(5/2)))+b*(-1/41472*(-d*(c^2*x^2-1))^(1/
2)*(256*x^10*c^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+688*c
^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-280*c^4*x^4+432*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+
9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccosh(c*x))*d/(c*x+1)/c^6/(c*x
-1)-1/25088*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-
25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c+1)*(-1+7*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)+1/3200*(-d*(c^2*x^2-1
))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c
^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c-1)*(-1+5*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(
1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)-3/256*(
-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arcco
sh(c*x))*d/(c*x+1)/c^6/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)+1/1152
*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/
c^6/(c*x-1)+1/3200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^6/
(c*x-1)-1/25088*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7
*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)-1/41472*(-d*(c^2*
x^2-1))^(1/2)*(-256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*x^10*c^10+576*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^5*c^5+688*c^6*x^6+120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4
-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d/(c*x+
1)/c^6/(c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21108, size = 568, normalized size = 1.77

$$-\frac{315\left(35\,bc^{10}dx^{10}-85\,bc^{8}dx^{8}+53\,bc^{6}dx^{6}+bc^{4}dx^{4}+4\,bc^{2}dx^{2}-8\,bd\right)\sqrt{-c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-\left(1225\,bc^{9}dx^{9}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] -1/99225*(315*(35*b*c^10*d*x^10 - 85*b*c^8*d*x^8 + 53*b*c^6*d*x^6 + b*c^4*d
*x^4 + 4*b*c^2*d*x^2 - 8*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
 1)) - (1225*b*c^9*d*x^9 - 2250*b*c^7*d*x^7 + 189*b*c^5*d*x^5 + 420*b*c^3*d
*x^3 + 2520*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 315*(35*a*c^1
0*d*x^10 - 85*a*c^8*d*x^8 + 53*a*c^6*d*x^6 + a*c^4*d*x^4 + 4*a*c^2*d*x^2 -
8*a*d)*sqrt(-c^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.82 $\int x^3 \left(d-c^2 d x^2\right)^{3/2} \left(a+b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=243

$$\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^4d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^4d}+\frac{bc^3dx^7\sqrt{d-c^2dx^2}}{49\sqrt{cx-1}\sqrt{cx+1}}-\frac{8bcdx^5\sqrt{d-c^2dx^2}}{175\sqrt{cx-1}\sqrt{cx+1}}+\frac{bd}{105c}$$

```
[Out] (2*b*d*x*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*
x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*b*c*d*x^
5*Sqrt[d - c^2*d*x^2])/(175*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^7*Sq
rt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/
2)*(a + b*ArcCosh[c*x]))/(5*c^4*d) + ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[
c*x]))/(7*c^4*d^2)
```

Rubi [A] time = 0.411973, antiderivative size = 272, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 100, 12, 74, 5733, 373}

$$-\frac{dx^2(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{2d(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4}+\frac{bc^3dx^7\sqrt{d-}}{49\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d*x*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*
x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*b*c*d*x^
5*Sqrt[d - c^2*d*x^2])/(175*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^7*Sq
rt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*d*(1 - c*x)^2*(1
+ c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*c^4) - (d*x^2*(1 - c
*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{2d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4} - \frac{dx^2(1-cx)^2(1+cx)^2}{}$$

$$= -\frac{2d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4} - \frac{dx^2(1-cx)^2(1+cx)^2}{}$$

$$= -\frac{2d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4} - \frac{dx^2(1-cx)^2(1+cx)^2}{}$$

$$= \frac{2bdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bdx^3\sqrt{d-c^2dx^2}}{105c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{8bcdx^5\sqrt{d-c^2dx^2}}{175\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 0.221202, size = 150, normalized size = 0.62

$$-\frac{d\sqrt{d-c^2dx^2}\left(5c^2x^2(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+2(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5}{7}bcx\left(c^2x^2-1\right.\right.}{35c^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*((-5*b*c*x*(-1 + c^2*x^2)^3)/7 - (19*b*c*(x - (2*c^
2*x^3)/3 + (c^4*x^5)/5))/7 + 2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcC
osh[c*x]) + 5*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x])
))/(35*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.31, size = 966, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/7*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(-c^2*d*x^2+d)^(5/2))+b*(
-1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c
^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c+1)*(-1+7*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/3200*(-d*(c^2*x^2-1))^(
1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x
^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-1)*(-1+5*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1/2)*
(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c
^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*
x))*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(
c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c
*x-1)+1/3200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^
5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1
)-1/6272*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64
*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c
^2*x^2+1)*(1+7*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21809, size = 482, normalized size = 1.98

$$-\frac{105\left(5\,bc^8dx^8-13\,bc^6dx^6+9\,bc^4dx^4+bc^2dx^2-2\,bd\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(75\,bc^7dx^7-168\,bc^5dx^5+\right.}{3675\left(c^6x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] -1/3675*(105*(5*b*c^8*d*x^8 - 13*b*c^6*d*x^6 + 9*b*c^4*d*x^4 + b*c^2*d*x^2
- 2*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^7*d*x^
7 - 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 + 210*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sq
rt(c^2*x^2 - 1) + 105*(5*a*c^8*d*x^8 - 13*a*c^6*d*x^6 + 9*a*c^4*d*x^4 + a*c
^2*d*x^2 - 2*a*d)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.83 $\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=165

$$-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2d}+\frac{bc^3dx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*d*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*x
^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^5*Sq
rt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/
2)*(a + b*ArcCosh[c*x]))/(5*c^2*d)
```

Rubi [A] time = 0.265324, antiderivative size = 178, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5798, 5718, 194}

$$-\frac{d(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}+\frac{bc^3dx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*x
^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^5*Sq
rt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(1 - c*x)^2*(1 +
c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
```

```
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2} + \frac{\left(bd\sqrt{d-c^2dx^2}\right)\int \left(-1 \cdots\right.}{5c\sqrt{-1+cx}\sqrt{1 \cdots}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2} + \frac{\left(bd\sqrt{d-c^2dx^2}\right)\int \left(1- \cdots\right.}{5c\sqrt{-1+cx}\cdots}$$

$$= \frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d(1-c \cdots}{\cdots}$$

Mathematica [A] time = 0.208018, size = 107, normalized size = 0.65

$$-\frac{d\sqrt{d-c^2dx^2}\left(15a\left(c^2x^2-1\right)^3+bcx\sqrt{cx-1}\sqrt{cx+1}\left(-3c^4x^4+10c^2x^2-15\right)+15b\left(c^2x^2-1\right)^3\cosh^{-1}(cx)\right)}{75c^2\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(15*a*(-1 + c^2*x^2)^3 + b*c*x*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(-15 + 10*c^2*x^2 - 3*c^4*x^4) + 15*b*(-1 + c^2*x^2)^3*ArcCosh[c*x
]))/(75*c^2*(-1 + c^2*x^2))
```

Maple [B] time = 0.217, size = 620, normalized size = 3.8

$$-\frac{a}{5c^2d}\left(-c^2dx^2+d\right)^{\frac{5}{2}}+b\left(-\frac{(-1+5\,\mathrm{arccosh}\,(cx))\,d}{(800\,cx+800)\,c^2\,(cx-1)}\sqrt{-d\left(c^2x^2-1\right)}\left(16\,c^6x^6-28\,c^4x^4+16\,\sqrt{cx+1}\sqrt{cx-1}x^5c^5+13\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/5*a/c^2/d*(-c^2*d*x^2+d)^(5/2)+b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*
x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arc
cosh(c*x))*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c
^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2
)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d/(c*x+1)/c
^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^
2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)
*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/800*(-d*(
c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1))
```

Maxima [A] time = 1.15021, size = 138, normalized size = 0.84

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}b\,\mathrm{arcosh}\,(cx)}{5\,c^2d}-\frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}a}{5\,c^2d}+\frac{\left(3\,c^4\sqrt{-d}d^2x^5-10\,c^2\sqrt{-d}d^2x^3+15\,\sqrt{-d}d^2x\right)b}{75\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/5*(-c^2*d*x^2 + d)^(5/2)*b*arccosh(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(
5/2)*a/(c^2*d) + 1/75*(3*c^4*sqrt(-d)*d^2*x^5 - 10*c^2*sqrt(-d)*d^2*x^3 + 1
5*sqrt(-d)*d^2*x)*b/(c*d)
```

Fricas [A] time = 2.16262, size = 398, normalized size = 2.41

$$-\frac{15\left(bc^6dx^6-3\,bc^4dx^4+3\,bc^2dx^2-bd\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(3\,bc^5dx^5-10\,bc^3dx^3+15\,bcdx\right)\sqrt{-c^2dx^2+}}{75\left(c^4x^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/75*(15*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*x
^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (3*b*c^5*d*x^5 - 10*b*c^3*d*x^3 + 15
*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(a*c^6*d*x^6 - 3*a*c^
4*d*x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.84 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=292

$$\frac{ibd\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{ibd\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)$$

```
[Out] (-4*b*c*d*x*Sqrt[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*
d*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]) + ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])
)/3 - (2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])
/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I
)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*d*Sqrt[d - c^2*d*x
^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.789303, antiderivative size = 304, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5745, 5743, 5761, 4180, 2279, 2391, 8}

$$\frac{ibd\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{ibd\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (-4*b*c*d*x*Sqrt[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*
d*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]) + (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/3 - (2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Arc
Tan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b*d*Sqrt[d - c^2*d
*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*
b*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{3}d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\sqrt{-1+cx}\sqrt{}}{}}{\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

Mathematica [A] time = 1.16786, size = 336, normalized size = 1.15

$$\frac{bd\sqrt{d-c^2dx^2}\left(i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)-cx+cx\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx)+\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx)\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -(a*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/3 - (b*d*Sqrt[d - c^2*d*x^2]*(9*c
*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcC
osh[c*x]]))/(36*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + a*d^(3/2)*Log[x] -
a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d*Sqrt[d - c^2*d*x^2]*(
-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 +
 c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[
c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*Poly
Log[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.218, size = 499, normalized size = 1.7

$$\frac{a}{3}\left(-c^2dx^2+d\right)^{\frac{3}{2}}-ad^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{-c^2dx^2+d}\right)\right)+a\sqrt{-c^2dx^2+d}d-\frac{4\,bd\text{arccosh}\,(cx)}{(3\,cx+3)\,(cx-1)}\sqrt{-d\left(c^2x^2-1\right)}+\frac{bdx^3c^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/3*(-c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2)
)/x)+a*(-c^2*d*x^2+d)^(1/2)*d-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1
)*arccosh(c*x)+1/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x
^3*c^3-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x*c+I*b*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2)))*d+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d-I*b*(-d*(c^2*x^2
-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2)))*d-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d-1/3*b*(-d*(c^2*x^2-1))^(1/2
)*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^4*c^4+5/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*
x+1)/(c*x-1)*arccosh(c*x)*x^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}\left(a + b\,\text{acosh}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)/x, x)
```

3.85 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=311

$$-\frac{3ibc^2d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3ibc^2d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh\right.$$

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2*d*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c
*x]))/(2*x^2) + (3*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^
ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((3*I)/2)*b*c^2*d*Sqrt[d -
 c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (((3*I)/2)*b*c^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.800931, antiderivative size = 323, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5740, 5743, 5761, 4180, 2279, 2391, 8, 14}

$$-\frac{3ibc^2d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3ibc^2d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2*d*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x]))/(2*x^2) + (3*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((3*I)
/2)*b*c^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) + (((3*I)/2)*b*c^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*
E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{\left(bcd\sqrt{d-c^2dx^2}\right)\int \frac{-1+c^2x^2}{x^2}}{2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{2x^2}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.$$

Mathematica [A] time = 1.46628, size = 500, normalized size = 1.61

$$\frac{1}{2}\left(\frac{bd^2(cx+1)\left(ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx)\right.}{x^2\sqrt{d-c^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-((a*d*(1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^2) - 3*a*c^2*d^(3/2)*Log[x]
+ 3*a*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (2*b*c^2*d*Sqrt[d
- c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(
-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]
] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[
c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)) + (b*d^2*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x] + c*
x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 -
I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1
```

```
 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[2, (-I)
/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[2, I/E^ArcC
osh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

Maple [A] time = 0.222, size = 542, normalized size = 1.7

$$-\frac{a}{2\,dx^2}\left(-c^2dx^2+d\right)^{\frac{5}{2}}-\frac{ac^2}{2}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+\frac{3\,ac^2}{2}d^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{-c^2dx^2+d}\right)\right)-\frac{3\,ac^2d}{2}\sqrt{-c^2dx^2+d}-\frac{bc^4d\mathrm{arc}}{(cx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(5/2)-1/2*a*c^2*(-c^2*d*x^2+d)^(3/2)+3/2*a*c^2*
d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-3/2*a*c^2*(-c^2*d*x^2+d)
^(1/2)*d-b*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2+b*
(-d*(c^2*x^2-1))^(1/2)*c^3*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x+1/2*b*(-d*(c^2*x
^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)*arccosh(c*x)-1/2*b*(-d*(c^2*x^2-1))^(1/2
)*d/x/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)*d/x^2/(c*x
+1)/(c*x-1)*arccosh(c*x)-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d+3/2*I
*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d+
3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right) + a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)/x^3, x)
```

3.86 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{x^5}\,dx$

Optimal. Leaf size=321

$$\frac{3ibc^4d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}-\frac{3ibc^4d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{8x^2}$$

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(12*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c
^3*d*Sqrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*c^2*d*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*x^2) - ((d - c^2*d*x^2)^(3/2)*(a
+ b*ArcCosh[c*x]))/(4*x^4) - (3*c^4*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*
x])*ArcTan[E^ArcCosh[c*x]])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (((3*I)/8)*b
*c^4*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - (((3*I)/8)*b*c^4*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^Arc
Cosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.836621, antiderivative size = 333, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5740, 5738, 30, 5761, 4180, 2279, 2391, 14}

$$\frac{3ibc^4d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}-\frac{3ibc^4d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(12*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c
^3*d*Sqrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*c^2*d*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*x^2) - (d*(1 - c*x)*(1 + c*x)*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(4*x^4) - (3*c^4*d*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(4*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (((3*I)/8)*b*c^4*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*
x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((3*I)/8)*b*c^4*d*Sqrt[d - c^2*d*x^2
]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
```

```
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4} - \frac{\left(bcd\sqrt{d-c^2dx^2}\right)\int \frac{-1+c^2x^2}{x^4}}{4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8x^2} - \frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{4x^4}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{8x^2}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{8x^2}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{8x^2}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{8x^2}$$

Mathematica [A] time = 1.18366, size = 574, normalized size = 1.79

$$\frac{-9ibc^4d^2x^4(cx-1)\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)+9ibc^4d^2x^4(cx-1)\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)-15ac^4d^2x^4\sqrt{\frac{cx-1}{cx+1}}+21ac^2d}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] (-2*b*c*d^2*x + 2*b*c^2*d^2*x^2 + 15*b*c^3*d^2*x^3 - 15*b*c^4*d^2*x^4 - 6*a
*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 21*a*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x
)] - 15*a*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 6*b*d^2*Sqrt[(-1 + c*x)/
(1 + c*x)]*ArcCosh[c*x] + 21*b*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCo
sh[c*x] - 15*b*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + (9*I)*
b*c^4*d^2*x^4*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (9*I)*b*c^5*d^2*x^5*
ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (9*I)*b*c^4*d^2*x^4*ArcCosh[c*x]*L
og[1 + I/E^ArcCosh[c*x]] + (9*I)*b*c^5*d^2*x^5*ArcCosh[c*x]*Log[1 + I/E^Arc
Cosh[c*x]] + 9*a*c^4*d^(3/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*
```

```
x^2]*Log[x] - 9*a*c^4*d^(3/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d
*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (9*I)*b*c^4*d^2*x^4*(-1 + c*x)
*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (9*I)*b*c^4*d^2*x^4*(-1 + c*x)*PolyLog[2
, I/E^ArcCosh[c*x]])/(24*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]
)
```

Maple [A] time = 0.241, size = 570, normalized size = 1.8

$$-\frac{a}{4\,dx^4}\left(-c^2dx^2+d\right)^{\frac{5}{2}}+\frac{ac^2}{8\,dx^2}\left(-c^2dx^2+d\right)^{\frac{5}{2}}+\frac{ac^4}{8}\left(-c^2dx^2+d\right)^{\frac{3}{2}}-\frac{3\,ac^4}{8}d^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{-c^2dx^2+d}\right)\right)+\frac{3\,ac^4d}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x)
```

```
[Out] -1/4*a/d/x^4*(-c^2*d*x^2+d)^(5/2)+1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(5/2)+1/8*
a*c^4*(-c^2*d*x^2+d)^(3/2)-3/8*a*c^4*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+
d)^(1/2))/x)+3/8*a*c^4*(-c^2*d*x^2+d)^(1/2)*d+5/8*b*d*(-d*(c^2*x^2-1))^(1/2
)/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+5/8*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(
1/2)/x/(c*x-1)^(1/2)*c^3-7/8*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/x^2/(c*x-1)
*arccosh(c*x)*c^2-1/12*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/x^3/(c*x-1)
^(1/2)*c+1/4*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/x^4/(c*x-1)*arccosh(c*x)+3/
8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d*c^4-3/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2)))*d*c^4+3/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dil
og(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d*c^4-3/8*I*b*(-d*(c^2*x^2-1))^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)*d*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right) + a\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)/x^5, x)
```

3.87 $\int x^4 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=454

$$\frac{1}{32}d^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{128c^2}-\frac{3d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{256c^4}-\frac{3d}{}$$

```
[Out] (3*b*d^2*x^2*Sqrt[d - c^2*d*x^2])/(512*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(b*d^2*x^4*Sqrt[d - c^2*d*x^2])/(512*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (31*
b*c*d^2*x^6*Sqrt[d - c^2*d*x^2])/(960*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (21*b
*c^3*d^2*x^8*Sqrt[d - c^2*d*x^2])/(640*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c
^5*d^2*x^10*Sqrt[d - c^2*d*x^2])/(100*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(256*c^4) - (d^2*x^3*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^2) + (d^2*x^5*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/32 + (d*x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]
))/16 + (x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/10 - (3*d^2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(512*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Rubi [A] time = 1.38481, antiderivative size = 485, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5745, 5743, 5759, 5676, 30, 14, 266, 43}

$$\frac{1}{32}d^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{10}d^2x^5(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{16}d^2x^5(1-cx)(cx+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d^2*x^2*Sqrt[d - c^2*d*x^2])/(512*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(b*d^2*x^4*Sqrt[d - c^2*d*x^2])/(512*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (31*
b*c*d^2*x^6*Sqrt[d - c^2*d*x^2])/(960*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (21*b
*c^3*d^2*x^8*Sqrt[d - c^2*d*x^2])/(640*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c
^5*d^2*x^10*Sqrt[d - c^2*d*x^2])/(100*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(256*c^4) - (d^2*x^3*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^2) + (d^2*x^5*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/32 + (d^2*x^5*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/16 + (d^2*x^5*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/10 - (3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
```

```
c*x])^2)/(512*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
```

```
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^4 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^4(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{10}d^2x^5(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int$$

$$= \frac{1}{16}d^2x^5(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{10}d^2x^5(1-cx)^2(1+$$

$$= \frac{1}{32}d^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{16}d^2x^5(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(\right.$$

$$= -\frac{31bcd^2x^6\sqrt{d-c^2dx^2}}{960\sqrt{-1+cx}\sqrt{1+cx}} + \frac{21bc^3d^2x^8\sqrt{d-c^2dx^2}}{640\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^{10}\sqrt{d-c^2dx^2}}{100\sqrt{-1+cx}\sqrt{1+cx}} - $$

$$= \frac{bd^2x^4\sqrt{d-c^2dx^2}}{512c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{31bcd^2x^6\sqrt{d-c^2dx^2}}{960\sqrt{-1+cx}\sqrt{1+cx}} + \frac{21bc^3d^2x^8\sqrt{d-c^2dx^2}}{640\sqrt{-1+cx}\sqrt{1+cx}} - $$

$$= \frac{3bd^2x^2\sqrt{d-c^2dx^2}}{512c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bd^2x^4\sqrt{d-c^2dx^2}}{512c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{31bcd^2x^6\sqrt{d-c^2dx^2}}{960\sqrt{-1+cx}\sqrt{1+cx}} + $$

Mathematica [A] time = 6.54835, size = 581, normalized size = 1.28

$$\sqrt{-d\left(c^2x^2-1\right)}\left(\frac{1}{10}ac^4d^2x^9 - \frac{21}{80}ac^2d^2x^7 - \frac{ad^2x^3}{128c^2} - \frac{3ad^2x}{256c^4} + \frac{31}{160}ad^2x^5\right) - \frac{3ad^{5/2}\tan^{-1}\left(\frac{cx\sqrt{-d(c^2x^2-1)}}{\sqrt{d}(c^2x^2-1)}\right)}{256c^5} + \frac{bd^2\sqrt{-d(cx-1}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((-3*a*d^2*x)/(256*c^4) - (a*d^2*x^3)/(128*c^2) +
 (31*a*d^2*x^5)/160 - (21*a*c^2*d^2*x^7)/80 + (a*c^4*d^2*x^9)/10) - (3*a*d^
(5/2)*ArcTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(25
6*c^5) + (b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*ArcCosh[c*x]] -
9*Cosh[4*ArcCosh[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCosh[c*x]] + 18*A
rcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 6
*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]])))/(2304*c^5*Sqrt[(-1 + c*x)/(1 + c*x)]*
(1 + c*x)) + (b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(1440*ArcCosh[c*x]^2 -
```

```
576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cosh[6*ArcCosh[c*x
]] + 9*Cosh[8*ArcCosh[c*x]] + 1152*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 576*
ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 384*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]] -
 72*ArcCosh[c*x]*Sinh[8*ArcCosh[c*x]]))/(36864*c^5*Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)) - (b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(50400*ArcCosh[c*x]^
2 - 25200*Cosh[2*ArcCosh[c*x]] + 3600*Cosh[4*ArcCosh[c*x]] + 2600*Cosh[6*Ar
cCosh[c*x]] + 675*Cosh[8*ArcCosh[c*x]] + 72*Cosh[10*ArcCosh[c*x]] + 50400*A
rcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 14400*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]
- 15600*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]] - 5400*ArcCosh[c*x]*Sinh[8*ArcCos
h[c*x]] - 720*ArcCosh[c*x]*Sinh[10*ArcCosh[c*x]]))/(3686400*c^5*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.49, size = 690, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/10*a*x^3*(-c^2*d*x^2+d)^(7/2)/c^2/d-3/80*a/c^4*x*(-c^2*d*x^2+d)^(7/2)/d+
1/160*a/c^4*x*(-c^2*d*x^2+d)^(5/2)+1/128*a/c^4*d*x*(-c^2*d*x^2+d)^(3/2)+3/2
56*a/c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+3/256*a/c^4*d^3/(c^2*d)^(1/2)*arctan((c
^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3/512*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)/c^5*arccosh(c*x)^2*d^2-1/100*b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^10+21/640*b*(-d*(c^2*x^2-1))^(1/2)*d^2
/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^8-31/960*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c
*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^6+1/512*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^
(1/2)/c/(c*x-1)^(1/2)*x^4+3/512*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/
c^3/(c*x-1)^(1/2)*x^2+1/10*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^6/(c*x-1)
*arccosh(c*x)*x^11-29/80*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^4/(c*x-1)*a
rccosh(c*x)*x^9+73/160*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^2/(c*x-1)*arc
cosh(c*x)*x^7-129/640*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(
c*x)*x^5-1/256*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x
)*x^3+3/256*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/c^4/(c*x-1)*arccosh(c*x)*x
-101/1228800*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c^5/(c*x-1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^4d^2x^8-2\,ac^2d^2x^6+ad^2x^4+\left(bc^4d^2x^8-2\,bc^2d^2x^6+bd^2x^4\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^8 - 2*a*c^2*d^2*x^6 + a*d^2*x^4 + (b*c^4*d^2*x^8 - 2*
b*c^2*d^2*x^6 + b*d^2*x^4)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arcosh}\left(cx\right)+a)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")

[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*x^4, x)
```

3.88 $\int x^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=371

$$\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{128c^2}-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{256bc^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}x^3$$

```
[Out] (5*b*d^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5
9*b*c*d^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (17
*b*c^3*d^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*c^5*d^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^2) + (5*d^2*x^3*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/64 + (5*d*x^3*(d - c^2*d*x^2)^(3/2)*(a
+ b*ArcCosh[c*x]))/48 + (x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/8
- (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.17067, antiderivative size = 402, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5745, 5743, 5759, 5676, 30, 14, 266, 43}

$$\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}d^2x^3(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{48}d^2x^3(1-cx)(cx+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (5*b*d^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5
9*b*c*d^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (17
*b*c^3*d^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*c^5*d^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^2) + (5*d^2*x^3*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/64 + (5*d^2*x^3*(1 - c*x)*(1 + c*x)*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/48 + (d^2*x^3*(1 - c*x)^2*(1 + c*x)^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (5*d^2*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{1}{8}d^2x^3(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(5d^2\sqrt{d-c^2dx^2}\right)}{}$$

$$=\frac{5}{48}d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}d^2x^3(1-cx)^2(1+$$

$$=\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{48}d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}$$

$$=-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{-1+cx}\sqrt{1+cx}}+\frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}}{288\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^5d^2x^8\sqrt{d-c^2dx^2}}{64\sqrt{-1+cx}\sqrt{1+cx}}-$$

$$=\frac{5bd^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{-1+cx}\sqrt{1+cx}}+\frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}}{288\sqrt{-1+cx}\sqrt{1+cx}}-$$

Mathematica [A] time = 4.4999, size = 415, normalized size = 1.12

$$\frac{192acd^2x\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(48c^6x^6-136c^4x^4+118c^2x^2-15\right)\sqrt{d-c^2dx^2}-2880ad^{5/2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (192*a*c*d^2*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-1
5 + 118*c^2*x^2 - 136*c^4*x^4 + 48*c^6*x^6) - 2880*a*d^(5/2)*Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*
x^2))] - 576*b*d^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c
*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 64*b*d^2*Sqrt[d - c^2*d*x^2]*
(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*
Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*
ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])) + b*d^2*Sqrt[d - c^2*d*x^2]*(-1440*A
rcCosh[c*x]^2 + 576*Cosh[2*ArcCosh[c*x]] - 144*Cosh[4*ArcCosh[c*x]] - 64*Co
sh[6*ArcCosh[c*x]] - 9*Cosh[8*ArcCosh[c*x]] + 24*ArcCosh[c*x]*(-48*Sinh[2*A
rcCosh[c*x]] + 24*Sinh[4*ArcCosh[c*x]] + 16*Sinh[6*ArcCosh[c*x]] + 3*Sinh[8
```

```
*ArcCosh[c*x]])))/(73728*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.335, size = 581, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/8*a*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a/c^2*x*(-c^2*d*x^2+d)^(5/2)+5/192
*a/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a/c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/12
8*a/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-59/7
68*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+5/256*b*(
-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2-1/64*b*(-d*(c^2
*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^8+17/288*b*(-d*(c^2*x^
2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-5/256*b*(-d*(c^2*x^2-1)
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*d^2+1/8*b*(-d*(c^2*x
^2-1))^(1/2)*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^9-23/48*b*(-d*(c^2*x^2-
1))^(1/2)*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^7+127/192*b*(-d*(c^2*x^2-1
))^(1/2)*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5-133/384*b*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+5/128*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x+359/73728*b*(-d*(c^2*x^2-1))^(1/2)*
d^2/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(ac^4d^2x^6 - 2\,ac^2d^2x^4 + ad^2x^2 + \left(bc^4d^2x^6 - 2\,bc^2d^2x^4 + bd^2x^2\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^6 - 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 - 2*
b*c^2*d^2*x^4 + b*d^2*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2 + d\right)^{\frac{5}{2}}(b\operatorname{arcosh}\left(cx\right) + a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*x^2, x)
```

3.89 $\int \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=293

$$\frac{5}{16}d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{32bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{24}d$$

```
[Out] (-25*b*c*d^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
5*b*c^3*d^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (5*d*x*(d - c^
2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/24 + (x*(d - c^2*d*x^2)^(5/2)*(a + b*A
rcCosh[c*x]))/6 - (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.542958, antiderivative size = 324, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5713, 5685, 5683, 5676, 30, 14, 261}

$$\frac{5}{16}d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}d^2x(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{24}d^2x(1-cx)(cx+1)\sqrt{d}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-25*b*c*d^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
5*b*c^3*d^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (5*d^2*x*(1 -
c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/24 + (d^2*x*(1 - c
*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (5*d^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
```

```
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{1}{6}d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(5d^2\sqrt{d-c^2dx^2}\right)\int(-1}{6}$$

$$=\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5}{24}d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.$$

$$=\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5}{16}d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{24}d^2x(1-$$

$$=-\frac{25bcd^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5bc^3d^2x^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}}+$$

Mathematica [A] time = 2.27664, size = 347, normalized size = 1.18

$$48acd^2x\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(8c^4x^4-26c^2x^2+33\right)\sqrt{d-c^2dx^2}-720ad^{5/2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-288bd^2\sqrt{d-c^2dx$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (48*a*c*d^2*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(33
- 26*c^2*x^2 + 8*c^4*x^4) - 720*a*d^(5/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 288*b*d^2*
Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] -
Sinh[2*ArcCosh[c*x]])) + 36*b*d^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + C
osh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + b*d^2*Sqrt[d -
 c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCos
```

```
h[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]]
 + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(2304*c*Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.204, size = 462, normalized size = 1.6

$$\frac{ax}{6}\left(-c^2dx^2+d\right)^{\frac{5}{2}}+\frac{5\,adx}{24}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+\frac{5\,ad^2x}{16}\sqrt{-c^2dx^2+d}+\frac{5\,ad^3}{16}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}-\frac{5\,d^2b\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/6*a*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*(
-c^2*d*x^2+d)^(1/2)+5/16*a*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d
*x^2+d)^(1/2))-5/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*
arccosh(c*x)^2*d^2-1/36*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*c^5*x^6+13/96*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2
)*c^3*x^4-11/32*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*
x^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^6*arccosh(c*x)*x^7-1
7/24*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)*x^5+59/4
8*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)*x^3-11/16*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x+299/2304*b*(-d*(
c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ac^4d^2x^4-2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4-2\,bc^2d^2x^2+bd^2\right)\mathrm{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a), x)
```

3.90 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=284

$$-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16b\sqrt{cx-1}\sqrt{cx+1}}-\frac{5}{4}c^2dx\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(c\right.$$

```
[Out] (9*b*c^3*d^2*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
b*c^5*d^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*
c^2*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (5*c^2*d*x*(d - c^2
*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/4 - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCo
sh[c*x]))/x + (15*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.678239, antiderivative size = 315, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5740, 5685, 5683, 5676, 30, 14, 266, 43}

$$-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5}{4}c^2d^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{15cd^2\sqrt{d-c^2dx^2}\left(a\right.}{16b\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (9*b*c^3*d^2*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
b*c^5*d^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*
c^2*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (5*c^2*d^2*x*(1 - c
*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (d^2*(1 - c*x)^
2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x + (15*c*d^2*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (b*c*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
```

```
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
```

```
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{\left(bcd^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1\cdots}{\cdots}}{\sqrt{-1+cx}\sqrt{1+c\cdots}}$$

$$= -\frac{5}{4}c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d^2(1-cx)^2(1+cx)^2\sqrt{\cdots}}{\cdots}$$

$$= -\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{5}{4}c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(\cdots\right.$$

$$= \frac{9bc^3d^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}} - \frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{\cdots}\right.$$

Mathematica [A] time = 1.66892, size = 305, normalized size = 1.07

$$\frac{1}{128}d^2\left(\frac{16a\left(2c^4x^4-9c^2x^2-8\right)\sqrt{d-c^2dx^2}}{x} + 240ac\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) + 64bc\sqrt{d-c^2dx^2}\left(\frac{2\log(cx)+\cosh^{-1}(cx\cdots}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (d^2*((16*a*Sqrt[d - c^2*d*x^2]*(-8 - 9*c^2*x^2 + 2*c^4*x^4))/x + 240*a*c*S
qrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 64*b*c*
Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x]
)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (32*b*c*Sqrt[d - c^2*d*x^2]*(2*
ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]
))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*c*Sqrt[d - c^2*d*x^2]*(8*Arc
Cosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/128
```

Maple [B] time = 0.255, size = 550, normalized size = 1.9

$$-\frac{a}{dx}\left(-c^2dx^2+d\right)^{\frac{7}{2}} - ac^2x\left(-c^2dx^2+d\right)^{\frac{5}{2}} - \frac{5\,dac^2x}{4}\left(-c^2dx^2+d\right)^{\frac{3}{2}} - \frac{15\,d^2ac^2x}{8}\sqrt{-c^2dx^2+d} - \frac{15\,ac^2d^3}{8}\arctan\left(x\sqrt{c^2d}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(7/2)-a*c^2*x*(-c^2*d*x^2+d)^(5/2)-5/4*a*c^2*d*x*(-c^
2*d*x^2+d)^(3/2)-15/8*a*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-15/8*a*c^2*d^3/(c^2*
d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-d*(c^2*x^2-1))^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c
*d^2+15/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x
)^2*c*d^2-1/16*b*(-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)
*x^4+9/16*b*(-d*(c^2*x^2-1))^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2-
b*(-d*(c^2*x^2-1))^(1/2)*c*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)+b*(
-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*d^2/(c*x+1)/(c*x-1)/x+1/4*b*(-d*(c^2*x^2
-1))^(1/2)*c^6*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^5-11/8*b*(-d*(c^2*x^2-1))
^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/8*b*(-d*(c^2*x^2-1))^(1/2
)*c^2*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x-33/128*b*(-d*(c^2*x^2-1))^(1/2)*c*
d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\text{arcosh}\,(cx)\right)\sqrt{-c^2dx^2 + d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)/x^2, x)
```

3.91 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=293

$$\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5c^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4b\sqrt{cx-1}\sqrt{cx+1}}+\frac{5c^2d\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3x}$$

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^
5*d^2*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^4*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 + (5*c^2*d*(d - c^2*d*x^2)^
(3/2)*(a + b*ArcCosh[c*x]))/(3*x) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c
*x]))/(3*x^3) - (5*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c^3*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(
3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.86047, antiderivative size = 324, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5740, 5683, 5676, 30, 14, 266, 43}

$$\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5c^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4b\sqrt{cx-1}\sqrt{cx+1}}+\frac{5c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\right.}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^
5*d^2*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^4*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 + (5*c^2*d^2*(1 - c*x)*(1 +
 c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x) - (d^2*(1 - c*x)^2*(1
 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^3) - (5*c^3*d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (7*b*c^3*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} + \frac{\left(bcd^2\sqrt{d-c^2dx^2}\right)\int \frac{(-}{}}{3\sqrt{-1+cx}\sqrt{1-}}$$

$$= \frac{5c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x} - \frac{d^2(1-cx)^2(1+cx)^2\sqrt{d}}{}$$

$$= \frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{5c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+\right.}{3x}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\left(a+b\cos\right.$$

Mathematica [A] time = 1.35995, size = 319, normalized size = 1.09

$$\frac{-4d^3\left(a\sqrt{\frac{cx-1}{cx+1}}\left(3c^6x^6+11c^4x^4-16c^2x^2+2\right)-14bc^3x^3(cx-1)\log(cx)+bcx(1-cx)\right)-60ac^3d^{5/2}x^3\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\,\mathrm{ta}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (30*b*c^3*d^3*x^3*(-1 + c*x)*ArcCosh[c*x]^2 - 60*a*c^3*d^(5/2)*x^3*Sqrt[(-1
 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sq
rt[d]*(-1 + c^2*x^2))] + 3*b*c^3*d^3*x^3*(-1 + c*x)*Cosh[2*ArcCosh[c*x]] -
4*d^3*(b*c*x*(1 - c*x) + a*Sqrt[(-1 + c*x)/(1 + c*x)]*(2 - 16*c^2*x^2 + 11*
c^4*x^4 + 3*c^6*x^6) - 14*b*c^3*x^3*(-1 + c*x)*Log[c*x]) - 2*b*d^3*(-1 + c*
x)*ArcCosh[c*x]*(4*Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 - c*x + 7*c^2*x^2 + 7*c^3
*x^3) + 3*c^3*x^3*Sinh[2*ArcCosh[c*x]]))/(24*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]
*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.265, size = 1407, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] 5/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c
*x-1)^(1/2)*c^3+4/3*a*c^2/d/x*(-c^2*d*x^2+d)^(7/2)+1/2*b*(-d*(c^2*x^2-1))^(
1/2)*c^6*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)*
c^4*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x+49/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(6
3*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-28/3*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+7/6*b*(-d*(c^2*x^2
-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4+1/3*b*(-d*(c
^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(
c*x)-21/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*c^5-1/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c
^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-7/3*b*(-d*(c^2*x^2-1))^(1/2)*d^
2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3-1/
3*a/d/x^3*(-c^2*d*x^2+d)^(7/2)+4/3*a*c^4*x*(-c^2*d*x^2+d)^(5/2)-147*b*(-d*(
c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(
1/2)*arccosh(c*x)*c^7+35*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^
2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5+147*b*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-
203*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x
-1)*arccosh(c*x)*c^6+190/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*
x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-23/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2
/(63*c^4*x^4-15*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2+5/3*a*c^4*d*x
*(-c^2*d*x^2+d)^(3/2)+5/2*a*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/2*a*c^4*d^3/(c
^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-7/3*b*(-d*(c^2*x^2
```

```
-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2+1)*c^3*d^2-5/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arcc
osh(c*x)^2*c^3*d^2+14/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)*arccosh(c*x)*c^3*d^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(
c*x-1)^(1/2)*x^2+1/8*b*(-d*(c^2*x^2-1))^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)
^(1/2)-49/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3*c^6+
7/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2 + d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)/x^4, x)
```

3.92 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=293

$$\frac{c^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{c^2d\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{(d$$

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(20*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (11*
b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(30*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c^4
*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x + (c^2*d*(d - c^2*d*x^2)^(
3/2)*(a + b*ArcCosh[c*x]))/(3*x^3) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[
c*x]))/(5*x^5) + (c^5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (23*b*c^5*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(
15*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.947257, antiderivative size = 324, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5740, 5738, 29, 5676, 14, 266, 43}

$$\frac{c^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(20*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (11*
b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(30*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c^4
*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x + (c^2*d^2*(1 - c*x)*(1 +
c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^3) - (d^2*(1 - c*x)^2*(
1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*x^5) + (c^5*d^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (23*b*c^5*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd^2\sqrt{d-c^2dx^2}\right)\int \frac{(-}{}}{5\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{d^2(1-cx)^2(1+cx)^2\sqrt{d}}{5}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\right.}{3x^3}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{11bc^3d^2\sqrt{d-c^2dx^2}}{30x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{x}$$

Mathematica [A] time = 3.18745, size = 400, normalized size = 1.37

$$\frac{d^2\left(8ad\sqrt{\frac{cx-1}{cx+1}}\left(c^2x^2-1\right)\left(23c^4x^4-11c^2x^2+3\right)+120ac^5\sqrt{d}x^5\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-60bc^4dx^4(1-cx)\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] (d^2*(8*a*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + c^2*x^2)*(3 - 11*c^2*x^2 + 23*
c^4*x^4) + 120*a*c^5*Sqrt[d]*x^5*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*
x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 40*b*c^2*
d*x^2*(1 - c*x)*(c*x - 2*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c
*x] + 2*c^3*x^3*Log[c*x]) - 60*b*c^4*d*x^4*(1 - c*x)*(2*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*x]^2 + 2*Log[c*x])) - b*d*(
1 - c*x)*(20*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + Cosh[5*Arc
Cosh[c*x]]*Log[c*x] + Cosh[3*ArcCosh[c*x]]*(-1 + 5*Log[c*x]) + c*x*(3 + 10*
Log[c*x]) - 5*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - ArcCosh[c*x]*Sinh[5*ArcCo
sh[c*x]])))/(120*x^5*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.275, size = 2429, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x)
```

```
[Out] -8/15*a*c^4/d/x*(-c^2*d*x^2+d)^(7/2)-1173*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(103
5*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^6/(c*x+1)^(1/2)/(c*x-1)^(
1/2)*arccosh(c*x)*c^11+1495/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-76
5*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh
(c*x)*c^9-115*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^
4*x^4-75*c^2*x^2+9)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7-1587*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^
2+9)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^14+3519*b*(-d*(c^2*x^2-1))^(1/2)*d^
2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^7/(c*x+1)/(c*x-1)*a
rccosh(c*x)*c^12-9595/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*
x^6+325*c^4*x^4-75*c^2*x^2+9)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10+5318/3*
b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x
^2+9)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-9602/15*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x/(c*x+1)/(c*x-1)*
arccosh(c*x)*c^6+777/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x
^6+325*c^4*x^4-75*c^2*x^2+9)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-117/5*b*(-d
*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)
/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-1/5*a/d/x^5*(-c^2*d*x^2+d)^(7/2)-8/15
*a*c^6*x*(-c^2*d*x^2+d)^(5/2)-9/20*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x
^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+
759/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75
```

```
*c^2*x^2+9)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^11-1329/4*b*(-d*(c^2*x^2-1))^
(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^4/(c*x+1)^(
1/2)/(c*x-1)^(1/2)*c^9+1889/12*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-7
65*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+69
/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^
2*x^2+9)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5+2/15*a*c^2/d/x^3*(-c^
2*d*x^2+d)^(7/2)+207/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x
^6+325*c^4*x^4-75*c^2*x^2+9)*x^3/(c*x+1)/(c*x-1)*c^8-69/20*b*(-d*(c^2*x^2-1
))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x/(c*x+1)/
(c*x-1)*c^6+9/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*
c^4*x^4-75*c^2*x^2+9)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)-5819/30*b*(-d*(c^2*x
^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^9/(c
*x+1)/(c*x-1)*c^14+18791/60*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*
c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^7/(c*x+1)/(c*x-1)*c^12-943/6*b*(-d*(c^2
*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^5/
(c*x+1)/(c*x-1)*c^10+141/20*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*
c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-7153/
60*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^
2*x^2+9)*x^5*c^10+759/20*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6
*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3*c^8-69/20*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(
1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x*c^6+5819/30*b*(-d*(c^2
*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^7*
c^12-2/3*a*c^6*d*x*(-c^2*d*x^2+d)^(3/2)-a*c^6*d^2*x*(-c^2*d*x^2+d)^(1/2)-a*
c^6*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1587*b*(
-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+
9)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^13-46/15*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^5*d^2+1/2*b*(-d*(c^2*x^
2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c^5*d^2+23/15*b*(-d*
(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))^2+1)*c^5*d^2-175/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*
c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4-2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4-2\,bc^2d^2x^2+bd^2\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)/x^6, x)
```

3.93 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=219

$$-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7dx^7}-\frac{3bc^5d^2\sqrt{d-c^2dx^2}}{14x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3d^2\sqrt{d-c^2dx^2}}{28x^4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd^2\sqrt{d-c^2dx^2}}{42x^6\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc^7d^2\log(}{7\sqrt{cx-}}$$

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(42*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b
*c^3*d^2*Sqrt[d - c^2*d*x^2])/(28*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*
c^5*d^2*Sqrt[d - c^2*d*x^2])/(14*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d -
c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*d*x^7) - (b*c^7*d^2*Sqrt[d - c^2*
d*x^2]*Log[x])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.378647, antiderivative size = 234, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5724, 266, 43}

$$-\frac{d^2(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7}-\frac{3bc^5d^2\sqrt{d-c^2dx^2}}{14x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3d^2\sqrt{d-c^2dx^2}}{28x^4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd^2\sqrt{d-c^2dx}}{42x^6\sqrt{cx-1}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(42*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b
*c^3*d^2*Sqrt[d - c^2*d*x^2])/(28*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*
c^5*d^2*Sqrt[d - c^2*d*x^2])/(14*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(
1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*x^7) -
(b*c^7*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} - \frac{\left(bcd^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1}{7\sqrt{-1+cx}\sqrt{1+}}$$

$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} - \frac{\left(bcd^2\sqrt{d-c^2dx^2}\right)\mathrm{Subs}}{14\sqrt{-1+c}}$$

$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} - \frac{\left(bcd^2\sqrt{d-c^2dx^2}\right)\mathrm{Subs}}{14\sqrt{}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3d^2\sqrt{d-c^2dx^2}}{28x^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3bc^5d^2\sqrt{d-c^2dx^2}}{14x^2\sqrt{-1+cx}\sqrt{1+cx}} -$$

Mathematica [A] time = 0.0949244, size = 105, normalized size = 0.48

$$\frac{d^2\sqrt{d-c^2dx^2}\left(12(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)-bcx\left(18c^4x^4-9c^2x^2+12c^6x^6\log(x)+2\right)\right)}{84x^7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(12*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCos
h[c*x]) - b*c*x*(2 - 9*c^2*x^2 + 18*c^4*x^4 + 12*c^6*x^6*Log[x])))/(84*x^7*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.301, size = 3775, normalized size = 17.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x)
```

```
[Out] -3/14*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*
c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^11*c^18+1/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/
(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^7
/(c*x+1)/(c*x-1)*arccosh(c*x)+3/14*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^1
2-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^13/(c*x+1)/(
c*x-1)*c^20-27/28*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35
*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^11/(c*x+1)/(c*x-1)*c^18-41/28
*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x
^6+21*c^4*x^4-7*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+23/84*b*(-d*
(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c
^4*x^4-7*c^2*x^2+1)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/42*b*(-d*(c^2*x^2
-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7
*c^2*x^2+1)/x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-1/7*b*(-d*(c^2*x^2-1))^(1/2)*
d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)
/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7-3/2*b*(-d*(c^2*x^2-1))^(1/2)*
d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)
*x^10/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^17+21/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7
*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^8/(
c*x+1)^(1/2)/(c*x-1)^(1/2)*c^15-119/12*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12
*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^6/(c*x+1
```

)^(1/2)/(c*x-1)^(1/2)*c^13+47/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^11-109/12*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9+73/42*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^9/(c*x+1)/(c*x-1)*c^16-67/42*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^7/(c*x+1)/(c*x-1)*c^14+11/14*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^12-17/84*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^10+1/42*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^8-7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^11/(c*x+1)/(c*x-1)*arccosh(c*x)*c^18+23*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^16-47*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^14+66*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12-66*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10+330/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-165/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+55/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-11/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11+b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^12/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^19+3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^10/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^17-5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^15+b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^13/(c*x+1)/(c*x-1)*arccosh(c*x)*c^20+5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^13-1/7*a/d/x^7*(-c^2*d*x^2+d)^(7/2)+3/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c

```
^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^9*c^1
6-83/84*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-3
5*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^7*c^14+17/28*b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*
x^5*c^12-5/28*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8
*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^3*c^10+1/42*b*(-d*(c^2*x^2-1))^(1
/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^
2+1)*x*c^8+55/12*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*
c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+
2/7*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^7*d
^2-1/7*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2+1)*c^7*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69859, size = 1473, normalized size = 6.73

$$\left[\frac{12\left(bc^8d^2x^8-4\,bc^6d^2x^6+6\,bc^4d^2x^4-4\,bc^2d^2x^2+bd^2\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+6\left(bc^9d^2x^9-bc^7d^2x^7\right)\sqrt{-d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="fricas"
)
```

```
[Out] [1/84*(12*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*
x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 6*(b*c^9*d
^2*x^9 - b*c^7*d^2*x^7)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(
-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2))
```

```
- (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x^7 - 9*b*c^3*d^2*x^
3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 12*(a*c^8*d^2*x^8
 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*sqrt(-c^2*d
*x^2 + d))/(c^2*x^9 - x^7), -1/84*(12*(b*c^9*d^2*x^9 - b*c^7*d^2*x^7)*sqrt(
d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x
^4 - (c^2 + 1)*d*x^2 + d)) - 12*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*
d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*
x^2 - 1)) + (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x^7 - 9*b*
c^3*d^2*x^3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 12*(a*c
^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*s
qrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)/x^8, x)
```

3.94 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx$

Optimal. Leaf size=314

$$-\frac{2c^2\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{63dx^7}-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{9dx^9}-\frac{bc^7d^2\sqrt{d-c^2dx^2}}{21x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^5d^2\sqrt{d-c^2dx^2}}{42x^4\sqrt{cx-1}\sqrt{cx-}}$$

```
[Out] -(b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(189*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*c^5*d^2*Sqrt[d - c^2*d*x^2])/(42*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c
^7*d^2*Sqrt[d - c^2*d*x^2])/(21*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^
2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(72*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(9*d*x^9) - (2*c^2*(d - c^
2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(63*d*x^7) - (2*b*c^9*d^2*Sqrt[d - c^2
*d*x^2]*Log[x])/(63*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.525799, antiderivative size = 448, normalized size of antiderivative = 1.43, number of steps used = 7, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 97, 12, 103, 95, 5733, 446, 78, 43}

$$\frac{2c^8d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63x}+\frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63x^3}-\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5}+\frac{5c^2d}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] -(b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(189*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*c^5*d^2*Sqrt[d - c^2*d*x^2])/(42*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c
^7*d^2*Sqrt[d - c^2*d*x^2])/(21*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^
2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(72*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (c^4*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(21*x^5) + (c^6*d^2*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(63*x^3) + (2*c^8*d^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(63*x) + (5*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(63*x^7) - (d^2*(1 - c*x)^2*(1 + c*x)
^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*x^9) - (2*b*c^9*d^2*Sqrt[d
- c^2*d*x^2]*Log[x])/(63*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
```

```
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63x^3} + \frac{2}{-}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63x^3} + \frac{2}{-}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63x^3} + \frac{2}{-}$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)^4\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\sqrt{1+cx}} - \frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5} + \frac{c^6d^2\sqrt{d}}{}$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)^4\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\sqrt{1+cx}} - \frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5} + \frac{c^6d^2\sqrt{d}}{}$$

$$= -\frac{bc^3d^2\sqrt{d-c^2dx^2}}{189x^6\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^5d^2\sqrt{d-c^2dx^2}}{42x^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^7d^2\sqrt{d-c^2dx^2}}{21x^2\sqrt{-1+cx}\sqrt{1+cx}} -$$

Mathematica [A] time = 0.162677, size = 147, normalized size = 0.47

$$\frac{d^2\sqrt{d-c^2dx^2}\left(48c^2x^2(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+168(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)-bcx\left(-12c^6x\right.\right.}{1512x^9\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(168*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCo
sh[c*x]) + 48*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x])
 - b*c*x*(21 - 76*c^2*x^2 + 90*c^4*x^4 - 12*c^6*x^6 + 48*c^8*x^8*Log[x])))/
(1512*x^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.398, size = 5006, normalized size = 15.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.923, size = 1715, normalized size = 5.46

$$\left[\frac{24\left(2\,bc^{10}d^{2}x^{10}-bc^{8}d^{2}x^{8}-16\,bc^{6}d^{2}x^{6}+34\,bc^{4}d^{2}x^{4}-26\,bc^{2}d^{2}x^{2}+7\,bd^{2}\right)\sqrt{-c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)+24\left(bc^{1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="fricas
")
```

```
[Out] [1/1512*(24*(2*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^
4*d^2*x^4 - 26*b*c^2*d^2*x^2 + 7*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt
(c^2*x^2 - 1)) + 24*(b*c^11*d^2*x^11 - b*c^9*d^2*x^9)*sqrt(-d)*log((c^2*d*x
^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*s
qrt(-d) - d)/(c^2*x^4 - x^2)) + (12*b*c^7*d^2*x^7 - 90*b*c^5*d^2*x^5 - (12*
b*c^7 - 90*b*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*b*c^3*d^2*x^3 - 21*b*c*d
^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 24*(2*a*c^10*d^2*x^10 - a*c^
8*d^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 26*a*c^2*d^2*x^2 + 7*a*d^
2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9), -1/1512*(48*(b*c^11*d^2*x^11 - b
*c^9*d^2*x^9)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 +
```

```
1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 24*(2*b*c^10*d^2*x^10 - b*c
^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^4*d^2*x^4 - 26*b*c^2*d^2*x^2 + 7*b*d
^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (12*b*c^7*d^2*x^7 -
 90*b*c^5*d^2*x^5 - (12*b*c^7 - 90*b*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*
b*c^3*d^2*x^3 - 21*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 24*(
2*a*c^10*d^2*x^10 - a*c^8*d^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 2
6*a*c^2*d^2*x^2 + 7*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**10,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)/x^10, x)
```

3.95 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx$

Optimal. Leaf size=385

$$-\frac{8c^4\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{693dx^7}-\frac{4c^2\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{99dx^9}-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{11dx^{11}}+\frac{}{69}$$

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(110*x^10*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2
3*b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(792*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
113*b*c^5*d^2*Sqrt[d - c^2*d*x^2])/(4158*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (b*c^7*d^2*Sqrt[d - c^2*d*x^2])/(924*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(2*b*c^9*d^2*Sqrt[d - c^2*d*x^2])/(693*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(11*d*x^11) - (4*c^2*(d - c^2*
d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(99*d*x^9) - (8*c^4*(d - c^2*d*x^2)^(7/2
)*(a + b*ArcCosh[c*x]))/(693*d*x^7) - (8*b*c^11*d^2*Sqrt[d - c^2*d*x^2]*Log
[x])/(693*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.578848, antiderivative size = 519, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 97, 12, 103, 95, 5733, 1251, 893}

$$\frac{8c^{10}d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693x}+\frac{4c^8d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693x^3}+\frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^5}-\frac{5c}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(110*x^10*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2
3*b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(792*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
113*b*c^5*d^2*Sqrt[d - c^2*d*x^2])/(4158*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (b*c^7*d^2*Sqrt[d - c^2*d*x^2])/(924*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(2*b*c^9*d^2*Sqrt[d - c^2*d*x^2])/(693*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(5*c^4*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(231*x^7) + (c^6*d^2*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(231*x^5) + (4*c^8*d^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(693*x^3) + (8*c^10*d^2*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/(693*x) + (5*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x]))/(99*x^9) - (d^2*(1 - c*x)^2*(1 + c*x)^2*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(11*x^11) - (8*b*c^11*d^2*Sqrt[d - c^2
```

```
*d*x^2]*Log[x])/(693*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^5} +$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^5} +$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^5} +$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^5} +$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{23bc^3d^2\sqrt{d-c^2dx^2}}{792x^8\sqrt{-1+cx}\sqrt{1+cx}} - \frac{113bc^5d^2\sqrt{d-c^2dx^2}}{4158x^6\sqrt{-1+cx}\sqrt{1+}}$$

Mathematica [A] time = 0.198293, size = 165, normalized size = 0.43

$$\frac{d^2\sqrt{d-c^2dx^2}\left(480c^2x^2(cx-1)^{7/2}\left(2c^2x^2+7\right)(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+7560(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)\right.}{83160x^{11}\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(7560*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcC
osh[c*x]) + 480*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(7 + 2*c^2*x^2)*(a
 + b*ArcCosh[c*x]) - b*c*x*(756 - 2415*c^2*x^2 + 2260*c^4*x^4 - 90*c^6*x^6
- 240*c^8*x^8 + 960*c^10*x^10*Log[x])))/(83160*x^11*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

Maple [B] time = 0.543, size = 6379, normalized size = 16.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.09624, size = 1979, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="fricas
")
```

```
[Out] [1/83160*(120*(8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 116*
b*c^6*d^2*x^6 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*sqrt(-c^2
*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 480*(b*c^13*d^2*x^13 - b*c^11*d^
2*x^11)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*
sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (240*b*c^9*d^2
*x^9 + 90*b*c^7*d^2*x^7 - (240*b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 -
 756*b*c)*d^2*x^11 - 2260*b*c^5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*
x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 120*(8*a*c^12*d^2*x^12 - 4*a*c^
10*d^2*x^10 - a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a
*c^2*d^2*x^2 + 63*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11), -1/83160*
(960*(b*c^13*d^2*x^13 - b*c^11*d^2*x^11)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d
)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) -
120*(8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 116*b*c^6*d^2*
```

```
x^6 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*sqrt(-c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 - 1)) - (240*b*c^9*d^2*x^9 + 90*b*c^7*d^2*x^7 - (2
40*b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 - 756*b*c)*d^2*x^11 - 2260*b*
c^5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 120*(8*a*c^12*d^2*x^12 - 4*a*c^10*d^2*x^10 - a*c^8*d^2*x^8
- 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a*c^2*d^2*x^2 + 63*a*d^2)*sqr
t(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**12,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="giac")

[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)/x^12, x)
```

3.96 $\int x^7 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=458

$$\frac{\left(d-c^2dx^2\right)^{13/2}\left(a+b\cosh^{-1}(cx)\right)}{13c^8d^4}-\frac{3\left(d-c^2dx^2\right)^{11/2}\left(a+b\cosh^{-1}(cx)\right)}{11c^8d^3}+\frac{\left(d-c^2dx^2\right)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^8d^2}-\frac{\left(d-c^2d\right.}{}$$

```
[Out] (16*b*d^2*x*Sqrt[d - c^2*d*x^2])/(3003*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(8*b*d^2*x^3*Sqrt[d - c^2*d*x^2])/(9009*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
 (2*b*d^2*x^5*Sqrt[d - c^2*d*x^2])/(5005*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (5*b*d^2*x^7*Sqrt[d - c^2*d*x^2])/(21021*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (53*b*c*d^2*x^9*Sqrt[d - c^2*d*x^2])/(3861*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (27*b*c^3*d^2*x^11*Sqrt[d - c^2*d*x^2])/(1573*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (b*c^5*d^2*x^13*Sqrt[d - c^2*d*x^2])/(169*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^8*d) + ((d - c^2*d*x^
2)^(9/2)*(a + b*ArcCosh[c*x]))/(3*c^8*d^2) - (3*(d - c^2*d*x^2)^(11/2)*(a +
 b*ArcCosh[c*x]))/(11*c^8*d^3) + ((d - c^2*d*x^2)^(13/2)*(a + b*ArcCosh[c*x
]))/(13*c^8*d^4)
```

Rubi [A] time = 0.51565, antiderivative size = 527, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 100, 12, 74, 5733, 1810}

$$-\frac{d^2x^6(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{13c^2}-\frac{6d^2x^4(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{143c^4}-\frac{8d^2x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^2*x*Sqrt[d - c^2*d*x^2])/(3003*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(8*b*d^2*x^3*Sqrt[d - c^2*d*x^2])/(9009*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
 (2*b*d^2*x^5*Sqrt[d - c^2*d*x^2])/(5005*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (5*b*d^2*x^7*Sqrt[d - c^2*d*x^2])/(21021*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (53*b*c*d^2*x^9*Sqrt[d - c^2*d*x^2])/(3861*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (27*b*c^3*d^2*x^11*Sqrt[d - c^2*d*x^2])/(1573*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (b*c^5*d^2*x^13*Sqrt[d - c^2*d*x^2])/(169*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (16*d^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
))/(3003*c^8) - (8*d^2*x^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a +
 b*ArcCosh[c*x]))/(429*c^6) - (6*d^2*x^4*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c
```

```
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(143*c^4) - (d^2*x^6*(1 - c*x)^3*(1 + c*x)^
3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(13*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^7 \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^7(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{16d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3003c^8} - \frac{8d^2x^2(1-cx)^3(1+\ldots}{\ldots}$$

$$= -\frac{16d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3003c^8} - \frac{8d^2x^2(1-cx)^3(1+\ldots}{\ldots}$$

$$= -\frac{16d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3003c^8} - \frac{8d^2x^2(1-cx)^3(1+\ldots}{\ldots}$$

$$= \frac{16bd^2x\sqrt{d-c^2dx^2}}{3003c^7\sqrt{-1+cx}\sqrt{1+cx}} + \frac{8bd^2x^3\sqrt{d-c^2dx^2}}{9009c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bd^2x^5\sqrt{d-c^2\ldots}}{5005c^3\sqrt{-1+cx}\ldots}$$

Mathematica [A] time = 0.246745, size = 193, normalized size = 0.42

$$\frac{d^2\sqrt{d-c^2dx^2}\left(231c^5x^6(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{2(cx-1)^{7/2}(cx+1)^{7/2}\left(63c^4x^4+28c^2x^2+8\right)\left(a+b\cosh^{-1}(cx)\right)}{c}+b\left(-\frac{231}{13}\ldots\right.\right.}{3003c^7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(b*(16*x + (8*c^2*x^3)/3 + (6*c^4*x^5)/5 + (5*c^6*
x^7)/7 - (371*c^8*x^9)/9 + (567*c^10*x^11)/11 - (231*c^12*x^13)/13) + 231*c
^5*x^6*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x]) + (2*(-1 + c*x
)^(7/2)*(1 + c*x)^(7/2)*(8 + 28*c^2*x^2 + 63*c^4*x^4)*(a + b*ArcCosh[c*x]))
/c))/(3003*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.5, size = 2374, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/13*x^6*(-c^2*d*x^2+d)^(7/2)/c^2/d+6/13/c^2*(-1/11*x^4*(-c^2*d*x^2+d)^
(7/2)/c^2/d+4/11/c^2*(-1/9*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(-c^2*
d*x^2+d)^(7/2))))+b*(1/1384448*(-d*(c^2*x^2-1))^(1/2)*(-1-16896*c^8*x^8+649
6*c^6*x^6-1204*c^4*x^4+85*c^2*x^2+4096*x^14*c^14-15360*x^12*c^12+22784*x^10
*c^10-364*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+13*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x*c+4096*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^13*c^13-13312*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^11*c^11+16640*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9-9984*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^7*c^7+2912*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5)*(-1+13*
arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)+1/991232*(-d*(c^2*x^2-1))^(1/2)*(1+40
96*c^8*x^8-2352*c^6*x^6+620*c^4*x^4-61*c^2*x^2+1024*x^12*c^12-3328*x^10*c^1
0+220*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c+1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11-2816*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*x^9*c^9+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-1232*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^5*c^5)*(-1+11*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)-1/110592*(-
d*(c^2*x^2-1))^(1/2)*(256*x^10*c^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^9*c^9+688*c^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-280*c^4*x^
4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccosh(c*x))*
d^2/(c*x+1)/c^8/(c*x-1)-3/200704*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6
*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1
)+3/40960*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)
+5/24576*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d^
2/(c*x+1)/c^8/(c*x-1)-5/2048*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)-5/2048*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))*d^2/(c*x+1)/c^8/(c*x-1)+5/24576*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*
x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)+3/40960*(-d*(c^2*x^2-1))^
(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x
^2-1)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)-3/200704*(-d*(c^2*x^2-1))^
(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+1
04*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x
))*d^2/(c*x+1)/c^8/(c*x-1)-1/110592*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(1
```

```
/2)*(c*x-1)^(1/2)*x^9*c^9+256*x^10*c^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7
*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*c^6*x^6+120*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)+1/991232*(-d*
(c^2*x^2-1))^(1/2)*(-1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11+1024*x^12*c
^12+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9-3328*x^10*c^10-2816*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^7*c^7+4096*c^8*x^8+1232*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5
*c^5-2352*c^6*x^6-220*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+620*c^4*x^4+11*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-61*c^2*x^2+1)*(1+11*arccosh(c*x))*d^2/(c*x+1)
/c^8/(c*x-1)+1/1384448*(-d*(c^2*x^2-1))^(1/2)*(-4096*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^13*c^13+4096*x^14*c^14+13312*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11-
15360*x^12*c^12-16640*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+22784*x^10*c^10+9
984*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-16896*c^8*x^8-2912*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^5*c^5+6496*c^6*x^6+364*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-12
04*c^4*x^4-13*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+85*c^2*x^2-1)*(1+13*arccosh(c
*x))*d^2/(c*x+1)/c^8/(c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.29234, size = 833, normalized size = 1.82

$$\frac{45045\left(231\,bc^{14}d^{2}x^{14} - 798\,bc^{12}d^{2}x^{12} + 938\,bc^{10}d^{2}x^{10} - 376\,bc^{8}d^{2}x^{8} - bc^{6}d^{2}x^{6} - 2\,bc^{4}d^{2}x^{4} - 8\,bc^{2}d^{2}x^{2} + 16\,bd^{2}\right)\sqrt{-c^{2}d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] 1/135270135*(45045*(231*b*c^14*d^2*x^14 - 798*b*c^12*d^2*x^12 + 938*b*c^10*
d^2*x^10 - 376*b*c^8*d^2*x^8 - b*c^6*d^2*x^6 - 2*b*c^4*d^2*x^4 - 8*b*c^2*d^
```

```
2*x^2 + 16*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (8004
15*b*c^13*d^2*x^13 - 2321865*b*c^11*d^2*x^11 + 1856855*b*c^9*d^2*x^9 - 3217
5*b*c^7*d^2*x^7 - 54054*b*c^5*d^2*x^5 - 120120*b*c^3*d^2*x^3 - 720720*b*c*d
^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 45045*(231*a*c^14*d^2*x^14 -
 798*a*c^12*d^2*x^12 + 938*a*c^10*d^2*x^10 - 376*a*c^8*d^2*x^8 - a*c^6*d^2*
x^6 - 2*a*c^4*d^2*x^4 - 8*a*c^2*d^2*x^2 + 16*a*d^2)*sqrt(-c^2*d*x^2 + d))/(
c^10*x^2 - c^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.97 $\int x^5 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=378

$$-\frac{\left(d-c^2dx^2\right)^{11/2}\left(a+b\cosh^{-1}(cx)\right)}{11c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^6d}-\frac{bc^5d^2x}{121\sqrt{c}}$$

```
[Out] (8*b*d^2*x*Sqrt[d - c^2*d*x^2])/(693*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4
*b*d^2*x^3*Sqrt[d - c^2*d*x^2])/(2079*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*d^2*x^5*Sqrt[d - c^2*d*x^2])/(1155*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (113
*b*c*d^2*x^7*Sqrt[d - c^2*d*x^2])/(4851*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (23
*b*c^3*d^2*x^9*Sqrt[d - c^2*d*x^2])/(891*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*c^5*d^2*x^11*Sqrt[d - c^2*d*x^2])/(121*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d
 - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^6*d) + (2*(d - c^2*d*x^2)^(9
/2)*(a + b*ArcCosh[c*x]))/(9*c^6*d^2) - ((d - c^2*d*x^2)^(11/2)*(a + b*ArcC
osh[c*x]))/(11*c^6*d^3)
```

Rubi [A] time = 0.468281, antiderivative size = 429, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 100, 12, 74, 5733, 1153}

$$-\frac{d^2x^4(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{11c^2}-\frac{4d^2x^2(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{99c^4}-\frac{8d^2(1}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*x*Sqrt[d - c^2*d*x^2])/(693*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4
*b*d^2*x^3*Sqrt[d - c^2*d*x^2])/(2079*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*d^2*x^5*Sqrt[d - c^2*d*x^2])/(1155*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (113
*b*c*d^2*x^7*Sqrt[d - c^2*d*x^2])/(4851*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (23
*b*c^3*d^2*x^9*Sqrt[d - c^2*d*x^2])/(891*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*c^5*d^2*x^11*Sqrt[d - c^2*d*x^2])/(121*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*
d^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(693*
c^6) - (4*d^2*x^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/(99*c^4) - (d^2*x^4*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(11*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^5 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^5(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{8d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693c^6} - \frac{4d^2x^2(1-cx)^3(1+$$

$$= -\frac{8d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693c^6} - \frac{4d^2x^2(1-cx)^3(1+$$

$$= -\frac{8d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693c^6} - \frac{4d^2x^2(1-cx)^3(1+$$

$$= \frac{8bd^2x\sqrt{d-c^2dx^2}}{693c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4bd^2x^3\sqrt{d-c^2dx^2}}{2079c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bd^2x^5\sqrt{d-c^2dx^2}}{1155c\sqrt{-1+cx}\sqrt{1+$$

Mathematica [A] time = 0.191586, size = 175, normalized size = 0.46

$$\frac{d^2\sqrt{d-c^2dx^2}\left(63c^3x^4(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{4(cx-1)^{7/2}(cx+1)^{7/2}\left(7c^2x^2+2\right)\left(a+b\cosh^{-1}(cx)\right)}{c}+b\left(-\frac{63}{11}c^{10}x^{11}+\right.\right.}{693c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (113*c^6
*x^7)/7 + (161*c^8*x^9)/9 - (63*c^10*x^11)/11) + 63*c^3*x^4*(-1 + c*x)^(7/2
)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x]) + (4*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2
)*(2 + 7*c^2*x^2)*(a + b*ArcCosh[c*x]))/c))/(693*c^5*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])
```

Maple [B] time = 0.415, size = 1840, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/11*x^4*(-c^2*d*x^2+d)^(7/2)/c^2/d+4/11/c^2*(-1/9*x^2*(-c^2*d*x^2+d)^(
7/2)/c^2/d-2/63/d/c^4*(-c^2*d*x^2+d)^(7/2)))+b*(1/247808*(-d*(c^2*x^2-1))^(
1/2)*(1+4096*c^8*x^8-2352*c^6*x^6+620*c^4*x^4-61*c^2*x^2+1024*x^12*c^12-332
8*x^10*c^10+220*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-11*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c+1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11-2816*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^9*c^9+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-1232*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^5*c^5)*(-1+11*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-1
/165888*(-d*(c^2*x^2-1))^(1/2)*(256*x^10*c^10-704*c^8*x^8+256*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^9*c^9+688*c^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-
280*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arcc
osh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/100352*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x
^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/
c^6/(c*x-1)+1/10240*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3
*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c
^6/(c*x-1)+5/9216*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh
(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/1024*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/1
024*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+
arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)+5/9216*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)+1/10240*(-d*(c^2*
x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
13*c^2*x^2-1)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/100352*(-d*(c^2*
x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arc
cosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-1/165888*(-d*(c^2*x^2-1))^(1/2)*(-256*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*x^10*c^10+576*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*c^6*x^
6+120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)+1/247
808*(-d*(c^2*x^2-1))^(1/2)*(-1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11+102
4*x^12*c^12+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9-3328*x^10*c^10-2816*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+4096*c^8*x^8+1232*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^5*c^5-2352*c^6*x^6-220*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+620*c^4*x
^4+11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-61*c^2*x^2+1)*(1+11*arccosh(c*x))*d^2
```

```
/(c*x+1)/c^6/(c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12527, size = 722, normalized size = 1.91

$$\frac{3465\left(63\,bc^{12}d^2x^{12} - 224\,bc^{10}d^2x^{10} + 274\,bc^8d^2x^8 - 116\,bc^6d^2x^6 - bc^4d^2x^4 - 4\,bc^2d^2x^2 + 8\,bd^2\right)\sqrt{-c^2dx^2 + d}\,\log\left(cx + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] 1/2401245*(3465*(63*b*c^12*d^2*x^12 - 224*b*c^10*d^2*x^10 + 274*b*c^8*d^2*x
^8 - 116*b*c^6*d^2*x^6 - b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 8*b*d^2)*sqrt(-c
^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (19845*b*c^11*d^2*x^11 - 61985
*b*c^9*d^2*x^9 + 55935*b*c^7*d^2*x^7 - 2079*b*c^5*d^2*x^5 - 4620*b*c^3*d^2*
x^3 - 27720*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 3465*(63*a*
c^12*d^2*x^12 - 224*a*c^10*d^2*x^10 + 274*a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6
 - a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + 8*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^8*x^
2 - c^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.98 $\int x^3 \left(d-c^2 d x^2\right)^{5/2} \left(a+b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=298

$$\frac{\left(d-c^2dx^2\right)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{9c^4d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^4d}-\frac{bc^5d^2x^9\sqrt{d-c^2dx^2}}{81\sqrt{cx-1}\sqrt{cx+1}}+\frac{19bc^3d^2x^7\sqrt{d-c^2dx^2}}{441\sqrt{cx-1}\sqrt{cx+1}}-$$

```
[Out] (2*b*d^2*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
d^2*x^3*Sqrt[d - c^2*d*x^2])/(189*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^
2*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (19*b*c^3*d^
2*x^7*Sqrt[d - c^2*d*x^2])/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*
x^9*Sqrt[d - c^2*d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^
2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^4*d) + ((d - c^2*d*x^2)^(9/2)*(a + b*Ar
cCosh[c*x]))/(9*c^4*d^2)
```

Rubi [A] time = 0.423745, antiderivative size = 331, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 100, 12, 74, 5733, 373}

$$-\frac{d^2x^2(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9c^2}-\frac{2d^2(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4}-\frac{bc^5d^2x^9}{81\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d^2*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
d^2*x^3*Sqrt[d - c^2*d*x^2])/(189*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^
2*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (19*b*c^3*d^
2*x^7*Sqrt[d - c^2*d*x^2])/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*
x^9*Sqrt[d - c^2*d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*d^2*(1 - c*
x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(63*c^4) - (d^2*
x^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*c^
2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
```

```
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4} - \frac{d^2x^2(1-cx)^3(1+c}{}$$

$$= -\frac{2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4} - \frac{d^2x^2(1-cx)^3(1+c}{}$$

$$= -\frac{2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4} - \frac{d^2x^2(1-cx)^3(1+c}{}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bd^2x^3\sqrt{d-c^2dx^2}}{189c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2x^5\sqrt{d-c^2dx^2}}{21\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 0.145952, size = 160, normalized size = 0.54

$$\frac{d^2\sqrt{d-c^2dx^2}\left(7c^2x^2(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+2(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)-\frac{7}{9}bcx\left(c^2x^2-\right.\right.}{63c^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*((-7*b*c*x*(-1 + c^2*x^2)^4)/9 + (25*b*c*(x - c^2*
x^3 + (3*c^4*x^5)/5 - (c^6*x^7)/7))/9 + 2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*
(a + b*ArcCosh[c*x]) + 7*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*Ar
cCosh[c*x])))/(63*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.317, size = 1102, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/9*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(-c^2*d*x^2+d)^(7/2))+b*(
1/41472*(-d*(c^2*x^2-1))^(1/2)*(256*x^10*c^10-704*c^8*x^8+256*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^9*c^9+688*c^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-
280*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arcc
osh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^
8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3
*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c
^4/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(
c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(
c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256
*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arc
cosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*
c^2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-
1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c
^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh
(c*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/41472*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*x^10*c^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*c^6*x^6+120
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17884, size = 602, normalized size = 2.02

$$\frac{63\left(7\,bc^{10}d^2x^{10} - 26\,bc^8d^2x^8 + 34\,bc^6d^2x^6 - 16\,bc^4d^2x^4 - bc^2d^2x^2 + 2\,bd^2\right)\sqrt{-c^2dx^2 + d}\,\log\left(cx + \sqrt{c^2x^2 - 1}\right) - \left(49\,bc^9d^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] 1/3969*(63*(7*b*c^10*d^2*x^10 - 26*b*c^8*d^2*x^8 + 34*b*c^6*d^2*x^6 - 16*b*
c^4*d^2*x^4 - b*c^2*d^2*x^2 + 2*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 - 1)) - (49*b*c^9*d^2*x^9 - 171*b*c^7*d^2*x^7 + 189*b*c^5*d^2*x^5 -
 21*b*c^3*d^2*x^3 - 126*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) +
 63*(7*a*c^10*d^2*x^10 - 26*a*c^8*d^2*x^8 + 34*a*c^6*d^2*x^6 - 16*a*c^4*d^2
*x^4 - a*c^2*d^2*x^2 + 2*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.99 $\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=218

$$-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2d}-\frac{bc^5d^2x^7\sqrt{d-c^2dx^2}}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3d^2x^5\sqrt{d-c^2dx^2}}{35\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{cx-1}\sqrt{cx+1}}+\frac{bd^2x\sqrt{d-c^2dx^2}}{7c\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*d^2*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2
*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*d^2*x
^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*x^7*
Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(
7/2)*(a + b*ArcCosh[c*x]))/(7*c^2*d)
```

Rubi [A] time = 0.281578, antiderivative size = 233, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5798, 5718, 194}

$$-\frac{d^2(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{bc^5d^2x^7\sqrt{d-c^2dx^2}}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3d^2x^5\sqrt{d-c^2dx^2}}{35\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{cx-1}\sqrt{cx+1}}+$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d^2*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2
*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*d^2*x
^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*x^7*
Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(1 - c*x)^3*(
1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{\left(bd^2\sqrt{d-c^2dx^2}\right)\int (}{7c\sqrt{-1+cx}}$$

$$=-\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{\left(bd^2\sqrt{d-c^2dx^2}\right)\int (}{7c\sqrt{-}}$$

$$=\frac{bd^2x\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3bc^3d^2x^5\sqrt{d-c^2dx^2}}{35\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^5d^2x}{49\sqrt{-1}}$$

Mathematica [A] time = 0.232665, size = 117, normalized size = 0.54

$$\frac{d^2\sqrt{d-c^2dx^2}\left(35a\left(c^2x^2-1\right)^4+bcx\sqrt{cx-1}\sqrt{cx+1}\left(-5c^6x^6+21c^4x^4-35c^2x^2+35\right)+35b\left(c^2x^2-1\right)^4\cosh^{-1}(cx)\right)}{245c^2\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(35*a*(-1 + c^2*x^2)^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(35 - 35*c^2*x^2 + 21*c^4*x^4 - 5*c^6*x^6) + 35*b*(-1 + c^2*x^2)^
```

```
4*ArcCosh[c*x]))/(245*c^2*(-1 + c^2*x^2))
```

Maple [B] time = 0.267, size = 956, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/7*a/c^2/d*(-c^2*d*x^2+d)^(7/2)+b*(1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*
x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)
/c^2/(c*x-1)-1/640*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*
c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^
2/(c*x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c
*x))*d^2/(c*x+1)/c^2/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-5/128*
(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arcc
osh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c
^2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-1/640*(-d*(c^2*x^2-1))
^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*
x^2-1)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/6272*(-d*(c^2*x^2-1))^(
1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+10
4*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x)
)*d^2/(c*x+1)/c^2/(c*x-1))
```

Maxima [A] time = 1.22272, size = 159, normalized size = 0.73

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}b\,\mathrm{arcosh}\left(cx\right)}{7\,c^2d}-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}a}{7\,c^2d}-\frac{\left(5\,c^6\sqrt{-d}d^3x^7-21\,c^4\sqrt{-d}d^3x^5+35\,c^2\sqrt{-d}d^3x^3-35\,\sqrt{-d}d^3x\right)b}{245\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*(-c^2*d*x^2 + d)^(7/2)*b*arccosh(c*x)/(c^2*d) - 1/7*(-c^2*d*x^2 + d)^(
7/2)*a/(c^2*d) - 1/245*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5 +
35*c^2*sqrt(-d)*d^3*x^3 - 35*sqrt(-d)*d^3*x)*b/(c*d)
```

Fricas [A] time = 2.12957, size = 502, normalized size = 2.3

$$\frac{35\left(bc^8d^2x^8-4\,bc^6d^2x^6+6\,bc^4d^2x^4-4\,bc^2d^2x^2+bd^2\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(5\,bc^7d^2x^7-21\,bc^5d^2x^5+3\right.}{245\left(c^4x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*
x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (5*b*c^7*d
^2*x^7 - 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 - 35*b*c*d^2*x)*sqrt(-c^2*d*x^
2 + d)*sqrt(c^2*x^2 - 1) + 35*(a*c^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^
2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.100 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=379

$$\frac{ibd^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{ibd^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

```
[Out] (-23*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (11
*b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^2])/(45*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*
c^5*d^2*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]) + (d*(d - c^2*d*x^2)^(3/2)*(a + b*Ar
cCosh[c*x]))/3 + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/5 - (2*d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (I*b*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh
[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*d^2*Sqrt[d - c^2*d*x^2]*PolyL
og[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.0589, antiderivative size = 410, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5745, 5743, 5761, 4180, 2279, 2391, 8, 194}

$$\frac{ibd^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{ibd^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (-23*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (11
*b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^2])/(45*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*
c^5*d^2*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]) + (d^2*(1 - c*x)*(1 + c*x)*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/3 + (d^2*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/5 - (2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b*d^2
*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) - (I*b*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{5}d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx}{}}{\sqrt{-1+c}}$$

$$= \frac{1}{3}d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{5}d^2(1-cx)^2(1+cx)^2\sqrt{d-}$$

$$= -\frac{8bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}} + d^2\sqrt{d}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}} + d^2\sqrt{d}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}} + d^2\sqrt{d}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}} + d^2\sqrt{d}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}} + d^2\sqrt{d}$$

Mathematica [A] time = 3.57599, size = 471, normalized size = 1.24

$$\frac{bd^2\sqrt{d-c^2dx^2}\left(i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)-cx+cx\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx)+\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (a*d^2*Sqrt[d - c^2*d*x^2]*(23 - 11*c^2*x^2 + 3*c^4*x^4))/15 - (b*d^2*Sqrt[
d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh
[c*x] - Cosh[3*ArcCosh[c*x]]))/(18*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) +
a*d^(5/2)*Log[x] - a*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d^2*
Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x
```

```
*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCo
sh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^A
rcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)) - (b*d^2*Sqrt[d - c^2*d*x^2]*(25*Cosh[3*ArcCosh[c*x]] + 9*(-50*
c*x + Cosh[5*ArcCosh[c*x]]) + 15*ArcCosh[c*x]*(30*Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x) - 5*Sinh[3*ArcCosh[c*x]] - 3*Sinh[5*ArcCosh[c*x]])))/(3600*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.258, size = 620, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/5*(-c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(-c^2*d*x^2+d)^(3/2)-a*d^(5/2)*ln((2*d+2
*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+a*(-c^2*d*x^2+d)^(1/2)*d^2+I*b*(-d*(c^2*x
^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))*d^2-1/25*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c
*x-1)^(1/2)*x^5*c^5+11/45*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1
)^(1/2)*x^3*c^3-23/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1
/2)*x*c+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+
1)/(c*x-1)*arccosh(c*x)*x^6*c^6-14/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/
(c*x-1)*arccosh(c*x)*x^4*c^4+34/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*
x-1)*arccosh(c*x)*x^2*c^2-23/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1
)*arccosh(c*x)-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog
(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
)))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4-2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4-2\,bc^2d^2x^2+bd^2\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)/x, x)
```

3.101 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=404

$$-\frac{5ibc^2d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{5ibc^2d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{5}{2}c^2d^2\sqrt{d-c^2dx^2}\left(a+b\right.$$

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*b*c^
3*d^2*x*Sqrt[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*
x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (5*c^2*d*(d - c^2*d*x^2)^(3/2)*(a
 + b*ArcCosh[c*x]))/6 - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(2*x^2
) + (5*c^2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*
x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((5*I)/2)*b*c^2*d^2*Sqrt[d - c^2*d*x
^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (((5*
I)/2)*b*c^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.06375, antiderivative size = 435, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {5798, 5740, 5745, 5743, 5761, 4180, 2279, 2391, 8, 270}

$$-\frac{5ibc^2d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{5ibc^2d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{5}{2}c^2d^2\sqrt{d-c^2dx^2}\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*b*c^
3*d^2*x*Sqrt[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*
x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (5*c^2*d^2*(1 - c*x)*(1 + c*x)*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (d^2*(1 - c*x)^2*(1 + c*x)^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*x^2) + (5*c^2*d^2*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (((5*I)/2)*b*c^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh
[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (((5*I)/2)*b*c^2*d^2*Sqrt[d - c^2*
```

```
d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
```

```
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{\left(bcd^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1\cdots}{}}{2\sqrt{-1+cx}\sqrt{1+\cdots}}$$

$$= -\frac{5}{6}c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d^2(1-cx)^2(1+cx)^2\sqrt{d\cdots}}{}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^3d^2x\sqrt{d-c^2dx^2}}{6\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5}{2}c^2d^2\sqrt{d\cdots}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5}{2}c^2d^2\sqrt{d\cdots}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5}{2}c^2d^2\sqrt{d\cdots}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5}{2}c^2d^2\sqrt{d\cdots}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5}{2}c^2d^2\sqrt{d\cdots}$$

Mathematica [A] time = 3.89555, size = 596, normalized size = 1.48

$$\frac{1}{36}d^2\left(-\frac{72bc^2\sqrt{d-c^2dx^2}\left(i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)-cx+cx\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx)+\sqrt{\frac{cx-1}{cx+1}}\cdots}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (d^2*((6*a*Sqrt[d - c^2*d*x^2]*(-3 - 14*c^2*x^2 + 2*c^4*x^4))/x^2 + (b*c^2*
Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*Ar
cCosh[c*x] - Cosh[3*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
- 90*a*c^2*Sqrt[d]*Log[x] + 90*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d
*x^2]] - (72*b*c^2*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]
```

```
*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x
]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*
PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)) + (18*b*d*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1
+ c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 +
c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1
 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)
/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1
+ c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2])))/36
```

Maple [A] time = 0.27, size = 667, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(7/2)-1/2*a*c^2*(-c^2*d*x^2+d)^(5/2)-5/6*a*c^2*
d*(-c^2*d*x^2+d)^(3/2)+5/2*a*c^2*d^(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(
1/2))/x)-5/2*a*c^2*(-c^2*d*x^2+d)^(1/2)*d^2+5/2*I*b*(-d*(c^2*x^2-1))^(1/2)/
(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2)))*c^2*d^2-1/9*b*(-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*x^3+7/3*b*(-d*(c^2*x^2-1))^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*
x-1/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/x/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-5/2*I*b*
(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))*c^2*d^2-5/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d^2
+11/6*b*(-d*(c^2*x^2-1))^(1/2)*c^2*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)+1/2*b*(
-d*(c^2*x^2-1))^(1/2)*d^2/x^2/(c*x+1)/(c*x-1)*arccosh(c*x)+1/3*b*(-d*(c^2*x
^2-1))^(1/2)*c^6*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^4-8/3*b*(-d*(c^2*x^2-1)
)^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2+5/2*I*b*(-d*(c^2*x^2-1))^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
))*c^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2 + d}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)/x^3, x)
```

3.102 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=407

$$\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}-\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{15}{8}c^4d^2\sqrt{d-c^2dx^2}\left(a+b\right.$$

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(12*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (9*b
*c^3*d^2*Sqrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d
^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*c^4*d^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (5*c^2*d*(d - c^2*d*x^2)^(3/2)*(a
 + b*ArcCosh[c*x]))/(8*x^2) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/
(4*x^4) - (15*c^4*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^Arc
Cosh[c*x]])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (((15*I)/8)*b*c^4*d^2*Sqrt[d
 - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (((15*I)/8)*b*c^4*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]]
)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.08268, antiderivative size = 438, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {5798, 5740, 5743, 5761, 4180, 2279, 2391, 8, 14, 270}

$$\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}-\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{15}{8}c^4d^2\sqrt{d-c^2dx^2}\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(12*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (9*b
*c^3*d^2*Sqrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d
^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*c^4*d^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (5*c^2*d^2*(1 - c*x)*(1 + c*x)*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*x^2) - (d^2*(1 - c*x)^2*(1 + c*x
)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(4*x^4) - (15*c^4*d^2*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(4*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + (((15*I)/8)*b*c^4*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)
*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((15*I)/8)*b*c^4*d^2*Sq
rt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*
```

x])

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4} + \frac{\left(bcd^2\sqrt{d-c^2dx^2}\right)\int \frac{(-}{4\sqrt{-1+cx}\sqrt{1-}}$$

$$= \frac{5c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8x^2} - \frac{d^2(1-cx)^2(1+cx)^2\sqrt{d}}{}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^5d^2x\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{15}{8}c^4$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{15}{8}c^4d$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{15}{8}c^4d$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{15}{8}c^4d$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{15}{8}c^4d$$

Mathematica [A] time = 1.39051, size = 660, normalized size = 1.62

$$\frac{-45ibc^4d^3x^4(cx-1)\mathrm{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right) + 45ibc^4d^3x^4(cx-1)\mathrm{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right) - 24ac^6d^3x^6\sqrt{\frac{cx-1}{cx+1}} - 3ac}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] (-2*b*c*d^3*x + 2*b*c^2*d^3*x^2 + 27*b*c^3*d^3*x^3 - 27*b*c^4*d^3*x^4 - 24*
b*c^5*d^3*x^5 + 24*b*c^6*d^3*x^6 - 6*a*d^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 33*
a*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] - 3*a*c^4*d^3*x^4*Sqrt[(-1 + c*x)/
(1 + c*x)] - 24*a*c^6*d^3*x^6*Sqrt[(-1 + c*x)/(1 + c*x)] - 6*b*d^3*Sqrt[(-1
 + c*x)/(1 + c*x)]*ArcCosh[c*x] + 33*b*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x
```

```
)]*ArcCosh[c*x] - 3*b*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] -
 24*b*c^6*d^3*x^6*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + (45*I)*b*c^4*d^
3*x^4*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (45*I)*b*c^5*d^3*x^5*ArcCosh
[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (45*I)*b*c^4*d^3*x^4*ArcCosh[c*x]*Log[1 +
 I/E^ArcCosh[c*x]] + (45*I)*b*c^5*d^3*x^5*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[
c*x]] + 45*a*c^4*d^(5/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]
*Log[x] - 45*a*c^4*d^(5/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^
2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (45*I)*b*c^4*d^3*x^4*(-1 + c*x)*P
olyLog[2, (-I)/E^ArcCosh[c*x]] + (45*I)*b*c^4*d^3*x^4*(-1 + c*x)*PolyLog[2,
 I/E^ArcCosh[c*x]])/(24*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [A] time = 0.283, size = 691, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x)
```

```
[Out] -1/4*a/d/x^4*(-c^2*d*x^2+d)^(7/2)+3/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(7/2)+3/8*
a*c^4*(-c^2*d*x^2+d)^(5/2)+5/8*a*c^4*d*(-c^2*d*x^2+d)^(3/2)-15/8*a*c^4*d^(5
/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+15/8*a*c^4*(-c^2*d*x^2+d)^(1
/2)*d^2+b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2-b
*(-d*(c^2*x^2-1))^(1/2)*d^2*c^5/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x+1/8*b*(-d*(c^
2*x^2-1))^(1/2)*d^2*c^4/(c*x+1)/(c*x-1)*arccosh(c*x)+9/8*b*d^2*(-d*(c^2*x^2
-1))^(1/2)/(c*x+1)^(1/2)/x/(c*x-1)^(1/2)*c^3-11/8*b*d^2*(-d*(c^2*x^2-1))^(1
/2)/(c*x+1)/x^2/(c*x-1)*arccosh(c*x)*c^2-1/12*b*d^2*(-d*(c^2*x^2-1))^(1/2)/
(c*x+1)^(1/2)/x^3/(c*x-1)^(1/2)*c+1/4*b*d^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/
x^4/(c*x-1)*arccosh(c*x)-15/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2*c^4-15/8*I*b*(-d
*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2*c^4+15/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2*c^4+1
5/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(
1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}\left(b\operatorname{arcosh}\left(cx\right) + a\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)/x^5, x)
```

3.103 $\int \sqrt{1-x^2} \cosh^{-1}(x) \, dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1-x}x^2}{4\sqrt{x-1}}+\frac{1}{2}\sqrt{1-x^2}x\cosh^{-1}(x)-\frac{\sqrt{1-x}\cosh^{-1}(x)^2}{4\sqrt{x-1}}$$

```
[Out] -(Sqrt[1 - x]*x^2)/(4*Sqrt[-1 + x]) + (x*Sqrt[1 - x^2]*ArcCosh[x])/2 - (Sqr
t[1 - x]*ArcCosh[x]^2)/(4*Sqrt[-1 + x])
```

Rubi [A] time = 0.104011, antiderivative size = 84, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5713, 5683, 5676, 30}

$$-\frac{\sqrt{1-x^2}x^2}{4\sqrt{x-1}\sqrt{x+1}}+\frac{1}{2}\sqrt{1-x^2}x\cosh^{-1}(x)-\frac{\sqrt{1-x^2}\cosh^{-1}(x)^2}{4\sqrt{x-1}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]*ArcCosh[x],x]
```

```
[Out] -(x^2*Sqrt[1 - x^2])/(4*Sqrt[-1 + x]*Sqrt[1 + x]) + (x*Sqrt[1 - x^2]*ArcCos
h[x])/2 - (Sqrt[1 - x^2]*ArcCosh[x]^2)/(4*Sqrt[-1 + x]*Sqrt[1 + x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
```

```
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1-x^2}\cosh^{-1}(x)\,dx &= \frac{\sqrt{1-x^2}\int \sqrt{-1+x}\sqrt{1+x}\cosh^{-1}(x)\,dx}{\sqrt{-1+x}\sqrt{1+x}}\\
&= \frac{1}{2}x\sqrt{1-x^2}\cosh^{-1}(x) - \frac{\sqrt{1-x^2}\int x\,dx}{2\sqrt{-1+x}\sqrt{1+x}} - \frac{\sqrt{1-x^2}\int \frac{\cosh^{-1}(x)}{\sqrt{-1+x}\sqrt{1+x}}\,dx}{2\sqrt{-1+x}\sqrt{1+x}}\\
&= -\frac{x^2\sqrt{1-x^2}}{4\sqrt{-1+x}\sqrt{1+x}} + \frac{1}{2}x\sqrt{1-x^2}\cosh^{-1}(x) - \frac{\sqrt{1-x^2}\cosh^{-1}(x)^2}{4\sqrt{-1+x}\sqrt{1+x}}
\end{aligned}
$$

Mathematica [A] time = 0.110636, size = 54, normalized size = 0.82

$$
-\frac{\sqrt{-(x-1)(x+1)}\left(\cosh\left(2\cosh^{-1}(x)\right)+2\cosh^{-1}(x)\left(\cosh^{-1}(x)-\sinh\left(2\cosh^{-1}(x)\right)\right)\right)}{8\sqrt{\frac{x-1}{x+1}}(x+1)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^2]*ArcCosh[x],x]
```

```
[Out] -(Sqrt[-((-1 + x)*(1 + x))]*(Cosh[2*ArcCosh[x]] + 2*ArcCosh[x]*(ArcCosh[x]
- Sinh[2*ArcCosh[x]])))/(8*Sqrt[(-1 + x)/(1 + x)]*(1 + x))
```

Maple [B] time = 0.147, size = 152, normalized size = 2.3

$$-\frac{\left(\operatorname{arccosh}(x)\right)^2}{4}\sqrt{-x^2+1}\frac{1}{\sqrt{-1+x}}\frac{1}{\sqrt{1+x}}+\frac{-1+2\operatorname{arccosh}(x)}{(-16+16x)(1+x)}\sqrt{-x^2+1}\left(2x^3-2x+2\sqrt{1+x}\sqrt{-1+x}x^2-\sqrt{-1+x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x)*(-x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-x^2+1)^(1/2)/(-1+x)^(1/2)/(1+x)^(1/2)*arccosh(x)^2+1/16*(-x^2+1)^(1/
2)*(2*x^3-2*x+2*(1+x)^(1/2)*(-1+x)^(1/2)*x^2-(-1+x)^(1/2)*(1+x)^(1/2))*(-1+
2*arccosh(x))/(-1+x)/(1+x)+1/16*(-x^2+1)^(1/2)*(-2*(1+x)^(1/2)*(-1+x)^(1/2)
*x^2+2*x^3+(-1+x)^(1/2)*(1+x)^(1/2)-2*x)*(1+2*arccosh(x))/(-1+x)/(1+x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^2+1}\operatorname{arcosh}(x), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*arccosh(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(x-1)(x+1)}\operatorname{acosh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))*acosh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2+1}\,\mathrm{arcosh}\,(x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)*arccosh(x), x)
```

3.104 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=236

$$-\frac{x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2d}-\frac{4x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4d}-\frac{8\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^6d}-\frac{bx^5\sqrt{cx-}}{25c\sqrt{d-}}$$

```
[Out] (-8*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (4*b*x
^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(45*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^5*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(25*c*Sqrt[d - c^2*d*x^2]) - (8*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(15*c^6*d) - (4*x^2*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(15*c^4*d) - (x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*
c^2*d)
```

Rubi [A] time = 0.708788, antiderivative size = 260, normalized size of antiderivative = 1.1, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5759, 5718, 8, 30}

$$-\frac{x^4(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{5c^2\sqrt{d-c^2dx^2}}-\frac{4x^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{15c^4\sqrt{d-c^2dx^2}}-\frac{8(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{15c^6\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-8*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (4*b*x
^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(45*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^5*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(25*c*Sqrt[d - c^2*d*x^2]) - (8*(1 - c*x)*(1 + c*
x)*(a + b*ArcCosh[c*x]))/(15*c^6*Sqrt[d - c^2*d*x^2]) - (4*x^2*(1 - c*x)*(1
 + c*x)*(a + b*ArcCosh[c*x]))/(15*c^4*Sqrt[d - c^2*d*x^2]) - (x^4*(1 - c*x)
*(1 + c*x)*(a + b*ArcCosh[c*x]))/(5*c^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^4(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{5c^2\sqrt{d-c^2dx^2}} + \frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{5c^2\sqrt{d-c^2dx^2}} - \frac{(b}{}$$

$$= -\frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{25c\sqrt{d-c^2dx^2}} - \frac{4x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{15c^4\sqrt{d-c^2dx^2}} - \frac{x^4(1-cx)(1+cx)\left(a+\right.}{5c^2\sqrt{d-c^2}}$$

$$= -\frac{4bx^3\sqrt{-1+cx}\sqrt{1+cx}}{45c^3\sqrt{d-c^2dx^2}} - \frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{25c\sqrt{d-c^2dx^2}} - \frac{8(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{15c^6\sqrt{d-c^2dx^2}} -$$

$$= -\frac{8bx\sqrt{-1+cx}\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{4bx^3\sqrt{-1+cx}\sqrt{1+cx}}{45c^3\sqrt{d-c^2dx^2}} - \frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{25c\sqrt{d-c^2dx^2}} - \frac{8(1-cx)(1+}{15c}$$

Mathematica [A] time = 0.253573, size = 140, normalized size = 0.59

$$\frac{\sqrt{d-c^2dx^2}\left(-15a\left(3c^6x^6+c^4x^4+4c^2x^2-8\right)+bcx\sqrt{cx-1}\sqrt{cx+1}\left(9c^4x^4+20c^2x^2+120\right)-15b\left(3c^6x^6+c^4x^4+4c^2x^2\right.\right.}{225c^6d(cx-1)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(120 + 20*c^2*x^2
+ 9*c^4*x^4) - 15*a*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6) - 15*b*(-8 + 4*c
^2*x^2 + c^4*x^4 + 3*c^6*x^6)*ArcCosh[c*x]))/(225*c^6*d*(-1 + c*x)*(1 + c*x
))
```

Maple [B] time = 0.314, size = 670, normalized size = 2.8

$$a\left(-\frac{x^4}{5c^2d}\sqrt{-c^2dx^2+d}+\frac{4}{5c^2}\left(-\frac{x^2}{3c^2d}\sqrt{-c^2dx^2+d}-\frac{2}{3dc^4}\sqrt{-c^2dx^2+d}\right)\right)+b\left(-\frac{-1+5\,\mathrm{arccosh}\,(cx)}{800\,dc^6\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\right(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(-1/5*x^4/c^2/d*(-c^2*d*x^2+d)^(1/2)+4/5/c^2*(-1/3*x^2/c^2/d*(-c^2*d*x^2+
d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2)))+b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*
(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(
-1+5*arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^
4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c+1)*(-1+3*arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/
2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^6/d/(c^2
*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x
^2-1)*(1+arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(-4*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/c^6/d/(c^2*x^2-1)-1/800*(-d*(c^2*x^2-1)
)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2
*x^2-1)*(1+5*arccosh(c*x))/c^6/d/(c^2*x^2-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13713, size = 382, normalized size = 1.62

$$-\frac{15\left(3\,bc^{6}x^{6}+bc^{4}x^{4}+4\,bc^{2}x^{2}-8\,b\right)\sqrt{-c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-\left(9\,bc^{5}x^{5}+20\,bc^{3}x^{3}+120\,bcx\right)\sqrt{-c^{2}dx^{2}+d}\sqrt{c^{2}}}{225\left(c^{8}dx^{2}-c^{6}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/225*(15*(3*b*c^6*x^6 + b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 +
d)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^5*x^5 + 20*b*c^3*x^3 + 120*b*c*x)*
```

```
sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(3*a*c^6*x^6 + a*c^4*x^4 + 4*a*
c^2*x^2 - 8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d*x^2 - c^6*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 (a + b \operatorname{acosh}(cx))}{\sqrt{-d (cx - 1) (cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^5}{\sqrt{-c^2 dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^5/sqrt(-c^2*d*x^2 + d), x)
```

3.105 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=212

$$-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4c^2d}-\frac{3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^4d}+\frac{3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^5\sqrt{d-c^2dx^2}}-\frac{bx^4\sqrt{cx}}{16c\sqrt{}}$$

```
[Out] (-3*b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^3*Sqrt[d - c^2*d*x^2]) - (b*x
^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c*Sqrt[d - c^2*d*x^2]) - (3*x*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*c^4*d) - (x^3*Sqrt[d - c^2*d*x^2]*(a +
 b*ArcCosh[c*x]))/(4*c^2*d) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCos
h[c*x])^2)/(16*b*c^5*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.647655, antiderivative size = 228, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5759, 5676, 30}

$$-\frac{x^3(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{4c^2\sqrt{d-c^2dx^2}}-\frac{3x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{8c^4\sqrt{d-c^2dx^2}}+\frac{3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^5\sqrt{d-c^2dx^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-3*b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^3*Sqrt[d - c^2*d*x^2]) - (b*x
^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c*Sqrt[d - c^2*d*x^2]) - (3*x*(1 - c*x
)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(8*c^4*Sqrt[d - c^2*d*x^2]) - (x^3*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(4*c^2*Sqrt[d - c^2*d*x^2]) + (3*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(16*b*c^5*Sqrt[d - c^2*d*x^2
])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^3(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{4c^2\sqrt{d-c^2dx^2}} + \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{4c^2\sqrt{d-c^2dx^2}} - \frac{(b}{}$$

$$= -\frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{16c\sqrt{d-c^2dx^2}} - \frac{3x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{8c^4\sqrt{d-c^2dx^2}} - \frac{x^3(1-cx)(1+cx)\left(a+\right.}{4c^2\sqrt{d-c^2}}$$

$$= -\frac{3bx^2\sqrt{-1+cx}\sqrt{1+cx}}{16c^3\sqrt{d-c^2dx^2}} - \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{16c\sqrt{d-c^2dx^2}} - \frac{3x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{8c^4\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.843036, size = 171, normalized size = 0.81

$$\frac{-\frac{16acx(2c^2x^2+3)\sqrt{d-c^2dx^2}}{d}-\frac{48a\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(c^2x^2-1)}\right)}{\sqrt{d}}+\frac{b\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(-16\cosh\left(2\cosh^{-1}(cx)\right)-\cosh\left(4\cosh^{-1}(cx)\right)+4\cosh^{-1}(cx)\left(6\cosh^{-1}(cx)+8\sinh\left(2\cosh^{-1}(cx)\right)+\sinh\left(4\cosh^{-1}(cx)\right)\right)\right)}{\sqrt{d-c^2dx^2}}}{128c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-16*a*c*x*(3 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/d - (48*a*ArcTan[(c*x*Sqrt
[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)*(-16*Cosh[2*ArcCosh[c*x]] - Cosh[4*ArcCosh[c*x]] + 4*Arc
Cosh[c*x]*(6*ArcCosh[c*x] + 8*Sinh[2*ArcCosh[c*x]] + Sinh[4*ArcCosh[c*x]]))
)/Sqrt[d - c^2*d*x^2])/(128*c^5)
```

Maple [B] time = 0.355, size = 408, normalized size = 1.9

$$-\frac{x^3a}{4c^2d}\sqrt{-c^2dx^2+d}-\frac{3\,ax}{8\,dc^4}\sqrt{-c^2dx^2+d}+\frac{3\,a}{8\,c^4}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}+\frac{bx^4}{16\,cd\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{c\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(-c^2*d*x^2+d)^(1/2)+3/
8*a/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/16*b*(
-d*(c^2*x^2-1))^(1/2)/d/c/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+3/16*
b*(-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-
3/16*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^5/(c^2*x^2-1)
*arccosh(c*x)^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*x^5
-1/8*b*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2-1)*arccosh(c*x)*x^3+3/8*b*(-d*
(c^2*x^2-1))^(1/2)/d/c^4/(c^2*x^2-1)*arccosh(c*x)*x-15/128*b*(-d*(c^2*x^2-1
))^(1/2)/d/c^5/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(bx^4 \operatorname{arcosh}(cx) + ax^4\right)\sqrt{-c^2dx^2 + d}}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^2*d*x^2 - d)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b \operatorname{acosh}(cx))}{\sqrt{-d (cx - 1) (cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^4}{\sqrt{-c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/sqrt(-c^2*d*x^2 + d), x)
```

3.106 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=156

$$-\frac{x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}-\frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^4d}-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{9c\sqrt{d-c^2dx^2}}-\frac{2bx\sqrt{cx-1}\sqrt{cx+1}}{3c^3\sqrt{d-c^2dx^2}}$$

```
[Out] (-2*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c*Sqrt[d - c^2*d*x^2]) - (2*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/(3*c^4*d) - (x^2*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(3*c^2*d)
```

Rubi [A] time = 0.49645, antiderivative size = 172, normalized size of antiderivative = 1.1, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5759, 5718, 8, 30}

$$-\frac{x^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^2\sqrt{d-c^2dx^2}}-\frac{2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^4\sqrt{d-c^2dx^2}}-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{9c\sqrt{d-c^2dx^2}}-\frac{2bx\sqrt{cx-1}\sqrt{cx+1}}{3c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c*Sqrt[d - c^2*d*x^2]) - (2*(1 - c*x)*(1 +
 c*x)*(a + b*ArcCosh[c*x]))/(3*c^4*Sqrt[d - c^2*d*x^2]) - (x^2*(1 - c*x)*(1
 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^2\sqrt{d-c^2dx^2}} + \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3c^2\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-\right.}{}$$

$$= -\frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{9c\sqrt{d-c^2dx^2}} - \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^4\sqrt{d-c^2dx^2}} - \frac{x^2(1-cx)(1+cx)\left(a+b\,c\right.}{3c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{2bx\sqrt{-1+cx}\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{9c\sqrt{d-c^2dx^2}} - \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^4\sqrt{d-c^2dx^2}} - x$$

Mathematica [A] time = 0.212323, size = 113, normalized size = 0.72

$$\frac{\sqrt{d-c^2dx^2}\left(-3a\left(c^4x^4+c^2x^2-2\right)+bcx\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2+6\right)-3b\left(c^4x^4+c^2x^2-2\right)\cosh^{-1}(cx)\right)}{9c^4d(cx-1)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2) - 3*
a*(-2 + c^2*x^2 + c^4*x^4) - 3*b*(-2 + c^2*x^2 + c^4*x^4)*ArcCosh[c*x]))/(9
*c^4*d*(-1 + c*x)*(1 + c*x))
```

Maple [B] time = 0.234, size = 382, normalized size = 2.5

$$a\left(-\frac{x^2}{3\,c^2d}\sqrt{-c^2dx^2+d}-\frac{2}{3\,dc^4}\sqrt{-c^2dx^2+d}\right)+b\left(-\frac{-1+3\,\mathrm{arccosh}\,(cx)}{72\,dc^4\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\left(4\,c^4x^4-5\,c^2x^2+4\,\sqrt{cx+1}\sqrt{cx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(-1/3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2))+b*(-
1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/c^4/d/
(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2
*x^2-1)*(-1+arccosh(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*(-(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/c^4/d/(c^2*x^2-1)
-1/72*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*
x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/c^4/d
/(c^2*x^2-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.46026, size = 308, normalized size = 1.97

$$-\frac{3\left(bc^4x^4+bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^3x^3+6\,bcx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}+3\left(ac^4x^4+ac^2x^2-\right.}{9\left(c^6dx^2-c^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/9*(3*(b*c^4*x^4 + b*c^2*x^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 - 1)) - (b*c^3*x^3 + 6*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
 + 3*(a*c^4*x^4 + a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^4*d
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\,(a+b\operatorname{acosh}(cx))}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)x^3}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^3/sqrt(-c^2*d*x^2 + d), x)
```

3.107 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=132

$$-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}$$

```
[Out] -(b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (x*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*c^2*d) + (Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.398236, antiderivative size = 140, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5759, 5676, 30}

$$-\frac{x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -(b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (x*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^2*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
```

```
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2c^2\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1}\right.}{2}$$

$$= -\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}} - \frac{x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{4bc^3\sqrt{d-c^2d}}$$

Mathematica [A] time = 0.610003, size = 141, normalized size = 1.07

$$\frac{-\frac{4acx\sqrt{d-c^2dx^2}}{d} - \frac{4a\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{\sqrt{d}} + \frac{b\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+\sinh\left(2\cosh^{-1}(cx)\right)\right)-\cosh\left(2\cosh^{-1}(cx)\right)\right)}{\sqrt{d-c^2dx^2}}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-4*a*c*x*Sqrt[d - c^2*d*x^2])/d - (4*a*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(
Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*(-Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] + Sinh[2*ArcCosh[c*x
]])))/Sqrt[d - c^2*d*x^2])/(8*c^3)
```

Maple [B] time = 0.225, size = 291, normalized size = 2.2

$$-\frac{ax}{2\,c^2d}\sqrt{-c^2dx^2+d}+\frac{a}{2\,c^2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}-\frac{b\,(\mathrm{arccosh}\,(cx))^2}{4\,c^3d\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}-\frac{bar}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a/c^2/(c^2*d)^(1/2)*arctan((c^2*d)^
(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/4*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d
/(c^2*x^2-1)*arccosh(c*x)*x^3+1/4*b*(-d*(c^2*x^2-1))^(1/2)/d/c/(c^2*x^2-1)*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2
-1)*arccosh(c*x)*x-1/8*b*(-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(bx^2\,\mathrm{arcosh}\,(cx)+ax^2\right)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^2*d*x^2 - d)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{acosh}(cx))}{\sqrt{-d (cx - 1) (cx + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^2}{\sqrt{-c^2 dx^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/sqrt(-c^2*d*x^2 + d), x)
```

3.108 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^2d}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

```
[Out] -((b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2])) - (Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(c^2*d)
```

Rubi [A] time = 0.210074, antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5798, 5718, 8}

$$-\frac{(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -((b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2])) - ((1 - c*x)*
(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
```

```
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\
&= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{\left(b\sqrt{-1+cx}\sqrt{1+cx}\right)\int 1\,dx}{c\sqrt{d-c^2dx^2}}\\
&= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}}-\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.158515, size = 85, normalized size = 1.18

$$\frac{\sqrt{d-c^2dx^2}\left(-ac^2x^2+a+\left(b-bc^2x^2\right)\cosh^{-1}(cx)+bcx\sqrt{cx-1}\sqrt{cx+1}\right)}{c^2d(cx-1)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] +
(b - b*c^2*x^2)*ArcCosh[c*x]))/(c^2*d*(-1 + c*x)*(1 + c*x))
```

Maple [B] time = 0.15, size = 158, normalized size = 2.2

$$-\frac{a}{c^2d}\sqrt{-c^2dx^2+d}+b\left(-\frac{-1+\operatorname{arccosh}(cx)}{2c^2d\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\left(\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)-\frac{1+\operatorname{arccosh}(cx)}{2c^2d\left(c^2x^2-1\right)}\sqrt{-d\left(c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/c^2/d*(-c^2*d*x^2+d)^(1/2)+b*(-1/2*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^2/d/(c^2*x^2-1)-1/2*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))/c^2/d/(c^2*x^2-1))
```

Maxima [A] time = 1.17786, size = 85, normalized size = 1.18

$$\frac{b\sqrt{-d}x}{cd} - \frac{\sqrt{-c^2dx^2 + d}b \operatorname{arcosh}(cx)}{c^2d} - \frac{\sqrt{-c^2dx^2 + d}a}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*sqrt(-d)*x/(c*d) - sqrt(-c^2*d*x^2 + d)*b*arccosh(c*x)/(c^2*d) - sqrt(-c^
2*d*x^2 + d)*a/(c^2*d)
```

Fricas [A] time = 2.08748, size = 236, normalized size = 3.28

$$\frac{\sqrt{-c^2dx^2 + d}\sqrt{c^2x^2 - 1}bcx - \left(bc^2x^2 - b\right)\sqrt{-c^2dx^2 + d}\log\left(cx + \sqrt{c^2x^2 - 1}\right) - \left(ac^2x^2 - a\right)\sqrt{-c^2dx^2 + d}}{c^4dx^2 - c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - (b*c^2*x^2 - b)*sqrt(-c^2*d
*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 +
d))/(c^4*d*x^2 - c^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a + b\operatorname{acosh}(cx)\right)}{\sqrt{-d\left(cx - 1\right)\left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)x}{\sqrt{-c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/sqrt(-c^2*d*x^2 + d), x)
```

3.109 $\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}$$

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d
*x^2])
```

Rubi [A] time = 0.121405, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d
*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.0340022, size = 53, normalized size = 1.

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d
*x^2])
```

Maple [A] time = 0.043, size = 89, normalized size = 1.7

$$a\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}-\frac{b\left(\operatorname{arccosh}(cx)\right)^2}{2\,cd\left(c^2x^2-1\right)}\sqrt{-\left(cx-1\right)\left(cx+1\right)d}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-(c*x-1
)*(c*x+1)*d)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(c^2*x^2-1)*arccosh(c*x)
^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{acosh}\,(cx)}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.110 $\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=151

$$-\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx-1}\sqrt{cx+1}\tan^{-1}\left(\right.}{\sqrt{d}}$$

```
[Out] (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]]
)/Sqrt[d - c^2*d*x^2] - (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E
^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Pol
yLog[2, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rubi [A] time = 0.333308, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5761, 4180, 2279, 2391}

$$-\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx-1}\sqrt{cx+1}\tan^{-1}\left(\right.}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```


```
[Out] (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]]
)/Sqrt[d - c^2*d*x^2] - (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E
^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Pol
yLog[2, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
```

```
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\
&= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int(a+bx)\mathbf{sech}(x)\,dx,x,\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\\
&= \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{\left(ib\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int\log\right.}{\sqrt{d-c^2dx}}\\
&= \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{\left(ib\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{\log(}{}\right.}{\sqrt{d-c^2dx^2}}\\
&= \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{ib\sqrt{-1+cx}\sqrt{1+cx}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx}\right.}{\sqrt{d-c^2dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.27329, size = 153, normalized size = 1.01

$$-\frac{ib\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+\cosh^{-1}(cx)\left(\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-\log\left(1\right.\right.\right.}{\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (I*
b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(Log[1 - I/E^ArcCosh[c
*x]] - Log[1 + I/E^ArcCosh[c*x]]) + PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyL
og[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2]
```

Maple [A] time = 0.195, size = 327, normalized size = 2.2

$$-a\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{-c^2dx^2+d}\right)\right)\frac{1}{\sqrt{d}}+\frac{ib\text{arccosh}\,(cx)}{d\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}\ln\left(1+i\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+I*b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)+I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*dilo
g(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{c^2dx^3-dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^3 - d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\text{acosh}\,(cx)}{x\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x), x)
```

3.111 $\int \frac{a+b\cosh^{-1}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{dx}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\log(x)}{\sqrt{d-c^2dx^2}}$$

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(d*x)) - (b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*Log[x])/Sqrt[d - c^2*d*x^2]
```

Rubi [A] time = 0.302531, antiderivative size = 79, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5798, 5724, 29}

$$-\frac{(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\log(x)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -(((1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(x*Sqrt[d - c^2*d*x^2])) - (b*
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/Sqrt[d - c^2*d*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
```

```
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh^{-1}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\ &= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}}-\frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x}\,dx}{\sqrt{d-c^2dx^2}}\\ &= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}\log(x)}{\sqrt{d-c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.0645657, size = 71, normalized size = 1.

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{x}-bc\log(x)\right)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x]))/x - b*c*Log[x]))/Sqrt[d - c^2*d*x^2]
```

Maple [B] time = 0.167, size = 219, normalized size = 3.1

$$-\frac{a}{dx}\sqrt{-c^2dx^2+d}-\frac{b\operatorname{arccosh}(cx)\,c}{d\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}-\frac{b\operatorname{arccosh}(cx)\,xc^2}{d\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}+\frac{b\operatorname{arccosh}(cx)}{\left(c^2x^2-1\right)dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*c-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*x/
(c^2*x^2-1)/d*c^2+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/x/(c^2*x^2-1)/d+b*(
-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*ln((c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.51823, size = 586, normalized size = 8.25

$$\left[-\frac{bc\sqrt{-d}x\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\left(x^4-1\right)\sqrt{-d}-d}{c^2x^4-x^2}\right)+2\sqrt{-c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2-1}\right)+2\sqrt{-c^2dx^2+d}a}{2\,dx}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/2*(b*c*sqrt(-d)*x*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2
+ d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + 2*sqrt(-c
^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)) + 2*sqrt(-c^2*d*x^2 + d)*a)/(d
*x), (b*c*sqrt(d)*x*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)
*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - sqrt(-c^2*d*x^2 + d)*b*log(c*
x + sqrt(c^2*x^2 - 1)) - sqrt(-c^2*d*x^2 + d)*a)/(d*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{acosh}\,(cx)}{x^2\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcosh}\,(cx) + a}{\sqrt{-c^2dx^2 + d}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

3.112 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=238

$$-\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{d-c^2dx^2}}+\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{2dx^2}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*d*x^2) + (c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] - ((I/2)
*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/Sqrt[d
 - c^2*d*x^2] + ((I/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^Ar
cCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rubi [A] time = 0.541038, antiderivative size = 246, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5748, 5761, 4180, 2279, 2391, 30}

$$-\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{d-c^2dx^2}}+\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cx+1)\left(a+b\right.}{2x^2\sqrt{d-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x*Sqrt[d - c^2*d*x^2]) - ((1 - c*x)*(
1 + c*x)*(a + b*ArcCosh[c*x]))/(2*x^2*Sqrt[d - c^2*d*x^2]) + (c^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/Sqrt[d - c
^2*d*x^2] - ((I/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^Arc
Cosh[c*x]])/Sqrt[d - c^2*d*x^2] + ((I/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*PolyLog[2, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{d-c^2dx^2}} + \frac{\left(c^2\sqrt{-1+cx}\sqrt{1+cx}\right)}{2\sqrt{d-c}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{\left(c^2\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int(\right.}{2\sqrt{d-}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-}\right.}{\sqrt{d-c^2dx}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-}\right.}{\sqrt{d-c^2dx}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-}\right.}{\sqrt{d-c^2dx}}$$

Mathematica [A] time = 1.03586, size = 309, normalized size = 1.3

$$\frac{1}{2}\left(\frac{b(cx+1)\left(-ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)+ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)-ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx)\right.}{x^2\sqrt{d-c}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (-((a*Sqrt[d - c^2*d*x^2])/(d*x^2)) + (a*c^2*Log[x])/Sqrt[d] - (a*c^2*Log[d
 + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] + (b*(1 + c*x)*(c*x*Sqrt[(-1 + c*x
)/(1 + c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] - I*c^2*x^2*Sqrt[(-1 + c*x)/
(1 + c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*
x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 +
 c*x)/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x
)/(1 + c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

Maple [B] time = 0.243, size = 489, normalized size = 2.1

$$-\frac{a}{2\,dx^{2}}\sqrt{-c^{2}dx^{2}+d}-\frac{ac^{2}}{2}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{-c^{2}dx^{2}+d}\right)\right)\frac{1}{\sqrt{d}}-\frac{b\mathrm{arccosh}\,(cx)\,c^{2}}{2\,d\left(c^{2}x^{2}-1\right)}\sqrt{-d\left(c^{2}x^{2}-1\right)}-\frac{bc}{2\left(c^{2}x^{2}-1\right)dx}\sqrt{-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(1/2)-1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2
*d*x^2+d)^(1/2))/x)-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)
*c^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)/x^2*arccosh(c*x)+1/2*I*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)
*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-1/2*I*b*(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2
-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*d
ilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^{2}dx^{2}+d}(b\,\mathrm{arcosh}\,(cx)+a)}{c^{2}dx^{5}-dx^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{acosh}(cx)}{x^3\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(cx)+a}{\sqrt{-c^2dx^2+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

3.113 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=155

$$-\frac{2c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3dx}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3dx^3}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6x^2\sqrt{d-c^2dx^2}}-\frac{2bc^3\sqrt{cx-1}\sqrt{cx+1}\log(x)}{3\sqrt{d-c^2dx^2}}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2*Sqrt[d - c^2*d*x^2]) - (Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*d*x^3) - (2*c^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x]))/(3*d*x) - (2*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/
(3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.505676, antiderivative size = 171, normalized size of antiderivative = 1.1, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5748, 5724, 29, 30}

$$-\frac{2c^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3x\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6x^2\sqrt{d-c^2dx^2}}-\frac{2bc^3\sqrt{cx-1}\sqrt{cx+1}}{3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2*Sqrt[d - c^2*d*x^2]) - ((1 - c*x)
*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*x^3*Sqrt[d - c^2*d*x^2]) - (2*c^2*(1 -
c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*x*Sqrt[d - c^2*d*x^2]) - (2*b*c^3*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/(3*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
```

```
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^3}\,dx}{3\sqrt{d-c^2dx^2}} + \frac{\left(2c^2\sqrt{-1+cx}\sqrt{1+cx}\right)}{3\sqrt{d-c}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}} - \frac{2c^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(c\right.}{3x\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}} - \frac{2c^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(c\right.}{3x\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.317444, size = 174, normalized size = 1.12

$$-\frac{\sqrt{d-c^2dx^2}\left(4ac^2x^2\sqrt{cx-1}\sqrt{cx+1}+2a\sqrt{cx-1}\sqrt{cx+1}+6bc^3x^3-4bc^3x^3\log(cx-1)-4bc^3x^3\log\left(\frac{1}{cx-1}+1\right)+2b\sqrt{cx}\right.}{6dx^3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -(Sqrt[d - c^2*d*x^2]*(b*c*x + 6*b*c^3*x^3 + 2*a*Sqrt[-1 + c*x]*Sqrt[1 + c*
x] + 4*a*c^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(1 + 2*c^2*x^2)*ArcCosh[c*x] - 4*b*c^3*x^3*Log[-1 + c*x] - 4*b*c^3*x^
3*Log[1 + (-1 + c*x)^(-1)]))/(6*d*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.215, size = 854, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a/d/x^3*(-c^2*d*x^2+d)^(1/2)-2/3*a*c^2/d/x*(-c^2*d*x^2+d)^(1/2)-4/3*b*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*
x)*c^3-2/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*(c*x+1)*(
```

```
c*x-1)*c^6+2/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*c^8+2
*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*arccosh(c*x)*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*c^5-2*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2
-1)*x^3*arccosh(c*x)*c^6-1/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^
2-1)*x*(c*x+1)*(c*x-1)*c^4-1/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*
x^2-1)*x^3*c^6+2/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*arcco
sh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c
^4*x^4-2*c^2*x^2-1)*x*arccosh(c*x)*c^4-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^
4*x^4-2*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-1/3*b*(-d*(c^2*x^2-1))^(
1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*c^4+4/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*
x^4-2*c^2*x^2-1)/x*arccosh(c*x)*c^2-1/6*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x
^4-2*c^2*x^2-1)/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1
/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x^3*arccosh(c*x)+2/3*b*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2+1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.59292, size = 999, normalized size = 6.45

$$\left[-\frac{2\left(2\,bc^4x^4 - bc^2x^2 - b\right)\sqrt{-c^2dx^2 + d}\,\log\left(cx + \sqrt{c^2x^2 - 1}\right) + 2\left(bc^5x^5 - bc^3x^3\right)\sqrt{-d}\,\log\left(\frac{c^2dx^6 + c^2dx^2 - dx^4 + \sqrt{-c^2dx^2 + d}\sqrt{c^2x^2 - 1}}{c^2x^4 - x^2}\right.}{6\left(c^2dx^5 - dx^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/6*(2*(2*b*c^4*x^4 - b*c^2*x^2 - b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 - 1)) + 2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x
^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)
/(c^2*x^4 - x^2)) - sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1
) + 2*(2*a*c^4*x^4 - a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^
3), 1/6*(4*(b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 2*(2*b
*c^4*x^4 - b*c^2*x^2 - b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))
 + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) - 2*(2*a*c^4*x^
4 - a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{acosh}(cx)}{x^4\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(cx) + a}{\sqrt{-c^2dx^2 + d}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^4), x)
```

3.114 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^6d^3}+\frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^6d^2}+\frac{a+b\cosh^{-1}(cx)}{c^6d\sqrt{d-c^2dx^2}}-\frac{bx^3\sqrt{d-c^2dx^2}}{9c^3d^2\sqrt{cx-1}\sqrt{cx+1}}-\cdots$$

```
[Out] (-5*b*x*Sqrt[d - c^2*d*x^2])/(3*c^5*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*
x^3*Sqrt[d - c^2*d*x^2])/(9*c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*
ArcCosh[c*x])/(c^6*d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x]))/(c^6*d^2) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(3*c
^6*d^3) - (b*Sqrt[d - c^2*d*x^2]*ArcTanh[c*x])/(c^6*d^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Rubi [A] time = 0.434414, antiderivative size = 262, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 98, 21, 100, 12, 74, 5733, 1153, 208}

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{4x^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^4d\sqrt{d-c^2dx^2}}+\frac{8(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^6d\sqrt{d-c^2dx^2}}+\frac{bx^3\sqrt{cx-1}\sqrt{c\cdots}}{9c^3d\sqrt{d-c^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (5*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^5*d*Sqrt[d - c^2*d*x^2]) + (b*x^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3*d*Sqrt[d - c^2*d*x^2]) + (x^4*(a + b*
ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) + (8*(1 - c*x)*(1 + c*x)*(a + b*
ArcCosh[c*x]))/(3*c^6*d*Sqrt[d - c^2*d*x^2]) + (4*x^2*(1 - c*x)*(1 + c*x)*(
a + b*ArcCosh[c*x]))/(3*c^4*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*ArcTanh[c*x])/(c^6*d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{8(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^6d\sqrt{d-c^2dx^2}} + \frac{4x^2(1-cx)(1+cx)\left(a+b\right.}{3c^4d\sqrt{d-c^2d\right.}}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{8(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^6d\sqrt{d-c^2dx^2}} + \frac{4x^2(1-cx)(1+cx)\left(a+b\right.}{3c^4d\sqrt{d-c^2d\right.}}$$

$$= \frac{5bx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{8(1-cx)(1+c}{3c^6d}$$

$$= \frac{5bx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{8(1-cx)(1+c}{3c^6d}$$

Mathematica [A] time = 0.101714, size = 145, normalized size = 0.62

$$\frac{-3ac^4x^4-12ac^2x^2+24a+bc^3x^3\sqrt{cx-1}\sqrt{cx+1}-3b\left(c^4x^4+4c^2x^2-8\right)\cosh^{-1}(cx)+15bcx\sqrt{cx-1}\sqrt{cx+1}+9b\sqrt{cx}}{9c^6d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (24*a - 12*a*c^2*x^2 - 3*a*c^4*x^4 + 15*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
+ b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*b*(-8 + 4*c^2*x^2 + c^4*x^4)*A
rcCosh[c*x] + 9*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(9*c^6*d*Sqrt[
d - c^2*d*x^2])
```

Maple [B] time = 0.285, size = 431, normalized size = 1.9

$$-\frac{x^4a}{3\,c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}-\frac{4\,ax^2}{3\,dc^4}\frac{1}{\sqrt{-c^2dx^2+d}}+\frac{8\,a}{3\,dc^6}\frac{1}{\sqrt{-c^2dx^2+d}}-\frac{8\,b\mathrm{arccosh}\,(cx)}{3\,d^2c^6\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}+\frac{b}{d^2c^6\left(c^2x^2-1\right)}\sqrt{-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/3*a*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)-4/3*a/c^4*x^2/d/(-c^2*d*x^2+d)^(1/2)+
8/3*a/c^6/d/(-c^2*d*x^2+d)^(1/2)-8/3*b*(-d*(c^2*x^2-1))^(1/2)/c^6/d^2/(c^2*
x^2-1)*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
6/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-1/9*b*(-d*(c^2*x^2-
1))^(1/2)/c^3/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3-5/3*b*(-d*(c^
2*x^2-1))^(1/2)/c^5/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/3*b*(-d
*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*x^4+4/3*b*(-d*(c^2*x^2
-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)*x^2-b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

[Out] Exception raised: ValueError

Fricas [A] time = 2.68376, size = 1046, normalized size = 4.49

$$\left[\frac{12\left(bc^4x^4+4\,bc^2x^2-8\,b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-9\left(bc^2x^2-b\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\left(c^3x^3+cx\right)\sqrt{-c^2}}{c^6x^6-3\,c^4x^4+3\,c^2x^2-}\right.}{36\left(c^8d^2x^2-c^6d^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/36*(12*(b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sq
rt(c^2*x^2 - 1)) - 9*(b*c^2*x^2 - b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4
 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*s
qrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*(b*c^3*x^3 + 15*b*c
*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 12*(a*c^4*x^4 + 4*a*c^2*x^2 -
8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^2*x^2 - c^6*d^2), -1/18*(9*(b*c^2*x^2 - b
)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*
d*x^4 - d)) - 6*(b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*
x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*x^3 + 15*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 6*(a*c^4*x^4 + 4*a*c^2*x^2 - 8*a)*sqrt(-c^2*d*x^2 + d))/(c^
8*d^2*x^2 - c^6*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(a+b\operatorname{acosh}(cx)\right)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^5}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^5/(-c^2*d*x^2 + d)^(3/2), x)
```

3.115 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=226

$$\frac{3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^5d\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+}}{4c^3d\sqrt{d-c^2dx^2}}$$

```
[Out] (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^3*d*Sqrt[d - c^2*d*x^2]) + (x^3*(
a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) + (3*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(2*c^4*d^2) - (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])^2)/(4*b*c^5*d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*Log[1 - c^2*x^2])/(2*c^5*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.680918, antiderivative size = 237, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5752, 5759, 5676, 30, 266, 43}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{3x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}-\frac{3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^5d\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\sqrt{c}}{4c^3d\sqrt{d-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^3*d*Sqrt[d - c^2*d*x^2]) + (x^3*(
a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) + (3*x*(1 - c*x)*(1 + c*x)
*(a + b*ArcCosh[c*x]))/(2*c^4*d*Sqrt[d - c^2*d*x^2]) - (3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^5*d*Sqrt[d - c^2*d*x^2]) - (b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*c^5*d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1+cx}\sqrt{1+cx}\cdots\right.}{cd\sqrt{d-c^2\cdots}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} - \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\int \cdots}{2c^4d\sqrt{d-c^2\cdots}}$$

$$= \frac{3bx^2\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} - \cdots$$

$$= \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} - \cdots$$

Mathematica [A] time = 1.4341, size = 192, normalized size = 0.85

$$\frac{-4acdx\left(c^2x^2-3\right)+12a\sqrt{d}\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+bd\left(8cx\cosh^{-1}(cx)-\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(8\log\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\right)\cdots\right.\right.}{8c^5d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-4*a*c*d*x*(-3 + c^2*x^2) + 12*a*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*S
qrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d*(8*c*x*ArcCosh[c*x] - S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(6*ArcCosh[c*x]^2 - Cosh[2*ArcCosh[c*x]
] + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + 2*ArcCosh[c*x]*Sinh[2*Arc
Cosh[c*x]])))/(8*c^5*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.306, size = 445, normalized size = 2.

$$-\frac{x^3 a}{2 c^2 d}\frac{1}{\sqrt{-c^2 d x^2+d}}+\frac{3 a x}{2 d c^4}\frac{1}{\sqrt{-c^2 d x^2+d}}-\frac{3 a}{2 d c^4}\arctan\left(x\sqrt{c^2 d}\frac{1}{\sqrt{-c^2 d x^2+d}}\right)\frac{1}{\sqrt{c^2 d}}+\frac{3 b\left(\operatorname{arccosh}\left(c x\right)\right)^2}{4 c^5 d^2\left(c^2 x^2-1\right)}\sqrt{-d\left(c^2 x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/2*a*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(-c^2*d*x^2+d)^(1/2)-3/
2*a/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3/4*b*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arcc
osh(c*x)^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccosh(c*x)*x^
3-1/4*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x
^2-1)*arccosh(c*x)-3/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arccosh
(c*x)*x+1/8*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2
*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b x^4 \operatorname{arcosh}\left(c x\right)+a x^4\right)\sqrt{-c^2 d x^2+d}}{c^4 d^2 x^4-2 c^2 d^2 x^2+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(a + b \operatorname{acosh}\left(cx\right)\right)}{\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}\left(cx\right) + a)x^4}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(-c^2*d*x^2 + d)^(3/2), x)
```

3.116 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2}+\frac{a+b\cosh^{-1}(cx)}{c^4d\sqrt{d-c^2dx^2}}-\frac{bx\sqrt{d-c^2dx^2}}{c^3d^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\sqrt{d-c^2dx^2}\tanh^{-1}(cx)}{c^4d^2\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -((b*x*Sqrt[d - c^2*d*x^2])/(c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (a +
b*ArcCosh[c*x])/(c^4*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x]))/(c^4*d^2) - (b*Sqrt[d - c^2*d*x^2]*ArcTanh[c*x])/(c^4*d^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.385017, antiderivative size = 163, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 98, 21, 74, 5733, 388, 208}

$$\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^4d\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{c^3d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) + (x^2*(a +
b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) + (2*(1 - c*x)*(1 + c*x)*(a +
b*ArcCosh[c*x]))/(c^4*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*ArcTanh[c*x])/(c^4*d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3 \left(a + b\cosh^{-1}(cx)\right)}{\left(d - c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^4d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{2-}{c^4-}}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^4d\sqrt{d-c^2dx^2}} + \frac{(bc}{}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^4d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{}}{}$$

Mathematica [A] time = 0.0690195, size = 97, normalized size = 0.65

$$\frac{-ac^2x^2 + 2a + b\left(2 - c^2x^2\right)\cosh^{-1}(cx) + bcx\sqrt{cx-1}\sqrt{cx+1} + b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*(2 - c^2*x^2)*Arc
Cosh[c*x] + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(c^4*d*Sqrt[d - c^
2*d*x^2])
```

Maple [B] time = 0.218, size = 313, normalized size = 2.1

$$-\frac{ax^2}{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}} + 2\frac{a}{dc^4\sqrt{-c^2dx^2+d}} + \frac{bx^2\operatorname{arccosh}(cx)}{c^2d^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)} - \frac{bx}{c^3d^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx+1}\sqrt{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a*x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*a/d/c^4/(-c^2*d*x^2+d)^(1/2)+b*(-d*(c^2
*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*x^2-b*(-d*(c^2*x^2-1))^(1/2
)/c^3/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-2*b*(-d*(c^2*x^2-1))^(1
```

```
/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-b*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(1
+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54527, size = 915, normalized size = 6.1

$$\left[-\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}bcx-4\left(bc^2x^2-2b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(bc^2x^2-b\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5c^4dx^4-}{}\right.}{4\left(c^6d^2x^2-c^4d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - 4*(b*c^2*x^2 - 2*b)
*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*x^2 - b)*sqrt(-
d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x
^2 - 1)) - 4*(a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d^2*x^2 - c^4*d^2
), -1/2*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + (b*c^2*x^2 - b)*s
qrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x
^4 - d)) - 2*(b*c^2*x^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2
- 1)) - 2*(a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d^2*x^2 - c^4*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b \operatorname{acosh}(cx))}{(-d (cx - 1) (cx + 1))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^3}{\left(-c^2 dx^2 + d\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^3/(-c^2*d*x^2 + d)^(3/2), x)
```

3.117 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2c^3d\sqrt{d-c^2dx^2}}$$

```
[Out] (x*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*c^3*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.453895, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5752, 5676, 260}

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2c^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*c^3*d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
```

```
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{}{-1+}}{cd\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}\log\left(1\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.65547, size = 159, normalized size = 1.11

$$\frac{2a\sqrt{d}\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+2acdx+bd\left(2cx\cosh^{-1}(cx)-\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(2\log\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\right)+\cosh^{-1}(cx)^2\right.\right.}{2c^3d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a*c*d*x + 2*a*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2
])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d*(2*c*x*ArcCosh[c*x] - Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)*(ArcCosh[c*x]^2 + 2*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)])))/(2*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.18, size = 279, normalized size = 2.

$$\frac{ax}{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}-\frac{a}{c^2d}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}+\frac{b\left(\operatorname{arccosh}(cx)\right)^2}{2\,d^2c^3\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}-\frac{b\operatorname{arc}}{d^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)
*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(bx^2\operatorname{arcosh}(cx)+ax^2\right)}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(a + b\operatorname{acosh}\left(cx\right)\right)}{\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arcosh}\left(cx\right) + a\right)x^2}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

3.118 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

```
[Out] (a + b*ArcCosh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*ArcTanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.248323, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5798, 5718, 207}

$$\frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (a + b*ArcCosh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*ArcTanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
```

```
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{-1+c^2x^2}\,dx}{cd\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.228695, size = 90, normalized size = 1.18

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^2d^2\left(c^2x^2-1\right)} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\tanh^{-1}(cx)}{c^2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(c^2*d^2*(-1 + c^2*x^2))) - (b
*Sqrt[-(d*(-1 + c^2*x^2))]*ArcTanh[c*x])/(c^2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

Maple [B] time = 0.136, size = 198, normalized size = 2.6

$$\frac{a}{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}} - \frac{b\text{arccosh}\,(cx)}{c^2d^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)} + \frac{b}{c^2d^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}\ln\left(cx+\sqrt{cx-1}\sqrt{cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a/c^2/d/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*a
rccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c
^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\left(\frac{\frac{\left(c\sqrt{d}x+\sqrt{cx+1}\sqrt{cx-1}\sqrt{d}\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\sqrt{-cx+1}}+\frac{\sqrt{cx+1}\sqrt{cx-1}\sqrt{d}}{\sqrt{-cx+1}}}{\sqrt{cx+1}c^3d^2x+(cx+1)\sqrt{cx-1}c^2d^2}-\int\frac{c^2x^3+cx^2e^{\left(\frac{1}{2}\log(\right.}}{\sqrt{-cx+1}\left(\left(c^2d^{\frac{3}{2}}x^2-d^{\frac{3}{2}}\right)e^{\left(\frac{3}{2}\log(cx+1)+\log(cx-1)\right)}+2\left(c^3d^{\frac{3}{2}}x^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(((c*sqrt(d)*x + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d))*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))/sqrt(-c*x + 1) + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d)/sq
rt(-c*x + 1))/(sqrt(c*x + 1)*c^3*d^2*x + (c*x + 1)*sqrt(c*x - 1)*c^2*d^2) -
 integrate((c^2*x^3 + c*x^2*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)) - x)/(s
qrt(-c*x + 1)*((c^2*d^(3/2)*x^2 - d^(3/2))*e^(3/2*log(c*x + 1) + log(c*x -
1)) + 2*(c^3*d^(3/2)*x^3 - c*d^(3/2)*x)*e^(log(c*x + 1) + 1/2*log(c*x - 1))
 + (c^4*d^(3/2)*x^4 - c^2*d^(3/2)*x^2)*sqrt(c*x + 1))), x)) + a/(sqrt(-c^2*
d*x^2 + d)*c^2*d)
```

Fricas [A] time = 2.81167, size = 697, normalized size = 9.17

$$\left[-\frac{4\sqrt{-c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(bc^2x^2-b\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5c^4dx^4-5c^2dx^2-4\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\sqrt{-d}-d}{c^6x^6-3c^4x^4+3c^2x^2-1}\right)+4}{4\left(c^4d^2x^2-c^2d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*x^2 -
 b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)) + 4*sqrt(-c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 - c^2*d^2), -1/
2*((b*c^2*x^2 - b)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*
c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*
x^2 - 1)) + 2*sqrt(-c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 - c^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(a+b\operatorname{acosh}(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(-c^2*d*x^2 + d)^(3/2), x)
```

3.119 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}$$

```
[Out] (x*(a + b*ArcCosh[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*Log[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.144078, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5713, 5688, 260}

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(3/2),x]
```


```
[Out] (x*(a + b*ArcCosh[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*Log[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
```

```
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}} \\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{1-c^2x^2}\,dx}{d\sqrt{d-c^2dx^2}} \\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0290793, size = 72, normalized size = 0.86

$$
\frac{2acx-b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)+2bcx\cosh^{-1}(cx)}{2cd\sqrt{d-c^2dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a*c*x + 2*b*c*x*ArcCosh[c*x] - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^
2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.102, size = 180, normalized size = 2.1

$$
\frac{ax}{d}\frac{1}{\sqrt{-c^2dx^2+d}}-\frac{b\operatorname{arccosh}(cx)}{cd^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}-\frac{b\operatorname{arccosh}(cx)\,x}{d^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}+\frac{b}{cd^2\left(c^2x^2-1\right)}\sqrt{-d
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a/d*x/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d
^2/(c^2*x^2-1)*x+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c
/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [A] time = 1.20032, size = 95, normalized size = 1.13

$$-\frac{bc\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)}{2\,d}+\frac{bx\operatorname{arcosh}(cx)}{\sqrt{-c^2dx^2+d}d}+\frac{ax}{\sqrt{-c^2dx^2+d}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*c*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b*x*arccosh(c*x)/(sqrt(-c^2*
d*x^2 + d)*d) + a*x/(sqrt(-c^2*d*x^2 + d)*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2
*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{acosh}(cx)}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(-c^2 d x^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

3.120 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx}}{}$$

```
[Out] (a + b*ArcCosh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) +
(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (I*
b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d -
 c^2*d*x^2]) + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^ArcCosh[c*x
]])/(d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.5906, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5756, 5761, 4180, 2279, 2391, 207}

$$-\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcCosh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) +
(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (I*
b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d -
 c^2*d*x^2]) + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^ArcCosh[c*x
]])/(d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{-1+c^2x^2}}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\tanh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)\text{se}\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1}}{d\sqrt{d}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1}}{d\sqrt{d}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1}}{d\sqrt{d}}$$

Mathematica [A] time = 2.37051, size = 301, normalized size = 1.31

$$-\frac{\frac{ibd\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+i\cosh^{-1}(cx)+\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-\sqrt{\frac{cx-}{cx+}}\right.}{\sqrt{d-c^2dx^2}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(((a*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2) - a*Sqrt[d]*Log[x] + a*Sqrt[d]*Lo
g[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (I*b*d*(I*ArcCosh[c*x] + Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - I*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + Sqrt[(-1 + c*x)
```

```
/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2])/d^2)
```

Maple [B] time = 0.217, size = 511, normalized size = 2.2

$$\frac{a}{d}\frac{1}{\sqrt{-c^2dx^2+d}}-a\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{-c^2dx^2+d}\right)\right)d^{-\frac{3}{2}}-\frac{b\text{arccosh}\left(cx\right)}{d^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}-\frac{ib\text{arccosh}\left(cx\right)}{d^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a/d/(-c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/
x)-b*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)-I*b*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)))+I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^
2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c
^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{c^4d^2x^5-2\,c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2
*x^3 + d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acosh}(cx)}{x\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

3.121 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{d^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\log\left(1-c^2x^2\right)}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -((a + b*ArcCosh[c*x])/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcCosh
[c*x]))/(d*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*Log[x])/(d^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^2*d*x^2]*Log[1 - c^2*x^2])/(2*
d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.396939, antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 103, 12, 39, 5733, 446, 72}

$$\frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\log(x)}{d\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcCosh
[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])
/(d*Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^
2])/(2*d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{-1+2c^2x^2}{x\left(1-c^2x^2\right)}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{-1+2c^2x}{x\left(1-c^2x\right)}\,dx, x, x\right.}{2d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\left(-\frac{1}{x}-\frac{c^2}{-1+c^2x}\right)\right.}{2d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\log(x)}{d\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\sqrt{1+}}{2d\sqrt{d-}}$$

Mathematica [A] time = 0.0839544, size = 114, normalized size = 0.72

$$\frac{4ac^2x^2-2a-bcx\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)+2b\left(2c^2x^2-1\right)\cosh^{-1}(cx)-2bcx\sqrt{cx-1}\sqrt{cx+1}\log(x)}{2dx\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-2*a + 4*a*c^2*x^2 + 2*b*(-1 + 2*c^2*x^2)*ArcCosh[c*x] - 2*b*c*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*Log[x] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2
*x^2])/(2*d*x*Sqrt[d - c^2*d*x^2])
```

Maple [A] time = 0.139, size = 242, normalized size = 1.5

$$-\frac{a}{dx}\frac{1}{\sqrt{-c^2dx^2+d}}+2\frac{ac^2x}{d\sqrt{-c^2dx^2+d}}-2\frac{b\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}\operatorname{arccosh}(cx)\,c}{d^2\left(c^2x^2-1\right)}-2\frac{b\sqrt{-d\left(c^2x^2-1\right)}\operatorname{arccosh}(c}{d^2\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a/d/x/(-c^2*d*x^2+d)^(1/2)+2*a*c^2/d*x/(-c^2*d*x^2+d)^(1/2)-2*b*(-d*(c^2*x
^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*c-2*b
*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*x/(c^2*x^2-1)/d^2*c^2+b*(-d*(c^2*x^2-1
))^(1/2)*arccosh(c*x)/x/(c^2*x^2-1)/d^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)
*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2 + d}(b \operatorname{arcosh}(cx) + a)}{c^4d^2x^6 - 2c^2d^2x^4 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2
*x^4 + d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^2 (-d(cx-1)(cx+1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(cx) + a}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

3.122 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=329

$$-\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2d\sqrt{d-c^2dx^2}}+\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2d\sqrt{d-c^2dx^2}}+\frac{3c^2\left(a+b\cosh^{-1}(c\right.}{2d\sqrt{d-c^2dx^2}}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x*Sqrt[d - c^2*d*x^2]) + (3*c^2*(a
+ b*ArcCosh[c*x]))/(2*d*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCosh[c*x])/(2*d*x^
2*Sqrt[d - c^2*d*x^2]) + (3*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (b*c^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (((3*I)/2)*b*c^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c
^2*d*x^2]) + (((3*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^A
rcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.862479, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5748, 5756, 5761, 4180, 2279, 2391, 207, 325}

$$-\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2d\sqrt{d-c^2dx^2}}+\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2d\sqrt{d-c^2dx^2}}+\frac{3c^2\left(a+b\cosh^{-1}(c\right.}{2d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```


```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x*Sqrt[d - c^2*d*x^2]) + (3*c^2*(a
+ b*ArcCosh[c*x]))/(2*d*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCosh[c*x])/(2*d*x^
2*Sqrt[d - c^2*d*x^2]) + (3*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (b*c^2*Sqrt[-1 + c
x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (((3*I)/2)*b*c^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c
^2*d*x^2]) + (((3*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^A
rcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2\left(-1+c^2x^2\right)}\,dx}{2d\sqrt{d-c^2dx^2}} - \frac{\left(3c^2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+cx)}}{2d\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{\left(3c^2\sqrt{-1+cx}\sqrt{1+cx}\right)\int}{2d\sqrt{d-c^2dx}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{bc^2\sqrt{-1+cx}\sqrt{1+cx}\tanh}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{3c^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{d\sqrt{}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{3c^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{d\sqrt{}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{3c^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{d\sqrt{}}$$

Mathematica [A] time = 4.29032, size = 405, normalized size = 1.23

$$\frac{1}{2}\left(-\frac{bc^2\left(3i\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-3i\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+\left(\frac{1}{c^2x^2}-1\right)\cosh^{-1}(cx)\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-((a*(-1 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(d^2*x^2*(-1 + c^2*x^2))) + (3*
a*c^2*Log[x])/d^(3/2) - (3*a*c^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/d^(3
/2) - (b*c^2*(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x)) + (-1 + 1/(c^
2*x^2))*ArcCosh[c*x] - 2*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 + (3*I)*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (3*
I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*
x]] + 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + (3
```

```
*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] -
(3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] + 2
*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^2))/(d*Sqrt[d - c^2*d*x^2]))/2
```

Maple [B] time = 0.227, size = 648, normalized size = 2.

$$-\frac{a}{2\,dx^2}\frac{1}{\sqrt{-c^2dx^2+d}}+\frac{3\,ac^2}{2\,d}\frac{1}{\sqrt{-c^2dx^2+d}}-\frac{3\,ac^2}{2}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{-c^2dx^2+d}\right)\right)d^{-\frac{3}{2}}-\frac{3\,b\mathrm{arccosh}\,(cx)\,c^2}{2\,d^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/2*a/d/x^2/(-c^2*d*x^2+d)^(1/2)+3/2*a*c^2/d/(-c^2*d*x^2+d)^(1/2)-3/2*a*c^
2/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-3/2*b*(-d*(c^2*x^2-1))
^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*c^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(c
^2*x^2-1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/
(c^2*x^2-1)/x^2*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/(c^2*x^2-1)/d^2*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c^2-b*(-d*(c^2
*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*ln(1+c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*c^2+3/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))*c^2-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x
^2-1)/d^2*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+3/2*I*
b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*dilog(
1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{c^4d^2x^7-2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2
*x^5 + d^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^3), x)
```

3.123 $\int \frac{a+b\cosh^{-1}(cx)}{x^4(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=250

$$\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}}-\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{d-c^2dx^2}}{6d^2x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bc^3\log(x)\sqrt{d-c^2dx^2}}{3d^2\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(6*d^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (a +
b*ArcCosh[c*x])/(3*d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcCosh[c*x])
)/(3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x]))/(3*d*Sqrt[d
- c^2*d*x^2]) + (5*b*c^3*Sqrt[d - c^2*d*x^2]*Log[x])/(3*d^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (b*c^3*Sqrt[d - c^2*d*x^2]*Log[1 - c^2*x^2])/(2*d^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.458335, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 103, 12, 39, 5733, 1251, 893}

$$\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}}-\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6dx^2\sqrt{d-c^2dx^2}}-\frac{5bc^3\sqrt{cx-1}\sqrt{cx+1}\log(x)}{3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2*Sqrt[d - c^2*d*x^2]) - (a + b*A
rcCosh[c*x])/(3*d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcCosh[c*x]))/(
3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x]))/(3*d*Sqrt[d - c
^2*d*x^2]) - (5*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/(3*d*Sqrt[d - c^
2*d*x^2]) - (b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*d*Sqrt
[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

)

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^4(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+c\right.}{d\sqrt{d-}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+c\right.}{3d\sqrt{d}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+c\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+c\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} -$$

Mathematica [A] time = 0.116255, size = 161, normalized size = 0.64

$$\frac{16ac^4x^4 - 8ac^2x^2 - 2a - 10bc^3x^3\sqrt{cx-1}\sqrt{cx+1}\log(x) - 3bc^3x^3\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right) + 2b\left(8c^4x^4 - 4c^2x^2 - 1\right)}{6dx^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-2*a - 8*a*c^2*x^2 + 16*a*c^4*x^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2
*b*(-1 - 4*c^2*x^2 + 8*c^4*x^4)*ArcCosh[c*x] - 10*b*c^3*x^3*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*Log[x] - 3*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2
*x^2])/(6*d*x^3*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.211, size = 1050, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/3*a/d/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*a*c^2/d/x/(-c^2*d*x^2+d)^(1/2)+8/3*a*
c^4/d*x/(-c^2*d*x^2+d)^(1/2)-16/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*c^3+32/3*b*(-d*(c^2*x^2-1))^(1/2)/
d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*(c*x+1)*(c*x-1)*c^8-32/3*b*(-d*(c^2*x^2-1))
^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^7*c^10-16/3*b*(-d*(c^2*x^2-1))^(1/2)/d
^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6+16*b*(-d*(c^2*x^2-1))^(1
/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*c^8+64/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(
8*c^4*x^4-7*c^2*x^2-1)*x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5-64/
3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)*c^6
-4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*(c*x+1)*(c*x-1)
*c^4-4*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*c^6+8/3*b*(
-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*c^3+8*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*
x*arccosh(c*x)*c^4-4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^
4-7*c^2*x^2-1)*x*c^4+4*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)
/x*arccosh(c*x)*c^2-1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1
)/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4
*x^4-7*c^2*x^2-1)/x^3*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*c^3+
5/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{c^4d^2x^8-2\,c^2d^2x^6+d^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2
*x^6 + d^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

3.124 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^6d^3}-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{3c^6d\left(d-c^2dx^2\right)^{3/2}}+\frac{bx\sqrt{d-c^2dx^2}}{c^5d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{bx\sqrt{d-}}{6c^5d^3\sqrt{cx-1}\sqrt{cx}}$$

```
[Out] (b*x*Sqrt[d - c^2*d*x^2])/(c^5*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*x*Sqr
t[d - c^2*d*x^2])/(6*c^5*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)) +
(a + b*ArcCosh[c*x])/(3*c^6*d*(d - c^2*d*x^2)^(3/2)) - (2*(a + b*ArcCosh[c*
x]))/(c^6*d^2*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*
x]))/(c^6*d^3) + (11*b*Sqrt[d - c^2*d*x^2]*ArcTanh[c*x])/(6*c^6*d^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.442693, antiderivative size = 280, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 98, 21, 74, 5733, 12, 1157, 388, 206}

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}-\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}}-\frac{8(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^6d^2\sqrt{d-c^2dx^2}}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{c^5d^2\sqrt{d-c^2dx^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -((b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^5*d^2*Sqrt[d - c^2*d*x^2])) + (b*x*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^5*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])
 - (4*x^2*(a + b*ArcCosh[c*x]))/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (x^4*(a +
 b*ArcCosh[c*x]))/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (8*
(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^6*d^2*Sqrt[d - c^2*d*x^2]) -
 (11*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*c^6*d^2*Sqrt[d - c^2*d
*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
```

```
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$=-\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}-\frac{8(1-cx)(1+cx)\left(a+b\cos\right.}{3c^6d^2\sqrt{d-c^2dx^2}}$$

$$=-\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}-\frac{8(1-cx)(1+cx)\left(a+b\cos\right.}{3c^6d^2\sqrt{d-c^2dx^2}}$$

$$=\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

$$=-\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{c^5d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{x^4\left(\right.}{3c^2d^2(1-}$$

$$=-\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{c^5d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{x^4\left(\right.}{3c^2d^2(1-}$$

Mathematica [A] time = 0.165248, size = 167, normalized size = 0.69

$$\frac{6ac^4x^4-24ac^2x^2+16a-6bc^3x^3\sqrt{cx-1}\sqrt{cx+1}+2b\left(3c^4x^4-12c^2x^2+8\right)\cosh^{-1}(cx)-11b\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2-1\right)}{6c^6d^2\left(c^2x^2-1\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (16*a - 24*a*c^2*x^2 + 6*a*c^4*x^4 + 5*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] -
 6*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*(8 - 12*c^2*x^2 + 3*c^4*x^4
)*ArcCosh[c*x] - 11*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*ArcTanh[c
*x])/(6*c^6*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.273, size = 466, normalized size = 1.9

$$-\frac{x^4a}{c^2d}\left(-c^2dx^2+d\right)^{-\frac{3}{2}}+4\frac{ax^2}{dc^4\left(-c^2dx^2+d\right)^{3/2}}-\frac{8a}{3dc^6}\left(-c^2dx^2+d\right)^{-\frac{3}{2}}-\frac{bx^2\mathrm{arccosh}\left(cx\right)}{c^4d^3\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}+\frac{bx}{c^5d^3\left(c^2x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a*x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)+4*a/c^4*x^2/d/(-c^2*d*x^2+d)^(3/2)-8/3*a/
c^6/d/(-c^2*d*x^2+d)^(3/2)-b*(-d*(c^2*x^2-1))^(1/2)/c^4/d^3/(c^2*x^2-1)*arc
cosh(c*x)*x^2+b*(-d*(c^2*x^2-1))^(1/2)/c^5/d^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x+b*(-d*(c^2*x^2-1))^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)+2*b
*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)*x^2+1/6*b*(-d*(c
^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-5/3*b*
(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^6*arccosh(c*x)-11/6*b*(-d*(c^2*x
^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*ln(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2)-1)+11/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c^6/d^3/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.6752, size = 1149, normalized size = 4.73

$$\left[-\frac{8\left(3\,bc^4x^4-12\,bc^2x^2+8\,b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+11\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2+}{c^6x^6}\right.}{24\left(c^{10}d^3x^4-2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [-1/24*(8*(3*b*c^4*x^4 - 12*b*c^2*x^2 + 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 - 1)) + 11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-d)*log(-(c^6*d
*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*s
qrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*(
6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 8*(3*a*c^4*
x^4 - 12*a*c^2*x^2 + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^3*x^4 - 2*c^8*d^3*x
^2 + c^6*d^3), 1/12*(11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt
(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) - 4*(3*b*c^
4*x^4 - 12*b*c^2*x^2 + 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1
)) + 2*(6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(
3*a*c^4*x^4 - 12*a*c^2*x^2 + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^3*x^4 - 2*c
^8*d^3*x^2 + c^6*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^5}{(-c^2dx^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^5/(-c^2*d*x^2 + d)^(5/2), x)
```

3.125 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^5d^2\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{3c^5d^2\sqrt{d-c^2dx^2}}$$

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^5*d*(d - c^2*d*x^2)^(3/2)) + (x^3*(a
+ b*ArcCosh[c*x]))/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) - (x*(a + b*ArcCosh[c*x]
))/(c^4*d^2*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*Arc
Cosh[c*x])^2)/(2*b*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (2*b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*Log[1 - c^2*x^2])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.757194, antiderivative size = 251, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 5752, 5676, 260, 266, 43}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^5d^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^5*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) - (x*(a + b*ArcCosh[c*x]))/(c^4*d^2*Sqrt[d - c^2*d*x^2]) + (x^3*(a + b*
ArcCosh[c*x]))/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^5*d^2*Sqrt[d - c^2*d
*x^2]) + (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(3*c^5*d^2*Sqr
t[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4 \left(a + b\cosh^{-1}(cx)\right)}{\left(d - c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^4(a+b\cosh^{-1}(cx))}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2(a+b\cosh^{-1}(cx))}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{c^2d^2\sqrt{d-c^2dx^2}} + \frac{\left(b\sqrt{-1+c\right.}{3}$$

$$= -\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int\frac{a+b\cosh}{\sqrt{-1+cx}}}{c^4d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{2bc^5d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + $$

Mathematica [A] time = 0.709556, size = 225, normalized size = 1.

$$\frac{\frac{2acx\left(4c^2x^2-3\right)\sqrt{d-c^2dx^2}}{\left(c^2x^2-1\right)^2} - 6a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) + \frac{bd\left(-\frac{\sqrt{\frac{cx-1}{cx+1}}(cx+1)+2cx\cosh^{-1}(cx)}{c^2x^2-1} - 8cx\cosh^{-1}(cx) + \sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(8\log\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\right)+3\cosh^{-}\right.\right.}{\sqrt{d-c^2dx^2}}}{6c^5d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((2*a*c*x*(-3 + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2)^2 - 6*a*Sqrt
[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*d*(-8*c
*x*ArcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh[c*x
])/(-1 + c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*ArcCosh[c*x]^2
+ 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)])))/Sqrt[d - c^2*d*x^2])/(6*c^
5*d^3)
```

Maple [B] time = 0.308, size = 1519, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a*x^3/c^2/d/(-c^2*d*x^2+d)^(3/2)-a/c^4/d^2*x/(-c^2*d*x^2+d)^(1/2)+a/c^4
/d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c
*x)^2+8/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2
*x^2-1)*arccosh(c*x)-32*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118
*c^4*x^4-71*c^2*x^2+16)*c/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^6+
32*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+1
6)*c^2/d^3*arccosh(c*x)*x^7-8/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6
*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*(c*x-1)*(c*x+1)*x^5+8/3*b*(-d*(c^2*x^2-
1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*x^7+84*
b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/
c/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4-76*b*(-d*(c^2*x^2-1))^(1
/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*arccosh(c*x)*x^5+
14/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2
+16)/c^2/d^3*(c*x-1)*(c*x+1)*x^3+4*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*
c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4-22
/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+1
6)/d^3*x^5-220/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^
4-71*c^2*x^2+16)/c^3/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2+181/3
*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)
/c^2/d^3*arccosh(c*x)*x^3-2*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6
+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*(c*x-1)*(c*x+1)*x-13/2*b*(-d*(c^2*x^2-1
))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^2+20/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^
6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*x^3+64/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*
c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*arccosh(c*x)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)-16*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*
c^4*x^4-71*c^2*x^2+16)/c^4/d^3*arccosh(c*x)*x+8/3*b*(-d*(c^2*x^2-1))^(1/2)/
(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*(c*
x+1)^(1/2)-2*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71
*c^2*x^2+16)/c^4/d^3*x-4/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d^3/c^5/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(bx^4 \operatorname{arcosh}(cx) + ax^4\right)\sqrt{-c^2dx^2 + d}}{c^6d^3x^6 - 3\,c^4d^3x^4 + 3\,c^2d^3x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\,(a + b\operatorname{acosh}(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)x^4}{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)*x^4/(-c^2*d*x^2 + d)^(5/2), x)
```

3.126 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{a+b\cosh^{-1}(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{3c^4d\left(d-c^2dx^2\right)^{3/2}}+\frac{bx\sqrt{d-c^2dx^2}}{6c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}+\frac{5b\sqrt{d-c^2dx^2}\tanh^{-1}(cx)}{6c^4d^3\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*x*Sqrt[d - c^2*d*x^2])/(6*c^3*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (a
 + b*ArcCosh[c*x])/(3*c^4*d*(d - c^2*d*x^2)^(3/2)) - (a + b*ArcCosh[c*x])/(
c^4*d^2*Sqrt[d - c^2*d*x^2]) + (5*b*Sqrt[d - c^2*d*x^2]*ArcTanh[c*x])/(6*c^
4*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.447681, antiderivative size = 243, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 94, 89, 21, 37, 5733, 12, 385, 206}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3cd^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{(1-cx)^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2(cx+1)\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{c^4d^2(cx+1)\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*
x^2]) - (a + b*ArcCosh[c*x])/(c^4*d^2*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (x^3
*(a + b*ArcCosh[c*x]))/(3*c*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) +
((1 - c*x)^2*(a + b*ArcCosh[c*x]))/(3*c^4*d^2*(1 + c*x)*Sqrt[d - c^2*d*x^2]
) - (5*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*c^4*d^2*Sqrt[d - c^2
*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3cd^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{(1-cx)^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2(1+cx)\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3cd^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{(1-cx)^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2(1+cx)\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3cd^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3cd^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.121852, size = 122, normalized size = 0.77

$$\frac{-6ac^2x^2+4a+b\left(4-6c^2x^2\right)\cosh^{-1}(cx)-5b\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2-1\right)\tanh^{-1}(cx)-bcx\sqrt{cx-1}\sqrt{cx+1}}{6c^4d^2\left(c^2x^2-1\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a - 6*a*c^2*x^2 - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*(4 - 6*c^2*x^2)
*ArcCosh[c*x] - 5*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*ArcTanh[c*x
])/(6*c^4*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.208, size = 313, normalized size = 2.

$$\frac{ax^2}{c^2d}\left(-c^2dx^2+d\right)^{-\frac{3}{2}}-\frac{2a}{3dc^4}\left(-c^2dx^2+d\right)^{-\frac{3}{2}}+\frac{bx^2\operatorname{arccosh}(cx)}{d^3\left(c^2x^2-1\right)^2c^2}\sqrt{-d\left(c^2x^2-1\right)}+\frac{bx}{6d^3\left(c^2x^2-1\right)^2c^3}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] a*x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3*a/d/c^4/(-c^2*d*x^2+d)^(3/2)+b*(-d*(c^
2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c*x)*x^2+1/6*b*(-d*(c^2*x^2-1
))^(1/2)/d^3/(c^2*x^2-1)^2/c^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-2/3*b*(-d*(c^2
*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)-5/6*b*(-d*(c^2*x^2-1))^(1
/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2)-1)+5/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3
/c^4/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [A] time = 1.27614, size = 236, normalized size = 1.49

$$\frac{1}{12}bc\left(\frac{2\sqrt{-d}x}{c^6d^3x^2-c^4d^3}+\frac{5\sqrt{-d}\log(cx+1)}{c^5d^3}-\frac{5\sqrt{-d}\log(cx-1)}{c^5d^3}\right)+\frac{1}{3}b\left(\frac{3x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}-\frac{2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)\text{arcosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/12*b*c*(2*sqrt(-d)*x/(c^6*d^3*x^2 - c^4*d^3) + 5*sqrt(-d)*log(c*x + 1)/(c
^5*d^3) - 5*sqrt(-d)*log(c*x - 1)/(c^5*d^3)) + 1/3*b*(3*x^2/((-c^2*d*x^2 +
d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arccosh(c*x) + 1/3*a*(3
*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))
```

Fricas [A] time = 2.58097, size = 1013, normalized size = 6.41

$$\left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}bcx+8\left(3\,bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-5\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-d}\log\left(-\frac{c^6}{}\right.}{24\left(c^8d^3x^4-2\,c^6d^3x^2+c^4d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + 8*(3*b*c^2*x^2 - 2*
b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 5*(b*c^4*x^4 - 2*b*c
^2*x^2 + b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x
^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3
*c^4*x^4 + 3*c^2*x^2 - 1)) + 8*(3*a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c
^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3), 1/12*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^
2*x^2 - 1)*b*c*x + 5*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(-c
^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^2*x
^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 4*(3*a*c^2*x^
2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(a+b\operatorname{acosh}\left(cx\right)\right)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arcosh}\left(cx\right)+a\right)x^3}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^3/(-c^2*d*x^2 + d)^(5/2), x)
```

3.127 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{6c^3d^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6c^3d\left(d-c^2dx^2\right)^{3/2}}$$

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^3*d*(d - c^2*d*x^2)^(3/2)) + (x^3*(a
+ b*ArcCosh[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (b*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*Log[1 - c^2*x^2])/(6*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.404869, antiderivative size = 160, normalized size of antiderivative = 1.2, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5724, 266, 43}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{6c^3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (x^3*(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*
x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(6*c^3*d^2*Sqrt[d
 - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
```

```
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{\left(-1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{x}{\left(-1+c^2x\right)^2}\,dx, x, x^2\right)}{6d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{1}{c^2\left(-1+c^2x\right)^2} + \frac{1}{c^2\left(-1+c^2x\right)}\right.\right.}{6d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\log\left(\right.}{6c^3d^2\sqrt{d-c^2dx}}$$

Mathematica [A] time = 0.198563, size = 101, normalized size = 0.76

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\frac{b\left(\frac{1}{1-c^2x^2}+\log\left(1-c^2x^2\right)\right)}{c^3}-\frac{2x^3\left(a+b\cosh^{-1}(cx)\right)}{(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{6d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((-2*x^3*(a + b*ArcCosh[c*x]))/((-1 + c*x)^(3
/2)*(1 + c*x)^(3/2)) + (b*((1 - c^2*x^2)^(-1) + Log[1 - c^2*x^2]))/c^3))/(6
*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.202, size = 1228, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a/c^2/d*x/(-c^2*d*x^2+d)^(3/2)-1/3*a/c^2/d^2*x/(-c^2*d*x^2+d)^(1/2)+2/3
*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*a
rccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*
x^2+1)*c^3/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6+b*(-d*(c^2*x^2-
1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*arccosh(c*x)
*x^7-1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2
+1)*c^2/d^3*(c*x+1)*(c*x-1)*x^5+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c
^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*x^7+2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^
8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^4-b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*
c^2*x^2+1)*c^2/d^3*arccosh(c*x)*x^5+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8
-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*(c*x+1)*(c*x-1)*x^3+1/2*b*(-d*(c^2*x
^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c/d^3*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^4-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c
^4*x^4-5*c^2*x^2+1)*c^2/d^3*x^5-4/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c
^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*x^2+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x
^2+1)/d^3*arccosh(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^
6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/6*b*(-d*(
```

```
c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*x^3+1/3*
b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d
^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3
*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*(c*x-1)^(1/2)*(c*x+1)^(1
/2)-1/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x
^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [A] time = 1.28269, size = 228, normalized size = 1.71

$$\frac{1}{6}\,bc\left(\frac{\sqrt{-d}}{c^6d^3x^2-c^4d^3}-\frac{\sqrt{-d}\log\left(cx+1\right)}{c^4d^3}-\frac{\sqrt{-d}\log\left(cx-1\right)}{c^4d^3}\right)-\frac{1}{3}\,b\left(\frac{x}{\sqrt{-c^2dx^2+d}c^2d^2}-\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)\operatorname{arcosh}\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*b*c*(sqrt(-d)/(c^6*d^3*x^2 - c^4*d^3) - sqrt(-d)*log(c*x + 1)/(c^4*d^3)
 - sqrt(-d)*log(c*x - 1)/(c^4*d^3)) - 1/3*b*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d^
2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arccosh(c*x) - 1/3*a*(x/(sqrt(-c^2*d
*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(bx^2\operatorname{arcosh}\left(cx\right)+ax^2\right)}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{acosh}(cx))}{(-d (cx - 1) (cx + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^2}{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(-c^2*d*x^2 + d)^(5/2), x)
```

3.128 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=127

$$\frac{a+b\cosh^{-1}(cx)}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}(cx)}{6c^2d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{6cd\left(d-c^2dx^2\right)^{3/2}}$$

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c*d*(d - c^2*d*x^2)^(3/2)) + (a + b*A
rcCosh[c*x])/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*ArcTanh[c*x])/(6*c^2*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.275824, antiderivative size = 154, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {5798, 5718, 199, 207}

$$\frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}(cx)}{6c^2d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (a + b*ArcCosh[c*x])/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^
2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*c^2*d^2*Sqrt[d - c^2
*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
```

```
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(b\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{\left(-1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{}{-1}}{6cd^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\tanh^{-}}{6c^2d^2\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.255342, size = 119, normalized size = 0.94

$$\frac{\sqrt{d-c^2dx^2}\left(2a+bcx\sqrt{cx-1}\sqrt{cx+1}+2b\cosh^{-1}(cx)\right)}{6c^2d^3\left(c^2x^2-1\right)^2} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\tanh^{-1}(cx)}{6c^2d^3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(2*a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*ArcCos
h[c*x]))/(6*c^2*d^3*(-1 + c^2*x^2)^2) - (b*Sqrt[-(d*(-1 + c^2*x^2))]*ArcTan
h[c*x])/(6*c^2*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.164, size = 249, normalized size = 2.

$$\frac{a}{3c^2d}\left(-c^2dx^2+d\right)^{-\frac{3}{2}}+\frac{bx}{6d^3\left(c^2x^2-1\right)^2c}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx+1}\sqrt{cx-1}+\frac{b\operatorname{arccosh}(cx)}{3d^3\left(c^2x^2-1\right)^2c^2}\sqrt{-d\left(c^2x^2-1\right)}+\frac{}{6c^2d^3(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-
1)^2/c*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*
x^2-1)^2/c^2*arccosh(c*x)+1/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-1/6*b*(-d
*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(1+c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{x\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx+\frac{a}{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(5/2)
, x) + 1/3*a/((-c^2*d*x^2 + d)^(3/2)*c^2*d)
```

Fricas [A] time = 2.59786, size = 910, normalized size = 7.17

$$\left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}bcx+8\sqrt{-c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^4x^4-2bc^2x^2+b\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5c^4dx^4-5c^2dx}{c}\right.}{24\left(c^6d^3x^4-2c^4d^3x^2+c^2d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + 8*sqrt(-c^2*d*x^2 +
 d)*b*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-d)
*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^2*
d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2
 - 1)) + 8*sqrt(-c^2*d*x^2 + d)*a)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3),
 1/12*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - (b*c^4*x^4 - 2*b*c^
2*x^2 + b)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d
)*x/(c^4*d*x^4 - d)) + 4*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)
) + 4*sqrt(-c^2*d*x^2 + d)*a)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\operatorname{acosh}\left(cx\right)\right)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}\left(cx\right)+a)x}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)*x/(-c^2*d*x^2 + d)^(5/2), x)
```

3.129 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=162

$$\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6cd\left(d-c^2dx^2\right)^{3/2}}$$

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c*d*(d - c^2*d*x^2)^(3/2)) + (x*(a + b*
ArcCosh[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcCosh[c*x]))/(3*
d^2*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2]
)/(3*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.268182, antiderivative size = 189, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5713, 5691, 5688, 260, 261}

$$\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]
) + (2*x*(a + b*ArcCosh[c*x]))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (x*(a + b*ArcC
osh[c*x]))/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(3*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
```

```
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{\right.}}{3d^2}$$

$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(2bc\right.}{}$$

$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{b\sqrt{-}}{}$$

Mathematica [A] time = 0.0835394, size = 132, normalized size = 0.81

$$\frac{4ac^3x^3-6acx-2b\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2-1\right)\log\left(1-c^2x^2\right)+2bcx\left(2c^2x^2-3\right)\cosh^{-1}(cx)-b\sqrt{cx-1}\sqrt{cx+1}}{6cd^2\left(c^2x^2-1\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (-6*a*c*x + 4*a*c^3*x^3 - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*c*x*(-3 + 2*
c^2*x^2)*ArcCosh[c*x] - 2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*Log
[1 - c^2*x^2])/(6*c*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.143, size = 1073, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a/d*x/(-c^2*d*x^2+d)^(3/2)+2/3*a/d^2*x/(-c^2*d*x^2+d)^(1/2)-4/3*b*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(c*x
)+2/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*
(c*x+1)*(c*x-1)*x^5-2/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c
^2*x^2-4)*c^6/d^3*x^7+2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c
^2*x^2-4)*c^3/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-2*b*(-d*(c^2
*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)*x^5
-5/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*(
c*x+1)*(c*x-1)*x^3+7/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^
2*x^2-4)*c^4/d^3*x^5-14/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11
*c^2*x^2-4)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+17/3*b*(-d*(
c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)*
x^3+b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*(c*x+1
)*(c*x-1)*x+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4
)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-8/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6
*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*x^3+8/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c
^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1
/2)-4*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*arcc
osh(c*x)*x-2/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)
/c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c
```

```
^4*x^4+11*c^2*x^2-4)/d^3*x+2/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/d^3/c/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [A] time = 1.22012, size = 212, normalized size = 1.31

$$\frac{1}{6}\,bc\left(\frac{\sqrt{-d}}{c^4d^3x^2-c^2d^3}+\frac{2\sqrt{-d}\log\left(cx+1\right)}{c^2d^3}+\frac{2\sqrt{-d}\log\left(cx-1\right)}{c^2d^3}\right)+\frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{-c^2dx^2+d}d^2}+\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}\right)\operatorname{arcosh}\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*c*(sqrt(-d)/(c^4*d^3*x^2 - c^2*d^3) + 2*sqrt(-d)*log(c*x + 1)/(c^2*d^
3) + 2*sqrt(-d)*log(c*x - 1)/(c^2*d^3)) + 1/3*b*(2*x/(sqrt(-c^2*d*x^2 + d)*
d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arccosh(c*x) + 1/3*a*(2*x/(sqrt(-c^2*d
*x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}\left(cx\right)+a)}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^
3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{acosh}\left(cx\right)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(5/2), x)
```

3.130 $\int \frac{a+b\cosh^{-1}(cx)}{x(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx}}{}$$

```
[Out] (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (a + b*ArcCosh[c*x])/(3*d*(d - c^2*d*x^2)^(3/2)) + (a + b*ArcCosh[c*x
])/(d^2*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCo
sh[c*x])*ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (7*b*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*d^2*Sqrt[d - c^2*d*x^2]) - (I*b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d
*x^2]) + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^ArcCosh[c*x]])/(d
^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.831696, antiderivative size = 332, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5756, 5761, 4180, 2279, 2391, 207, 199}

$$-\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(cx+1)\sqrt{d-}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (a + b*ArcCosh[c*x])/(d^2*Sqrt[d - c^2*d*x^2]) + (a + b*ArcCosh[c*x])
/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^
2]) + (7*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*d^2*Sqrt[d - c^2*d
*x^2]) - (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])
/(d^2*Sqrt[d - c^2*d*x^2]) + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I
*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d^2\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{\cdots}\right.}{3d^2\cdots}$$

$$= \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\cdots\right.}{\cdots}$$

$$= \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{7b\sqrt{-1+c\cdots}}{6d\cdots}$$

$$= \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\cdots}{\cdots}$$

$$= \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\cdots}{\cdots}$$

$$= \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\cdots}{\cdots}$$

Mathematica [A] time = 7.0114, size = 377, normalized size = 1.19

$$\frac{b\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(-24i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)+24i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)-24i\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(a/(3*d^3*(-1 + c^2*x^2)^2) - a/(d^3*(-1 + c^2*x^
2))) + (a*Log[x])/d^(5/2) - (a*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/
d^(5/2) + (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(14*ArcCosh[c*x]*Coth[Arc
Cosh[c*x]/2] - Csch[ArcCosh[c*x]/2]^2 - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)*ArcCosh[c*x]*Csch[ArcCosh[c*x]/2]^4)/2 - (24*I)*ArcCosh[c*x]*Log[1 - I/E
^ArcCosh[c*x]] + (24*I)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - 28*Log[Tan
h[ArcCosh[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (24*I)*PolyLo
g[2, I/E^ArcCosh[c*x]] - Sech[ArcCosh[c*x]/2]^2 - (8*ArcCosh[c*x]*Sinh[ArcC
osh[c*x]/2]^4)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3) - 14*ArcCosh[c*x]
*Tanh[ArcCosh[c*x]/2]))/(24*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

Maple [A] time = 0.243, size = 619, normalized size = 2.

$$\frac{a}{3\,d}\left(-c^2dx^2+d\right)^{-\frac{3}{2}}+\frac{a}{d^2}\frac{1}{\sqrt{-c^2dx^2+d}}-a\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{-c^2dx^2+d}\right)\right)d^{-\frac{5}{2}}-\frac{bx^2\operatorname{arccosh}(cx)\,c^2}{d^3\left(c^2x^2-1\right)^2}\sqrt{-d\left(c^2x^2-1\right)}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a/d/(-c^2*d*x^2+d)^(3/2)+a/d^2/(-c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+2
*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^
2*arccosh(c*x)*x^2*c^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*c+4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2*a
rccosh(c*x)+7/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c
^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-7/6*b*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))+I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^
2-1)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x
```

```
-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d^3/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)
*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{c^6d^3x^7-3\,c^4d^3x^5+3\,c^2d^3x^3-d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^
3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

3.131 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=248

$$\frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{d-c^2dx^2}}{6d^3\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)}+\frac{bc\log(x)\sqrt{d}}{d^3\sqrt{cx-1}\sqrt{}}$$

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(6*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2
)) - (a + b*ArcCosh[c*x])/(d*x*(d - c^2*d*x^2)^(3/2)) + (4*c^2*x*(a + b*Arc
Cosh[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (8*c^2*x*(a + b*ArcCosh[c*x]))/(3
*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*Log[x])/(d^3*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) + (5*b*c*Sqrt[d - c^2*d*x^2]*Log[1 - c^2*x^2])/(6*d^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.44195, antiderivative size = 279, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 103, 12, 40, 39, 5733, 1251, 893}

$$\frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]
) + (8*c^2*x*(a + b*ArcCosh[c*x]))/(3*d^2*Sqrt[d - c^2*d*x^2]) - (a + b*Arc
Cosh[c*x])/(d^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (4*c^2*x*(a +
b*ArcCosh[c*x]))/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/(d^2*Sqrt[d - c^2*d*x^2]) - (5*b*c*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(6*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} -$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} -$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} -$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} -$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4}{3d^2(}$$

Mathematica [A] time = 0.336904, size = 147, normalized size = 0.59

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\frac{4c^2x\left(2c^2x^2-3\right)\left(a+b\cosh^{-1}(cx)\right)}{3(cx-1)^{3/2}(cx+1)^{3/2}}+\frac{a+b\cosh^{-1}(cx)}{x(cx-1)^{3/2}(cx+1)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+6\log(x)\right)\right)}{d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/(x*(-1 + c*x)^(3/2)*(1
+ c*x)^(3/2)) + (4*c^2*x*(-3 + 2*c^2*x^2)*(a + b*ArcCosh[c*x]))/(3*(-1 + c*
x)^(3/2)*(1 + c*x)^(3/2)) - (b*c*((-1 + c^2*x^2)^(-1) + 6*Log[x] + 5*Log[1
- c^2*x^2]))/6))/(d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.194, size = 1350, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a/d/x/(-c^2*d*x^2+d)^(3/2)+4/3*a*c^2/d*x/(-c^2*d*x^2+d)^(3/2)+8/3*a*c^2/d^
2*x/(-c^2*d*x^2+d)^(1/2)-16/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*c+32/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*(c*x+1)*(c*x-1)*c^8-32/3*b*(-d*(c^2*x
^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^9*c^10-80/3*b*(-d*(c
^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*(c*x+1)*(c*x-1
)*c^6+112/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9
)*x^7*c^8+64/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^
2-9)*x^4*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5-64/3*b*(-d*(c^2*x^2-1
))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*arccosh(c*x)*c^6+20*b*
(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*(c*x+1)*
(c*x-1)*c^4-140/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2
*x^2-9)*x^5*c^6-136/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26
*c^2*x^2-9)*x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+56*b*(-d*(c^2*
x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*arccosh(c*x)*c^4-
4*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*(c*x+1
)*(c*x-1)*c^2+4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2
*x^2-9)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+24*b*(-d*(c^2*x^2-1))^(1/2)/d^3
/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*c^4+24*b*(-d*(c^2*x^2-1))^(1/2)/d^
3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*c-44*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x
*arccosh(c*x)*c^2-3/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26
*c^2*x^2-9)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-4*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*c^2+9*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c
^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/x*arccosh(c*x)+5/3*b*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2-1)*c+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2
```

```
*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{c^6d^3x^8-3\,c^4d^3x^6+3\,c^2d^3x^4-d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^
3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

3.132 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=479

$$-\frac{5ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2d^2\sqrt{d-c^2dx^2}}+\frac{5ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2d^2\sqrt{d-c^2dx^2}}+\frac{5c^2\left(a+b\cosh^{-1}(c\right.}{2d^2\sqrt{d-c^2dx^2}}$$

```
[Out] (3*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*d^2*x*Sqrt[d - c^2*d*x^2]) - (b*c*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(4*d^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) +
(5*b*c^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(12*d^2*(1 - c^2*x^2)*Sqrt[d - c^2
*d*x^2]) + (5*c^2*(a + b*ArcCosh[c*x]))/(6*d*(d - c^2*d*x^2)^(3/2)) - (a +
b*ArcCosh[c*x])/(2*d*x^2*(d - c^2*d*x^2)^(3/2)) + (5*c^2*(a + b*ArcCosh[c*x
]))/(2*d^2*Sqrt[d - c^2*d*x^2]) + (5*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (13*b*c
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*d^2*Sqrt[d - c^2*d*x^2]) -
 (((5*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*
x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (((5*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*PolyLog[2, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.14279, antiderivative size = 509, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {5798, 5748, 5756, 5761, 4180, 2279, 2391, 207, 199, 290, 325}

$$-\frac{5ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2d^2\sqrt{d-c^2dx^2}}+\frac{5ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2d^2\sqrt{d-c^2dx^2}}+\frac{5c^2\left(a+b\cosh\right.}{6d^2(1-cx)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (3*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*d^2*x*Sqrt[d - c^2*d*x^2]) - (b*c*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(4*d^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) +
(5*b*c^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(12*d^2*(1 - c^2*x^2)*Sqrt[d - c^2
*d*x^2]) + (5*c^2*(a + b*ArcCosh[c*x]))/(2*d^2*Sqrt[d - c^2*d*x^2]) + (5*c^
2*(a + b*ArcCosh[c*x]))/(6*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (
a + b*ArcCosh[c*x])/(2*d^2*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (
5*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*
x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (13*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Arc
```

```
Tanh[c*x])/(6*d^2*Sqrt[d - c^2*d*x^2]) - (((5*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + ((
(5*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^ArcCosh[c*x]])/(
d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
```

```
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2d^2x^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2\left(-1+c^2x^2\right)^2}\,dx}{2d^2\sqrt{d-c^2dx^2}} + \frac{\left(5c^2\sqrt{-1+cx}\sqrt{\cdots}\right.}{2\cdots}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{6d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2(1-cx)(1+cx)\sqrt{d-c^2\cdots}}$$

$$= \frac{3bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\sqrt{1+cx}}{12d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cdots\right.}{2d^2\sqrt{d\cdots}}$$

$$= \frac{3bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\sqrt{1+cx}}{12d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cdots\right.}{2d^2\sqrt{d\cdots}}$$

$$= \frac{3bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\sqrt{1+cx}}{12d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cdots\right.}{2d^2\sqrt{d\cdots}}$$

$$= \frac{3bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\sqrt{1+cx}}{12d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cdots\right.}{2d^2\sqrt{d\cdots}}$$

$$= \frac{3bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\sqrt{1+cx}}{12d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cdots\right.}{2d^2\sqrt{d\cdots}}$$

Mathematica [A] time = 7.20668, size = 500, normalized size = 1.04

$$\frac{bc^2\left(-30i\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)+30i\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+\frac{6(cx-1)(cx+1)\cosh^{-1}(cx)}{c^2x^2}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(-a/(2*d^3*x^2) + (a*c^2)/(3*d^3*(-1 + c^2*x^2)^2
) - (2*a*c^2)/(d^3*(-1 + c^2*x^2))) + (5*a*c^2*Log[x])/(2*d^(5/2)) - (5*a*c
^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/(2*d^(5/2)) + (b*c^2*((6*Sqr
t[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x) + (6*(-1 + c*x)*(1 + c*x)*ArcCosh[
c*x])/(c^2*x^2) + 26*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 - Coth[ArcCosh[c*x
]/2] - ArcCosh[c*x]*Coth[ArcCosh[c*x]/2]^2 - (30*I)*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + (30*I)*Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - 26*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] - (30*I)*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (30*I)*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] - 26*ArcCosh[c*x
]*Sinh[ArcCosh[c*x]/2]^2 - Tanh[ArcCosh[c*x]/2] - ArcCosh[c*x]*Tanh[ArcCosh
[c*x]/2]^2))/(12*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

Maple [A] time = 0.285, size = 801, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/2*a/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/6*a*c^2/d/(-c^2*d*x^2+d)^(3/2)+5/2*a*c^
2/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+
d)^(1/2))/x)-5/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^2*x^2+1)*x^2*arc
cosh(c*x)*c^4-1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^2*x^2+1)*x*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*c^3+10/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^
2*x^2+1)*arccosh(c*x)*c^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^2*x
^2+1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4
*x^4-2*c^2*x^2+1)/x^2*arccosh(c*x)+13/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c^2
-13/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*
ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^2-5/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2)))*c^2+5/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))*c^2+5/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c
^2*x^2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-5/2*I*b*(-d*(c^2
```

```
*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*dilog(1-I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)}{c^6d^3x^9-3\,c^4d^3x^7+3\,c^2d^3x^5-d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^
3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(-c^2 d x^2 + d\right)^{\frac{5}{2}} x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

3.133 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=338

$$\frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}-\frac{bc^3\sqrt{d-c^2}}{6d^3\sqrt{cx-1}\sqrt{cx+}}$$

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(6*d^3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^
3*Sqrt[d - c^2*d*x^2])/(6*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)) -
 (a + b*ArcCosh[c*x])/(3*d*x^3*(d - c^2*d*x^2)^(3/2)) - (2*c^2*(a + b*ArcCo
sh[c*x]))/(d*x*(d - c^2*d*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcCosh[c*x]))/(3*d
*(d - c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcCosh[c*x]))/(3*d^2*Sqrt[d - c
^2*d*x^2]) + (8*b*c^3*Sqrt[d - c^2*d*x^2]*Log[x])/(3*d^3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (4*b*c^3*Sqrt[d - c^2*d*x^2]*Log[1 - c^2*x^2])/(3*d^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.541376, antiderivative size = 383, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 103, 12, 40, 39, 5733, 1799, 1620}

$$\frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}-\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(cx+1)\sqrt{d-}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d^2*x^2*Sqrt[d - c^2*d*x^2]) + (b*c^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) +
(16*c^4*x*(a + b*ArcCosh[c*x]))/(3*d^2*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCos
h[c*x])/(3*d^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (2*c^2*(a + b
*ArcCosh[c*x]))/(d^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*
(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (8*
b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/(3*d^2*Sqrt[d - c^2*d*x^2]) - (4
*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(3*d^2*Sqrt[d - c^2*d
*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^4(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6d^2x^2\sqrt{d-c^2dx^2}} + \frac{bc^3\sqrt{-1+cx}\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\,\mathrm{c}}{3d^2x^3(1-cx)(1}$$

Mathematica [A] time = 0.397771, size = 169, normalized size = 0.5

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\frac{2c^2\left(8c^4x^4-12c^2x^2+3\right)\left(a+b\cosh^{-1}(cx)\right)}{x(cx-1)^{3/2}(cx+1)^{3/2}} + \frac{a+b\cosh^{-1}(cx)}{x^3(cx-1)^{3/2}(cx+1)^{3/2}} - bc\left(\frac{1}{2x^2\left(c^2x^2-1\right)} + 4c^2\log\left(1-c^2x^2\right) + 8c^2\log(x)\right)\right)}{3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/(x^3*(-1 + c*x)^(3/2)*(
1 + c*x)^(3/2)) + (2*c^2*(3 - 12*c^2*x^2 + 8*c^4*x^4)*(a + b*ArcCosh[c*x]))
/(x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - b*c*(1/(2*x^2*(-1 + c^2*x^2)) + 8*c
^2*Log[x] + 4*c^2*Log[1 - c^2*x^2])))/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.209, size = 1878, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -8/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*
x^2-1)*x*c^4+8/3*a*c^4/d*x/(-c^2*d*x^2+d)^(3/2)+16/3*a*c^4/d^2*x/(-c^2*d*x^
2+d)^(1/2)+16/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*
x^4-10*c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+128/3*b*(-d*
(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^9*
(c*x+1)*(c*x-1)*c^12-320/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*
x^6+35*c^4*x^4-10*c^2*x^2-1)*x^7*(c*x+1)*(c*x-1)*c^10+80*b*(-d*(c^2*x^2-1))
^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^5*(c*x+1)*(c*x
-1)*c^8-40/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4
-10*c^2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6-8/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12
*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x*(c*x+1)*(c*x-1)*c^4+2*b*(-d*
(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^2*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5-1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^
8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-32/
3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arcc
osh(c*x)*c^3+8/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(
c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)*c^3-2*a*c^2/d/x/(-c^2*
d*x^2+d)^(3/2)-1/3*a/d/x^3/(-c^2*d*x^2+d)^(3/2)+64*b*(-d*(c^2*x^2-1))^(1/2)
/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^6*arccosh(c*x)*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*c^9-128*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*
c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^4*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*c^7+176/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4
-10*c^2*x^2-1)*x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5-2*b*(-d*(c^
2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*c^3-64*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6
*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^7*arccosh(c*x)*c^10+160*b*(-d*(c^2*x^2-1))^
(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^5*arccosh(c*x)*
```

```
c^8-344/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10
*c^2*x^2-1)*x^3*arccosh(c*x)*c^6+12*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^
8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x*arccosh(c*x)*c^4+6*b*(-d*(c^2*x^2-1
))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/x*arccosh(c*x)
*c^2+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*
c^2*x^2-1)/x^3*arccosh(c*x)-128/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-
36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^11*c^14+448/3*b*(-d*(c^2*x^2-1))^(1/2
)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^9*c^12-560/3*b*(-d*
(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^7*
c^10+280/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-1
0*c^2*x^2-1)*x^5*c^8-32/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x
^6+35*c^4*x^4-10*c^2*x^2-1)*x^3*c^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{c^6d^3x^{10}-3\,c^4d^3x^8+3\,c^2d^3x^6-d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d
^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(-c^2dx^2 + d\right)^{\frac{5}{2}} x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^4), x)
```

3.134 $\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=246

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}}{15ac^3(1-a^2x^2)\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}}{20ac^3(1-a^2x^2)^2\sqrt{c-a^2cx^2}}-\frac{4\sqrt{ax-1}\sqrt{ax+1}\log(1-a^2x^2)}{15ac^3\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}+\frac{4x}{15c^2}$$

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(20*a*c^3*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2
]) + (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(15*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*
c*x^2]) + (x*ArcCosh[a*x])/(5*c*(c - a^2*c*x^2)^(5/2)) + (4*x*ArcCosh[a*x])
/(15*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x*ArcCosh[a*x])/(15*c^3*Sqrt[c - a^2*c
*x^2]) - (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[1 - a^2*x^2])/(15*a*c^3*Sqrt[c
 - a^2*c*x^2])
```

Rubi [A] time = 0.339178, antiderivative size = 276, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5713, 5691, 5688, 260, 261}

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}}{15ac^3(1-a^2x^2)\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}}{20ac^3(1-a^2x^2)^2\sqrt{c-a^2cx^2}}-\frac{4\sqrt{ax-1}\sqrt{ax+1}\log(1-a^2x^2)}{15ac^3\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}+\frac{}{15c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(20*a*c^3*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2
]) + (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(15*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*
c*x^2]) + (8*x*ArcCosh[a*x])/(15*c^3*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh[a*x]
)/(5*c^3*(1 - a*x)^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]) + (4*x*ArcCosh[a*x])/
(15*c^3*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (4*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*Log[1 - a^2*x^2])/(15*a*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
```

```
!IntegerQ[p]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{(-1+ax)^{7/2}(1+ax)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{\left(4\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{5c^3\sqrt{c-a^2cx^2}} - \frac{\left(a\sqrt{-1+ax}\sqrt{1+\right.}{5c^3\sqrt{c}}$$

$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{4x\cosh^{-1}(ax)}{15c^3(1-ax)(1+ax)\sqrt{c-a^2cx}}$$

$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{-1+ax}\sqrt{1+ax}}{15ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}} + \frac{x\cosh}{5c^3(1-ax)^2(1+}$$

$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{-1+ax}\sqrt{1+ax}}{15ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}} + \frac{x\cosh}{5c^3(1-ax)^2(1+}$$

Mathematica [A] time = 0.0882938, size = 116, normalized size = 0.47

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\left(-8a^2x^2-16\left(a^2x^2-1\right)^2\log\left(1-a^2x^2\right)+11\right)+4ax\left(8a^4x^4-20a^2x^2+15\right)\cosh^{-1}(ax)}{60ac^3\left(a^2x^2-1\right)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (4*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcCosh[a*x] + Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*(11 - 8*a^2*x^2 - 16*(-1 + a^2*x^2)^2*Log[1 - a^2*x^2]))/(60*a*c^3*(-
1 + a^2*x^2)^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.229, size = 419, normalized size = 1.7

$$-\frac{16\,\mathrm{arccosh}\,(ax)}{15\,ac^4\left(a^2x^2-1\right)}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{ax-1}\sqrt{ax+1}-\frac{1}{\left(2400\,a^{10}x^{10}-12900\,x^8a^8+28140\,x^6a^6-31020\,x^4a^4+17220\,a^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -16/15*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)
*arccosh(a*x)-1/60*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-20*x^3*a^3-8*(a*x+1)^(
1/2)*(a*x-1)^(1/2)*x^4*a^4+15*a*x+16*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-8*
(a*x-1)^(1/2)*(a*x+1)^(1/2))*(-64*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^7*a^7-64*x^
8*a^8+248*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^5*a^5+280*x^6*a^6+160*arccosh(a*x)*
x^4*a^4-340*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)-456*x^4*a^4-380*a^2*x^2*arc
cosh(a*x)+165*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+328*a^2*x^2+256*arccosh(a*x)-
88)/(40*a^10*x^10-215*a^8*x^8+469*a^6*x^6-517*a^4*x^4+287*a^2*x^2-64)/a/c^4
+8/15*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*
ln((a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2-1)
```

Maxima [A] time = 1.27007, size = 258, normalized size = 1.05

$$-\frac{1}{60}a\left(\frac{16\sqrt{-\frac{1}{a^4c}}\log\left(x^2-\frac{1}{a^2}\right)}{c^3}+\frac{3}{\left(a^6c^3x^4\sqrt{-\frac{1}{c}}-2a^4c^3x^2\sqrt{-\frac{1}{c}}+a^2c^3\sqrt{-\frac{1}{c}}\right)c}-\frac{8}{\left(a^4c^2x^2\sqrt{-\frac{1}{c}}-a^2c^2\sqrt{-\frac{1}{c}}\right)c^2}\right)+\frac{1}{15}\left(\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/60*a*(16*sqrt(-1/(a^4*c))*log(x^2 - 1/a^2)/c^3 + 3/((a^6*c^3*x^4*sqrt(-1
/c) - 2*a^4*c^3*x^2*sqrt(-1/c) + a^2*c^3*sqrt(-1/c))*c) - 8/((a^4*c^2*x^2*s
qrt(-1/c) - a^2*c^2*sqrt(-1/c))*c^2)) + 1/15*(8*x/(sqrt(-a^2*c*x^2 + c)*c^3
) + 4*x/((-a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((-a^2*c*x^2 + c)^(5/2)*c))*arcc
osh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\text{arcosh}\,(ax)}{a^8c^4x^8-4\,a^6c^4x^6+6\,a^4c^4x^4-4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 + 6
*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45557, size = 192, normalized size = 0.78

$$\frac{1}{60}\sqrt{-c}\left(\frac{16\log\left(\left|a^2x^2-1\right|\right)}{ac^4}-\frac{24\,a^4x^4-56\,a^2x^2+35}{\left(a^2x^2-1\right)^2ac^4}\right)-\frac{\sqrt{-a^2cx^2+c}\left(4\left(\frac{2\,a^4x^2}{c}-\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2-1}\right)}{15\left(a^2cx^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(-c)*(16*log(abs(a^2*x^2 - 1))/(a*c^4) - (24*a^4*x^4 - 56*a^2*x^2
+ 35)/((a^2*x^2 - 1)^2*a*c^4)) - 1/15*sqrt(-a^2*c*x^2 + c)*(4*(2*a^4*x^2/c
- 5*a^2/c)*x^2 + 15/c)*x*log(a*x + sqrt(a^2*x^2 - 1))/(a^2*c*x^2 - c)^3
```

3.135 $\int \frac{x^4 \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=145

$$-\frac{3x^2\sqrt{ax-1}}{16a^3\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{8a^4}+\frac{3\sqrt{ax-1}\cosh^{-1}(ax)^2}{16a^5\sqrt{1-ax}}-\frac{x^4\sqrt{ax-1}}{16a\sqrt{1-ax}}$$

```
[Out] (-3*x^2*Sqrt[-1 + a*x])/(16*a^3*Sqrt[1 - a*x]) - (x^4*Sqrt[-1 + a*x])/(16*a
*Sqrt[1 - a*x]) - (3*x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/(8*a^4) - (x^3*Sqrt[
1 - a^2*x^2]*ArcCosh[a*x])/(4*a^2) + (3*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(16*
a^5*Sqrt[1 - a*x])
```

Rubi [A] time = 0.500843, antiderivative size = 206, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5798, 5759, 5676, 30}

$$-\frac{x^4\sqrt{ax-1}\sqrt{ax+1}}{16a\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{ax-1}\sqrt{ax+1}}{16a^3\sqrt{1-a^2x^2}}-\frac{x^3(1-ax)(ax+1)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}}-\frac{3x(1-ax)(ax+1)\cosh^{-1}(ax)}{8a^4\sqrt{1-a^2x^2}}+\frac{3\sqrt{ax-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-3*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(16*a^3*Sqrt[1 - a^2*x^2]) - (x^4*Sqr
t[-1 + a*x]*Sqrt[1 + a*x])/(16*a*Sqrt[1 - a^2*x^2]) - (3*x*(1 - a*x)*(1 + a
*x)*ArcCosh[a*x])/(8*a^4*Sqrt[1 - a^2*x^2]) - (x^3*(1 - a*x)*(1 + a*x)*ArcC
osh[a*x])/(4*a^2*Sqrt[1 - a^2*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCo
sh[a*x]^2)/(16*a^5*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^4\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3(1-ax)(1+ax)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)}{4a\sqrt{1-a^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{16a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1\right.}}{}$$

$$= -\frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}}{16a^3\sqrt{1-a^2x^2}} - \frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{16a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+}{4a^2\sqrt{1}}$$

Mathematica [A] time = 0.254521, size = 93, normalized size = 0.64

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(-16\cosh\left(2\cosh^{-1}(ax)\right)-\cosh\left(4\cosh^{-1}(ax)\right)+4\cosh^{-1}(ax)\left(6\cosh^{-1}(ax)+8\sinh\left(2\cosh^{-1}(ax)\right)+\right.\right.}{128a^5\sqrt{-(ax-1)(ax+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-16*Cosh[2*ArcCosh[a*x]] - Cosh[4*Ar
cCosh[a*x]] + 4*ArcCosh[a*x]*(6*ArcCosh[a*x] + 8*Sinh[2*ArcCosh[a*x]] + Sin
h[4*ArcCosh[a*x]])))/(128*a^5*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

Maple [B] time = 0.299, size = 456, normalized size = 3.1

$$-\frac{3\,(\mathrm{arccosh}\,(ax))^2}{16\,a^5\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}-\frac{-1+4\,\mathrm{arccosh}\,(ax)}{256\,a^5\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\Big(8\,x^5a^5-12\,x^3a^3+8\,\sqrt{ax+1}\sqrt{ax-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -3/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arccos
h(a*x)^2-1/256*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5-12*x^3*a^3+8*(a*x+1)^(1/2)*(a*
x-1)^(1/2)*x^4*a^4+4*a*x-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2
)*(a*x+1)^(1/2))*(-1+4*arccosh(a*x))/a^5/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2
)*(2*x^3*a^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-(a*x-1)^(1/2)*(a*x
+1)^(1/2))*(-1+2*arccosh(a*x))/a^5/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2)*(2*x
^3*a^3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2)*(a*x+1)^(1
/2))*(1+2*arccosh(a*x))/a^5/(a^2*x^2-1)-1/256*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5
-12*x^3*a^3-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^4*a^4+4*a*x+8*(a*x+1)^(1/2)*(a*
x-1)^(1/2)*x^2*a^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+4*arccosh(a*x))/a^5/(a^2
*x^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^4\operatorname{arcosh}(ax)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

3.136 $\int \frac{x^3 \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=110

$$-\frac{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{3a^2}-\frac{2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{3a^4}-\frac{2x\sqrt{ax-1}}{3a^3\sqrt{1-ax}}-\frac{x^3\sqrt{ax-1}}{9a\sqrt{1-ax}}$$

```
[Out] (-2*x*Sqrt[-1 + a*x])/(3*a^3*Sqrt[1 - a*x]) - (x^3*Sqrt[-1 + a*x])/(9*a*Sqr
t[1 - a*x]) - (2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/(3*a^4) - (x^2*Sqrt[1 - a^
2*x^2]*ArcCosh[a*x])/(3*a^2)
```

Rubi [A] time = 0.392274, antiderivative size = 158, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5798, 5759, 5718, 8, 30}

$$-\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{9a\sqrt{1-a^2x^2}}-\frac{2x\sqrt{ax-1}\sqrt{ax+1}}{3a^3\sqrt{1-a^2x^2}}-\frac{x^2(1-ax)(ax+1)\cosh^{-1}(ax)}{3a^2\sqrt{1-a^2x^2}}-\frac{2(1-ax)(ax+1)\cosh^{-1}(ax)}{3a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a^3*Sqrt[1 - a^2*x^2]) - (x^3*Sqrt[-
1 + a*x]*Sqrt[1 + a*x])/(9*a*Sqrt[1 - a^2*x^2]) - (2*(1 - a*x)*(1 + a*x)*Ar
cCosh[a*x])/(3*a^4*Sqrt[1 - a^2*x^2]) - (x^2*(1 - a*x)*(1 + a*x)*ArcCosh[a*
x])/(3*a^2*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
```

```
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^3\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{3a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)}{3a\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{9a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)}{3a^2\sqrt{1-a^2x^2}} - \frac{\left(2\sqrt{-1\right.}}{3}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a^3\sqrt{1-a^2x^2}} - \frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{9a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-ax)(1+ax}{3a^2\sqrt{1-}}$$

Mathematica [A] time = 0.122945, size = 74, normalized size = 0.67

$$-\frac{ax\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2+6\right)-3\left(a^4x^4+a^2x^2-2\right)\cosh^{-1}(ax)}{9a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(6 + a^2*x^2) - 3*(-2 + a^2*x^2 + a^4*x^
4)*ArcCosh[a*x])/(9*a^4*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.207, size = 311, normalized size = 2.8

$$-\frac{-1+3\,\mathrm{arccosh}\,(ax)}{72\,a^4\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\left(4\,x^4a^4-5\,a^2x^2+4\,a^3x^3\sqrt{ax-1}\sqrt{ax+1}-3\,\sqrt{ax+1}\sqrt{ax-1}ax+1\right)-\frac{-3+3\,\mathrm{arcco}}{8\,a^4\left(a^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/72*(-a^2*x^2+1)^(1/2)*(4*x^4*a^4-5*a^2*x^2+4*a^3*x^3*(a*x-1)^(1/2)*(a*x+
1)^(1/2)-3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(-1+3*arccosh(a*x))/a^4/(a^2*
x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(
-1+arccosh(a*x))/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1
/2)*(a*x-1)^(1/2)*a*x-1)*(1+arccosh(a*x))/a^4/(a^2*x^2-1)-1/72*(-a^2*x^2+1)
^(1/2)*(4*x^4*a^4-5*a^2*x^2-4*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+3*(a*x+1)
^(1/2)*(a*x-1)^(1/2)*a*x+1)*(1+3*arccosh(a*x))/a^4/(a^2*x^2-1)
```

Maxima [C] time = 1.92775, size = 84, normalized size = 0.76

$$\frac{1}{9}\,a\left(\frac{i\,x^3}{a^2}+\frac{6i\,x}{a^4}\right)-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2}+\frac{2\,\sqrt{-a^2x^2+1}}{a^4}\right)\mathrm{arcosh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*a*(I*x^3/a^2 + 6*I*x/a^4) - 1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a
^2*x^2 + 1)/a^4)*arccosh(a*x)
```

Fricas [A] time = 2.1322, size = 209, normalized size = 1.9

$$-\frac{3\left(a^4x^4+a^2x^2-2\right)\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(a^3x^3+6\,ax\right)\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}}{9\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(3*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 -
 1)) - (a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a
^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [C] time = 1.22301, size = 89, normalized size = 0.81

$$\frac{-i\,a^2x^3-6i\,x}{9\,a^3}+\frac{\left(\left(-a^2x^2+1\right)^{\frac{3}{2}}-3\sqrt{-a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2-1}\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*(-I*a^2*x^3 - 6*I*x)/a^3 + 1/3*((-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2
+ 1))*log(a*x + sqrt(a^2*x^2 - 1))/a^4
```

3.137 $\int \frac{x^2 \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=88

$$-\frac{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{2a^2}+\frac{\sqrt{ax-1}\cosh^{-1}(ax)^2}{4a^3\sqrt{1-ax}}-\frac{x^2\sqrt{ax-1}}{4a\sqrt{1-ax}}$$

```
[Out] -(x^2*Sqrt[-1 + a*x])/(4*a*Sqrt[1 - a*x]) - (x*Sqrt[1 - a^2*x^2]*ArcCosh[a*
x])/(2*a^2) + (Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(4*a^3*Sqrt[1 - a*x])
```

Rubi [A] time = 0.324336, antiderivative size = 125, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5798, 5759, 5676, 30}

$$-\frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{4a\sqrt{1-a^2x^2}}-\frac{x(1-ax)(ax+1)\cosh^{-1}(ax)}{2a^2\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{4a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a^2*x^2]) - (x*(1 - a*x)*
(1 + a*x)*ArcCosh[a*x])/(2*a^2*Sqrt[1 - a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*ArcCosh[a*x]^2)/(4*a^3*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
```

```
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{x(1-ax)(1+ax)\cosh^{-1}(ax)}{2a^2\sqrt{1-a^2x^2}}+\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2a^2\sqrt{1-a^2x^2}}-\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int}{2a\sqrt{1-a^2x^2}}\\
&= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{4a\sqrt{1-a^2x^2}}-\frac{x(1-ax)(1+ax)\cosh^{-1}(ax)}{2a^2\sqrt{1-a^2x^2}}+\frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4a^3\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.155927, size = 75, normalized size = 0.85

$$
-\frac{\sqrt{-(ax-1)(ax+1)}\left(2\cosh^{-1}(ax)\left(\cosh^{-1}(ax)+\sinh\left(2\cosh^{-1}(ax)\right)\right)-\cosh\left(2\cosh^{-1}(ax)\right)\right)}{8a^3\sqrt{\frac{ax-1}{ax+1}}(ax+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(Sqrt[-((-1 + a*x)*(1 + a*x))]*(-Cosh[2*ArcCosh[a*x]] + 2*ArcCosh[a*x]*(Ar
cCosh[a*x] + Sinh[2*ArcCosh[a*x]])))/(8*a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 +
```

```
a*x))
```

Maple [B] time = 0.186, size = 223, normalized size = 2.5

$$-\frac{(\operatorname{arccosh}(ax))^2}{4a^3(a^2x^2-1)}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}-\frac{-1+2\operatorname{arccosh}(ax)}{16a^3(a^2x^2-1)}\sqrt{-a^2x^2+1}\left(2x^3a^3-2ax+2\sqrt{ax+1}\sqrt{ax-1}x^2a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccosh
(a*x)^2-1/16*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1
/2)*x^2*a^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(-1+2*arccosh(a*x))/a^3/(a^2*x^2-1
)-1/16*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^
2*a^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+2*arccosh(a*x))/a^3/(a^2*x^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^2\operatorname{arcosh}(ax)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

3.138 $\int \frac{x \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{a^2}-\frac{x\sqrt{ax-1}}{a\sqrt{1-ax}}$$

```
[Out] -((x*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x])) - (Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/
a^2
```

Rubi [A] time = 0.176885, antiderivative size = 73, normalized size of antiderivative = 1.49, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5798, 5718, 8}

$$-\frac{x\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{1-a^2x^2}}-\frac{(1-ax)(ax+1)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -((x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[1 - a^2*x^2])) - ((1 - a*x)*(1 +
 a*x)*ArcCosh[a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
```

```
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int 1\,dx}{a\sqrt{1-a^2x^2}}\\ &= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{1-a^2x^2}}-\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0842637, size = 55, normalized size = 1.12

$$\frac{\left(a^2x^2-1\right)\cosh^{-1}(ax)-ax\sqrt{ax-1}\sqrt{ax+1}}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (-1 + a^2*x^2)*ArcCosh[a*x])/(a^2*Sq
rt[1 - a^2*x^2])
```

Maple [B] time = 0.12, size = 123, normalized size = 2.5

$$-\frac{-1+\operatorname{arccosh}(ax)}{2a^2\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\left(\sqrt{ax+1}\sqrt{ax-1}ax+a^2x^2-1\right)-\frac{1+\operatorname{arccosh}(ax)}{2a^2\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\left(a^2x^2-\sqrt{ax+1}\sqrt{ax-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(-1+arc
cosh(a*x))/a^2/(a^2*x^2-1)-1/2*(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a
*x-1)^(1/2)*a*x-1)*(1+arccosh(a*x))/a^2/(a^2*x^2-1)
```

Maxima [C] time = 1.16619, size = 38, normalized size = 0.78

$$\frac{ix}{a} - \frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] I*x/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)/a^2
```

Fricas [A] time = 2.06582, size = 151, normalized size = 3.08

$$\frac{\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}ax + \left(-a^2x^2+1\right)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^4x^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x + (-a^2*x^2 + 1)^(3/2)*log(a*x +
sqrt(a^2*x^2 - 1)))/(a^4*x^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [C] time = 1.186, size = 54, normalized size = 1.1

$$-\frac{ix}{a}-\frac{\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -I*x/a - sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))/a^2
```

3.139 $\int \frac{\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{ax-1}\cosh^{-1}(ax)^2}{2a\sqrt{1-ax}}$$

```
[Out] (Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(2*a*Sqrt[1 - a*x])
```

Rubi [A] time = 0.098098, antiderivative size = 45, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5713, 5676}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{2a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*a*Sqrt[1 - a^2*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0195566, size = 45, normalized size = 1.41

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{2a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.034, size = 51, normalized size = 1.6

$$-\frac{(\operatorname{arccosh}(ax))^2}{2a\left(a^2x^2-1\right)}\sqrt{-(ax-1)(ax+1)}\sqrt{ax-1}\sqrt{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*arc
cosh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{acosh}\,(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{arcosh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

3.140 $\int \frac{\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{i\sqrt{ax-1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{i\sqrt{ax-1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{2\sqrt{ax-1}\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}$$

```
[Out] (2*Sqrt[-1 + a*x]*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a*x] - (I*S
qrt[-1 + a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a*x] + (I*Sqrt[-1 +
 a*x]*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.276047, antiderivative size = 142, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5798, 5761, 4180, 2279, 2391}

$$-\frac{i\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{i\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(a}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/Sqrt[1
 - a^2*x^2] - (I*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x
]])/Sqrt[1 - a^2*x^2] + (I*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, I*E^ArcC
osh[a*x]])/Sqrt[1 - a^2*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
```

```
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int x\,\text{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(i\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, \right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(i\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{co}}\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{i\sqrt{-1+ax}\sqrt{1+ax}\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} + \frac{i\sqrt{-1+}}{}$$

Mathematica [A] time = 0.141795, size = 113, normalized size = 1.1

$$\frac{i\sqrt{-(ax-1)(ax+1)}\left(\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(ax)}\right)+\cosh^{-1}(ax)\left(\log\left(1-ie^{-\cosh^{-1}(ax)}\right)-\log\right.\right.}{\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (I*Sqrt[-((-1 + a*x)*(1 + a*x))]*(ArcCosh[a*x]*(Log[1 - I/E^ArcCosh[a*x]] -
 Log[1 + I/E^ArcCosh[a*x]]) + PolyLog[2, (-I)/E^ArcCosh[a*x]] - PolyLog[2,
I/E^ArcCosh[a*x]]))/(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [B] time = 0.142, size = 270, normalized size = 2.6

$$\frac{i\text{arccosh}\,(ax)}{a^2x^2-1}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}\ln\left(1+i\left(ax+\sqrt{ax-1}\sqrt{ax+1}\right)\right)-\frac{i\text{arccosh}\,(ax)}{a^2x^2-1}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*l
n(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)
*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*ln(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2)))+I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*dilog(1
+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a
*x+1)^(1/2)/(a^2*x^2-1)*dilog(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcosh}\,(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{acosh}\,(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{arcosh}\,(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.141 $\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{x}-\frac{a\sqrt{ax-1}\log(x)}{\sqrt{1-ax}}$$

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/x) - (a*Sqrt[-1 + a*x]*Log[x])/Sqrt[1 -
a*x]
```

Rubi [A] time = 0.254105, antiderivative size = 72, normalized size of antiderivative = 1.5, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5798, 5724, 29}

$$-\frac{a\sqrt{ax-1}\sqrt{ax+1}\log(x)}{\sqrt{1-a^2x^2}}-\frac{(1-ax)(ax+1)\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(((1 - a*x)*(1 + a*x)*ArcCosh[a*x])/(x*Sqrt[1 - a^2*x^2])) - (a*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]*Log[x])/Sqrt[1 - a^2*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
```

```
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{x}\,dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{a\sqrt{-1+ax}\sqrt{1+ax}\log(x)}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0302518, size = 57, normalized size = 1.19

$$\frac{\left(a^2x^2-1\right)\cosh^{-1}(ax)-ax\sqrt{ax-1}\sqrt{ax+1}\log(x)}{x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((-1 + a^2*x^2)*ArcCosh[a*x] - a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[x])/(x*
Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.136, size = 168, normalized size = 3.5

$$-2\,\frac{\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}\operatorname{arccosh}(ax)\,a}{a^2x^2-1}-\frac{\operatorname{arccosh}(ax)}{\left(a^2x^2-1\right)x}\sqrt{-a^2x^2+1}\left(a^2x^2-\sqrt{ax+1}\sqrt{ax-1}ax-1\right)+\frac{a}{a^2x^2-1}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*
a-(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*arccosh(a*
x)/x/(a^2*x^2-1)+(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)
*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a
```

Maxima [C] time = 1.78803, size = 99, normalized size = 2.06

$$-\frac{1}{2}\left(a^{2}\sqrt{-\frac{1}{a^{4}}}\log\left(x^{2}-\frac{1}{a^{2}}\right)+i\left(-1\right)^{-2\,a^{2}x^{2}+2}\log\left(-2\,a^{2}+\frac{2}{x^{2}}\right)\right)a-\frac{\sqrt{-a^{2}x^{2}+1}\,\operatorname{arcosh}\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*sqrt(-1/a^4)*log(x^2 - 1/a^2) + I*(-1)^(-2*a^2*x^2 + 2)*log(-2*a^
2 + 2/x^2))*a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)/x
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acosh}\left(ax\right)}{x^{2}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**2/(-a**2*x**2+1)**(1/2),x)
```

[Out] Integral(acosh(a*x)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)

Giac [C] time = 1.20838, size = 116, normalized size = 2.42

$$-\frac{1}{2}i\,a\log\left(-i\,a^{2}x^{2}\right)+\frac{1}{2}\left(\frac{a^{4}x}{\left(\sqrt{-a^{2}x^{2}+1}|a|+a\right)|a|}-\frac{\sqrt{-a^{2}x^{2}+1}|a|+a}{x|a|}\right)\log\left(ax+\sqrt{a^{2}x^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*a*log(-I*a^2*x^2) + 1/2*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(
a)) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a)))*log(a*x + sqrt(a^2*x^2 -
1))
```

3.142 $\int \frac{\cosh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{ia^2\sqrt{ax-1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{ia^2\sqrt{ax-1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{2x^2}+\frac{a^2\sqrt{ax-1}\cos$$

```
[Out] (a*Sqrt[-1 + a*x])/(2*x*Sqrt[1 - a*x]) - (Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/(
2*x^2) + (a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 -
a*x] - ((I/2)*a^2*Sqrt[-1 + a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 -
a*x] + ((I/2)*a^2*Sqrt[-1 + a*x]*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a*x
]
```

Rubi [A] time = 0.471289, antiderivative size = 230, normalized size of antiderivative = 1.38, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5798, 5748, 5761, 4180, 2279, 2391, 30}

$$-\frac{ia^2\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-a^2x^2}}+\frac{ia^2\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-a^2x^2}}+\frac{a\sqrt{ax-1}\sqrt{ax+1}}{2x\sqrt{1-a^2x^2}}-\frac{(1$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*x*Sqrt[1 - a^2*x^2]) - ((1 - a*x)*(1 +
a*x)*ArcCosh[a*x])/(2*x^2*Sqrt[1 - a^2*x^2]) + (a^2*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] - ((I/2)*a^2*S
qrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x
^2] + ((I/2)*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, I*E^ArcCosh[a*x]])
/Sqrt[1 - a^2*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x^3\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(}{x\sqrt{-1+ax}\,}}{2\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int x\,\mathrm{sech}(x)\,dx\right.}{2\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh}\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh}\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh}\right.}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.29017, size = 234, normalized size = 1.4

$$\frac{(ax+1)\left(-ia^2x^2\sqrt{\frac{ax-1}{ax+1}}\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(ax)}\right)+ia^2x^2\sqrt{\frac{ax-1}{ax+1}}\text{PolyLog}\left(2,ie^{-\cosh^{-1}(ax)}\right)-ia^2x^2\sqrt{\frac{ax-1}{ax+1}}\cosh^{-1}(ax)\log\right.}{2x^2\sqrt{1-a^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((1 + a*x)*(a*x*Sqrt[(-1 + a*x)/(1 + a*x)] - ArcCosh[a*x] + a*x*ArcCosh[a*x
] - I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a
*x]] + I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCos
h[a*x]] - I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, (-I)/E^ArcCosh[a*
x]] + I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, I/E^ArcCosh[a*x]]))/(
2*x^2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.206, size = 349, normalized size = 2.1

$$-\frac{1}{\left(2\,a^{2}x^{2}-2\right)x^{2}}\left(a^{2}x^{2}\mathrm{arccosh}\left(ax\right)+\sqrt{ax+1}\sqrt{ax-1}ax-\mathrm{arccosh}\left(ax\right)\right)\sqrt{-a^{2}x^{2}+1}+\frac{i\mathrm{arccosh}\left(ax\right)a^{2}}{2\,a^{2}x^{2}-2}\sqrt{-a^{2}x^{2}+1}\sqrt{ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(a^2*x^2*arccosh(a*x)+(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-arccosh(a*x))*(-
a^2*x^2+1)^(1/2)/(a^2*x^2-1)/x^2+I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)
^(1/2)*arccosh(a*x)*ln(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^
2-2)-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)*ln(1-I*(
a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^2-2)+I*(-a^2*x^2+1)^(1/2)*(a
*x-1)^(1/2)*(a*x+1)^(1/2)*dilog(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/
(2*a^2*x^2-2)-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*dilog(1-I*(a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^2-2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arcosh}\left(ax\right)}{\sqrt{-a^{2}x^{2}+1}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^{2}x^{2}+1}\,\mathrm{arcosh}\left(ax\right)}{a^{2}x^{5}-x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.143 $\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=98

$$\frac{4bc\sqrt{cx-1}(fx)^{7/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2x^2\right)}{35f^2\sqrt{1-cx}}+\frac{2(fx)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)\left(a+\right.}{5f}$$

```
[Out] (2*(f*x)^(5/2)*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^
2])/(5*f) + (4*b*c*(f*x)^(7/2)*Sqrt[-1 + c*x]*HypergeometricPFQ[{1, 7/4, 7/
4}, {9/4, 11/4}, c^2*x^2])/(35*f^2*Sqrt[1 - c*x])
```

Rubi [A] time = 0.357704, antiderivative size = 111, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5798, 5763}

$$\frac{4bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{1-c^2x^2}}+\frac{2(fx)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (2*(f*x)^(5/2)*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^
2])/(5*f) + (4*b*c*(f*x)^(7/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricP
FQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2])/(35*f^2*Sqrt[1 - c^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
```

```
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$
$$= \frac{2(fx)^{5/2}\left(a+b\cosh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)}{5f} + \frac{4bc(fx)^{7/2}\sqrt{-1+cx}\sqrt{1+cx}\,{}_3F_2\left(1,\right.}{35f^2\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.106495, size = 100, normalized size = 1.02

$$\frac{2}{35}x(fx)^{3/2}\left(\frac{2bcx\sqrt{cx-1}\sqrt{cx+1}\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2x^2\right)}{\sqrt{1-c^2x^2}}+7\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (2*x*(f*x)^(3/2)*(7*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c
^2*x^2] + (2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 7/4,
7/4}, {9/4, 11/4}, c^2*x^2])/Sqrt[1 - c^2*x^2]))/35
```

Maple [F] time = 0.277, size = 0, normalized size = 0.

$$\int (a+b\,\mathrm{arccosh}\,(cx))\left(fx\right)^{\frac{3}{2}}\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^{\frac{3}{2}}\,(b\,\mathrm{arcosh}\,(cx)+a)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}\left(bfx\,\mathrm{arcosh}\,(cx)+afx\right)\sqrt{fx}}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*f*x*arccosh(c*x) + a*f*x)*sqrt(f*x)/(c^2*x^
2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx)+a)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*x^2 + 1), x)
```

3.144 $\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=141

$$\frac{4bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{7/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2x^2\right)}{35f^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{1-c^2x^2}(fx)^{5/2}\text{Hypergeometric2F1}\left(\right.}{5f\sqrt{d-c^2dx}}$$

```
[Out] (2*(f*x)^(5/2)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, 5/4, 9/4, c^2*x^2])/(5*f*Sqrt[d - c^2*d*x^2]) + (4*b*c*(f*x)^(7/2)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^
2])/(35*f^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.382549, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {5798, 5763}

$$\frac{4bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{1-c^2x^2}(fx)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{5f\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2*(f*x)^(5/2)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, 5/4, 9/4, c^2*x^2])/(5*f*Sqrt[d - c^2*d*x^2]) + (4*b*c*(f*x)^(7/2)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^
2])/(35*f^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{2(fx)^{5/2}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)}{5f\sqrt{d-c^2dx^2}} + \frac{4bc(fx)^{7/2}\sqrt{-1+cx}\sqrt{1+}}{35f^2\sqrt{d}}$$

Mathematica [A] time = 0.044996, size = 115, normalized size = 0.82

$$\frac{2x(fx)^{3/2}\left(2bcx\sqrt{cx-1}\sqrt{cx+1}\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2x^2\right)+7\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\right.\right.}{35\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2*x*(f*x)^(3/2)*(7*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F
1[1/2, 5/4, 9/4, c^2*x^2] + 2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeome
tricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2]))/(35*Sqrt[d - c^2*d*x^2])
```

Maple [F] time = 0.363, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(cx))(fx)^{\frac{3}{2}}\frac{1}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(bfx\operatorname{arcosh}(cx)+afx\right)\sqrt{fx}}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*f*x*arccosh(c*x) + a*f*x)*sqrt(f*x)/(c^2*
d*x^2 - d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^{\frac{3}{2}}(b\operatorname{arcosh}(cx) + a)}{\sqrt{-c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.145 $\int (fx)^m \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=429

$$-\frac{3bcd^3\left(35m^3+455m^2+1813m+2161\right)\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{f^2(m+1)(m+2)(m+3)^2(m+5)^2(m+7)^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3c^2d^3(fx)^{m+3}\left(a\right.}{f^3(m}$$

```
[Out] -((b*c*d^3*(2271 + 1329*m + 284*m^2 + 27*m^3 + m^4)*(f*x)^(2 + m)*(1 - c^2*
x^2))/(f^2*(3 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (
b*c^3*d^3*(9 + m)*(13 + 2*m)*(f*x)^(4 + m)*(1 - c^2*x^2))/(f^4*(5 + m)^2*(7
 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^3*(f*x)^(6 + m)*(1 - c^2*x
^2))/(f^6*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*(f*x)^(1 + m)*(a +
 b*ArcCosh[c*x]))/(f*(1 + m)) - (3*c^2*d^3*(f*x)^(3 + m)*(a + b*ArcCosh[c*x
]))/(f^3*(3 + m)) + (3*c^4*d^3*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))/(f^5*(5
+ m)) - (c^6*d^3*(f*x)^(7 + m)*(a + b*ArcCosh[c*x]))/(f^7*(7 + m)) - (3*b*c
*d^3*(2161 + 1813*m + 455*m^2 + 35*m^3)*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hyp
ergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2 + m)*(3
 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 2.75505, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5731, 12, 1610, 1809, 1267, 459, 365, 364}

$$-\frac{3c^2d^3(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{3c^4d^3(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}-\frac{c^6d^3(fx)^{m+7}\left(a+b\cosh^{-1}(cx)\right)}{f^7(m+7)}+\frac{d^3(fx)^{m+1}\left(a\right.}{f(m}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```


```
[Out] -((b*c*d^3*(2271 + 1329*m + 284*m^2 + 27*m^3 + m^4)*(f*x)^(2 + m)*(1 - c^2*
x^2))/(f^2*(3 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (
b*c^3*d^3*(9 + m)*(13 + 2*m)*(f*x)^(4 + m)*(1 - c^2*x^2))/(f^4*(5 + m)^2*(7
 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^3*(f*x)^(6 + m)*(1 - c^2*x
^2))/(f^6*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*(f*x)^(1 + m)*(a +
 b*ArcCosh[c*x]))/(f*(1 + m)) - (3*c^2*d^3*(f*x)^(3 + m)*(a + b*ArcCosh[c*x
]))/(f^3*(3 + m)) + (3*c^4*d^3*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))/(f^5*(5
+ m)) - (c^6*d^3*(f*x)^(7 + m)*(a + b*ArcCosh[c*x]))/(f^7*(7 + m)) - (3*b*c
*d^3*(2161 + 1813*m + 455*m^2 + 35*m^3)*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hyp
ergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2 + m)*(3
```

```
 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
```

```
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{3c^2d^3(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{3c^4d}{}$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{3c^2d^3(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{3c^4d}{}$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{3c^2d^3(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{3c^4d}{}$$

$$= -\frac{bc^5d^3(fx)^{6+m}\left(1-c^2x^2\right)}{f^6(7+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{3c^2d^3(}{}$$

$$= \frac{bc^3d^3(9+m)(13+2m)(fx)^{4+m}\left(1-c^2x^2\right)}{f^4(5+m)^2(7+m)^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^5d^3(fx)^{6+m}\left(1-c^2x^2\right)}{f^6(7+m)^2\sqrt{-1+cx}\sqrt{1+c}}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d}{f^4}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d}{f^4}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d}{f^4}$$

Mathematica [A] time = 1.16857, size = 387, normalized size = 0.9

$$d^3x(fx)^m\left(\frac{3bc^3x^3\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+4}{2},\frac{m+6}{2},c^2x^2\right)}{\left(m^2+7m+12\right)\sqrt{cx-1}\sqrt{cx+1}} - \frac{bcx\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m}{}\right.}{\left(m^2+3m+2\right)\sqrt{cx-1}\sqrt{cx+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] d^3*x*(f*x)^m*((a + b*ArcCosh[c*x])/(1 + m) - (3*c^2*x^2*(a + b*ArcCosh[c*x
]))/(3 + m) + (3*c^4*x^4*(a + b*ArcCosh[c*x]))/(5 + m) - (c^6*x^6*(a + b*Ar
```

```
cCosh[c*x]))/(7 + m) + (b*c^7*x^7*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2,
4 + m/2, 5 + m/2, c^2*x^2])/((7 + m)*(8 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2
*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*x^3*Sqrt[1
 - c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((12 + 7
*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*x^5*Sqrt[1 - c^2*x^2]*Hy
pergeometric2F1[1/2, (6 + m)/2, (8 + m)/2, c^2*x^2])/((5 + m)*(6 + m)*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]))
```

Maple [F] time = 3.132, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(-c^{2}dx^{2}+d\right)^{3} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(ac^{6}d^{3}x^{6}-3\,ac^{4}d^{3}x^{4}+3\,ac^{2}d^{3}x^{2}-ad^{3}+\left(bc^{6}d^{3}x^{6}-3\,bc^{4}d^{3}x^{4}+3\,bc^{2}d^{3}x^{2}-bd^{3}\right)\mathrm{arcosh}\left(cx\right)\right)\left(fx\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c
^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))*(f*x)
^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.146 $\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=307

$$-\frac{bcd^2\left(15m^2+100m+149\right)\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{f^2(m+1)(m+2)(m+3)^2(m+5)^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{2c^2d^2(fx)^{m+3}\left(a+b\cosh^{-1}(\right.}{f^3(m+3)}$$

```
[Out] -((b*c*d^2*(38 + 13*m + m^2)*(f*x)^(2 + m)*(1 - c^2*x^2))/(f^2*(3 + m)^2*(5
 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (b*c^3*d^2*(f*x)^(4 + m)*(1 - c^2*
x^2))/(f^4*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*(f*x)^(1 + m)*(a
+ b*ArcCosh[c*x]))/(f*(1 + m)) - (2*c^2*d^2*(f*x)^(3 + m)*(a + b*ArcCosh[c*
x]))/(f^3*(3 + m)) + (c^4*d^2*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))/(f^5*(5 +
 m)) - (b*c*d^2*(149 + 100*m + 15*m^2)*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hype
rgeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2 + m)*(3
+ m)^2*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.500995, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {270, 5731, 12, 520, 1267, 459, 365, 364}

$$-\frac{2c^2d^2(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{c^4d^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}+\frac{d^2(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}-\frac{bcd^2\left(15m^2+10\right.}{f^2(m+}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```


```
[Out] -((b*c*d^2*(38 + 13*m + m^2)*(f*x)^(2 + m)*(1 - c^2*x^2))/(f^2*(3 + m)^2*(5
 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (b*c^3*d^2*(f*x)^(4 + m)*(1 - c^2*
x^2))/(f^4*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*(f*x)^(1 + m)*(a
+ b*ArcCosh[c*x]))/(f*(1 + m)) - (2*c^2*d^2*(f*x)^(3 + m)*(a + b*ArcCosh[c*
x]))/(f^3*(3 + m)) + (c^4*d^2*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))/(f^5*(5 +
 m)) - (b*c*d^2*(149 + 100*m + 15*m^2)*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hype
rgeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2 + m)*(3
+ m)^2*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{2c^2d^2(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{c^4d^2(f}{}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{2c^2d^2(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{c^4d^2(f}{}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{2c^2d^2(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{c^4d^2(f}{}$$

$$= \frac{bc^3d^2(fx)^{4+m}\left(1-c^2x^2\right)}{f^4(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{2c^2d^2(fx)}{}$$

$$= -\frac{bcd^2\left(38+13m+m^2\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d^2(fx)^{4+m}\left(1-c^2x^2\right)}{f^4(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{bcd^2\left(38+13m+m^2\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d^2(fx)^{4+m}\left(1-c^2x^2\right)}{f^4(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{bcd^2\left(38+13m+m^2\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d^2(fx)^{4+m}\left(1-c^2x^2\right)}{f^4(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.436913, size = 290, normalized size = 0.94

$$d^2x(fx)^m\left(\frac{2bc^3x^3\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+4}{2},\frac{m+6}{2},c^2x^2\right)}{(m^2+7m+12)\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\right.}{(m^2+3m+2)\sqrt{cx-1}\sqrt{cx+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] d^2*x*(f*x)^m*((a + b*ArcCosh[c*x])/(1 + m) - (2*c^2*x^2*(a + b*ArcCosh[c*x
]))/(3 + m) + (c^4*x^4*(a + b*ArcCosh[c*x]))/(5 + m) - (b*c*x*Sqrt[1 - c^2*
x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((2 + 3*m + m^2
)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*x^3*Sqrt[1 - c^2*x^2]*Hypergeome
tric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((12 + 7*m + m^2)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - (b*c^5*x^5*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (6
+ m)/2, (8 + m)/2, c^2*x^2])/((5 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
)
```

Maple [F] time = 2.434, size = 0, normalized size = 0.

$$\int (fx)^m \left(-c^2dx^2+d\right)^2 (a+b\operatorname{arccosh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\text{arcosh}\,(cx)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.147 $\int (fx)^m \left(d - c^2dx^2\right)\left(a + b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=184

$$-\frac{bcd(3m+7)\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{f^2(m+1)(m+2)(m+3)^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{c^2d(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d(fx)^{m+1}}{}$$

```
[Out] (b*c*d*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(f^2*(3 + m)^2) + (d*(f*
x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) - (c^2*d*(f*x)^(3 + m)*(a + b*
ArcCosh[c*x]))/(f^3*(3 + m)) - (b*c*d*(7 + 3*m)*(f*x)^(2 + m)*Sqrt[1 - c^2*
x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2
 + m)*(3 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.258224, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {14, 5731, 12, 460, 126, 365, 364}

$$-\frac{c^2d(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}-\frac{bcd(3m+7)\sqrt{1-c^2x^2}(fx)^{m+2}{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{f^2(m+1)(m+2)(m+3)^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (b*c*d*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(f^2*(3 + m)^2) + (d*(f*
x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) - (c^2*d*(f*x)^(3 + m)*(a + b*
ArcCosh[c*x]))/(f^3*(3 + m)) - (b*c*d*(7 + 3*m)*(f*x)^(2 + m)*Sqrt[1 - c^2*
x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2
 + m)*(3 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
```

```
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)dx = \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{c^2d(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} - (bc)\int \frac{a}{f(}$$

$$= \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{c^2d(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} - \frac{(bcd)\int \frac{(}{}}{f}$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{c^2d(fx)^{3+}}{}$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{c^2d(fx)^{3+}}{}$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{c^2d(fx)^{3+}}{}$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{c^2d(fx)^{3+}}{}$$

Mathematica [A] time = 0.230071, size = 191, normalized size = 1.04

$$dx(fx)^m\left(\frac{bc^3x^3\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+4}{2},\frac{m+6}{2},c^2x^2\right)}{\left(m^2+7m+12\right)\sqrt{cx-1}\sqrt{cx+1}} - \frac{bcx\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2}\right.}{\left(m^2+3m+2\right)\sqrt{cx-1}\sqrt{cx+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] d*x*(f*x)^m*((a + b*ArcCosh[c*x])/(1 + m) - (c^2*x^2*(a + b*ArcCosh[c*x]))/
(3 + m) - (b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m
)/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*x^3*
Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((
12 + 7*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Maple [F] time = 2.286, size = 0, normalized size = 0.

$$\int (fx)^m \left(-c^2dx^2+d\right)(a+b\text{arccosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\text{arcosh}\,(cx)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*(f*x)^m, x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d\left(\int -a\left(fx\right)^m\,dx + \int -b\left(fx\right)^m\text{acosh}\,(cx)\,dx + \int ac^2x^2\left(fx\right)^m\,dx + \int bc^2x^2\left(fx\right)^m\text{acosh}\,(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] -d*(Integral(-a*(f*x)**m, x) + Integral(-b*(f*x)**m*acosh(c*x), x) + Integr
al(a*c**2*x**2*(f*x)**m, x) + Integral(b*c**2*x**2*(f*x)**m*acosh(c*x), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.148 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x]
```

Rubi [A] time = 0.0720581, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$$

Mathematica [A] time = 3.9553, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x]
```

Maple [A] time = 0.484, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b\operatorname{arccosh}(cx))}{-c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{c^2dx^2 - d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{a(fx)^m}{c^2x^2-1}\,dx + \int \frac{b(fx)^m \operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*(f*x)**m/(c**2*x**2 - 1), x) + Integral(b*(f*x)**m*acosh(c*x)/
(c**2*x**2 - 1), x))/d
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{c^2dx^2 - d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

3.149 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=160

$$\frac{(1-m)\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2},x\right)}{2d}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{2d^2f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}+\frac{(fx)^m}{}$$

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(2*d^2*f*(1 - c^2*x^2)) - (b*c*(f*x)^(
2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, c^2*x
^2])/(2*d^2*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - m)*Unintegrab
le[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x])/(2*d)
```

Rubi [A] time = 0.207492, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(2*d^2*f*(1 - c^2*x^2)) - (b*c*(f*x)^(
2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, c^2*x
^2])/(2*d^2*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - m)*Defer[Int]
[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x])/(2*d)
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx = \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)} + \frac{(bc)\int \frac{(fx)^{1+m}}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2f} + \frac{(1-m)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}}{2d}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)} + \frac{(1-m)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{2d} + \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{(f}{(-1+}}{2d^2f\sqrt{-1+cx}\sqrt{1}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)} + \frac{(1-m)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{2d} - \frac{\left(bc\sqrt{1-c^2x^2}\right)\int \frac{(fx)^1}{(1-c^2x}}{2d^2f\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)} - \frac{bc(fx)^{2+m}\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};c^2x^2\right)}{2d^2f^2(2+m)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(1-m)\int}{}$$

Mathematica [A] time = 5.89835, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2, x]
```

Maple [A] time = 0.562, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a+b\,\mathrm{arccosh}(cx))}{\left(-c^2dx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)\left(fx\right)^m}{\left(c^2dx^2-d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(b\operatorname{arcosh}(cx)+a)\left(fx\right)^m}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas"
)
```

```
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\left(fx\right)^m}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{b\left(fx\right)^m\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*(f*x)**m/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*(f*x)**
m*acosh(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{(c^2dx^2 - d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^2, x)
```

3.150 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=293

$$\frac{(1-m)(3-m)\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2},x\right)}{8d^2}-\frac{bc(3-m)\sqrt{1-c^2x^2}(fx)^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2}\right.}{8d^3f^2(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(4*d^3*f*(1 - c^2*x^2)^2) + ((3 - m)*(
f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(8*d^3*f*(1 - c^2*x^2)) - (b*c*(3 - m)*(
f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2,
 c^2*x^2])/(8*d^3*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(f*x)^(2
 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, c^2*x^
2])/(4*d^3*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - m)*(3 - m)*Uni
ntegrable[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x])/(8*d^2)
```

Rubi [A] time = 0.335463, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(4*d^3*f*(1 - c^2*x^2)^2) + ((3 - m)*(
f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(8*d^3*f*(1 - c^2*x^2)) - (b*c*(3 - m)*(
f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2,
 c^2*x^2])/(8*d^3*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(f*x)^(2
 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, c^2*x^
2])/(4*d^3*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - m)*(3 - m)*Def
er[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x])/(8*d^2)
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2} - \frac{(bc)\int \frac{(fx)^{1+m}}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3f} + \frac{(3-m)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}}{4d}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2} + \frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)} + \frac{(bc(3-m))\int \frac{}{(-1+c}}{8d^3f}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2} + \frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)} + \frac{((1-m)(3-m))\int}{8}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2} + \frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)} - \frac{bc(fx)^{2+m}\sqrt{1-c^2x}}{4d^3f^2(2+m}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2} + \frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)} - \frac{bc(3-m)(fx)^{2+m}}{8d^3f^2(2}$$

Mathematica [A] time = 6.52498, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]

[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3, x]
```

Maple [A] time = 0.569, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a+b\operatorname{arccosh}(cx))}{\left(-c^2dx^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{(c^2dx^2 - d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{c^6d^3x^6 - 3c^4d^3x^4 + 3c^2d^3x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas"
)
```

```
[Out] integral(-(b*arccosh(c*x) + a)*(f*x)^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 - d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{(c^2dx^2 - d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^3, x)
```

3.151 $\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=723

$$-\frac{15bcd^2\sqrt{d-c^2dx^2}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)(m+6)\sqrt{cx-1}\sqrt{cx+1}}+\frac{15d^2\sqrt{d-c^2dx^2}(fx)^{m+}}{f(m}$$

```
[Out] -((b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)*(6 + m)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])) - (15*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^
2*(2 + m)^2*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d^2*(f*x
)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (5*b*c^3*d^2*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2])/(f^4*(4 +
m)^2*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d^2*(f*x)^(4 + m)*Sqr
t[d - c^2*d*x^2])/(f^4*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c
^5*d^2*(f*x)^(6 + m)*Sqrt[d - c^2*d*x^2])/(f^6*(6 + m)^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (15*d^2*(f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
))/(f*(6 + m)*(8 + 6*m + m^2)) + (5*d*(f*x)^(1 + m)*(d - c^2*d*x^2)^(3/2)*(
a + b*ArcCosh[c*x]))/(f*(4 + m)*(6 + m)) + ((f*x)^(1 + m)*(d - c^2*d*x^2)^(
5/2)*(a + b*ArcCosh[c*x]))/(f*(6 + m)) + (15*d^2*(f*x)^(1 + m)*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c
^2*x^2])/(f*(4 + m)*(6 + m)*(2 + 3*m + m^2)*Sqrt[1 - c*x]*Sqrt[1 + c*x]) -
(15*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2
, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)^2*(4 + m)*
(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.38827, antiderivative size = 764, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5798, 5745, 5743, 5763, 32, 14, 270}

$$-\frac{15bcd^2\sqrt{d-c^2dx^2}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)(m+6)\sqrt{cx-1}\sqrt{cx+1}}+\frac{15d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\right.}{f(m+4)(m+6)\left(m^2+3m+2\right)(}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)*(6 + m)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])) - (15*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^
2*(2 + m)^2*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d^2*(f*x
```

```
)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (5*b*c^3*d^2*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2])/(f^4*(4 +
m)^2*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d^2*(f*x)^(4 + m)*Sqr
t[d - c^2*d*x^2])/(f^4*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c
^5*d^2*(f*x)^(6 + m)*Sqrt[d - c^2*d*x^2])/(f^6*(6 + m)^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (15*d^2*(f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
))/(f*(6 + m)*(8 + 6*m + m^2)) + (5*d^2*(f*x)^(1 + m)*(1 - c*x)*(1 + c*x)*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*(4 + m)*(6 + m)) + (d^2*(f*x)^(
1 + m)*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f
*(6 + m)) + (15*d^2*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f
*(4 + m)*(6 + m)*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x)) - (15*b*c*d^2*(f*x)^(
2 + m)*Sqrt[d - c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 +
m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)^2*(4 + m)*(6 + m)*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
```

```
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (fx)^m(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{d^2(fx)^{1+m}(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(6+m)} - \frac{\left(5d^2\sqrt{d-c^2 \right.}{}$$

$$= \frac{5d^2(fx)^{1+m}(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(4+m)(6+m)} + \frac{d^2(fx)^{1+m}(1}{}$$

$$= -\frac{bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^3d^2(fx)^{4+m}\sqrt{d-c^2dx^2}}{f^4(4+m)(6+m)\sqrt{-1+cx}\sqrt{1+}}$$

$$= -\frac{bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{15bcd^2(fx)^{2+m}\sqrt{d-c^2d}}{f^2(2+m)^2(4+m)(6+m)\sqrt{-1+}}$$

Mathematica [A] time = 1.42059, size = 350, normalized size = 0.48

$$d^2x\sqrt{d-c^2dx^2}(fx)^m \left(\frac{15\left(-\frac{(m+2)\left(\frac{bcx\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{m+2}+\frac{\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{m+1}+(\ldots\right)}{(m+2)^2(m+4)} \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((5*b*c*x*(-(2 + m)^(-1) + (c^2*x^2)/(4
+ m)))/(4 + m) - b*c*x*((2 + m)^(-1) - (2*c^2*x^2)/(4 + m) + (c^4*x^4)/(6 +
 m)) - (5*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]))/(4 + m) +
(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x]) + (15*(-(b*c*x) + (2
+ m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]) - ((2 + m)*((Sqrt[1
- c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2
, c^2*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*x*HypergeometricPFQ[{1, 1
 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(2 + m)))/(1 + m)))/((2 +
```

```
m)^2*(4 + m))))/((6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [F] time = 1.507, size = 0, normalized size = 0.

$$\int (fx)^m \left(-c^2 d x^2 + d\right)^{\frac{5}{2}} (a + b\operatorname{arccosh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2 d x^2 + d\right)^{\frac{5}{2}} (b \operatorname{arcosh}(cx) + a) (fx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2 + d}\,(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.152 $\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=455

$$-\frac{3bcd\sqrt{d-c^2dx^2}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\sqrt{d-c^2dx^2}(fx)^{m+1}\text{Hy}}{f(m}$$

```
[Out] (-3*b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*(4 + m)*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2
+ m)*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*(f*x)^(4 + m)*Sqrt[d
- c^2*d*x^2])/(f^4*(4 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*(f*x)^(1
+ m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*(8 + 6*m + m^2)) + ((f*x)
^(1 + m)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(f*(4 + m)) + (3*d*(f*
x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2,
(1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(4 + m)*(2 + 3*m + m^2)*Sqrt[1 - c*x]*Sq
rt[1 + c*x]) - (3*b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*HypergeometricPFQ
[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m
)^2*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.904162, antiderivative size = 477, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5798, 5745, 5743, 5763, 32, 14}

$$-\frac{3bcd\sqrt{d-c^2dx^2}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\right.}{f(m+4)\left(m^2+3m+2\right)(1-cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*(4 + m)*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2
+ m)*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*(f*x)^(4 + m)*Sqrt[d
- c^2*d*x^2])/(f^4*(4 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*(f*x)^(1
+ m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*(8 + 6*m + m^2)) + (d*(f*
x)^(1 + m)*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f
*(4 + m)) + (3*d*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(4
 + m)*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x)) - (3*b*c*d*(f*x)^(2 + m)*Sqrt[d
```

```
- c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2},
 c^2*x^2])/(f^2*(1 + m)*(2 + m)^2*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
```

```
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int (fx)^m(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{d(fx)^{1+m}(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(4+m)} + \frac{\left(3d\sqrt{d-c^2d\right.}{}$$

$$= \frac{3d(fx)^{1+m}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f\left(8+6m+m^2\right)} + \frac{d(fx)^{1+m}(1-cx)(1+cx)\sqrt{}}{f(4}$$

$$= -\frac{3bcd(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)(4+m)\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A] time = 0.79709, size = 274, normalized size = 0.6

$$-\frac{dx\sqrt{d-c^2dx^2}(fx)^m\left(\frac{3bcx\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{(m+1)(m+2)^2} + \frac{3\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\cosh}{(m+1)(m+2)\sqrt{cx-1}\sqrt{cx+1}}\right.}{(m+4)\sqrt{cx-1}\sqrt{c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d*x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((3*b*c*x)/(2 + m)^2 + b*c*x*((2 + m)^(-
1) - (c^2*x^2)/(4 + m)) - (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*
```

```
x]))/(2 + m) + (-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + (3*S
qrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, c^2*x^2])/((1 + m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c*x
*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(
(1 + m)*(2 + m)^2)))/((4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Maple [F] time = 1.326, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(-c^{2} d x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^{2} d x^{2}+d\right)^{\frac{3}{2}} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right) \left(fx\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(ac^{2} d x^{2}-ad+\left(bc^{2} d x^{2}-bd\right)\mathrm{arcosh}\left(cx\right)\right)\sqrt{-c^{2} d x^{2}+d}\left(fx\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.153 $\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=278

$$-\frac{bc\sqrt{d-c^2dx^2}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{f^2(m+1)(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}(fx)^{m+1}\text{Hyperge}}{f\left(m^2\right.}$$

```
[Out] -((b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*
(2 + m)) + ((f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Hypergeo
metric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(2 + 3*m + m^2)*Sqrt[1 -
c*x]*Sqrt[1 + c*x]) - (b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*Hypergeometric
PFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2
+ m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.576508, antiderivative size = 288, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5798, 5743, 5763, 32}

$$-\frac{bc\sqrt{d-c^2dx^2}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)(}{f\left(m^2+3m+2\right)(1-cx)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*
(2 + m)) + ((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(2 + 3*
m + m^2)*(1 - c*x)*(1 + c*x)) - (b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*Hype
rgeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(
1 + m)*(2 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\sqrt{d-c^2dx^2}\int (fx)^m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{(fx)^{1+m}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(2+m)}-\frac{\sqrt{d-c^2dx^2}\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,d}{(2+m)\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{bc(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{(fx)^{1+m}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(2+m)}+$$

Mathematica [A] time = 0.282914, size = 223, normalized size = 0.8

$$x\sqrt{d-c^2dx^2}(fx)^m\left(-bcx\sqrt{cx-1}\sqrt{cx+1}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)-(m+2)\sqrt{1-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((1 + m)*(-(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*ArcCosh[c*x]) - (
2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometri
cPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1 + m)*(2 +
m)^2*(-1 + c*x)*(1 + c*x))
```

Maple [F] time = 1.338, size = 0, normalized size = 0.

$$\int (fx)^m \sqrt{-c^2dx^2+d}\,(a+b\operatorname{arccosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(fx\right)^m\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((f*x)**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.154 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=176

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\text{Hyperg}}{}$$

```
[Out] ((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f
*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 +
 m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*
x^2])
```

Rubi [A] time = 0.352383, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {5798, 5763}

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\,\text{co}\right.}{f(m+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f
*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 +
 m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*
x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$=\frac{(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{f(1+m)\sqrt{d-c^2dx^2}}+\frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{}}{f}$$

Mathematica [A] time = 0.0756048, size = 147, normalized size = 0.84

$$\frac{x(fx)^m\left(bcx\sqrt{cx-1}\sqrt{cx+1}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)+(m+2)\sqrt{1-c^2x^2}\text{Hyp}\right.}{(m+1)(m+2)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (x*(f*x)^m*((2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F
1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/(
(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

Maple [F] time = 0.45, size = 0, normalized size = 0.

$$\int (fx)^m\,(a+b\,\mathrm{arccosh}\,(cx))\,\frac{1}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)(fx)^m}{c^2dx^2-d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m(a+b\operatorname{acosh}(cx))}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)(fx)^m}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

3.155 $\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{bcm\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{df^2(m+1)(m+2)\sqrt{d-c^2dx^2}}-\frac{m\sqrt{1-c^2x^2}(fx)^{m+1}\text{Hy}}{}$$

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d*f*Sqrt[d - c^2*d*x^2]) - (m*(f*x)^(
1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, c^2*x^2])/(d*f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f*x)^(2
 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, c^2*x^2])/(d*f^2*(2 + m)*Sqrt[d - c^2*d*x^2]) - (b*c*m*(f*x)^(2 + m)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m
/2, 2 + m/2}, c^2*x^2])/(d*f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.660031, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5798, 5756, 5763, 364}

$$-\frac{bcm\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{df^2(m+1)(m+2)\sqrt{d-c^2dx^2}}-\frac{m\sqrt{1-c^2x^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+\right.}{df(m+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```


```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d*f*Sqrt[d - c^2*d*x^2]) - (m*(f*x)^(
1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, c^2*x^2])/(d*f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f*x)^(2
 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, c^2*x^2])/(d*f^2*(2 + m)*Sqrt[d - c^2*d*x^2]) - (b*c*m*(f*x)^(2 + m)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m
/2, 2 + m/2}, c^2*x^2])/(d*f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m \left(a + b\cosh^{-1}(cx)\right)}{\left(d - c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{(fx)^{1+m}\left(a + b\cosh^{-1}(cx)\right)}{df\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^{1+m}}{-1+c^2x^2}\,dx}{df\sqrt{d-c^2dx^2}} - \frac{\left(m\sqrt{-1+cx}\sqrt{1+c\right.}{d\sqrt{}}$$

$$= \frac{(fx)^{1+m}\left(a + b\cosh^{-1}(cx)\right)}{df\sqrt{d-c^2dx^2}} - \frac{m(fx)^{1+m}\sqrt{1-c^2x^2}\left(a + b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2}\right.}{df(1+m)\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.241915, size = 216, normalized size = 0.72

$$\frac{x(fx)^m\left(-bcmx\sqrt{cx-1}\sqrt{cx+1}\,\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right) - m(m+2)\sqrt{1-c^2x^2}\text{H}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(-(m*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2]) + (1 + m)*((2 + m)*(a + b*ArcCos
h[c*x]) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, 1 + m/2,
2 + m/2, c^2*x^2]) - b*c*m*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ
[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/(d*(1 + m)*(2 + m)
*Sqrt[d - c^2*d*x^2])
```

Maple [F] time = 0.598, size = 0, normalized size = 0.

$$\int \left(fx\right)^m \left(a + b\text{arccosh}\left(cx\right)\right)\left(-c^2dx^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{(-c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2 + d}(b \operatorname{arcosh}(cx) + a)(fx)^m}{c^4d^2x^4 - 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{acosh}(cx))}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)\left(fx\right)^{m}}{\left(-c^{2}dx^{2} + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

3.156 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=450

$$-\frac{bc(2-m)m\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{3d^2f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}-\frac{(2-m)m\sqrt{1-}}{}$$

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d*f*(d - c^2*d*x^2)^(3/2)) + ((2 -
m)*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d^2*f*Sqrt[d - c^2*d*x^2]) - ((2
- m)*m*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2
F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(3*d^2*f*(1 + m)*Sqrt[d - c^2*d*x^2
]) + (b*c*(2 - m)*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric
2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d^2*f^2*(2 + m)*Sqrt[d - c^2*d*x^
2]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[2,
(2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d^2*f^2*(2 + m)*Sqrt[d - c^2*d*x^2]) - (
b*c*(2 - m)*m*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[
{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(3*d^2*f^2*(1 + m)*(
2 + m)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.993008, antiderivative size = 465, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5798, 5756, 5763, 364}

$$-\frac{bc(2-m)m\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{3d^2f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}-\frac{(2-m)m\sqrt{1-c^2x^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+}{2}\right.}{3d^2f(m+1)\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((2 - m)*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d^2*f*Sqrt[d - c^2*d*x^2])
+ ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d^2*f*(1 - c*x)*(1 + c*x)*Sqrt[d
- c^2*d*x^2]) - ((2 - m)*m*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c
*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(3*d^2*f*(1 + m
)*Sqrt[d - c^2*d*x^2]) + (b*c*(2 - m)*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d^2*f^2*(2 +
m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*H
ypergeometric2F1[2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d^2*f^2*(2 + m)*Sqrt
```

```
[d - c^2*d*x^2]) - (b*c*(2 - m)*m*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/
(3*d^2*f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^{1+m}}{\left(-1+c^2x^2\right)^2}\,dx}{3d^2f\sqrt{d-c^2dx^2}} + \frac{\left((-2+m)\right.}{}$$

$$= \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f\sqrt{d-c^2dx^2}} + \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{bc(fx)^{2+m}\sqrt{}}{}$$

$$= \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f\sqrt{d-c^2dx^2}} + \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{(2-m)m(f}{}$$

Mathematica [A] time = 0.71891, size = 319, normalized size = 0.71

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}(fx)^m\left(\frac{(m-2)\left(bcmx\sqrt{cx-1}\sqrt{cx+1}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)+m(m+2)\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\right.}{(m+1)(m+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/((-1 + c*x)
^(3/2)*(1 + c*x)^(3/2))) + (b*c*x*Hypergeometric2F1[2, 1 + m/2, 2 + m/2, c^
2*x^2])/(2 + m) + ((-2 + m)*(m*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x
])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] - (1 + m)*((2 + m)
*(a + b*ArcCosh[c*x]) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F
1[1, 1 + m/2, 2 + m/2, c^2*x^2]) + b*c*m*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hyp
ergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1
+ m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [F] time = 0.582, size = 0, normalized size = 0.

$$\int \left(fx\right)^m (a+b\operatorname{arccosh}(cx))\left(-c^2dx^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{(-c^2dx^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2 + d}(b \operatorname{arcosh}(cx) + a)(fx)^m}{c^6d^3x^6 - 3\,c^4d^3x^4 + 3\,c^2d^3x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{(-c^2dx^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

3.157 $\int (fx)^m(\mathbf{d1}+c\mathbf{d1}x)^{5/2}(\mathbf{d2}-c\mathbf{d2}x)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=817

$$\frac{(cxd1+d1)^{5/2}(d2-cd2x)^{5/2}\left(a+b\cosh^{-1}(cx)\right)(fx)^{m+1}}{f(m+6)}+\frac{5d1d2(cxd1+d1)^{3/2}(d2-cd2x)^{3/2}\left(a+b\cosh^{-1}(cx)\right)(fx)^{\prime}}{f(m+4)(m+6)}$$

```
[Out] -((b*c*d1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2
 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - (15*b*c*d1^2*d2^2*(f*x)^(2 +
 m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*(4 + m)*(6 + m)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*
x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (5*b*c^3*d1^2*d2^2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x
])/(f^4*(4 + m)^2*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d1^2*d2^
2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^4*(4 + m)*(6 + m)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d1^2*d2^2*(f*x)^(6 + m)*Sqrt[d1 + c*d
1*x]*Sqrt[d2 - c*d2*x])/(f^6*(6 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*
d1^2*d2^2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[
c*x]))/(f*(6 + m)*(8 + 6*m + m^2)) + (5*d1*d2*(f*x)^(1 + m)*(d1 + c*d1*x)^(
3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]))/(f*(4 + m)*(6 + m)) + ((f*x)
^(1 + m)*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*x]))/(f*(
6 + m)) + (15*d1^2*d2^2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(
a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/
(f*(4 + m)*(6 + m)*(2 + 3*m + m^2)*Sqrt[1 - c*x]*Sqrt[1 + c*x]) - (15*b*c*d
1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*HypergeometricPF
Q[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 +
m)^2*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.57982, antiderivative size = 827, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5745, 5743, 5763, 32, 14, 270}

$$\frac{(cxd1+d1)^{5/2}(d2-cd2x)^{5/2}\left(a+b\cosh^{-1}(cx)\right)(fx)^{m+1}}{f(m+6)}+\frac{5d1d2(cxd1+d1)^{3/2}(d2-cd2x)^{3/2}\left(a+b\cosh^{-1}(cx)\right)(fx)^{\prime}}{f(m+4)(m+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*d1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2
 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - (15*b*c*d1^2*d2^2*(f*x)^(2 +
```

```
m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*(4 + m)*(6 + m)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*
x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (5*b*c^3*d1^2*d2^2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x
])/(f^4*(4 + m)^2*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d1^2*d2^
2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^4*(4 + m)*(6 + m)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d1^2*d2^2*(f*x)^(6 + m)*Sqrt[d1 + c*d
1*x]*Sqrt[d2 - c*d2*x])/(f^6*(6 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*
d1^2*d2^2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[
c*x]))/(f*(6 + m)*(8 + 6*m + m^2)) + (5*d1*d2*(f*x)^(1 + m)*(d1 + c*d1*x)^(
3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]))/(f*(4 + m)*(6 + m)) + ((f*x)
^(1 + m)*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*x]))/(f*(
6 + m)) + (15*d1^2*d2^2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*S
qrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, c^2*x^2])/(f*(4 + m)*(6 + m)*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x)) -
 (15*b*c*d1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*Hyperg
eometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1
+ m)*(2 + m)^2*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{f(6+m)}$$

$$= \frac{5\mathrm{d1}\mathrm{d2}(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}\left(a+b\cosh^{-}\right.}{f(4+m)(6+m)}$$

$$= -\frac{bc\mathrm{d1}^2\mathrm{d2}^2(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^3\mathrm{d1}^2\mathrm{d}}{f^4(4}$$

$$= -\frac{bc\mathrm{d1}^2\mathrm{d2}^2(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{15bc\mathrm{d1}^2\mathrm{d}}{f^2(2+m}$$

Mathematica [A] time = 2.57504, size = 387, normalized size = 0.47

$$\frac{\mathrm{d1}^2\mathrm{d2}^2x\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(fx)^m\left(\frac{5\left(\frac{3\left(-bcx\sqrt{cx-1}\sqrt{cx+1}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)-(m+2)\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\right.}{(m+1)(m+2)^2(c}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*
x]),x]
```

```
[Out] (d1^2*d2^2*x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(-((b*c*x*((2 + m)
^(-1) - (2*c^2*x^2)/(4 + m) + (c^4*x^4)/(6 + m)))/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])) + (-1 + c^2*x^2)^2*(a + b*ArcCosh[c*x]) + (5*((b*c*x*(-(2 + m)^(-1)
+ (c^2*x^2)/(4 + m)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (-1 + c*x)*(1 + c*x)
*(a + b*ArcCosh[c*x]) + (3*((1 + m)*(-(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*ArcCosh[c*x]) - (2 +
m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ
[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1 + m)*(2 + m)^2
*(-1 + c*x)*(1 + c*x))))/(4 + m)))/(6 + m)
```

Maple [F] time = 2.239, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m}\left(c\mathrm{d1}\,x+\mathrm{d1}\right)^{\frac{5}{2}}\left(-c\mathrm{d2}\,x+\mathrm{d2}\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cd_1x+d_1\right)^{\frac{5}{2}}\left(-cd_2x+d_2\right)^{\frac{5}{2}}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)\left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x
, algorithm="maxima")
```

```
[Out] integrate((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^(5/2)*(b*arccosh(c*x) + a)*(f*
x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^4d_1^2d_2^2x^4 - 2\,ac^2d_1^2d_2^2x^2 + ad_1^2d_2^2 + \left(bc^4d_1^2d_2^2x^4 - 2\,bc^2d_1^2d_2^2x^2 + bd_1^2d_2^2\right)\text{arcosh}\,(cx)\right)\sqrt{cd_1x + d_1}\sqrt{-cd_2x + d_2}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x
, algorithm="fricas")
```

```
[Out] integral((a*c^4*d1^2*d2^2*x^4 - 2*a*c^2*d1^2*d2^2*x^2 + a*d1^2*d2^2 + (b*c^
4*d1^2*d2^2*x^4 - 2*b*c^2*d1^2*d2^2*x^2 + b*d1^2*d2^2)*arccosh(c*x))*sqrt(c
*d1*x + d1)*sqrt(-c*d2*x + d2)*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c*d1*x+d1)**(5/2)*(-c*d2*x+d2)**(5/2)*(a+b*acosh(c*x)),
x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.158 $\int (fx)^m(\mathbf{d1}+c\mathbf{d1}x)^{3/2}(\mathbf{d2}-c\mathbf{d2}x)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=503

$$-\frac{3bc\mathrm{d1}\mathrm{d2}\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(fx)^{m+2}\mathrm{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\mathrm{d1}\mathrm{d2}\sqrt{}}{}$$

```
[Out] (-3*b*c*d1*d2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 +
m)^2*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d1*d2*(f*x)^(2 + m)*Sqrt[
d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*c^3*d1*d2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(
f^4*(4 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d1*d2*(f*x)^(1 + m)*Sqrt[d
1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]))/(f*(8 + 6*m + m^2)) + (
(f*x)^(1 + m)*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]))
/(f*(4 + m)) + (3*d1*d2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(
a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/
(f*(4 + m)*(2 + 3*m + m^2)*Sqrt[1 - c*x]*Sqrt[1 + c*x]) - (3*b*c*d1*d2*(f*x
)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*HypergeometricPFQ[{1, 1 + m/2
, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)^2*(4 + m)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.990924, antiderivative size = 513, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5745, 5743, 5763, 32, 14}

$$-\frac{3bc\mathrm{d1}\mathrm{d2}\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\mathrm{d1}\mathrm{d2}\sqrt{1-c^2x^2}\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d}}}{f(m+}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*c*d1*d2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 +
m)^2*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d1*d2*(f*x)^(2 + m)*Sqrt[
d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*c^3*d1*d2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(
f^4*(4 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d1*d2*(f*x)^(1 + m)*Sqrt[d
1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]))/(f*(8 + 6*m + m^2)) + (
(f*x)^(1 + m)*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]))
/(f*(4 + m)) + (3*d1*d2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*S
```

```
qrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, c^2*x^2])/(f*(4 + m)*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x)) - (3*b*c*
d1*d2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*HypergeometricPFQ[{
1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)^
2*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
```

```
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f(4+m)}$$

$$= \frac{3\mathrm{d1}\mathrm{d2}(fx)^{1+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\left(a+b\cosh^{-1}(cx)\right.}{f\left(8+6m+m^2\right)}$$

$$= -\frac{3bc\mathrm{d1}\mathrm{d2}(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc\mathrm{d1}\mathrm{d2}(f}{f^2(2+}$$

Mathematica [A] time = 1.04371, size = 288, normalized size = 0.57

$$\frac{\mathrm{d1}\mathrm{d2}x\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(fx)^m\left(-\frac{3bcx\,\mathrm{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{(m+1)(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3\sqrt{1-c^2x^2}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+}{2}\right.}{(m+1)(m+2)(cx-}\right.}{m+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*
x]),x]
```

```
[Out] (d1*d2*x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((-3*b*c*x)/((2 + m)^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*x*(-(2 + m)^(-1) + (c^2*x^2)/(4 + m))
)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*(a + b*ArcCosh[c*x]))/(2 + m) - (-1 +
 c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]) - (3*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)*(2 +
m)*(-1 + c*x)*(1 + c*x)) - (3*b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}
, {3/2 + m/2, 2 + m/2}, c^2*x^2])/((1 + m)*(2 + m)^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])))/(4 + m)
```

Maple [F] time = 1.931, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(\mathrm{cd}1\, x + \mathrm{d}1\right)^{\frac{3}{2}} \left(-\mathrm{cd}2\, x + \mathrm{d}2\right)^{\frac{3}{2}} \left(a + b\operatorname{arccosh}\left(cx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cd_1x + d_1)^{\frac{3}{2}}(-cd_2x + d_2)^{\frac{3}{2}}(b\operatorname{arcosh}(cx) + a)\left(fx\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x
, algorithm="maxima")
```

```
[Out] integrate((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^(3/2)*(b*arccosh(c*x) + a)*(f*
x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(ac^{2}d_{1}d_{2}x^{2} - ad_{1}d_{2} + \left(bc^{2}d_{1}d_{2}x^{2} - bd_{1}d_{2}\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{cd_{1}x + d_{1}}\sqrt{-cd_{2}x + d_{2}}\left(fx\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x
, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d1*d2*x^2 - a*d1*d2 + (b*c^2*d1*d2*x^2 - b*d1*d2)*arccosh(
c*x))*sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c*d1*x+d1)**(3/2)*(-c*d2*x+d2)**(3/2)*(a+b*acosh(c*x)),
x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.159 $\int (fx)^m \sqrt{\mathbf{d1}+c\mathbf{d1}x}\sqrt{\mathbf{d2}-c\mathbf{d2}x}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=302

$$-\frac{bc\sqrt{cd1x+d1}\sqrt{d2-cd2x}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{f^2(m+1)(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{cd1x+d1}}{}$$

```
[Out] -((b*c*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]*(a + b*ArcCosh[c*x]))/(f*(2 + m)) + ((f*x)^(1 + m)*Sqrt[d1 + c*d1*x]
*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (
3 + m)/2, c^2*x^2])/(f*(2 + 3*m + m^2)*Sqrt[1 - c*x]*Sqrt[1 + c*x]) - (b*c*
(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*HypergeometricPFQ[{1, 1 +
 m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)^2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.557134, antiderivative size = 312, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {5743, 5763, 32}

$$-\frac{bc\sqrt{cd1x+d1}\sqrt{d2-cd2x}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\sqrt{cd1x+d1}\sqrt{d2-cd2x}(fx}{f\left(m^2+3m\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]*(a + b*ArcCosh[c*x]))/(f*(2 + m)) + ((f*x)^(1 + m)*Sqrt[d1 + c*d1*x]
*Sqrt[d2 - c*d2*x]*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1
[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x
)) - (b*c*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*HypergeometricP
FQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 +
 m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
```

```
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{(fx)^{1+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\left(a+b\cosh^{-1}(cx)\right)}{f(2+m)}-\frac{\left(\sqrt{\mathrm{d1}\ldots\right.}{}$$

$$=-\frac{bc(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{(fx)^{1+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\ldots}{\ldots}$$

Mathematica [A] time = 0.21783, size = 229, normalized size = 0.76

$$\frac{x\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(fx)^m\left(-bcx\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]),
x]
```

```
[Out] (x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((1 + m)*(-(b*c*x*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*A
rcCosh[c*x]) - (2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometri
c2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])
)/((1 + m)*(2 + m)^2*(-1 + c*x)*(1 + c*x))
```

Maple [F] time = 1.584, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \sqrt{c\mathrm{d1}\,x+\mathrm{d1}}\sqrt{-c\mathrm{d2}\,x+\mathrm{d2}}\,(a+b\mathrm{arccosh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}(b\,\mathrm{arcosh}\,(cx)+a)\left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}(b\,\mathrm{arcosh}\,(cx)+a)\left(fx\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c*d1*x+d1)**(1/2)*(-c*d2*x+d2)**(1/2)*(a+b*acosh(c*x)),
x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.160 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{\mathbf{d1}+c\mathbf{d1}x}\sqrt{\mathbf{d2}-c\mathbf{d2}x}}\,dx$

Optimal. Leaf size=188

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{cd1x+d1}\sqrt{d2-cd2x}}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\text{Hype}}{f}$$

```
[Out] ((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*
d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[
{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)
*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rubi [A] time = 0.558932, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {5765, 5763}

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{cd1x+d1}\sqrt{d2-cd2x}}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\,c\right.}{f(m+1)\sqrt{cd1x+d1}\sqrt{d2-cd2x}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]),x]
```

```
[Out] ((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*
d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[
{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)
*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rule 5765

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(Sqrt[1 + c
*x]*Sqrt[-1 + c*x])/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[((f*x)^m*(a + b*
ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c,
d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[
n, 0] &&  !(GtQ[d1, 0] && LtQ[d2, 0]) && (IntegerQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}$$

$$= \frac{(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{f(1+m)\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}} + \frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1}}{f^2(1+m)}$$

Mathematica [C] time = 6.0086, size = 264, normalized size = 1.4

$$\frac{2^{-m-3}\sqrt{c\mathrm{d1}x+\mathrm{d1}}\left(\frac{cx}{cx+1}\right)^{1-m}(fx)^m\left(bm\left(\frac{cx}{cx+1}\right)^m\sinh\left(2\cosh^{-1}(cx)\right)\left(\sqrt{\pi}c(m+1)x\sqrt{\frac{cx-1}{cx+1}}\mathrm{Gamma}(m+1)\,{}_3\tilde{F}_2\left(1,\frac{m+2}{2},\frac{m+}{2}\right.\right.\right.}{c^2\mathrm{d1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*
x]),x]
```

```
[Out] (2^(-3 - m)*(f*x)^m*((c*x)/(1 + c*x))^(1 - m)*Sqrt[d1 + c*d1*x]*(2^(3 + m)*
a*(1 + m)*(-1 + c*x)*AppellF1[-m, -m, 1/2, 1 - m, (1 + c*x)^(-1), 2/(1 + c*
x)] + b*m*((c*x)/(1 + c*x))^m*(-(2^(2 + m)*(-1 + c*x)*ArcCosh[c*x]*Hypergeo
metric2F1[1, (2 + m)/2, (3 + m)/2, c^2*x^2]) + c*(1 + m)*Sqrt[Pi]*x*Sqrt[(-
1 + c*x)/(1 + c*x)]*Gamma[1 + m]*HypergeometricPFQRegularized[{1, (2 + m)/2
, (2 + m)/2}, {(3 + m)/2, (4 + m)/2}, c^2*x^2])*Sinh[2*ArcCosh[c*x]]))/(c^2
*d1*m*(1 + m)*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d2 - c*d2*x])
```

Maple [F] time = 0.505, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right) \frac{1}{\sqrt{c\mathrm{d1}\,x+\mathrm{d1}}}\frac{1}{\sqrt{-c\mathrm{d2}\,x+\mathrm{d2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)\left(fx\right)^{m}}{\sqrt{cd_{1}x+d_{1}}\sqrt{-cd_{2}x+d_{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2
)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{cd_{1}x+d_{1}}\sqrt{-cd_{2}x+d_{2}}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)\left(fx\right)^{m}}{c^{2}d_{1}d_{2}x^{2}-d_{1}d_{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
/(c^2*d1*d2*x^2 - d1*d2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{acosh}(cx))}{\sqrt{d_1 (cx + 1)}\sqrt{-d_2 (cx - 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(1/2)/(-c*d2*x+d2)**(1/2),
x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/(sqrt(d1*(c*x + 1))*sqrt(-d2*(c*x - 1)
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{\sqrt{cd_1 x + d_1}\sqrt{-cd_2 x + d_2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2
)), x)
```

3.161 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(\mathbf{d1}+c\mathbf{d1}x)^{3/2}(\mathbf{d2}-c\mathbf{d2}x)^{3/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{bcm\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{\text{d1}\text{d2}f^2(m+1)(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}-\frac{m\sqrt{1-c^2x^2}(fx)^{m+1}\text{H}}{}$$

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d1*d2*f*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]) - (m*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(d1*d2*f*(1 + m)*Sqrt[d1 + c*
d1*x]*Sqrt[d2 - c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d1*d2*f^2*(2 + m)*Sqr
t[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - (b*c*m*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c
^2*x^2])/(d1*d2*f^2*(1 + m)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rubi [A] time = 0.943476, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5756, 5765, 5763, 364}

$$-\frac{bcm\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{\text{d1}\text{d2}f^2(m+1)(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}-\frac{m\sqrt{1-c^2x^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)(a}{\text{d1}\text{d2}f(m+1)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)
),x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d1*d2*f*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]) - (m*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(d1*d2*f*(1 + m)*Sqrt[d1 + c*
d1*x]*Sqrt[d2 - c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d1*d2*f^2*(2 + m)*Sqr
t[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - (b*c*m*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c
^2*x^2])/(d1*d2*f^2*(1 + m)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5765

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(Sqrt[1 + c
*x]*Sqrt[-1 + c*x])/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[((f*x)^m*(a + b*
ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c,
d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[
n, 0] &&  !(GtQ[d1, 0] && LtQ[d2, 0]) && (IntegerQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m \left(a + b\cosh^{-1}(cx)\right)}{(\text{d1} + c\text{d1}x)^{3/2}(\text{d2} - c\text{d2}x)^{3/2}}\,dx = \frac{(fx)^{1+m}\left(a + b\cosh^{-1}(cx)\right)}{\text{d1d2}f\sqrt{\text{d1} + c\text{d1}x}\sqrt{\text{d2} - c\text{d2}x}} - \frac{m\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}\,dx}{\text{d1d2}} - \frac{\left(bc\sqrt{-1+cx}\sqrt{1\right.}{\text{d1d2}f\sqrt{\text{d1}+c}$$

$$= \frac{(fx)^{1+m}\left(a + b\cosh^{-1}(cx)\right)}{\text{d1d2}f\sqrt{\text{d1} + c\text{d1}x}\sqrt{\text{d2} - c\text{d2}x}} + \frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,{}_2F_1\left(1, \frac{2+m}{2}; \frac{4+m}{2};\right.}{\text{d1d2}f^2(2+m)\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}}$$

$$= \frac{(fx)^{1+m}\left(a + b\cosh^{-1}(cx)\right)}{\text{d1d2}f\sqrt{\text{d1} + c\text{d1}x}\sqrt{\text{d2} - c\text{d2}x}} - \frac{m(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\right.}{\text{d1d2}f(1+m)\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-}$$

Mathematica [F] time = 2.45387, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a + b\cosh^{-1}(cx)\right)}{(\text{d1} + c\text{d1}x)^{3/2}(\text{d2} - c\text{d2}x)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)
^(3/2)),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)
^(3/2)), x]
```

Maple [F] time = 0.659, size = 0, normalized size = 0.

$$\int \left(fx\right)^m \left(a + b\operatorname{arccosh}(cx)\right)\left(c\text{d1}\,x + \text{d1}\right)^{-\frac{3}{2}}\left(-c\text{d2}\,x + \text{d2}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arcosh}(cx) + a\right)\left(fx\right)^m}{\left(cd_1x + d_1\right)^{\frac{3}{2}}\left(-cd_2x + d_2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^
(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{c^4d_1^2d_2^2x^4-2\,c^2d_1^2d_2^2x^2+d_1^2d_2^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m/
(c^4*d1^2*d2^2*x^4 - 2*c^2*d1^2*d2^2*x^2 + d1^2*d2^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(3/2)/(-c*d2*x+d2)**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{(cd_1x+d_1)^{\frac{3}{2}}(-cd_2x+d_2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^
(3/2)), x)
```

3.162 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(\mathbf{d1}+c\mathbf{d1}x)^{5/2}(\mathbf{d2}-c\mathbf{d2}x)^{5/2}}\,dx$

Optimal. Leaf size=504

$$-\frac{bc(2-m)m\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+1)(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}-\frac{(2-m)m\sqrt{1-}}{}$$

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d1*d2*f*(d1 + c*d1*x)^(3/2)*(d2 - c
*d2*x)^(3/2)) + ((2 - m)*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d1^2*d2^2*f
*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - ((2 - m)*m*(f*x)^(1 + m)*Sqrt[1 - c
^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c
^2*x^2])/(3*d1^2*d2^2*f*(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) + (b*c
*(2 - m)*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2
 + m)/2, (4 + m)/2, c^2*x^2])/(3*d1^2*d2^2*f^2*(2 + m)*Sqrt[d1 + c*d1*x]*Sq
rt[d2 - c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeo
metric2F1[2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d1^2*d2^2*f^2*(2 + m)*Sqrt[
d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - (b*c*(2 - m)*m*(f*x)^(2 + m)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, c^2*x^2])/(3*d1^2*d2^2*f^2*(1 + m)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 -
 c*d2*x])
```

Rubi [A] time = 1.45193, antiderivative size = 504, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5756, 5765, 5763, 364}

$$-\frac{bc(2-m)m\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+1)(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}-\frac{(2-m)m\sqrt{1-c^2x^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\right.}{3\text{d1}^2\text{d2}^2f(m+1)\sqrt{c\text{d1}x+}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)
),x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d1*d2*f*(d1 + c*d1*x)^(3/2)*(d2 - c
*d2*x)^(3/2)) + ((2 - m)*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d1^2*d2^2*f
*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - ((2 - m)*m*(f*x)^(1 + m)*Sqrt[1 - c
^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c
^2*x^2])/(3*d1^2*d2^2*f*(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) + (b*c
*(2 - m)*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2
```

```
 + m)/2, (4 + m)/2, c^2*x^2])/(3*d1^2*d2^2*f^2*(2 + m)*Sqrt[d1 + c*d1*x]*Sq
rt[d2 - c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeo
metric2F1[2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d1^2*d2^2*f^2*(2 + m)*Sqrt[
d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - (b*c*(2 - m)*m*(f*x)^(2 + m)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, c^2*x^2])/(3*d1^2*d2^2*f^2*(1 + m)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 -
 c*d2*x])
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5765

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(Sqrt[1 + c
*x]*Sqrt[-1 + c*x])/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[((f*x)^m*(a + b*
ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c,
d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[
n, 0] &&  !(GtQ[d1, 0] && LtQ[d2, 0]) && (IntegerQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
```

```
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(\text{d1}+c\text{d1}x)^{5/2}(\text{d2}-c\text{d2}x)^{5/2}}\,dx = \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}\text{d2}f(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}} + \frac{(2-m)\int\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}\,dx}{3\text{d1}\text{d2}} + \frac{\left(bc\right.}{3\text{d}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}\text{d2}f(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}} + \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}^2\text{d2}^2f\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}} + \frac{bc}{-}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}\text{d2}f(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}} + \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}^2\text{d2}^2f\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}} + \frac{bc}{-}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}\text{d2}f(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}} + \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}^2\text{d2}^2f\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}} - \frac{(2}{-}$$

Mathematica [F] time = 2.48476, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(\text{d1}+c\text{d1}x)^{5/2}(\text{d2}-c\text{d2}x)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)
^(5/2)),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)
^(5/2)), x]
```

Maple [F] time = 0.677, size = 0, normalized size = 0.

$$\int (fx)^m\,(a+b\mathrm{arccosh}\,(cx))\,(c\text{d1}\,x+\text{d1})^{-\frac{5}{2}}\,(-c\text{d2}\,x+\text{d2})^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{(cd_1x + d_1)^{\frac{5}{2}}(-cd_2x + d_2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^
(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{cd_1x + d_1}\sqrt{-cd_2x + d_2}(b \operatorname{arcosh}(cx) + a)(fx)^m}{c^6d_1^3d_2^3x^6 - 3\,c^4d_1^3d_2^3x^4 + 3\,c^2d_1^3d_2^3x^2 - d_1^3d_2^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
/(c^6*d1^3*d2^3*x^6 - 3*c^4*d1^3*d2^3*x^4 + 3*c^2*d1^3*d2^3*x^2 - d1^3*d2^3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(5/2)/(-c*d2*x+d2)**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)\left(fx\right)^m}{(cd_1x + d_1)^{\frac{5}{2}}(-cd_2x + d_2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^
(5/2)), x)
```

3.163 $\int \frac{(fx)^m \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=128

$$\frac{a\sqrt{ax-1}(fx)^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},a^2x^2\right)}{f^2(m+1)(m+2)\sqrt{1-ax}}+\frac{\cosh^{-1}(ax)(fx)^{m+1}\text{Hypergeome}}{f(m+}$$

```
[Out] ((f*x)^(1 + m)*ArcCosh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^
2*x^2])/(f*(1 + m)) + (a*(f*x)^(2 + m)*Sqrt[-1 + a*x]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, a^2*x^2])/(f^2*(1 + m)*(2 + m)*Sq
rt[1 - a*x])
```

Rubi [A] time = 0.286776, antiderivative size = 141, normalized size of antiderivative = 1.1, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 5763}

$$\frac{a\sqrt{ax-1}\sqrt{ax+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;a^2x^2\right)}{f^2(m+1)(m+2)\sqrt{1-a^2x^2}}+\frac{\cosh^{-1}(ax)(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((f*x)^(1 + m)*ArcCosh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^
2*x^2])/(f*(1 + m)) + (a*(f*x)^(2 + m)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Hyperge
ometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, a^2*x^2])/(f^2*(1 +
 m)*(2 + m)*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
```

```
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^m \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{(fx)^m \cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{(fx)^{1+m}\cosh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{f(1+m)} + \frac{a(fx)^{2+m}\sqrt{-1+ax}\sqrt{1+ax}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2}\right.}{f^2(1+m)(2+m)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0765463, size = 124, normalized size = 0.97

$$\frac{x(fx)^m\left(\frac{ax\sqrt{ax-1}\sqrt{ax+1}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},a^2x^2\right)}{(m+2)\sqrt{1-a^2x^2}}+\cosh^{-1}(ax)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right.\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (x*(f*x)^m*(ArcCosh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x
^2] + (a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*HypergeometricPFQ[{1, 1 + m/2, 1 +
m/2}, {3/2 + m/2, 2 + m/2}, a^2*x^2])/((2 + m)*Sqrt[1 - a^2*x^2])))/(1 + m)
```

Maple [F] time = 0.368, size = 0, normalized size = 0.

$$\int \left(fx\right)^m \operatorname{arccosh}\left(ax\right)\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m \operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x)^m*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(fx)^m \operatorname{arcosh}(ax)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(f*x)^m*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m \operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((f*x)**m*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m \operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

3.164 $\int \left(c-a^2cx^2\right)^3 \cosh^{-1}(ax)^2\,dx$

Optimal. Leaf size=266

$$-\frac{2}{343}a^6c^3x^7+\frac{234a^4c^3x^5}{6125}-\frac{1514a^2c^3x^3}{11025}+\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\cosh^{-1}(ax)^2+\frac{6}{35}c^3x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{8}{35}c^3x\left(1-\right.$$

```
[Out] (4322*c^3*x)/3675 - (1514*a^2*c^3*x^3)/11025 + (234*a^4*c^3*x^5)/6125 - (2*
a^6*c^3*x^7)/343 - (32*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(35*a
) + (16*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])/(105*a) - (12*c^
3*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x])/(175*a) + (2*c^3*(-1 + a*x
)^(7/2)*(1 + a*x)^(7/2)*ArcCosh[a*x])/(49*a) + (16*c^3*x*ArcCosh[a*x]^2)/35
 + (8*c^3*x*(1 - a^2*x^2)*ArcCosh[a*x]^2)/35 + (6*c^3*x*(1 - a^2*x^2)^2*Arc
Cosh[a*x]^2)/35 + (c^3*x*(1 - a^2*x^2)^3*ArcCosh[a*x]^2)/7
```

Rubi [A] time = 0.676279, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5681, 5718, 194, 5654, 8}

$$-\frac{2}{343}a^6c^3x^7+\frac{234a^4c^3x^5}{6125}-\frac{1514a^2c^3x^3}{11025}+\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\cosh^{-1}(ax)^2+\frac{6}{35}c^3x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{8}{35}c^3x\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3*ArcCosh[a*x]^2,x]
```


```
[Out] (4322*c^3*x)/3675 - (1514*a^2*c^3*x^3)/11025 + (234*a^4*c^3*x^5)/6125 - (2*
a^6*c^3*x^7)/343 - (32*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(35*a
) + (16*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])/(105*a) - (12*c^
3*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x])/(175*a) + (2*c^3*(-1 + a*x
)^(7/2)*(1 + a*x)^(7/2)*ArcCosh[a*x])/(49*a) + (16*c^3*x*ArcCosh[a*x]^2)/35
 + (8*c^3*x*(1 - a^2*x^2)*ArcCosh[a*x]^2)/35 + (6*c^3*x*(1 - a^2*x^2)^2*Arc
Cosh[a*x]^2)/35 + (c^3*x*(1 - a^2*x^2)^3*ArcCosh[a*x]^2)/7
```


Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^3 \cosh^{-1}(ax)^2\,dx = \frac{1}{7}c^3x\left(1-a^2x^2\right)^3\cosh^{-1}(ax)^2 + \frac{1}{7}(6c)\int \left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^2\,dx + \frac{1}{7}\left(2ac^3\right)\int x($$
$$= \frac{2c^3(-1+ax)^{7/2}(1+ax)^{7/2}\cosh^{-1}(ax)}{49a} + \frac{6}{35}c^3x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2 + \frac{1}{7}c^3x\left(1-a^2\right.$$
$$= -\frac{12c^3(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)}{175a} + \frac{2c^3(-1+ax)^{7/2}(1+ax)^{7/2}\cosh^{-1}(ax)}{49a} + \frac{8}{35}$$
$$= \frac{2c^3x}{49} - \frac{2}{49}a^2c^3x^3 + \frac{6}{245}a^4c^3x^5 - \frac{2}{343}a^6c^3x^7 + \frac{16c^3(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}{105a}$$
$$= \frac{962c^3x}{3675} - \frac{1514a^2c^3x^3}{11025} + \frac{234a^4c^3x^5}{6125} - \frac{2}{343}a^6c^3x^7 - \frac{32c^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{35a}$$
$$= \frac{4322c^3x}{3675} - \frac{1514a^2c^3x^3}{11025} + \frac{234a^4c^3x^5}{6125} - \frac{2}{343}a^6c^3x^7 - \frac{32c^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax}{35a}$$

Mathematica [A] time = 0.280084, size = 125, normalized size = 0.47

$$\frac{c^3\left(-2250a^7x^7+14742a^5x^5-52990a^3x^3-11025ax\left(5a^6x^6-21a^4x^4+35a^2x^2-35\right)\cosh^{-1}(ax)^2+210\sqrt{ax-1}\sqrt{ax+1}\right.}{385875a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3*ArcCosh[a*x]^2,x]
```

```
[Out] (c^3*(453810*a*x - 52990*a^3*x^3 + 14742*a^5*x^5 - 2250*a^7*x^7 + 210*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*(-2161 + 757*a^2*x^2 - 351*a^4*x^4 + 75*a^6*x^6)*Ar
cCosh[a*x] - 11025*a*x*(-35 + 35*a^2*x^2 - 21*a^4*x^4 + 5*a^6*x^6)*ArcCosh[
a*x]^2))/(385875*a)
```

Maple [A] time = 0.148, size = 188, normalized size = 0.7

$$-\frac{c^3}{385875\,a}\left(55125\,\left(\mathrm{arccosh}\,(ax)\right)^2a^7x^7-15750\,\mathrm{arccosh}\,(ax)\,\sqrt{ax-1}\sqrt{ax+1}a^6x^6-231525\,\left(\mathrm{arccosh}\,(ax)\right)^2a^5x^5+73\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x)
```

```
[Out] -1/385875/a*c^3*(55125*arccosh(a*x)^2*a^7*x^7-15750*arccosh(a*x)*(a*x-1)^(1
/2)*(a*x+1)^(1/2)*a^6*x^6-231525*arccosh(a*x)^2*a^5*x^5+73710*arccosh(a*x)*
```

```
a^4*x^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)+2250*a^7*x^7+385875*arccosh(a*x)^2*a^3*
x^3-158970*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^2*x^2-14742*x^5*a^5-3
85875*arccosh(a*x)^2*a*x+453810*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+52
990*x^3*a^3-453810*a*x)
```

Maxima [A] time = 1.20855, size = 240, normalized size = 0.9

$$-\frac{2}{343}a^6c^3x^7+\frac{234}{6125}a^4c^3x^5-\frac{1514}{11025}a^2c^3x^3+\frac{4322}{3675}c^3x+\frac{2}{3675}\left(75\sqrt{a^2x^2-1}a^4c^3x^6-351\sqrt{a^2x^2-1}a^2c^3x^4+757\sqrt{a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/343*a^6*c^3*x^7 + 234/6125*a^4*c^3*x^5 - 1514/11025*a^2*c^3*x^3 + 4322/3
675*c^3*x + 2/3675*(75*sqrt(a^2*x^2 - 1)*a^4*c^3*x^6 - 351*sqrt(a^2*x^2 - 1
)*a^2*c^3*x^4 + 757*sqrt(a^2*x^2 - 1)*c^3*x^2 - 2161*sqrt(a^2*x^2 - 1)*c^3/
a^2)*a*arccosh(a*x) - 1/35*(5*a^6*c^3*x^7 - 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3
 - 35*c^3*x)*arccosh(a*x)^2
```

Fricas [A] time = 2.03693, size = 416, normalized size = 1.56

$$-\frac{2250\,a^7c^3x^7-14742\,a^5c^3x^5+52990\,a^3c^3x^3-453810\,ac^3x+11025\left(5\,a^7c^3x^7-21\,a^5c^3x^5+35\,a^3c^3x^3-35\,ac^3x\right)\log\left(ax\right.}{385875\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/385875*(2250*a^7*c^3*x^7 - 14742*a^5*c^3*x^5 + 52990*a^3*c^3*x^3 - 45381
0*a*c^3*x + 11025*(5*a^7*c^3*x^7 - 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 - 35*a*c
^3*x)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 210*(75*a^6*c^3*x^6 - 351*a^4*c^3*x^
4 + 757*a^2*c^3*x^2 - 2161*c^3)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 -
1)))/a
```

Sympy [A] time = 15.0503, size = 243, normalized size = 0.91

$$\begin{cases} -\frac{a^6c^3x^7\operatorname{acosh}^2(ax)}{7} - \frac{2a^6c^3x^7}{343} + \frac{2a^5c^3x^6\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{49} + \frac{3a^4c^3x^5\operatorname{acosh}^2(ax)}{5} + \frac{234a^4c^3x^5}{6125} - \frac{234a^3c^3x^4\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{1225} - a^2c^3x^3\operatorname{aco} \\ -\frac{\pi^2c^3x}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*acosh(a*x)**2,x)
```

```
[Out] Piecewise((-a**6*c**3*x**7*acosh(a*x)**2/7 - 2*a**6*c**3*x**7/343 + 2*a**5*
c**3*x**6*sqrt(a**2*x**2 - 1)*acosh(a*x)/49 + 3*a**4*c**3*x**5*acosh(a*x)**
2/5 + 234*a**4*c**3*x**5/6125 - 234*a**3*c**3*x**4*sqrt(a**2*x**2 - 1)*acos
h(a*x)/1225 - a**2*c**3*x**3*acosh(a*x)**2 - 1514*a**2*c**3*x**3/11025 + 15
14*a*c**3*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)/3675 + c**3*x*acosh(a*x)**2 +
 4322*c**3*x/3675 - 4322*c**3*sqrt(a**2*x**2 - 1)*acosh(a*x)/(3675*a), Ne(a
, 0)), (-pi**2*c**3*x/4, True))
```

Giac [A] time = 1.18543, size = 227, normalized size = 0.85

$$-\frac{2}{385875}\left(1125\,a^6x^7 - 7371\,a^4x^5 + 26495\,a^2x^3 - 226905\,x - \frac{105\left(75\left(a^2x^2-1\right)^{\frac{7}{2}} - 126\left(a^2x^2-1\right)^{\frac{5}{2}} + 280\left(a^2x^2-1\right)^{\frac{3}{2}} - \right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] -2/385875*(1125*a^6*x^7 - 7371*a^4*x^5 + 26495*a^2*x^3 - 226905*x - 105*(75
*(a^2*x^2 - 1)^(7/2) - 126*(a^2*x^2 - 1)^(5/2) + 280*(a^2*x^2 - 1)^(3/2) -
1680*sqrt(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x^2 - 1))/a)*c^3 - 1/35*(5*a^6*c
^3*x^7 - 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 - 35*c^3*x)*log(a*x + sqrt(a^2*x^2
 - 1))^2
```

3.165 $\int \left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=195

$$\frac{2}{125}a^4c^2x^5-\frac{76}{675}a^2c^2x^3+\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{4}{15}c^2x\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{8}{15}c^2x\cosh^{-1}(ax)^2-\frac{2c^2(ax-$$

```
[Out] (298*c^2*x)/225 - (76*a^2*c^2*x^3)/675 + (2*a^4*c^2*x^5)/125 - (16*c^2*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(15*a) + (8*c^2*(-1 + a*x)^(3/2)*(1
+ a*x)^(3/2)*ArcCosh[a*x])/(45*a) - (2*c^2*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2)
*ArcCosh[a*x])/(25*a) + (8*c^2*x*ArcCosh[a*x]^2)/15 + (4*c^2*x*(1 - a^2*x^2
)*ArcCosh[a*x]^2)/15 + (c^2*x*(1 - a^2*x^2)^2*ArcCosh[a*x]^2)/5
```

Rubi [A] time = 0.46169, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5681, 5718, 194, 5654, 8}

$$\frac{2}{125}a^4c^2x^5-\frac{76}{675}a^2c^2x^3+\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{4}{15}c^2x\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{8}{15}c^2x\cosh^{-1}(ax)^2-\frac{2c^2(ax-$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2*ArcCosh[a*x]^2,x]
```


```
[Out] (298*c^2*x)/225 - (76*a^2*c^2*x^3)/675 + (2*a^4*c^2*x^5)/125 - (16*c^2*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(15*a) + (8*c^2*(-1 + a*x)^(3/2)*(1
+ a*x)^(3/2)*ArcCosh[a*x])/(45*a) - (2*c^2*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2)
*ArcCosh[a*x])/(25*a) + (8*c^2*x*ArcCosh[a*x]^2)/15 + (4*c^2*x*(1 - a^2*x^2
)*ArcCosh[a*x]^2)/15 + (c^2*x*(1 - a^2*x^2)^2*ArcCosh[a*x]^2)/5
```


Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^2\,dx = \frac{1}{5}c^2x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{1}{5}(4c)\int\left(c-a^2cx^2\right)\cosh^{-1}(ax)^2\,dx-\frac{1}{5}\left(2ac^2\right)\int x(-$$

$$= -\frac{2c^2(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)}{25a}+\frac{4}{15}c^2x\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{1}{5}c^2x\left(1-a\right.$$

$$= \frac{8c^2(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}{45a}-\frac{2c^2(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)}{25a}+\frac{8}{15}c^2$$

$$= \frac{58c^2x}{225}-\frac{76}{675}a^2c^2x^3+\frac{2}{125}a^4c^2x^5-\frac{16c^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{15a}+\frac{8c^2(-1+ax)}{}$$

$$= \frac{298c^2x}{225}-\frac{76}{675}a^2c^2x^3+\frac{2}{125}a^4c^2x^5-\frac{16c^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{15a}+\frac{8c^2(-1+ax}{}$$

Mathematica [A] time = 0.222418, size = 101, normalized size = 0.52

$$\frac{c^2\left(54a^5x^5-380a^3x^3+225ax\left(3a^4x^4-10a^2x^2+15\right)\cosh^{-1}(ax)^2-30\sqrt{ax-1}\sqrt{ax+1}\left(9a^4x^4-38a^2x^2+149\right)\cosh^{-1}(\right.}{3375a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2*ArcCosh[a*x]^2,x]
```

```
[Out] (c^2*(4470*a*x - 380*a^3*x^3 + 54*a^5*x^5 - 30*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*(149 - 38*a^2*x^2 + 9*a^4*x^4)*ArcCosh[a*x] + 225*a*x*(15 - 10*a^2*x^2 + 3
*a^4*x^4)*ArcCosh[a*x]^2))/(3375*a)
```

Maple [A] time = 0.047, size = 140, normalized size = 0.7

$$\frac{c^2}{3375\,a}\left(675\,(\operatorname{arccosh}(ax))^2 a^5x^5 - 270\operatorname{arccosh}(ax)\,a^4x^4\sqrt{ax-1}\sqrt{ax+1} - 2250\,(\operatorname{arccosh}(ax))^2 a^3x^3 + 1140\operatorname{arccosh}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x)
```

```
[Out] 1/3375/a*c^2*(675*arccosh(a*x)^2*a^5*x^5-270*arccosh(a*x)*a^4*x^4*(a*x-1)^(
1/2)*(a*x+1)^(1/2)-2250*arccosh(a*x)^2*a^3*x^3+1140*arccosh(a*x)*(a*x-1)^(1
/2)*(a*x+1)^(1/2)*a^2*x^2+54*x^5*a^5+3375*arccosh(a*x)^2*a*x-4470*arccosh(a
*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)-380*x^3*a^3+4470*a*x)
```

Maxima [A] time = 1.24127, size = 181, normalized size = 0.93

$$\frac{2}{125}\,a^4c^2x^5 - \frac{76}{675}\,a^2c^2x^3 + \frac{298}{225}\,c^2x - \frac{2}{225}\left(9\sqrt{a^2x^2-1}a^2c^2x^4 - 38\sqrt{a^2x^2-1}c^2x^2 + \frac{149\sqrt{a^2x^2-1}c^2}{a^2}\right)a\operatorname{arcosh}(ax) + \frac{1}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2/125*a^4*c^2*x^5 - 76/675*a^2*c^2*x^3 + 298/225*c^2*x - 2/225*(9*sqrt(a^2*
x^2 - 1)*a^2*c^2*x^4 - 38*sqrt(a^2*x^2 - 1)*c^2*x^2 + 149*sqrt(a^2*x^2 - 1)
*c^2/a^2)*a*arccosh(a*x) + 1/15*(3*a^4*c^2*x^5 - 10*a^2*c^2*x^3 + 15*c^2*x)
*arccosh(a*x)^2
```

Fricas [A] time = 2.02822, size = 321, normalized size = 1.65

$$\frac{54\,a^5c^2x^5 - 380\,a^3c^2x^3 + 4470\,ac^2x + 225\left(3\,a^5c^2x^5 - 10\,a^3c^2x^3 + 15\,ac^2x\right)\log\left(ax + \sqrt{a^2x^2-1}\right)^2 - 30\left(9\,a^4c^2x^4 - 38\,a^2\right.}{3375\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(54*a^5*c^2*x^5 - 380*a^3*c^2*x^3 + 4470*a*c^2*x + 225*(3*a^5*c^2*x^
5 - 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 30*(9*a^4
*c^2*x^4 - 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x
^2 - 1)))/a
```

Sympy [A] time = 4.98755, size = 182, normalized size = 0.93

$$\begin{cases} \frac{a^4c^2x^5\operatorname{acosh}^2(ax)}{5} + \frac{2a^4c^2x^5}{125} - \frac{2a^3c^2x^4\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{25} - \frac{2a^2c^2x^3\operatorname{acosh}^2(ax)}{3} - \frac{76a^2c^2x^3}{675} + \frac{76ac^2x^2\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{225} + c^2x\operatorname{acosh}^2(ax \\ -\frac{\pi^2c^2x}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*acosh(a*x)**2,x)
```

```
[Out] Piecewise((a**4*c**2*x**5*acosh(a*x)**2/5 + 2*a**4*c**2*x**5/125 - 2*a**3*c
**2*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)/25 - 2*a**2*c**2*x**3*acosh(a*x)**2
/3 - 76*a**2*c**2*x**3/675 + 76*a*c**2*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)/
225 + c**2*x*acosh(a*x)**2 + 298*c**2*x/225 - 298*c**2*sqrt(a**2*x**2 - 1)*
acosh(a*x)/(225*a), Ne(a, 0)), (-pi**2*c**2*x/4, True))
```

Giac [A] time = 1.19807, size = 184, normalized size = 0.94

$$\frac{2}{3375}\left(27\,a^4x^5 - 190\,a^2x^3 + 2235\,x - \frac{15\left(9\left(a^2x^2-1\right)^{\frac{5}{2}} - 20\left(a^2x^2-1\right)^{\frac{3}{2}} + 120\sqrt{a^2x^2-1}\right)\log\left(ax + \sqrt{a^2x^2-1}\right)}{a}\right)c^2 + \frac{1}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] 2/3375*(27*a^4*x^5 - 190*a^2*x^3 + 2235*x - 15*(9*(a^2*x^2 - 1)^(5/2) - 20*
(a^2*x^2 - 1)^(3/2) + 120*sqrt(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x^2 - 1))/a
)*c^2 + 1/15*(3*a^4*c^2*x^5 - 10*a^2*c^2*x^3 + 15*c^2*x)*log(a*x + sqrt(a^2
*x^2 - 1))^2
```

3.166 $\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^2\,dx$

Optimal. Leaf size=112

$$-\frac{2}{27}a^2cx^3+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{2}{3}cx\cosh^{-1}(ax)^2+\frac{2c(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)}{9a}-\frac{4c\sqrt{ax-1}\sqrt{ax+1}}{3a}$$

```
[Out] (14*c*x)/9 - (2*a^2*c*x^3)/27 - (4*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a
*x])/(3*a) + (2*c*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])/(9*a) + (2
*c*x*ArcCosh[a*x]^2)/3 + (c*x*(1 - a^2*x^2)*ArcCosh[a*x]^2)/3
```

Rubi [A] time = 0.263092, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5681, 5718, 5654, 8}

$$-\frac{2}{27}a^2cx^3+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{2}{3}cx\cosh^{-1}(ax)^2+\frac{2c(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)}{9a}-\frac{4c\sqrt{ax-1}\sqrt{ax+1}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)*ArcCosh[a*x]^2,x]
```


```
[Out] (14*c*x)/9 - (2*a^2*c*x^3)/27 - (4*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a
*x])/(3*a) + (2*c*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])/(9*a) + (2
*c*x*ArcCosh[a*x]^2)/3 + (c*x*(1 - a^2*x^2)*ArcCosh[a*x]^2)/3
```


Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
```

```
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^2\,dx &= \frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{1}{3}(2c)\int \cosh^{-1}(ax)^2\,dx+\frac{1}{3}(2ac)\int x\sqrt{-1+ax}\sqrt{1+ax}\,\text{co}\\ &= \frac{2c(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}{9a}+\frac{2}{3}cx\cosh^{-1}(ax)^2+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^2-\\ &= \frac{2cx}{9}-\frac{2}{27}a^2cx^3-\frac{4c\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3a}+\frac{2c(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax}{9a}\\ &= \frac{14cx}{9}-\frac{2}{27}a^2cx^3-\frac{4c\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3a}+\frac{2c(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(a}{9a}\end{aligned}$$

Mathematica [A] time = 0.10888, size = 73, normalized size = 0.65

$$\frac{c\left(-2a^3x^3-9ax\left(a^2x^2-3\right)\cosh^{-1}(ax)^2+6\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2-7\right)\cosh^{-1}(ax)+42ax\right)}{27a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)*ArcCosh[a*x]^2,x]
```

```
[Out] (c*(42*a*x - 2*a^3*x^3 + 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-7 + a^2*x^2)*ArcC
osh[a*x] - 9*a*x*(-3 + a^2*x^2)*ArcCosh[a*x]^2))/(27*a)
```

Maple [A] time = 0.04, size = 90, normalized size = 0.8

$$-\frac{c}{27\,a}\left(9\,\left(\text{arccosh}\,(ax)\right)^2a^3x^3-6\,\text{arccosh}\,(ax)\,\sqrt{ax-1}\sqrt{ax+1}a^2x^2-27\,\left(\text{arccosh}\,(ax)\right)^2ax+42\,\text{arccosh}\,(ax)\,\sqrt{ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*arccosh(a*x)^2,x)
```

```
[Out] -1/27/a*c*(9*arccosh(a*x)^2*a^3*x^3-6*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1
/2)*a^2*x^2-27*arccosh(a*x)^2*a*x+42*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)+2*x^3*a^3-42*a*x)
```

Maxima [A] time = 1.15525, size = 103, normalized size = 0.92

$$-\frac{2}{27}a^2cx^3+\frac{2}{9}\left(\sqrt{a^2x^2-1}cx^2-\frac{7\sqrt{a^2x^2-1}c}{a^2}\right)a\operatorname{arcosh}(ax)-\frac{1}{3}\left(a^2cx^3-3cx\right)\operatorname{arcosh}(ax)^2+\frac{14}{9}cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/27*a^2*c*x^3 + 2/9*(sqrt(a^2*x^2 - 1)*c*x^2 - 7*sqrt(a^2*x^2 - 1)*c/a^2)
*a*arccosh(a*x) - 1/3*(a^2*c*x^3 - 3*c*x)*arccosh(a*x)^2 + 14/9*c*x
```

Fricas [A] time = 2.08833, size = 216, normalized size = 1.93

$$-\frac{2a^3cx^3-42acx+9\left(a^3cx^3-3acx\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2-6\left(a^2cx^2-7c\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{27a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/27*(2*a^3*c*x^3 - 42*a*c*x + 9*(a^3*c*x^3 - 3*a*c*x)*log(a*x + sqrt(a^2*
x^2 - 1))^2 - 6*(a^2*c*x^2 - 7*c)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2
- 1)))/a
```

Sympy [A] time = 1.25717, size = 105, normalized size = 0.94

$$\begin{cases}-\frac{a^2cx^3\operatorname{acosh}^2(ax)}{3}-\frac{2a^2cx^3}{27}+\frac{2acx^2\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{9}+cx\operatorname{acosh}^2(ax)+\frac{14cx}{9}-\frac{14c\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{9a} & \text{for}\ a\neq 0\\ -\frac{\pi^2cx}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*acosh(a*x)**2,x)
```

```
[Out] Piecewise((-a**2*c*x**3*acosh(a*x)**2/3 - 2*a**2*c*x**3/27 + 2*a*c*x**2*sqr
t(a**2*x**2 - 1)*acosh(a*x)/9 + c*x*acosh(a*x)**2 + 14*c*x/9 - 14*c*sqrt(a*
*2*x**2 - 1)*acosh(a*x)/(9*a), Ne(a, 0)), (-pi**2*c*x/4, True))
```

Giac [A] time = 1.19174, size = 127, normalized size = 1.13

$$-\frac{1}{3}\left(a^2cx^3 - 3\,cx\right)\log\left(ax + \sqrt{a^2x^2 - 1}\right)^2 - \frac{2}{27}\left(a^2x^3 - 21\,x - \frac{3\left(\left(a^2x^2 - 1\right)^{\frac{3}{2}} - 6\,\sqrt{a^2x^2 - 1}\right)\log\left(ax + \sqrt{a^2x^2 - 1}\right)}{a}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] -1/3*(a^2*c*x^3 - 3*c*x)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 2/27*(a^2*x^3 - 2
1*x - 3*((a^2*x^2 - 1)^(3/2) - 6*sqrt(a^2*x^2 - 1))*log(a*x + sqrt(a^2*x^2
- 1))/a)*c
```

3.167 $\int \frac{\cosh^{-1}(ax)^2}{c-a^2cx^2}\,dx$

Optimal. Leaf size=98

$$\frac{2\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\text{PolyLo}}{}$$

```
[Out] (2*ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(a*c) + (2*ArcCosh[a*x]*PolyLog[
2, -E^ArcCosh[a*x]])/(a*c) - (2*ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(a
*c) - (2*PolyLog[3, -E^ArcCosh[a*x]])/(a*c) + (2*PolyLog[3, E^ArcCosh[a*x]]
)/(a*c)
```

Rubi [A] time = 0.0978093, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5694, 4182, 2531, 2282, 6589}

$$\frac{2\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2),x]
```


```
[Out] (2*ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(a*c) + (2*ArcCosh[a*x]*PolyLog[
2, -E^ArcCosh[a*x]])/(a*c) - (2*ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(a
*c) - (2*PolyLog[3, -E^ArcCosh[a*x]])/(a*c) + (2*PolyLog[3, E^ArcCosh[a*x]]
)/(a*c)
```


Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{c-a^2cx^2}\,dx = -\frac{\text{Subst}\left(\int x^2\,\mathbf{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac}$$

$$= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\,\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \cosh^{-1}(ax)\right)}{ac} - \frac{2\,\text{Subst}\left(\int x\,\text{lo}\right.}{}$$

$$= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\cosh^{-1}(ax)\mathbf{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{2\cosh^{-1}(ax)\mathbf{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

$$= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\cosh^{-1}(ax)\mathbf{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{2\cosh^{-1}(ax)\mathbf{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

$$= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\cosh^{-1}(ax)\mathbf{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{2\cosh^{-1}(ax)\mathbf{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

Mathematica [A] time = 0.0794439, size = 95, normalized size = 0.97

$$\frac{2\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)-2\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)-2\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)+2\text{PolyLog}}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2),x]
```

```
[Out] (-(ArcCosh[a*x]^2*Log[1 - E^ArcCosh[a*x]]) + ArcCosh[a*x]^2*Log[1 + E^ArcCo
sh[a*x]] + 2*ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]] - 2*ArcCosh[a*x]*Poly
Log[2, E^ArcCosh[a*x]] - 2*PolyLog[3, -E^ArcCosh[a*x]] + 2*PolyLog[3, E^Arc
Cosh[a*x]])/(a*c)
```

Maple [A] time = 0.043, size = 201, normalized size = 2.1

$$-\frac{(\text{arccosh}(ax))^2}{ac}\ln\left(1-ax-\sqrt{ax-1}\sqrt{ax+1}\right)-2\frac{\text{arccosh}(ax)\,\text{polylog}\left(2,ax+\sqrt{ax-1}\sqrt{ax+1}\right)}{ac}+2\frac{\text{polylog}\left(3,ax\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c),x)
```

```
[Out] -1/a/c*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-2*arccosh(a*x)*
polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+2*polylog(3,a*x+(a*x-1)^(1/2
)*(a*x+1)^(1/2))/a/c+1/a/c*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1
/2))+2*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-2*polyl
og(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(\log(ax+1)-\log(ax-1))\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{2\,ac}-\int\frac{\left((ax\log(ax+1)-ax\log(ax-1))\sqrt{ax+1}\sqrt{ax-1}+\left(a^2\right.\right.}{a^3cx^3-acx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1) - log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/
(a*c) - integrate(((a*x*log(a*x + 1) - a*x*log(a*x - 1))*sqrt(a*x + 1)*sqrt
(a*x - 1) + (a^2*x^2 - 1)*log(a*x + 1) - (a^2*x^2 - 1)*log(a*x - 1))*log(a*
x + sqrt(a*x + 1)*sqrt(a*x - 1))/(a^3*c*x^3 - a*c*x + (a^2*c*x^2 - c)*sqrt(
a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{arcosh}(ax)^2}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^2/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\operatorname{acosh}^2(ax)}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(acosh(a*x)**2/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arcosh}(ax)^2}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)^2/(a^2*c*x^2 - c), x)
```

3.168 $\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=163

$$\frac{\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac^2}+\frac{\text{PolyLog}\left(3,\right.}{ac^{}}$$

```
[Out] -(ArcCosh[a*x]/(a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])) + (x*ArcCosh[a*x]^2)/(
2*c^2*(1 - a^2*x^2)) + (ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) - A
rcTanh[a*x]/(a*c^2) + (ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]])/(a*c^2) -
(ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(a*c^2) - PolyLog[3, -E^ArcCosh[a
*x]]/(a*c^2) + PolyLog[3, E^ArcCosh[a*x]]/(a*c^2)
```

Rubi [A] time = 0.287199, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5689, 5718, 207, 5694, 4182, 2531, 2282, 6589}

$$\frac{\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac^2}+\frac{\text{PolyLog}\left(3,\right.}{ac^{}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2)^2,x]
```


```
[Out] -(ArcCosh[a*x]/(a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])) + (x*ArcCosh[a*x]^2)/(
2*c^2*(1 - a^2*x^2)) + (ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) - A
rcTanh[a*x]/(a*c^2) + (ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]])/(a*c^2) -
(ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(a*c^2) - PolyLog[3, -E^ArcCosh[a
*x]]/(a*c^2) + PolyLog[3, E^ArcCosh[a*x]]/(a*c^2)
```


Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
```

```
egerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^2}\,dx = \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{a\int \frac{x\cosh^{-1}(ax)}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c^2} + \frac{\int \frac{\cosh^{-1}(ax)^2}{c-a^2cx^2}\,dx}{2c}$$

$$= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{c^2} - \frac{\text{Subst}\left(\int x^2\mathbf{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{2ac^2}$$

$$= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \frac{\tanh^{-1}(ax)}{ac^2} + \frac{\text{Sub}\ldots}{}$$

$$= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \frac{\tanh^{-1}(ax)}{ac^2} + \frac{\cosh\ldots}{}$$

$$= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \frac{\tanh^{-1}(ax)}{ac^2} + \frac{\cosh\ldots}{}$$

$$= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \frac{\tanh^{-1}(ax)}{ac^2} + \frac{\cosh\ldots}{}$$

Mathematica [A] time = 0.918435, size = 191, normalized size = 1.17

$$\frac{-8\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\cosh^{-1}(ax)}\right) + 8\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{-\cosh^{-1}(ax)}\right) - 8\text{PolyLog}\left(3,-e^{-\cosh^{-1}(ax)}\right) + 8\text{Po}\ldots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-4*ArcCosh[a*x]*Coth[ArcCosh[a*x]/2] - ArcCosh[a*x]^2*Csch[ArcCosh[a*x]/2]
^2 - 4*ArcCosh[a*x]^2*Log[1 - E^(-ArcCosh[a*x])] + 4*ArcCosh[a*x]^2*Log[1 +
 E^(-ArcCosh[a*x])] + 8*Log[Tanh[ArcCosh[a*x]/2]] - 8*ArcCosh[a*x]*PolyLog[
2, -E^(-ArcCosh[a*x])] + 8*ArcCosh[a*x]*PolyLog[2, E^(-ArcCosh[a*x])] - 8*P
olyLog[3, -E^(-ArcCosh[a*x])] + 8*PolyLog[3, E^(-ArcCosh[a*x])] - ArcCosh[a
*x]^2*Sech[ArcCosh[a*x]/2]^2 + 4*ArcCosh[a*x]*Tanh[ArcCosh[a*x]/2])/(8*a*c^
2)
```

Maple [A] time = 0.084, size = 288, normalized size = 1.8

$$-\frac{x\left(\operatorname{arccosh}(ax)\right)^{2}}{\left(2a^{2}x^{2}-2\right)c^{2}}-\frac{\operatorname{arccosh}(ax)}{a\left(a^{2}x^{2}-1\right)c^{2}}\sqrt{ax-1}\sqrt{ax+1}-\frac{\left(\operatorname{arccosh}(ax)\right)^{2}}{2ac^{2}}\ln\left(1-ax-\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{\operatorname{arccosh}(ax)}{ac^{2}}\mathrm{po}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/(a^2*x^2-1)*arccosh(a*x)^2/c^2*x-1/a/(a^2*x^2-1)*arccosh(a*x)/c^2*(a*x
-1)^(1/2)*(a*x+1)^(1/2)-1/2/a/c^2*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*
x+1)^(1/2))-arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+p
olylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+1/2/a/c^2*arccosh(a*x)^2*ln
(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/
2)*(a*x+1)^(1/2))/a/c^2-polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-2
/a/c^2*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2ax-\left(a^{2}x^{2}-1\right)\log(ax+1)+\left(a^{2}x^{2}-1\right)\log(ax-1)\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^{2}}{4\left(a^{3}c^{2}x^{2}-ac^{2}\right)}-\int-\frac{\left(2a^{3}x^{3}+\left(2a^{2}x^{2}-\left(a^{3}x^{3}-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(a*x - 1))*log(
a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(a^3*c^2*x^2 - a*c^2) - integrate(-1/2
*(2*a^3*x^3 + (2*a^2*x^2 - (a^3*x^3 - a*x)*log(a*x + 1) + (a^3*x^3 - a*x)*l
og(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x - 1) - 2*a*x - (a^4*x^4 - 2*a^2*x^2 + 1
```

```
)*log(a*x + 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x
 + 1)*sqrt(a*x - 1))/(a^5*c^2*x^5 - 2*a^3*c^2*x^3 + a*c^2*x + (a^4*c^2*x^4
- 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcosh}(ax)^2}{a^4c^2x^4 - 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^2/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{acosh}^2(ax)}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(acosh(a*x)**2/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^2}{\left(a^2cx^2 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/(a^2*c*x^2 - c)^2, x)
```

3.169 $\int \frac{\cosh^{-1}(ax)^2}{(c-a^2cx^2)^3} dx$

Optimal. Leaf size=258

$$\frac{3\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{3\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{3\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{4ac^3}+\frac{3\text{PolyLog}}{}$$

```
[Out] -x/(12*c^3*(1 - a^2*x^2)) + ArcCosh[a*x]/(6*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x
)^(3/2)) - (3*ArcCosh[a*x])/(4*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Arc
Cosh[a*x]^2)/(4*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x]^2)/(8*c^3*(1 - a^2
*x^2)) + (3*ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(4*a*c^3) - (5*ArcTanh[
a*x])/(6*a*c^3) + (3*ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]])/(4*a*c^3) -
(3*ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(4*a*c^3) - (3*PolyLog[3, -E^Ar
cCosh[a*x]])/(4*a*c^3) + (3*PolyLog[3, E^ArcCosh[a*x]])/(4*a*c^3)
```

Rubi [A] time = 0.49265, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5689, 5718, 199, 207, 5694, 4182, 2531, 2282, 6589}

$$\frac{3\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{3\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{3\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{4ac^3}+\frac{3\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2)^3,x]
```


```
[Out] -x/(12*c^3*(1 - a^2*x^2)) + ArcCosh[a*x]/(6*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x
)^(3/2)) - (3*ArcCosh[a*x])/(4*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Arc
Cosh[a*x]^2)/(4*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x]^2)/(8*c^3*(1 - a^2
*x^2)) + (3*ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(4*a*c^3) - (5*ArcTanh[
a*x])/(6*a*c^3) + (3*ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]])/(4*a*c^3) -
(3*ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(4*a*c^3) - (3*PolyLog[3, -E^Ar
cCosh[a*x]])/(4*a*c^3) + (3*PolyLog[3, E^ArcCosh[a*x]])/(4*a*c^3)
```


Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
```

```
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^3}\,dx = \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} - \frac{a\int \frac{x\cosh^{-1}(ax)}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{2c^3} + \frac{3\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} + \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)^2}{8c^3\left(1-a^2x^2\right)} - \frac{\int \frac{1}{\left(-1+a^2x^2\right)^2}\,dx}{6c^3} + \frac{(3a)\int \frac{x\cos}{(-1+ax)}}{4c}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\,c}{8c^3}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\,c}{8c^3}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\,c}{8c^3}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\,c}{8c^3}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\,c}{8c^3}$$

Mathematica [A] time = 4.8041, size = 319, normalized size = 1.24

$$-\frac{72\left(2\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\cosh^{-1}(ax)}\right) - 2\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{-\cosh^{-1}(ax)}\right) + 2\text{PolyLog}\left(3,-e^{-\cosh^{-1}(ax)}\right) - \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2)^3,x]
```

```
[Out] -(80*ArcCosh[a*x]*Coth[ArcCosh[a*x]/2] + 2*(-2 + 9*ArcCosh[a*x]^2)*Csch[Arc
Cosh[a*x]/2]^2 - 2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]*Csch[A
rcCosh[a*x]/2]^4 - 3*ArcCosh[a*x]^2*Csch[ArcCosh[a*x]/2]^4 - 160*Log[Tanh[A
rcCosh[a*x]/2]] + 72*(ArcCosh[a*x]^2*Log[1 - E^(-ArcCosh[a*x])] - ArcCosh[a
*x]^2*Log[1 + E^(-ArcCosh[a*x])] + 2*ArcCosh[a*x]*PolyLog[2, -E^(-ArcCosh[a
*x])] - 2*ArcCosh[a*x]*PolyLog[2, E^(-ArcCosh[a*x])] + 2*PolyLog[3, -E^(-Ar
```

```
cCosh[a*x])] - 2*PolyLog[3, E^(-ArcCosh[a*x])]) + 2*(-2 + 9*ArcCosh[a*x]^2)
*Sech[ArcCosh[a*x]/2]^2 + 3*ArcCosh[a*x]^2*Sech[ArcCosh[a*x]/2]^4 - (32*Arc
Cosh[a*x]*Sinh[ArcCosh[a*x]/2]^4)/(((-1 + a*x)/(1 + a*x))^(3/2)*(1 + a*x)^3
) - 80*ArcCosh[a*x]*Tanh[ArcCosh[a*x]/2])/(192*a*c^3)
```

Maple [A] time = 0.143, size = 443, normalized size = 1.7

$$-\frac{3a^2(\operatorname{arccosh}(ax))^2x^3}{(8x^4a^4-16a^2x^2+8)c^3}-\frac{3a\operatorname{arccosh}(ax)x^2}{(4x^4a^4-8a^2x^2+4)c^3}\sqrt{ax-1}\sqrt{ax+1}+\frac{a^2x^3}{(12x^4a^4-24a^2x^2+12)c^3}+\frac{5x(\operatorname{arccosh}(ax}{(8x^4a^4-16a^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -3/8*a^2/(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)^2*x^3-3/4*a/(a^4*x^4-2*a^2*
x^2+1)/c^3*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*x^2+1/12*a^2/(a^4*x^4-2
*a^2*x^2+1)/c^3*x^3+5/8/(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)^2*x+11/12/a/
(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/12/(a^
4*x^4-2*a^2*x^2+1)/c^3*x-5/3/a/c^3*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
-3/8/a/c^3*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-3/4*arccosh
(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+3/4*polylog(3,a*x+(a
*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+3/8/a/c^3*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))+3/4*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^
(1/2))/a/c^3-3/4*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(6a^3x^3-10ax-3\left(a^4x^4-2a^2x^2+1\right)\log(ax+1)+3\left(a^4x^4-2a^2x^2+1\right)\log(ax-1)\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{16\left(a^5c^3x^4-2a^3c^3x^2+ac^3\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)
)^2/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3) - integrate(-1/8*(6*a^5*x^5 - 16*
a^3*x^3 + (6*a^4*x^4 - 10*a^2*x^2 - 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x +
```

```
 1) + 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x -
1) + 10*a*x - 3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) + 3*(a^6
*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqr
t(a*x - 1))/(a^7*c^3*x^7 - 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 - a*c^3*x + (a^6*c
^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(a*x + 1)*sqrt(a*x - 1)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{arcosh}(ax)^2}{a^6c^3x^6 - 3\,a^4c^3x^4 + 3\,a^2c^3x^2 - c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^2/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\operatorname{acosh}^2(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(acosh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**
3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arcosh}(ax)^2}{\left(a^2cx^2 - c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")

[Out] integrate(-arccosh(a*x)^2/(a^2*c*x^2 - c)^3, x)
```

3.170 $\int x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=371

$$\frac{4abx\sqrt{d-c^2dx^2}}{15c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{2bx^3\sqrt{d-c^2dx^2}\left(a\right.}{45c\sqrt{cx-\right.}}$$

```
[Out] (-856*b^2*Sqrt[d - c^2*d*x^2])/(3375*c^4) + (22*b^2*x^2*Sqrt[d - c^2*d*x^2]
)/(3375*c^2) + (2*b^2*x^4*Sqrt[d - c^2*d*x^2])/125 + (4*a*b*x*Sqrt[d - c^2*
d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b^2*x*Sqrt[d - c^2*d*x^2
]*ArcCosh[c*x])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*x^3*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*
c*x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^4) - (x^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^2) + (x^4*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^2)/5
```

Rubi [A] time = 1.05567, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {5798, 5743, 5759, 5718, 5654, 74, 5662, 100, 12}

$$\frac{4abx\sqrt{d-c^2dx^2}}{15c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{2bx^3\sqrt{d-c^2dx^2}\left(a\right.}{45c\sqrt{cx-\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```


```
[Out] (-856*b^2*Sqrt[d - c^2*d*x^2])/(3375*c^4) + (22*b^2*x^2*Sqrt[d - c^2*d*x^2]
)/(3375*c^2) + (2*b^2*x^4*Sqrt[d - c^2*d*x^2])/125 + (4*a*b*x*Sqrt[d - c^2*
d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b^2*x*Sqrt[d - c^2*d*x^2
]*ArcCosh[c*x])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*x^3*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*
c*x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^4) - (x^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^2) + (x^4*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^2)/5
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
```

```
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\sqrt{d-c^2dx^2}\int x^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{5}x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(2bc\right.}{}$$

$$= -\frac{2bcx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{15c^2} + \frac{1}{5}$$

$$= \frac{2}{125}b^2x^4\sqrt{d-c^2dx^2} + \frac{2bx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{45c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcx^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+}}$$

$$= -\frac{2b^2x^2\sqrt{d-c^2dx^2}}{135c^2} + \frac{2}{125}b^2x^4\sqrt{d-c^2dx^2} + \frac{4abx\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bx^3\sqrt{d}}{4}$$

$$= \frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2} + \frac{2}{125}b^2x^4\sqrt{d-c^2dx^2} + \frac{4abx\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4b^2x\sqrt{d}}{15c^3\sqrt{}}$$

$$= -\frac{8b^2\sqrt{d-c^2dx^2}}{27c^4} + \frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2} + \frac{2}{125}b^2x^4\sqrt{d-c^2dx^2} + \frac{4abx\sqrt{d-c}}{15c^3\sqrt{-1+cx}}$$

$$= -\frac{856b^2\sqrt{d-c^2dx^2}}{3375c^4} + \frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2} + \frac{2}{125}b^2x^4\sqrt{d-c^2dx^2} + \frac{4abx\sqrt{d}}{15c^3\sqrt{-1+}}$$

Mathematica [A] time = 0.427265, size = 237, normalized size = 0.64

$$\frac{\sqrt{d-c^2dx^2}\left(225a^2\left(c^2x^2-1\right)^2\left(3c^2x^2+2\right)-30abcx\sqrt{cx-1}\sqrt{cx+1}\left(9c^4x^4-5c^2x^2-30\right)+30b\cosh^{-1}(cx)\left(15a\left(3c^2x^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(225*a^2*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2) - 30*a*b*c*x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-30 - 5*c^2*x^2 + 9*c^4*x^4) + 2*b^2*(428 -
439*c^2*x^2 - 16*c^4*x^4 + 27*c^6*x^6) + 30*b*(15*a*(-1 + c^2*x^2)^2*(2 + 3
*c^2*x^2) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(30 + 5*c^2*x^2 - 9*c^4*x^4)
)*ArcCosh[c*x] + 225*b^2*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2)*ArcCosh[c*x]^2))/
(3375*c^4*(-1 + c^2*x^2))
```

Maple [B] time = 0.544, size = 1284, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(-1/5*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(-c^2*d*x^2+d)^(3/2))+b
^2*(1/4000*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*arccosh(c*x)^2-10*arccosh(c*x)+2)/(c*x
+1)/c^4/(c*x-1)+1/864*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccos
h(c*x)^2-6*arccosh(c*x)+2)/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*
((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+
2)/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)/(c*x+1)/c^4/(c*x-1)+
1/864*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*
x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arcc
osh(c*x)+2)/(c*x+1)/c^4/(c*x-1)+1/4000*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c
^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c
*x)^2+10*arccosh(c*x)+2)/(c*x+1)/c^4/(c*x-1))+2*a*b*(1/800*(-d*(c^2*x^2-1))
^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2
*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c-1)*(-1+5*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/288*(-d*(c^2*x^2-1))^(1/2)*
(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*
x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))
/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/288*(-d*(c^2*x^2-
1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/80
0*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^
6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))/(c*x+1)/c^4/(c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27616, size = 753, normalized size = 2.03

$$\frac{225\left(3\,b^2c^6x^6-4\,b^2c^4x^4-b^2c^2x^2+2\,b^2\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-30\left(9\,abc^5x^5-5\,abc^3x^3-30\,abcx\right)\sqrt{-c^2dx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 - 4*b^2*c^4*x^4 - b^2*c^2*x^2 + 2*b^2)*sqrt(-c^2
*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(9*a*b*c^5*x^5 - 5*a*b*c^3*
x^3 - 30*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30*((9*b^2*c^5*x
^5 - 5*b^2*c^3*x^3 - 30*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 1
5*(3*a*b*c^6*x^6 - 4*a*b*c^4*x^4 - a*b*c^2*x^2 + 2*a*b)*sqrt(-c^2*d*x^2 + d
))*log(c*x + sqrt(c^2*x^2 - 1)) + (27*(25*a^2 + 2*b^2)*c^6*x^6 - 4*(225*a^2
 + 8*b^2)*c^4*x^4 - (225*a^2 + 878*b^2)*c^2*x^2 + 450*a^2 + 856*b^2)*sqrt(-
c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.171 $\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=319

$$-\frac{bcx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{cx-1}\sqrt{cx+1}}-\frac{x\sqrt{d-}}{}$$

```
[Out] -(b^2*x*Sqrt[d - c^2*d*x^2])/(64*c^2) + (b^2*x^3*Sqrt[d - c^2*d*x^2])/32 -
(b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(64*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) + (b*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (b*c*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(8
*c^2) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/4 - (Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x])^3)/(24*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.93039, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {5798, 5743, 5759, 5676, 5662, 90, 52, 100, 12}

$$-\frac{bcx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{cx-1}\sqrt{cx+1}}-\frac{x\sqrt{d-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```


```
[Out] -(b^2*x*Sqrt[d - c^2*d*x^2])/(64*c^2) + (b^2*x^3*Sqrt[d - c^2*d*x^2])/32 -
(b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(64*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) + (b*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (b*c*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(8
*c^2) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/4 - (Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x])^3)/(24*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
```

```
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\sqrt{d-c^2dx^2}\int x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\int\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(b}{}$$

$$= -\frac{bcx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{8c^2} + \frac{1}{4}$$

$$= \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} + \frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcx^4\sqrt{d-c^2dx^2}\,(a}{8\sqrt{-1+cx}}$$

$$= -\frac{b^2x\sqrt{d-c^2dx^2}}{16c^2} + \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} + \frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{b^2x\sqrt{d-c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} - \frac{b^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{16c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bx^2\sqrt{d}}{8}$$

$$= -\frac{b^2x\sqrt{d-c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} - \frac{b^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{64c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bx^2\sqrt{d}}{8}$$

Mathematica [A] time = 1.99127, size = 241, normalized size = 0.76

$$-\frac{-96a^2cx\left(2c^2x^2-1\right)\sqrt{d-c^2dx^2} + 96a^2\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) + \frac{12ab\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx)\sinh(}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}}{768c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -(-96*a^2*c*x*(-1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2] + 96*a^2*Sqrt[d]*ArcTan[
(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (12*a*b*Sqrt[d - c^2*
d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*Arc
Cosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b^2*Sqrt[d - c^2*d*x
^2]*(32*ArcCosh[c*x]^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*Ar
cCosh[c*x]^2)*Sinh[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
)/(768*c^3)
```

Maple [B] time = 0.376, size = 767, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a^2/c^2*x*(-c^2*d*x^2+d)^(1/2)+1/
8*a^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/8*
b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*arccosh(c*x)*x^2-1
/8*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*arccosh(c*x)*x^
4-1/24*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c
*x)^3-1/64*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)*arcco
sh(c*x)+1/32*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)*c^2/(c*x-1)*x^5-3/64*b^2*(-
d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*x^3+1/64*b^2*(-d*(c^2*x^2-1))^(1/2)/(c
*x+1)/c^2/(c*x-1)*x+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)*c^2/(c*x-1)*arcc
osh(c*x)^2*x^5-3/8*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)^
2*x^3+1/8*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)^2*x-1
/8*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^
2-1/8*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+1/8*a*b*
(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+1/2*a*b*(-d*(c^2*x
^2-1))^(1/2)/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5-3/4*a*b*(-d*(c^2*x^2-1))^
(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+
1)/c^2/(c*x-1)*arccosh(c*x)*x-1/64*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)
/c^3/(c*x-1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^2\operatorname{arcosh}(cx)^2 + 2\,abx^2\operatorname{arcosh}(cx) + a^2x^2\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(-
c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.172 $\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=186

$$-\frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d}+\frac{2}{27}b^2x$$

```
[Out] (-14*b^2*Sqrt[d - c^2*d*x^2])/(27*c^2) + (2*b^2*x^2*Sqrt[d - c^2*d*x^2])/27
 + (2*b*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (2*b*c*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/(
3*c^2*d)
```

Rubi [A] time = 0.370248, antiderivative size = 194, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 5718, 5680, 12, 460, 74}

$$-\frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-14*b^2*Sqrt[d - c^2*d*x^2])/(27*c^2) + (2*b^2*x^2*Sqrt[d - c^2*d*x^2])/27
 + (2*b*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (2*b*c*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - ((1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^2)/(3*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\sqrt{d-c^2dx^2}\int x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2} - \frac{\left(2b\sqrt{d-c^2dx^2}\right)\int\left(-1+c^2x\right.}{3c\sqrt{-1+cx}}$$

$$= \frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(1}{}$$

$$= \frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(1}{}$$

$$= \frac{2}{27}b^2x^2\sqrt{d-c^2dx^2} + \frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcx^3\sqrt{d-c^2dx^2}\left(a\right.}{9\sqrt{-1+cx}}$$

$$= -\frac{14b^2\sqrt{d-c^2dx^2}}{27c^2} + \frac{2}{27}b^2x^2\sqrt{d-c^2dx^2} + \frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\sqrt{1+cx}} -$$

Mathematica [A] time = 0.347932, size = 181, normalized size = 0.97

$$\frac{\sqrt{d-c^2dx^2}\left(9a^2\left(c^2x^2-1\right)^2 - 6abcx\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2-3\right) + 6b\cosh^{-1}(cx)\left(3a\left(c^2x^2-1\right)^2 + bcx\sqrt{cx-1}\sqrt{cx+1}\left(3-\right.\right.\right.}{27c^2\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(-6*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-3 + c^2*x^2
) + 9*a^2*(-1 + c^2*x^2)^2 + 2*b^2*(7 - 8*c^2*x^2 + c^4*x^4) + 6*b*(b*c*x*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - c^2*x^2) + 3*a*(-1 + c^2*x^2)^2)*ArcCosh[c
*x] + 9*b^2*(-1 + c^2*x^2)^2*ArcCosh[c*x]^2))/(27*c^2*(-1 + c^2*x^2))
```

Maple [B] time = 0.366, size = 726, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a^2/c^2/d*(-c^2*d*x^2+d)^(3/2)+b^2*(1/216*(-d*(c^2*x^2-1))^(1/2)*(4*c^
4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)/(c*x+1)/c^2/(c*x-1)-1/8
*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccos
h(c*x)^2-2*arccosh(c*x)+2)/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(
-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+
2)/(c*x+1)/c^2/(c*x-1)+1/216*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(
9*arccosh(c*x)^2+6*arccosh(c*x)+2)/(c*x+1)/c^2/(c*x-1))+2*a*b*(1/72*(-d*(c^
2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-
3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^2/(c*x-1
)-1/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-
1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)+1/72*
(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c*x+1)/c^2
/(c*x-1))
```

Maxima [A] time = 1.13214, size = 275, normalized size = 1.48

$$\frac{2}{27}\,b^2\left(\frac{\sqrt{c^2x^2-1}\sqrt{-d}dx^2-\frac{7\sqrt{c^2x^2-1}\sqrt{-d}d}{c^2}}{d}-\frac{3\left(c^2\sqrt{-d}dx^3-3\sqrt{-d}dx\right)\operatorname{arcosh}(cx)}{cd}\right)-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}b^2\operatorname{arcosh}(cx)^2}{3\,c^2d}-\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/27*b^2*((sqrt(c^2*x^2 - 1)*sqrt(-d)*d*x^2 - 7*sqrt(c^2*x^2 - 1)*sqrt(-d)*
d/c^2)/d - 3*(c^2*sqrt(-d)*d*x^3 - 3*sqrt(-d)*d*x)*arccosh(c*x)/(c*d)) - 1/
3*(-c^2*d*x^2 + d)^(3/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/3*(-c^2*d*x^2 + d)^
(3/2)*a*b*arccosh(c*x)/(c^2*d) - 2/9*(c^2*sqrt(-d)*d*x^3 - 3*sqrt(-d)*d*x)*
a*b/(c*d) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

Fricas [A] time = 2.27491, size = 591, normalized size = 3.18

$$\frac{9\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-6\left(abc^3x^3-3\,abcx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}-6\left(\left(b^2c^3x^3-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*sqrt(-c^2*d*x^2 + d)*log(c*x +
sqrt(c^2*x^2 - 1))^2 - 6*(a*b*c^3*x^3 - 3*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqr
t(c^2*x^2 - 1) - 6*((b^2*c^3*x^3 - 3*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2
*x^2 - 1) - 3*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*sqrt(-c^2*d*x^2 + d))*log
(c*x + sqrt(c^2*x^2 - 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 - 2*(9*a^2 + 8*b^2)*c^
2*x^2 + 9*a^2 + 14*b^2)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.173 $\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}b^2x\sqrt{d}$$

```
[Out] (b^2*x*Sqrt[d - c^2*d*x^2])/4 + (b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(4*c
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(6*b*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.348636, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5713, 5683, 5676, 5662, 90, 52}

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}b^2x\sqrt{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d - c^2*d*x^2])/4 + (b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(4*c
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(6*b*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=\frac{\sqrt{d-c^2dx^2}\int\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{\sqrt{d-c^2dx^2}\int\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(bc\sqrt{d\right.}$$

$$=-\frac{bcx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-$$

$$=\frac{1}{4}b^2x\sqrt{d-c^2dx^2}-\frac{bcx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\right.$$

$$=\frac{1}{4}b^2x\sqrt{d-c^2dx^2}+\frac{b^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.03956, size = 235, normalized size = 1.15

$$\frac{1}{24}\left(12a^2x\sqrt{d-c^2dx^2}-\frac{12a^2\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{c}-\frac{6ab\sqrt{d-c^2dx^2}\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)-2\cosh^{-1}(\right.}{c\sqrt{\frac{cx-1}{cx+1}}(cx+1)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (12*a^2*x*Sqrt[d - c^2*d*x^2] - (12*a^2*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*
x^2])/(Sqrt[d]*(-1 + c^2*x^2))]])/c - (6*a*b*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[
c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(c*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b^2*Sqrt[d - c^2*d*x^2]*(-4*ArcCosh[
c*x]^3 - 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] + (3 + 6*ArcCosh[c*x]^2)*Sinh[
2*ArcCosh[c*x]]))/(c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/24
```

Maple [B] time = 0.224, size = 528, normalized size = 2.6

$$\frac{xa^2}{2}\sqrt{-c^2dx^2+d}+\frac{a^2d}{2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}-\frac{b^2\left(\operatorname{arccosh}(cx)\right)^3}{6c}\sqrt{-d\left(c^2x^2-1\right)}\frac{1}{\sqrt{cx-1}}\frac{1}{\sqrt{cx+1}}-\frac{b^2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*x*a^2*(-c^2*d*x^2+d)^(1/2)+1/2*a^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))-1/6*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)/c*arccosh(c*x)^3-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*c*arccosh(c*x)*x^2+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)
*c^2*arccosh(c*x)^2*x^3-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arcc
osh(c*x)^2*x+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*c^2*x^3-1/4*b^2
*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*x+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/(c
*x+1)^(1/2)/(c*x-1)^(1/2)/c*arccosh(c*x)-1/2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2+a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1
)/(c*x-1)*c^2*arccosh(c*x)*x^3-1/2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)
/(c*x-1)^(1/2)*c*x^2-a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x
)*x+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.174 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=402

$$\frac{2ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] 2*b^2*Sqrt[d - c^2*d*x^2] - (2*a*b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - (2*b^2*c*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2 - (2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + ((2*I)*b*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLo
g[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - ((2*I)*b^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*E^Arc
Cosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^2*Sqrt[d - c^2*d*x^2]
*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.786137, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {5798, 5743, 5761, 4180, 2531, 2282, 6589, 5654, 74}

$$\frac{2ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ib\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] 2*b^2*Sqrt[d - c^2*d*x^2] - (2*a*b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - (2*b^2*c*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2 - (2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + ((2*I)*b*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLo
g[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - ((2*I)*b^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*E^Arc
Cosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^2*Sqrt[d - c^2*d*x^2]
```

```
*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(2bc\sqrt{d-\right.}{}$$

$$= -\frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int(\right.}{\sqrt{-}}$$

$$= -\frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-}\right.$$

$$= 2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2d}$$

$$= 2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2d}$$

$$= 2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2d}$$

Mathematica [A] time = 1.22036, size = 449, normalized size = 1.12

$$\frac{2ab\sqrt{d-c^2dx^2}\left(i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right) - i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right) - cx + cx\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx) + \sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] a^2*Sqrt[d - c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[d
]*Sqrt[d - c^2*d*x^2]] + (2*a*b*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*
x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] +
I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCo
sh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x
]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + b^2*Sqrt[d - c^2*d*x^2]*(2 +
(2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])/(1 - c*x) + ArcCosh[c*x]^2
+ (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x]^2*Log[1 + I/E
```

```
^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - 2*ArcCosh
[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[c*x]] - 2*
PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))
```

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{arccosh}(cx))^2}{x}\sqrt{-c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^2dx^2 + d}\left(b^2\operatorname{arcosh}(cx)^2 + 2\,ab\operatorname{arcosh}(cx) + a^2\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2/x, x)
```

3.175 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=234

$$-\frac{b^2c\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{cx-1}\sqrt{cx+1}}+\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d}}{}$$

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x) + (c*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x])^3)/(3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (b^2*c*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(-2*Arc
Cosh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.631433, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5798, 5738, 5660, 3718, 2190, 2279, 2391, 5676}

$$\frac{b^2c\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{cx-1}\sqrt{cx+1}}-\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x) - (c*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x])^3)/(3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (b^2*c*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(2*ArcCo
sh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} + \frac{\left(2bc\sqrt{d-c^2dx^2}\right)\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d}\right.}{}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} + \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(2bc\sqrt{d}\right.}{}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c\sqrt{d-c^2}}{3b}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c\sqrt{d-c^2}}{3b}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c\sqrt{d-c^2}}{3b}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c\sqrt{d-c^2}}{3b}$$

Mathematica [A] time = 1.62442, size = 270, normalized size = 1.15

$$\frac{1}{3}b^2c\sqrt{d-c^2dx^2}\left(\frac{3\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{1-cx}+\cosh^{-1}(cx)\left(\frac{\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+3\right)+6\log\left(e^{-2\cosh^{-1}(cx)}+\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[d - c^2*d*x^2])/x) + a^2*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x
^2])/(Sqrt[d]*(-1 + c^2*x^2))] + a*b*c*Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x
])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x))) + (b^2*c*Sqrt[d - c^2*d*x^2]*(ArcCosh[c*x]*((-3*ArcCosh[c*x])/(c*x) +
 (ArcCosh[c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])/(Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[2,
 -E^(-2*ArcCosh[c*x])])/(1 - c*x)))/3
```

Maple [B] time = 0.336, size = 582, normalized size = 2.5

$$-\frac{a^2}{dx}\left(-c^2dx^2+d\right)^{\frac{3}{2}}-a^2c^2x\sqrt{-c^2dx^2+d}-a^2c^2d\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}+\frac{b^2\left(\operatorname{arccosh}(cx)\right)^3c}{3}\sqrt{-d\left(c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] -a^2/d/x*(-c^2*d*x^2+d)^(3/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(1/2)-a^2*c^2*d/(c^2
*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b^2*(-d*(c^2*x^2
-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^3*c-b^2*(-d*(c^2*x^2-1)
)^(1/2)*arccosh(c*x)^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-b^2*(-d*(c^2*x^2-1))^(
1/2)*arccosh(c*x)^2/(c*x+1)/(c*x-1)*x*c^2+b^2*(-d*(c^2*x^2-1))^(1/2)*arccos
h(c*x)^2/(c*x+1)/(c*x-1)/x+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c+b^2*(-d*(
c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)*c+a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
*arccosh(c*x)^2*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*
arccosh(c*x)*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+1)/(c*x-1)*x*
c^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+1)/(c*x-1)/x+2*a*b*(-d*(
c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2+1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2/x^2, x)
```

3.176 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=427

$$-\frac{ibc^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{ibc^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -((b*c*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*x^2) + (c^2*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b^2*c^2*Sqrt[d - c^2*d*x^2]*ArcTan[Sqrt[-1 + c*x]*Sqrt
[1 + c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*c^2*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (I*b*c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E
^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b^2*c^2*Sqrt[d - c^2*d*
x^2]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b
^2*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sq
rt[1 + c*x])
```

Rubi [A] time = 0.876632, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5798, 5738, 5662, 92, 205, 5761, 4180, 2531, 2282, 6589}

$$-\frac{ibc^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{ibc^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^3,x]
```


```
[Out] -((b*c*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*x^2) + (c^2*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b^2*c^2*Sqrt[d - c^2*d*x^2]*ArcTan[Sqrt[-1 + c*x]*Sqrt
[1 + c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*c^2*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (I*b*c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E
^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b^2*c^2*Sqrt[d - c^2*d*
x^2]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b
```

```
^2*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sq
rt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d}\right.}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{\left(c^2\sqrt{d}\right.}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{c^2\sqrt{d-}}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{c^2\sqrt{d-}}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{c^2\sqrt{d-}}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{c^2\sqrt{d-}}{}$$

Mathematica [A] time = 79.7534, size = 547, normalized size = 1.28

$$\frac{1}{2}a\left(\frac{2bd(cx+1)\left(ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(c\right.}{x^2\sqrt{d-}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] (a*(-((a*Sqrt[d - c^2*d*x^2])/x^2) - a*c^2*Sqrt[d]*Log[x] + a*c^2*Sqrt[d]*L
og[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*b*d*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)
/(1 + c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(
1 + c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x
)/(1 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 +
c*x)/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)
/(1 + c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2])))/2 +
(b^2*c^2*Sqrt[d - c^2*d*x^2]*((2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])/(
```

```
c*x - c^2*x^2) - ArcCosh[c*x]^2/(c^2*x^2) - (I*((4*I)*ArcTan[Tanh[ArcCosh[c
*x]/2]] + ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x]^2*Log[1 +
 I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - 2*Arc
Cosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[c*x]]
- 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))
/2
```

Maple [F] time = 0.348, size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^2}{x^3}\sqrt{-c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x)
```

```
[Out] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\,\mathrm{arcosh}\,(cx)^2+2\,ab\,\mathrm{arcosh}\,(cx)+a^2\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2/x^3, x)
```

3.177 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=336

$$\frac{b^2c^3\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{c^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{3x^2\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b^2*c^2*Sqrt[d - c^2*d*x^2])/(3*x) - (b^2*c^3*Sqrt[d - c^2*d*x^2]*ArcCosh[
c*x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(1 - c^2*x^2)*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c^3*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/(3*d*x^3) - (2*b*c^3*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (b^2*c^3*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(-2*ArcC
osh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.582752, antiderivative size = 344, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {5798, 5724, 5729, 97, 12, 52, 5660, 3718, 2190, 2279, 2391}

$$-\frac{b^2c^3\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{3x^2\sqrt{cx-1}\sqrt{cx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (b^2*c^2*Sqrt[d - c^2*d*x^2])/(3*x) - (b^2*c^3*Sqrt[d - c^2*d*x^2]*ArcCosh[
c*x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(1 - c^2*x^2)*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^3*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
((1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*x^3) -
(2*b*c^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x]
)])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b^2*c^3*Sqrt[d - c^2*d*x^2]*PolyLog
[2, -E^(2*ArcCosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
```

```
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3} - \frac{\left(2bc\sqrt{d-c^2dx^2}\right)\int \frac{(-1+c^2x^2}{}}{3\sqrt{-1+cx}\sqrt{1}}$$

$$= -\frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,(a}{3x^3}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(1-cx)(1+c}{}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^3\sqrt{d-c^2dx^2}}{3\sqrt{-1}}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\,(a+}{3x^2\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\,(a+}{3x^2\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\,(a+}{3x^2\sqrt{-1+cx}\sqrt{1+}}$$

Mathematica [A] time = 0.990811, size = 304, normalized size = 0.9

$$-\frac{d(cx+1)\left(b^2c^3x^3\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+a^2c^3x^3-a^2c^2x^2-a^2cx+a^2-2abc^3x^3\sqrt{\frac{cx-1}{cx+1}}\log(cx)-b\cosh^{-1}(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] -(d*(1 + c*x)*(a^2 - a^2*c*x - a^2*c^2*x^2 - b^2*c^2*x^2 + a^2*c^3*x^3 + b^
2*c^3*x^3 - a*b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - b^2*(-1 + c*x + c^2*x^2 +
c^3*x^3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)]))*ArcCosh[c*x]^2 - b*ArcCosh[c*x]*
(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - 2*a*(-1 + c*x)^2*(1 + c*x) + 2*b*c^3*x^
3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[1 + E^(-2*ArcCosh[c*x])]) - 2*a*b*c^3*x^3*
Sqrt[(-1 + c*x)/(1 + c*x)]*Log[c*x] + b^2*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)
```

```
]*PolyLog[2, -E^(-2*ArcCosh[c*x])]))/(3*x^3*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.394, size = 2633, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x)
```

```
[Out] -6*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*a
rccosh(c*x)*c^6+20/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(
c*x+1)/(c*x-1)*arccosh(c*x)*c^4-10/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-
3*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-2*a*b*(-d*(c^2*x^2-1))^(1/2
)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+
2*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*arccosh(c*x)*c^5-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(3/2)+b^2*(-d*(c^2*
x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcco
sh(c*x)^2*c^5-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^
4*x^4-3*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c-b^2*(-d*(
c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*a
rccosh(c*x)^2*c^7-a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c
*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c
^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3-1/3*a*b*(-d*(c^2*x^2
-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+2/3*a*
b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccos
h(c*x)+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*
x-1)*c^4+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)
/(c*x-1)*c^8-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*
x+1)/(c*x-1)*c^6+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*
x+1)/(c*x-1)*arccosh(c*x)^2*c^8-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3
*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6-5/3*b^2*(-d*(c^2*x^2-1))^(
1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^2+10/3*b^2*
(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*
x)^2*c^4+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(
c*x-1)*arccosh(c*x)*c^4+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2
+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^
4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^6+2*a*b*(-d*(c^2*x^
2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8+1/
3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3*c^6+b^2*(-d*(c^2*x
^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-2/
```

```
3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c^3+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*
c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)
/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+4/3*b^2*(-d*(c^2*x^2-1))^(
1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-1/3*b^2*(-d*(c^2*x^2-1))
^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*c^2+1/3*b^2*(-d*(c^2*x^2-1
))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^2+4/3*a*b
*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^3-2/3*a*
b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))^2+1)*c^3+a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/
(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3
*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^3+b^2*(-d*(c^2*x^2
-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)
*c^3-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(
c*x-1)^(1/2)*c^7-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*
x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^
2*x^2+1)*x^3*arccosh(c*x)*c^6+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c
^2*x^2+1)*x*arccosh(c*x)*c^4-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^
2*x^2+1)*x^3*c^6+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x*c
^4+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^
2*c^3-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,
-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2/x^4, x)
```

3.178 $\int x^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=495

$$\frac{4abdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bc^3dx^7\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{49\sqrt{cx-1}\sqrt{cx+1}}-\frac{16bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{175\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{7}x^4\left(d-c^2d\right.$$

```
[Out] (-37384*b^2*d*Sqrt[d - c^2*d*x^2])/(385875*c^4) + (3358*b^2*d*x^2*Sqrt[d -
c^2*d*x^2])/(385875*c^2) + (484*b^2*d*x^4*Sqrt[d - c^2*d*x^2])/42875 - (2*b
^2*c^2*d*x^6*Sqrt[d - c^2*d*x^2])/343 + (4*a*b*d*x*Sqrt[d - c^2*d*x^2])/(35
*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b^2*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh
[c*x])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d*x^3*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (16*b*c*d*x
^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(175*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (2*b*c^3*d*x^7*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(49*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(
35*c^4) - (d*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(35*c^2) + (3*
d*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/35 + (x^4*(d - c^2*d*x^2)
^(3/2)*(a + b*ArcCosh[c*x])^2)/7
```

Rubi [A] time = 1.67374, antiderivative size = 507, normalized size of antiderivative = 1.02, number of steps used = 26, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5745, 5743, 5759, 5718, 5654, 74, 5662, 100, 12, 14, 5731, 460}

$$\frac{4abdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bc^3dx^7\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{49\sqrt{cx-1}\sqrt{cx+1}}-\frac{16bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{175\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{35}dx^4\sqrt{d-c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```


```
[Out] (-37384*b^2*d*Sqrt[d - c^2*d*x^2])/(385875*c^4) + (3358*b^2*d*x^2*Sqrt[d -
c^2*d*x^2])/(385875*c^2) + (484*b^2*d*x^4*Sqrt[d - c^2*d*x^2])/42875 - (2*b
^2*c^2*d*x^6*Sqrt[d - c^2*d*x^2])/343 + (4*a*b*d*x*Sqrt[d - c^2*d*x^2])/(35
*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b^2*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh
[c*x])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d*x^3*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (16*b*c*d*x
^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(175*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (2*b*c^3*d*x^7*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(49*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(
```

```
35*c^4) - (d*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(35*c^2) + (3*
d*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/35 + (d*x^4*(1 - c*x)*(1
+ c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/7
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
```

```
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2 dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{7}dx^4(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{\left(3d\sqrt{d-c^2dx^2}\right)\int$$

$$= -\frac{2bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^3dx^7\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{49\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{16bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{175\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^3dx^7\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{49\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{6}{875}b^2dx^4\sqrt{d-c^2dx^2} - \frac{2}{343}b^2c^2dx^6\sqrt{d-c^2dx^2} + \frac{2bdx^3\sqrt{d-c^2dx^2}\left(a+b\right.}{105c\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{2b^2dx^2\sqrt{d-c^2dx^2}}{315c^2} + \frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875} - \frac{2}{343}b^2c^2dx^6\sqrt{d-c^2dx^2} +$$

$$= \frac{22b^2dx^2\sqrt{d-c^2dx^2}}{7875c^2} + \frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875} - \frac{2}{343}b^2c^2dx^6\sqrt{d-c^2dx^2} +$$

$$= -\frac{8b^2d\sqrt{d-c^2dx^2}}{63c^4} + \frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2} + \frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875} - \frac{2}{343}$$

$$= -\frac{856b^2d\sqrt{d-c^2dx^2}}{7875c^4} + \frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2} + \frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875} -$$

$$= -\frac{37384b^2d\sqrt{d-c^2dx^2}}{385875c^4} + \frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2} + \frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875} -$$

Mathematica [A] time = 0.587769, size = 262, normalized size = 0.53

$$-\frac{d\sqrt{d-c^2dx^2}\left(11025a^2\left(5c^2x^2+2\right)\left(c^2x^2-1\right)^3 - 210abcx\sqrt{cx-1}\sqrt{cx+1}\left(75c^6x^6-168c^4x^4+35c^2x^2+210\right) - 210b\,\mathrm{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(11025*a^2*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2) - 210*a
*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^
6*x^6) + 2*b^2*(-18692 + 20371*c^2*x^2 + 499*c^4*x^4 - 3303*c^6*x^6 + 1125*
c^8*x^8) - 210*b*(-105*a*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2) + b*c*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^6*x^6))*ArcCosh[
c*x] + 11025*b^2*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2)*ArcCosh[c*x]^2))/(385875*
c^4*(-1 + c^2*x^2))
```

Maple [B] time = 0.536, size = 1952, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] a^2*(-1/7*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(-c^2*d*x^2+d)^(5/2))+b
^2*(-1/43904*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5
-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+1)*(49*arccosh(c*x)^2-14*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)+1/
16000*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*arccosh(c*x)^2-10*arccosh(c*x)+2)*d/(c*x+1)
/c^4/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(
c*x)^2-6*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)
*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)
+2)*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c
*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3
+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2
+6*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)+1/16000*(-d*(c^2*x^2-1))^(1/2)*(-1
6*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25
*arccosh(c*x)^2+10*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)-1/43904*(-d*(c^2*x
^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(49*arcco
sh(c*x)^2+14*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1))+2*a*b*(-1/6272*(-d*(c^2
*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c
^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1
```

```
)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*ar
ccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/3200*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6
-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccos
h(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2
*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)
*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d/(c*x+1)/c^
4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^
2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1/2
)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/3200*(-d
*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)-1/6272*(-d*(c^
2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*a
rccosh(c*x))*d/(c*x+1)/c^4/(c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.37675, size = 996, normalized size = 2.01

$$-\frac{11025\left(5\,b^2c^8dx^8-13\,b^2c^6dx^6+9\,b^2c^4dx^4+b^2c^2dx^2-2\,b^2d\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-210\left(75\,abc^7dx^7-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] -1/385875*(11025*(5*b^2*c^8*d*x^8 - 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 + b^
2*c^2*d*x^2 - 2*b^2*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2
- 210*(75*a*b*c^7*d*x^7 - 168*a*b*c^5*d*x^5 + 35*a*b*c^3*d*x^3 + 210*a*b*c*
d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 210*((75*b^2*c^7*d*x^7 - 168*
b^2*c^5*d*x^5 + 35*b^2*c^3*d*x^3 + 210*b^2*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 105*(5*a*b*c^8*d*x^8 - 13*a*b*c^6*d*x^6 + 9*a*b*c^4*d*x^4 +
 a*b*c^2*d*x^2 - 2*a*b*d)*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)
) + (1125*(49*a^2 + 2*b^2)*c^8*d*x^8 - 9*(15925*a^2 + 734*b^2)*c^6*d*x^6 +
(99225*a^2 + 998*b^2)*c^4*d*x^4 + (11025*a^2 + 40742*b^2)*c^2*d*x^2 - 2*(11
025*a^2 + 18692*b^2)*d)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.179 $\int x^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=441

$$\frac{bc^3dx^6\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18\sqrt{cx-1}\sqrt{cx+1}}-\frac{7bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2$$

```
[Out] (7*b^2*d*x*Sqrt[d - c^2*d*x^2])/(1152*c^2) + (43*b^2*d*x^3*Sqrt[d - c^2*d*x
^2])/1728 - (b^2*c^2*d*x^5*Sqrt[d - c^2*d*x^2])/108 + (7*b^2*d*Sqrt[d - c^2
*d*x^2]*ArcCosh[c*x])/(1152*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*x^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 - (7*b*c*d*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(48*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + (b*c^3*d*x^6*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(
18*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^2)/(16*c^2) + (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/8 +
(x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/6 - (d*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])^3)/(48*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.4774, antiderivative size = 453, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5745, 5743, 5759, 5676, 5662, 90, 52, 100, 12, 14, 5731, 460}

$$\frac{bc^3dx^6\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18\sqrt{cx-1}\sqrt{cx+1}}-\frac{7bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (7*b^2*d*x*Sqrt[d - c^2*d*x^2])/(1152*c^2) + (43*b^2*d*x^3*Sqrt[d - c^2*d*x
^2])/1728 - (b^2*c^2*d*x^5*Sqrt[d - c^2*d*x^2])/108 + (7*b^2*d*Sqrt[d - c^2
*d*x^2]*ArcCosh[c*x])/(1152*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*x^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 - (7*b*c*d*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(48*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + (b*c^3*d*x^6*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(
18*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^2)/(16*c^2) + (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/8 +
(d*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/6 -
(d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(48*b*c^3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2 \, dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{6}dx^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{\left(d\sqrt{d-c^2dx^2}\right)\int x$$

$$= -\frac{bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{12\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx^6\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{18\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{7bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx^6\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{18\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{64}b^2dx^3\sqrt{d-c^2dx^2} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2} + \frac{bdx^2\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{16c\sqrt{-1+cx}\sqrt{1+}}$$

$$= -\frac{b^2dx\sqrt{d-c^2dx^2}}{32c^2} + \frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2} + \frac{bdx^2}{}$$

$$= -\frac{b^2dx\sqrt{d-c^2dx^2}}{128c^2} + \frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2} - \frac{b^2d}{32}$$

$$= \frac{7b^2dx\sqrt{d-c^2dx^2}}{1152c^2} + \frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2} - \frac{b^2d}{128}$$

$$= \frac{7b^2dx\sqrt{d-c^2dx^2}}{1152c^2} + \frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2} + \frac{7b^2d}{115}$$

Mathematica [A] time = 4.28346, size = 485, normalized size = 1.1

$$\frac{-864a^2d^{3/2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - 288a^2cdx\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(8c^4x^4-14c^2x^2+3\right)\sqrt{d-c^2dx^2} - 216abd\sqrt{d-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-288*a^2*c*d*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(3
 - 14*c^2*x^2 + 8*c^4*x^4) - 864*a^2*d^(3/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
```

```
+ c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 216*a*b
*d*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh
[c*x]*Sinh[4*ArcCosh[c*x]]) - 18*b^2*d*Sqrt[d - c^2*d*x^2]*(32*ArcCosh[c*x]
^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*Sinh[4
*ArcCosh[c*x]]) - 12*a*b*d*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cos
h[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*Ar
cCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCo
sh[c*x]])) + b^2*d*Sqrt[d - c^2*d*x^2]*(288*ArcCosh[c*x]^3 + 12*ArcCosh[c*x
]*(-18*Cosh[2*ArcCosh[c*x]] + 9*Cosh[4*ArcCosh[c*x]] + 2*Cosh[6*ArcCosh[c*x
]]) + 108*Sinh[2*ArcCosh[c*x]] - 27*Sinh[4*ArcCosh[c*x]] - 4*Sinh[6*ArcCosh
[c*x]] - 72*ArcCosh[c*x]^2*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]
] + Sinh[6*ArcCosh[c*x]]))))/(13824*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
)
```

Maple [B] time = 0.444, size = 1021, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/16*a^2/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))
+1/18*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-7/48
*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+11/24*b^2*(
-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)^2*x^5-1/6*b^2*(-d*
(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)^2*x^7-1/16*a*b*(-d*(c
^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*d-17/24*a*b
*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/16*b^2*(-d*(c^
2*x^2-1))^(1/2)*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)^2*x+1/16*a*b*(-d*(c^2*x^
2-1))^(1/2)*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+1/18*b^2*(-d*(c^2*x^2-1))^(
1/2)*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*arccosh(c*x)*x^6-7/48*b^2*(-d*(c^2*x
^2-1))^(1/2)*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*arccosh(c*x)*x^4+1/16*b^2*(-d*
(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*arccosh(c*x)*x^2+59/1728
*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^2/(c*x-1)*x^5-7/1152*b^2*(-d*(c^2*x
^2-1))^(1/2)*d/(c*x+1)/c^2/(c*x-1)*x-17/48*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*
x+1)/(c*x-1)*arccosh(c*x)^2*x^3+1/24*a^2/c^2*x*(-c^2*d*x^2+d)^(3/2)+1/16*a^
2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/8*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/c^2
/(c*x-1)*arccosh(c*x)*x-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^4/(c*x-1
)*arccosh(c*x)*x^7+11/12*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^2/(c*x-1)*a
rccosh(c*x)*x^5+7/1152*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c^3/(c*x-
1)^(1/2)*arccosh(c*x)+7/1152*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c^3
```

```
/(c*x-1)^(1/2)-1/48*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/
c^3*arccosh(c*x)^3*d-1/108*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^4/(c*x-1)
*x^7-1/6*a^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d-65/3456*b^2*(-d*(c^2*x^2-1))^(1/2
)*d/(c*x+1)/(c*x-1)*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2c^2dx^4 - a^2dx^2 + \left(b^2c^2dx^4 - b^2dx^2\right)\text{arcosh}\left(cx\right)^2 + 2\left(abc^2dx^4 - abdx^2\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^4 - a^2*d*x^2 + (b^2*c^2*d*x^4 - b^2*d*x^2)*arccosh(
c*x)^2 + 2*(a*b*c^2*d*x^4 - a*b*d*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.180 $\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=348

$$\frac{2bc^3dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{cx-1}\sqrt{cx+1}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{cx-1}\sqrt{cx+1}}-$$

```
[Out] (-16*b^2*d*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(75*c^2*(1 - c*x)*(1 + c*x))
- (8*b^2*d*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(225*c^2*(1 - c*x)*(1 + c*x
)) - (2*b^2*d*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(125*c^2*(1 - c*x)*(1 +
c*x)) + (2*b*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (4*b*c*d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d*x^5*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/
2)*(a + b*ArcCosh[c*x])^2)/(5*c^2*d)
```

Rubi [A] time = 0.554161, antiderivative size = 361, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5718, 194, 5680, 12, 520, 1247, 698}

$$\frac{2bc^3dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{cx-1}\sqrt{cx+1}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{cx-1}\sqrt{cx+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-16*b^2*d*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(75*c^2*(1 - c*x)*(1 + c*x))
- (8*b^2*d*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(225*c^2*(1 - c*x)*(1 + c*x
)) - (2*b^2*d*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(125*c^2*(1 - c*x)*(1 +
c*x)) + (2*b*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (4*b*c*d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d*x^5*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(1 - c*x)^2*(1 +
c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(5*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2}+\frac{\left(2bd\sqrt{d-c^2dx^2}\right)\int}{5c}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{15\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{15\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{15\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{15\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{15\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{16b^2d\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{75c^2(1-cx)(1+cx)}-\frac{8b^2d\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}}{225c^2(1-cx)(1+cx)}-\frac{2b^2d\left(1-c\right.}{125c^2(1}$$

Mathematica [A] time = 0.496966, size = 208, normalized size = 0.6

$$-\frac{d\sqrt{d-c^2dx^2}\left(225a^2\left(c^2x^2-1\right)^3-30abcx\sqrt{cx-1}\sqrt{cx+1}\left(3c^4x^4-10c^2x^2+15\right)-30b\cosh^{-1}(cx)\left(bcx\sqrt{cx-1}\sqrt{cx+1}\right.\right.}{1125c^2\left(c^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(225*a^2*(-1 + c^2*x^2)^3 - 30*a*b*c*x*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(-149 + 187*c^2*x^2
- 47*c^4*x^4 + 9*c^6*x^6) - 30*b*(-15*a*(-1 + c^2*x^2)^3 + b*c*x*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(15 - 10*c^2*x^2 + 3*c^4*x^4))*ArcCosh[c*x] + 225*b^2*(-
1 + c^2*x^2)^3*ArcCosh[c*x]^2))/(1125*c^2*(-1 + c^2*x^2))
```

Maple [B] time = 0.382, size = 1270, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/5*a^2/c^2/d*(-c^2*d*x^2+d)^(5/2)+b^2*(-1/4000*(-d*(c^2*x^2-1))^(1/2)*(16
*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*
arccosh(c*x)^2-10*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*x-1)+1/288*(-d*(c^2*x^2-
1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)*d/(c*x+1
)/c^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*x-1)-1/16*(-d
*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c
*x)^2+2*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*x-1)+1/288*(-d*(c^2*x^2-1))^(1/2)*
(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*
x-1)-1/4000*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5
+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c*x)^2+10*arccosh(c*x)+2)*d/
(c*x+1)/c^2/(c*x-1))+2*a*b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^
4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x)
)*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-
1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1
)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1
)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*
(1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x
```

```
+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/800*(-d*(c^2*x^2-1
))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^
2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1))
```

Maxima [A] time = 1.17623, size = 375, normalized size = 1.08

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}b^2\operatorname{arcosh}(cx)^2}{5\,c^2d}-\frac{2}{1125}\,b^2\left(\frac{9\,\sqrt{c^2x^2-1}c^2\sqrt{-d}d^2x^4-38\,\sqrt{c^2x^2-1}\sqrt{-d}d^2x^2+\frac{149\,\sqrt{c^2x^2-1}\sqrt{-d}d^2}{c^2}}{d}-\frac{15\left(3\,c^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -1/5*(-c^2*d*x^2 + d)^(5/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/1125*b^2*((9*sqr
t(c^2*x^2 - 1)*c^2*sqrt(-d)*d^2*x^4 - 38*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2*x^2
 + 149*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2/c^2)/d - 15*(3*c^4*sqrt(-d)*d^2*x^5 -
 10*c^2*sqrt(-d)*d^2*x^3 + 15*sqrt(-d)*d^2*x)*arccosh(c*x)/(c*d)) - 2/5*(-c
^2*d*x^2 + d)^(5/2)*a*b*arccosh(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(5/2)*a
^2/(c^2*d) + 2/75*(3*c^4*sqrt(-d)*d^2*x^5 - 10*c^2*sqrt(-d)*d^2*x^3 + 15*sq
rt(-d)*d^2*x)*a*b/(c*d)
```

Fricas [A] time = 2.27358, size = 801, normalized size = 2.3

$$-\frac{225\left(b^2c^6dx^6-3\,b^2c^4dx^4+3\,b^2c^2dx^2-b^2d\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-30\left(3\,abc^5dx^5-10\,abc^3dx^3+15\,abc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] -1/1125*(225*(b^2*c^6*d*x^6 - 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 - b^2*d)*sq
rt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(3*a*b*c^5*d*x^5 - 1
0*a*b*c^3*d*x^3 + 15*a*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30
```

```
*((3*b^2*c^5*d*x^5 - 10*b^2*c^3*d*x^3 + 15*b^2*c*d*x)*sqrt(-c^2*d*x^2 + d)*
sqrt(c^2*x^2 - 1) - 15*(a*b*c^6*d*x^6 - 3*a*b*c^4*d*x^4 + 3*a*b*c^2*d*x^2 -
 a*b*d)*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (9*(25*a^2 + 2
*b^2)*c^6*d*x^6 - (675*a^2 + 94*b^2)*c^4*d*x^4 + (675*a^2 + 374*b^2)*c^2*d*
x^2 - (225*a^2 + 298*b^2)*d)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.181 $\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=336

$$-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{8bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{bd(\ldots}{\ldots}$$

```
[Out] (15*b^2*d*x*Sqrt[d - c^2*d*x^2])/64 + (b^2*d*x*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2])/32 + (9*b^2*d*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(64*c*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) - (3*b*c*d*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x
]))/(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*x*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/8 + (x*(d - c^2*d*x^2)^(3/2)*(a + b
*ArcCosh[c*x])^2)/4 - (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(8*b*c
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.608932, antiderivative size = 348, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5713, 5685, 5683, 5676, 5662, 90, 52, 5716, 38}

$$-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{8bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{1}{4}dx(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (15*b^2*d*x*Sqrt[d - c^2*d*x^2])/64 + (b^2*d*x*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2])/32 + (9*b^2*d*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(64*c*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) - (3*b*c*d*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x
]))/(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*x*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/8 + (d*x*(1 - c*x)*(1 + c*x)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/4 - (d*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x])^3)/(8*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
```

```
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
```

```
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2 \, dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}dx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{\left(3d\sqrt{d-c^2dx^2}\right)\int\sqrt{-1}}{4\sqrt{}}$$

$$= \frac{bd\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.$$

$$= \frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2} - \frac{3bcdx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= \frac{15}{64}b^2dx\sqrt{d-c^2dx^2} + \frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2} - \frac{3bcdx^2\sqrt{d-c^2dx^2}}{8\sqrt{-1+c}}$$

$$= \frac{15}{64}b^2dx\sqrt{d-c^2dx^2} + \frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2} + \frac{9b^2d\sqrt{d-c^2dx^2}\,\mathrm{co}}{64c\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A] time = 2.92394, size = 374, normalized size = 1.11

$$\frac{-288a^2d^{3/2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - 96a^2cdx\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(2c^2x^2-5\right)\sqrt{d-c^2dx^2} - 192abd\sqrt{d-c^2dx^2}\left(\cosh\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-96*a^2*c*d*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-5 + 2*c^2*x^2)*Sqrt[d
 - c^2*d*x^2] - 288*a^2*d^(3/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan
[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 192*a*b*d*Sqrt[d - c
^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*Arc
Cosh[c*x]])) - 32*b^2*d*Sqrt[d - c^2*d*x^2]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c
*x]*Cosh[2*ArcCosh[c*x]] - 3*(1 + 2*ArcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]) +
 12*a*b*d*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*
ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + b^2*d*Sqrt[d - c^2*d*x^2]*(32*ArcCosh[
c*x]^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*Si
nh[4*ArcCosh[c*x]]))/(768*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] time = 0.263, size = 775, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/4*x*(-c^2*d*x^2+d)^(3/2)*a^2+3/8*a^2*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a^2*d^2
/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/32*b^2*(-d*(c
^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*c^4*x^5+19/64*b^2*(-d*(c^2*x^2-1))^(1/2)
*d/(c*x+1)/(c*x-1)*c^2*x^3-17/64*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-
1)*x-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)^2*x^
5+7/8*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)^2*x^3-5
/8*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x+1/8*b^2*(-
d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3*arccosh(c*x)*x^4-5/8
*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*arccosh(c*x)*x^
2-1/8*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)
^3*d+17/64*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c*arcco
sh(c*x)-1/2*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)*x
^5+7/4*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)*x^3-5/
4*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x+17/64*a*b*(-d
*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c+1/8*a*b*(-d*(c^2*x^2-1)
)^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3*x^4-5/8*a*b*(-d*(c^2*x^2-1))^(1/2
)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x^2-3/8*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2c^2dx^2 - a^2d + \left(b^2c^2dx^2 - b^2d\right)\operatorname{arcosh}\left(cx\right)^2 + 2\left(abc^2dx^2 - abd\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}\left(a + b\operatorname{acosh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.182 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=573

$$\frac{2ibd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ibd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(c\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (68*b^2*d*Sqrt[d - c^2*d*x^2])/27 - (2*b^2*c^2*d*x^2*Sqrt[d - c^2*d*x^2])/2
7 - (2*a*b*c*d*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b
^2*c*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
 (2*b*c*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (2*b*c^3*d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2
 + ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/3 - (2*d*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) + ((2*I)*b*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I
)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b*d*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - ((2*I)*b^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*E^ArcCosh
[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^2*d*Sqrt[d - c^2*d*x^2]*P
olyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.2549, antiderivative size = 585, normalized size of antiderivative = 1.02, number of steps used = 18, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5745, 5743, 5761, 4180, 2531, 2282, 6589, 5654, 74, 5680, 12, 460}

$$\frac{2ibd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ibd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(c\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] (68*b^2*d*Sqrt[d - c^2*d*x^2])/27 - (2*b^2*c^2*d*x^2*Sqrt[d - c^2*d*x^2])/2
7 - (2*a*b*c*d*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b
^2*c*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
 (2*b*c*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (2*b*c^3*d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*S
```

```
qrt[-1 + c*x]*Sqrt[1 + c*x]) + d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2
 + (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/3 - (
2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2
*I)*b*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x
]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b^2*d*Sqrt[d - c^2*d*x^2]*PolyL
og[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^2*d*S
qrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{3}d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\sqrt{-1+cx}}{}}{\sqrt{-1+cx}}$$

$$= -\frac{2bcdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^3dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{9\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^3dx^3\sqrt{d-}}{9\sqrt{-}}$$

$$= -\frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cdx\sqrt{d-c^2dx^2}\cosh^{-1}(c}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{68}{27}b^2d\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cdx\sqrt{}}{\sqrt{-}}$$

$$= \frac{68}{27}b^2d\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cdx\sqrt{}}{\sqrt{-}}$$

$$= \frac{68}{27}b^2d\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cdx\sqrt{}}{\sqrt{-}}$$

Mathematica [A] time = 2.70997, size = 650, normalized size = 1.13

$$\frac{2abd\sqrt{d-c^2dx^2}\left(i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)-cx+cx\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx)+\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] -(a^2*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/3 - (b^2*d*Sqrt[d - c^2*d*x^2]*
(2*(-13 + Cosh[2*ArcCosh[c*x]]) + 9*ArcCosh[c*x]^2*(-1 + Cosh[2*ArcCosh[c*x
]]) + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*(9*c*x - Cosh[3*ArcCosh[c*
x]]))/(-1 + c*x)))/54 - (a*b*d*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/
```

```
(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]))/(18*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + a^2*d^(3/2)*Log[c*x] - a^2*d^(3/2)*Log[
d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*d*Sqrt[d - c^2*d*x^2]*(-(c*x) + S
qrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Arc
Cosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1
 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E
^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + b^2*d*Sqrt[d - c^
2*d*x^2]*(2 + (2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])/(1 - c*x) + A
rcCosh[c*x]^2 + (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x]
^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x
]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^ArcC
osh[c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)))
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}}{x}\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2c^2dx^2-a^2d+\left(b^2c^2dx^2-b^2d\right)\text{arcosh}\left(cx\right)^2+2\left(abc^2dx^2-abd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="fricas"
)
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\text{acosh}\left(cx\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2/x, x)
```

3.183 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=453

$$-\frac{b^2cd\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.$$

```
[Out] -(b^2*c^2*d*x*Sqrt[d - c^2*d*x^2])/4 - (5*b^2*c*d*Sqrt[d - c^2*d*x^2]*ArcCo
sh[c*x])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*d*x^2*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d*(1 - c^
2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) - (3*c^2*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 + (c*d*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((
d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x + (c*d*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])^3)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c*d*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (b^2*c*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(-2*ArcCo
sh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.965913, antiderivative size = 465, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.483, Rules used = {5798, 5740, 5683, 5676, 5662, 90, 52, 5727, 5660, 3718, 2190, 2279, 2391, 38}

$$\frac{b^2cd\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] -(b^2*c^2*d*x*Sqrt[d - c^2*d*x^2])/4 - (5*b^2*c*d*Sqrt[d - c^2*d*x^2]*ArcCo
sh[c*x])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*d*x^2*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d*(1 - c^
2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) - (3*c^2*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (c*d*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d
*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x + (c*d*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x
```

```
]) + (2*b*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh
[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b^2*c*d*Sqrt[d - c^2*d*x^2]*Poly
Log[2, -E^(2*ArcCosh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{\left(2bcd\sqrt{d-c^2dx^2}\right)\int \frac{(-1+}{}}{\sqrt{-1+cx}\sqrt{}}$$

$$= \frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{}\right.$$

$$= \frac{1}{2}b^2c^2dx\sqrt{d-c^2dx^2} + \frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bcd\left(1-c^2x^2\right)}{}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a\right.}{2\sqrt{-1+cx}\sqrt{1}}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{5b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a\right.}{2\sqrt{-1+cx}}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{5b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a\right.}{2\sqrt{-1+cx}}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{5b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a\right.}{2\sqrt{-1+cx}}$$

Mathematica [A] time = 3.98464, size = 433, normalized size = 0.96

$$\frac{-8b^2d\sqrt{d-c^2dx^2}\left(3cx\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+\cosh^{-1}(cx)\left(3\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)-cx\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] (-12*a^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(2 + c^2*x^2)*Sqrt[d - c^2*
d*x^2] + 36*a^2*c*d^(3/2)*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 24*a*b*d*Sqrt[d - c^2*d*
x^2]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*
x]^2 + 2*Log[c*x])) - 8*b^2*d*Sqrt[d - c^2*d*x^2]*(ArcCosh[c*x]*(3*Sqrt[(-1
```

```
 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*x]*(3 + ArcCosh[
c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])) + 3*c*x*PolyLog[2, -E^(-2*ArcCosh[
c*x])]) + 6*a*b*c*d*x*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh
[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + b^2*c*d*x*Sqrt[d - c^2*d*x^2
]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] - 3*(1 + 2*ArcCos
h[c*x]^2)*Sinh[2*ArcCosh[c*x]]))/(24*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
)
```

Maple [B] time = 0.329, size = 942, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x)
```

```
[Out] -a^2/d/x*(-c^2*d*x^2+d)^(5/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a^2*c^2*d*
x*(-c^2*d*x^2+d)^(1/2)-3/2*a^2*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-c^2*d*x^2+d)^(1/2))-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)
*x^3+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)*x+1/2*b^2*(-d*(c^
2*x^2-1))^(1/2)*c^3*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*x^2+1/2*b^2*
(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^3*c*d+b^2*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2)*c*d-b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)*arccosh(c*x)^2*c*d-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x
-1)*arccosh(c*x)^2*x^3-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)
*arccosh(c*x)^2*x+b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)^2*d/(c*x+1)/(c*x-
1)/x-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh
(c*x)+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)
*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c*d+3/2*a*b*(-d*(c^2*x^2-1))^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c*d-a*b*(-d*(c^2*x^2-1))^(1/2
)*c^4*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/2*a*b*(-d*(c^2*x^2-1))^(1/2)*c^3
*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)
^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)-a*b*(-d*(c^2*x^2-1))^(1/2)*c^2*d/(c*x+1)/
(c*x-1)*arccosh(c*x)*x-1/4*a*b*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*
x-1)^(1/2)+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*d/(c*x+1)/(c*x-1)/x+2*
a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2+1)*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2c^2dx^2 - a^2d + \left(b^2c^2dx^2 - b^2d\right)\text{arcosh}\left(cx\right)^2 + 2\left(abc^2dx^2 - abd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}\left(a + b\,\text{acosh}\left(cx\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2/x^2, x)
```

3.184 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=630

$$-\frac{3ibc^2d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{3ibc^2d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -2*b^2*c^2*d*Sqrt[d - c^2*d*x^2] + (3*a*b*c^3*d*x*Sqrt[d - c^2*d*x^2])/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (3*b^2*c^3*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x
])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x]))/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2*d*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh
[c*x])^2)/(2*x^2) + (3*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*Arc
Tan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b^2*c^2*d*Sqrt[d - c
^2*d*x^2]*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - ((3*I)*b*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, (-
I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((3*I)*b*c^2*d*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + ((3*I)*b^2*c^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*
E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((3*I)*b^2*c^2*d*Sqrt[d -
 c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.37413, antiderivative size = 642, normalized size of antiderivative = 1.02, number of steps used = 18, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {5798, 5740, 5743, 5761, 4180, 2531, 2282, 6589, 5654, 74, 14, 5731, 460, 92, 205}

$$-\frac{3ibc^2d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{3ibc^2d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] -2*b^2*c^2*d*Sqrt[d - c^2*d*x^2] + (3*a*b*c^3*d*x*Sqrt[d - c^2*d*x^2])/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (3*b^2*c^3*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x
])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x]))/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d*x*Sqrt[d - c^2*d*x^2]
```

```
*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2*d*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(2*x^2) + (3*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b^2*c^2*d*Sqrt[d - c^2*d*x^2]*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - ((3*I)*b*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((
3*I)*b*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCos
h[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((3*I)*b^2*c^2*d*Sqrt[d - c^2*d*x
^2]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((3*I
)*b^2*c^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
```

```
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} - \frac{\left(bcd\sqrt{d-c^2dx^2}\right)\int \frac{(-1+c^2}{}}{\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^3dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} -$$

$$= b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} -$$

$$= b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc}{}$$

$$= -2b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} -$$

$$= -2b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} -$$

$$= -2b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} -$$

Mathematica [A] time = 168.74, size = 1129, normalized size = 1.79

$$\frac{1}{2}d\sqrt{d-c^2dx^2}\left(\frac{4x^2\cosh^{-1}(cx)c^4}{(cx-1)^{3/2}\sqrt{cx+1}} - \frac{2x\cosh^{-1}(cx)^2c^3}{cx-1} - \frac{4x\cosh^{-1}(cx)c^3}{(cx-1)^{3/2}\sqrt{cx+1}} - \frac{2x\tan^{-1}\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^3}{(cx-1)\sqrt{c^2x^2-1}} - \frac{4xc^3}{cx-1} + \frac{2\cosh^{-1}}{cx-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] (-(a^2*c^2*d) - (a^2*d)/(2*x^2))*Sqrt[-(d*(-1 + c^2*x^2))] - (3*a^2*c^2*d^(
3/2)*Log[x])/2 + (3*a^2*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2)
)]])/2 - 2*a*b*c^2*d*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-((c*x)/(Sqrt[(-1 + c
```

```
*x)/(1 + c*x)]*(1 + c*x))) + ArcCosh[c*x] + (I*ArcCosh[c*x]*(Log[1 - I/E^Ar
cCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)) + (I*(PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]])
)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (I*a*b*c^2*d^2*(((-I)*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x))/(c*x) - (I*(-1 + c*x)*(1 + c*x)*ArcCosh[c*x])/
(c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^A
rcCosh[c*x]] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/
E^ArcCosh[c*x]] + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^Ar
cCosh[c*x]] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c
*x]]))/Sqrt[-(d*(-1 + c*x)*(1 + c*x))] + (b^2*d*Sqrt[d - c^2*d*x^2]*((4*c^2
)/(-1 + c*x) - (4*c^3*x)/(-1 + c*x) - (2*c^2*ArcCosh[c*x])/((-1 + c*x)^(3/2
)*Sqrt[1 + c*x]) + (2*c*ArcCosh[c*x])/(x*(-1 + c*x)^(3/2)*Sqrt[1 + c*x]) -
(4*c^3*x*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (4*c^4*x^2*ArcCos
h[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (2*c^2*ArcCosh[c*x]^2)/(-1 + c*x
) + ArcCosh[c*x]^2/(x^2*(-1 + c*x)) - (2*c^3*x*ArcCosh[c*x]^2)/(-1 + c*x) +
 (c*ArcCosh[c*x]^2)/(x - c*x^2) + (2*c^2*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1
 + c*x)*Sqrt[-1 + c^2*x^2]) - (2*c^3*x*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1 +
 c*x)*Sqrt[-1 + c^2*x^2]) - ((3*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c
*x]^2*Log[1 - I/E^ArcCosh[c*x]])/(-1 + c*x) + ((3*I)*c^2*Sqrt[(-1 + c*x)/(1
 + c*x)]*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]])/(-1 + c*x) - ((6*I)*c^2*
Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]])/(-
1 + c*x) + ((6*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*PolyLog[2, I/
E^ArcCosh[c*x]])/(-1 + c*x) - ((6*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog
[3, (-I)/E^ArcCosh[c*x]])/(-1 + c*x) + ((6*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)
]*PolyLog[3, I/E^ArcCosh[c*x]])/(-1 + c*x)))/2
```

Maple [F] time = 0.35, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{arccosh}(cx))^2}{x^3} \left(-c^2 d x^2 + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2c^2dx^2-a^2d+\left(b^2c^2dx^2-b^2d\right)\text{arcosh}\left(cx\right)^2+2\left(abc^2dx^2-abd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\text{acosh}\left(cx\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**3,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2/x^3, x)
```

3.185 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=426

$$\frac{4b^2c^3d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{c^3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{cx-1}\sqrt{cx+1}}-\frac{4c^3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b^2*c^2*d*Sqrt[d - c^2*d*x^2])/(3*x) - (b^2*c^3*d*Sqrt[d - c^2*d*x^2]*ArcC
osh[c*x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*(1 - c^2*x^2)*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^
2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x - (4*c^3*d*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*
d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/(3*x^3) - (c^3*d*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])^3)/(3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*b*c^3*d*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(3*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (4*b^2*c^3*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(
-2*ArcCosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.16697, antiderivative size = 438, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5740, 5738, 5660, 3718, 2190, 2279, 2391, 5676, 5729, 97, 12, 52}

$$-\frac{4b^2c^3d\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{c^3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{cx-1}\sqrt{cx+1}}+\frac{4c^3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{cx-1}\sqrt{cx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (b^2*c^2*d*Sqrt[d - c^2*d*x^2])/(3*x) - (b^2*c^3*d*Sqrt[d - c^2*d*x^2]*ArcC
osh[c*x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*(1 - c^2*x^2)*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^
2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x + (4*c^3*d*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(1 - c*
x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*x^3) - (c^3*d*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (8*b*c^3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCo
sh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b^2*c^3*d*Sqrt[d - c^2*d*x
```

```
^2]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
```

```
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3} - \frac{\left(2bcd\sqrt{d-c^2dx^2}\right)\int \frac{(-1}{3\sqrt{-1+cx}}$$

$$= -\frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{x}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2d\sqrt{d-}}{}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2d\sqrt{d-}}{}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3d\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx}}{3x^2\sqrt{-1+cx}}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3d\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx}}{3x^2\sqrt{-1+cx}}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3d\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx}}{3x^2\sqrt{-1+cx}}$$

Mathematica [A] time = 2.05603, size = 583, normalized size = 1.37

$$-4b^2c^3d^2x^3(cx-1)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-4a^2c^4d^2x^4\sqrt{\frac{cx-1}{cx+1}}+5a^2c^2d^2x^2\sqrt{\frac{cx-1}{cx+1}}-3a^2c^3d^{3/2}x^3\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\tan$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (-(a*b*c*d^2*x) + a*b*c^2*d^2*x^2 - a^2*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 5*
a^2*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + b^2*c^2*d^2*x^2*Sqrt[(-1 + c*x
)/(1 + c*x)] - 4*a^2*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - b^2*c^4*d^2*x
^4*Sqrt[(-1 + c*x)/(1 + c*x)] - b*d^2*(-1 + c*x)*(-3*a*c^3*x^3 + b*(-Sqrt[(
-1 + c*x)/(1 + c*x)] - c*x*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*c^2*x^2*Sqrt[(-1
+ c*x)/(1 + c*x)] + 4*c^3*x^3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)])))*ArcCosh[c
*x]^2 + b^2*c^3*d^2*x^3*(-1 + c*x)*ArcCosh[c*x]^3 - 3*a^2*c^3*d^(3/2)*x^3*S
qrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d^2*(-1 + c*x)*ArcCosh[c*x]*(b*c*x + 2*a*
Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x - 4*c^2*x^2 - 4*c^3*x^3) + 8*b*c^3*x^3*
Log[1 + E^(-2*ArcCosh[c*x])]) - 8*a*b*c^3*d^2*x^3*Log[c*x] + 8*a*b*c^4*d^2*
x^4*Log[c*x] - 4*b^2*c^3*d^2*x^3*(-1 + c*x)*PolyLog[2, -E^(-2*ArcCosh[c*x])
])/(3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.342, size = 2879, normalized size = 6.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x)
```

```
[Out] 2/3*a^2*c^2/d/x*(-c^2*d*x^2+d)^(5/2)+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^3*d-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c^3*d+20
/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1
)*c^8-29/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1
)/(c*x-1)*c^6+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/
(c*x+1)/(c*x-1)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+
1)/x/(c*x+1)/(c*x-1)*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^2-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(
```

c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2+1)*c^3*d+3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3-8*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c
^4*x^4-9*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+3*b^2*(-d*(c^2*x^2-
1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-4/
3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*arccosh(c*x)^2*c^3+3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^
2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c^3*d-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(5/2
)+2/3*a^2*c^4*x*(-c^2*d*x^2+d)^(3/2)+64*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^
4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-104*a*b*(-d*(c^2*x^
2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6
+146/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x
-1)*arccosh(c*x)*c^4+8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)*arccosh(c*x)^2*c^3*d-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^
2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(
24*c^4*x^4-9*c^2*x^2+1)*x*arccosh(c*x)*c^4-16/3*a*b*(-d*(c^2*x^2-1))^(1/2)*
d/(24*c^4*x^4-9*c^2*x^2+1)*x^3*c^6+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4
*x^4-9*c^2*x^2+1)*x*c^4-16/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)*x^3*arccosh(c*x)*c^6-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3*d-1/3*b^2*
(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^3*c^3*d-64*
a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*
x-1)^(1/2)*arccosh(c*x)*c^7+24*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c
^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-28/3*a*b*(-d*(c^
2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c
^2-52*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*
x-1)*arccosh(c*x)^2*c^6-20/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+73/3*b^2*(-d*(c^2*x^2-1))^(1/2
)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^4+4/3*b^2*(
-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(
c*x)*c^4-14/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x/(c*x+
1)/(c*x-1)*arccosh(c*x)^2*c^2+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4
-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-20/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24
*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+4/3*a*b*(-d*(c^2*x^2-1))^(1/2
)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4+2/3*a*b*(-d*(c^2*x^2-1))
^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)+12*b^2*(
-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*arccosh(c*x)^2*c^5-8*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x
^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-1/3*b^2*(-d*(c^2*x^2
-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcco
sh(c*x)*c-32*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4/(c*x
+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^7-8*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(
24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-8/3*a*b*(-d*(c^
2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcc

```
osh(c*x)*c^3-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^2/
(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+a^2*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+a^2*c^4*d^2/
(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+4/3*b^2*(-d*(c^2
*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3*c^6+32*b^2*(-d*(c^2*x^2-1))^(
1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^8+16/3
*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*
arccosh(c*x)*c^8
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2c^2dx^2 - a^2d + \left(b^2c^2dx^2 - b^2d\right)\text{arcosh}\left(cx\right)^2 + 2\left(abc^2dx^2 - abd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\text{acosh}\left(cx\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**4,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="giac"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2/x^4, x)
```

3.186 $\int x^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=880

$$-\frac{2bc^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^9}{81\sqrt{cx-1}\sqrt{cx+1}}+\frac{38bc^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^7}{441\sqrt{cx-1}\sqrt{cx+1}}-\frac{10b^2c^2d^2\sqrt{d-c^2dx^2}x^6}{3087}-\frac{2bcd^2\sqrt{}}{}$$

```
[Out] (-37384*b^2*d^2*Sqrt[d - c^2*d*x^2])/(694575*c^4) + (3358*b^2*d^2*x^2*Sqrt[
d - c^2*d*x^2])/(694575*c^2) + (484*b^2*d^2*x^4*Sqrt[d - c^2*d*x^2])/77175
- (10*b^2*c^2*d^2*x^6*Sqrt[d - c^2*d*x^2])/3087 + (4*a*b*d^2*x*Sqrt[d - c^2
*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (16*b^2*d^2*(1 - c^2*x^2)*
Sqrt[d - c^2*d*x^2])/(2835*c^4*(1 - c*x)*(1 + c*x)) + (8*b^2*d^2*(1 - c^2*x
^2)^2*Sqrt[d - c^2*d*x^2])/(8505*c^4*(1 - c*x)*(1 + c*x)) + (2*b^2*d^2*(1 -
 c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(4725*c^4*(1 - c*x)*(1 + c*x)) - (20*b^2*d
^2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(3969*c^4*(1 - c*x)*(1 + c*x)) + (2
*b^2*d^2*(1 - c^2*x^2)^5*Sqrt[d - c^2*d*x^2])/(729*c^4*(1 - c*x)*(1 + c*x))
 + (4*b^2*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(63*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (2*b*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(189*
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d^2*x^5*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x]))/(21*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (38*b*c^3*d^2*x^7*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
2*b*c^5*d^2*x^9*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(81*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(63*c
^4) - (d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(63*c^2) + (d^2*
x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/21 + (5*d*x^4*(d - c^2*d*x^
2)^(3/2)*(a + b*ArcCosh[c*x])^2)/63 + (x^4*(d - c^2*d*x^2)^(5/2)*(a + b*Arc
Cosh[c*x])^2)/9
```

Rubi [A] time = 2.34468, antiderivative size = 911, normalized size of antiderivative = 1.04, number of steps used = 34, number of rules used = 18, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.621, Rules used = {5798, 5745, 5743, 5759, 5718, 5654, 74, 5662, 100, 12, 14, 5731, 460, 270, 520, 1251, 897, 1153}

$$-\frac{2bc^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^9}{81\sqrt{cx-1}\sqrt{cx+1}}+\frac{38bc^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^7}{441\sqrt{cx-1}\sqrt{cx+1}}-\frac{10b^2c^2d^2\sqrt{d-c^2dx^2}x^6}{3087}-\frac{2bcd^2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-37384*b^2*d^2*Sqrt[d - c^2*d*x^2])/(694575*c^4) + (3358*b^2*d^2*x^2*Sqrt[
d - c^2*d*x^2])/(694575*c^2) + (484*b^2*d^2*x^4*Sqrt[d - c^2*d*x^2])/77175
- (10*b^2*c^2*d^2*x^6*Sqrt[d - c^2*d*x^2])/3087 + (4*a*b*d^2*x*Sqrt[d - c^2
*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (16*b^2*d^2*(1 - c^2*x^2)*
Sqrt[d - c^2*d*x^2])/(2835*c^4*(1 - c*x)*(1 + c*x)) + (8*b^2*d^2*(1 - c^2*x
^2)^2*Sqrt[d - c^2*d*x^2])/(8505*c^4*(1 - c*x)*(1 + c*x)) + (2*b^2*d^2*(1 -
 c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(4725*c^4*(1 - c*x)*(1 + c*x)) - (20*b^2*d
^2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(3969*c^4*(1 - c*x)*(1 + c*x)) + (2
*b^2*d^2*(1 - c^2*x^2)^5*Sqrt[d - c^2*d*x^2])/(729*c^4*(1 - c*x)*(1 + c*x))
 + (4*b^2*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(63*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (2*b*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(189*
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d^2*x^5*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x]))/(21*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (38*b*c^3*d^2*x^7*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
2*b*c^5*d^2*x^9*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(81*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(63*c
^4) - (d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(63*c^2) + (d^2*
x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/21 + (5*d^2*x^4*(1 - c*x)*(
1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/63 + (d^2*x^4*(1 - c*x
)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/9
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
```

```
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
```

```
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2 dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{9}d^2x^4(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\left(5d^2\sqrt{d-c^2dx^2}\right.}{}$$

$$= -\frac{2bcd^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{45\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4bc^3d^2x^7\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{63\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{8bcd^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105\sqrt{-1+cx}\sqrt{1+cx}} + \frac{38bc^3d^2x^7\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{441\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{2bcd^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21\sqrt{-1+cx}\sqrt{1+cx}} + \frac{38bc^3d^2x^7\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{441\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{2}{525}b^2d^2x^4\sqrt{d-c^2dx^2} - \frac{10b^2c^2d^2x^6\sqrt{d-c^2dx^2}}{3087} + \frac{2bd^2x^3\sqrt{d-c^2dx^2}\left(a+\right.}{189c\sqrt{-1+cx}}$$

$$= -\frac{2b^2d^2x^2\sqrt{d-c^2dx^2}}{567c^2} + \frac{484b^2d^2x^4\sqrt{d-c^2dx^2}}{77175} - \frac{10b^2c^2d^2x^6\sqrt{d-c^2dx^2}}{3087} +$$

$$= \frac{22b^2d^2x^2\sqrt{d-c^2dx^2}}{14175c^2} + \frac{484b^2d^2x^4\sqrt{d-c^2dx^2}}{77175} - \frac{10b^2c^2d^2x^6\sqrt{d-c^2dx^2}}{3087} +$$

$$= -\frac{40b^2d^2\sqrt{d-c^2dx^2}}{567c^4} + \frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2} + \frac{484b^2d^2x^4\sqrt{d-c^2dx^2}}{77175} -$$

$$= -\frac{856b^2d^2\sqrt{d-c^2dx^2}}{14175c^4} + \frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2} + \frac{484b^2d^2x^4\sqrt{d-c^2dx^2}}{77175}$$

$$= -\frac{37384b^2d^2\sqrt{d-c^2dx^2}}{694575c^4} + \frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2} + \frac{484b^2d^2x^4\sqrt{d-c^2dx}}{77175}$$

Mathematica [A] time = 0.707941, size = 288, normalized size = 0.33

$$\frac{d^2\sqrt{d-c^2dx^2}\left(3969a^2\left(7c^2x^2+2\right)\left(c^2x^2-1\right)^4 - 126abcx\sqrt{cx-1}\sqrt{cx+1}\left(49c^8x^8-171c^6x^6+189c^4x^4-21c^2x^2-126\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(3969*a^2*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^2) - 126*a
*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-126 - 21*c^2*x^2 + 189*c^4*x^4 - 171*
c^6*x^6 + 49*c^8*x^8) + 2*b^2*(6140 - 7039*c^2*x^2 - 106*c^4*x^4 + 2152*c^6
*x^6 - 1490*c^8*x^8 + 343*c^10*x^10) + 126*b*(63*a*(-1 + c^2*x^2)^4*(2 + 7*
c^2*x^2) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(126 + 21*c^2*x^2 - 189*c^4*x
^4 + 171*c^6*x^6 - 49*c^8*x^8))*ArcCosh[c*x] + 3969*b^2*(-1 + c^2*x^2)^4*(2
 + 7*c^2*x^2)*ArcCosh[c*x]^2))/(250047*c^4*(-1 + c^2*x^2))
```

Maple [B] time = 0.536, size = 2224, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] a^2*(-1/9*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(-c^2*d*x^2+d)^(7/2))+b
^2*(1/373248*(-d*(c^2*x^2-1))^(1/2)*(256*x^10*c^10-704*c^8*x^8+256*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^9*c^9+688*c^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7
*c^7-280*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(81*a
rccosh(c*x)^2-18*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)-3/175616*(-d*(c^2*
x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^
7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(49*arcco
sh(c*x)^2-14*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)+1/1728*(-d*(c^2*x^2-1)
)^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)*d^2/(c*x+1
)/c^4/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)-3/256
*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arcco
sh(c*x)^2+2*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)+1/1728*(-d*(c^2*x^2-1))
^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)*d^2/(c*x+1
)/c^4/(c*x-1)-3/175616*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-
56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c-25*c^2*x^2+1)*(49*arccosh(c*x)^2+14*arccosh(c*x)+2)*d^2/(c*x+1)/c
^4/(c*x-1)+1/373248*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^9*c^9+256*x^10*c^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-704*c^8*x^8
```

```
-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*c^6*x^6+120*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+41*c^2*x^2-
1)*(81*arccosh(c*x)^2+18*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1))+2*a*b*(1/
41472*(-d*(c^2*x^2-1))^(1/2)*(256*x^10*c^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^9*c^9+688*c^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-28
0*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccos
h(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-
144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c
^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c^4
/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*
x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256*(
-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arcco
sh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^
2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-1)
)^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3
+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c
*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/41472*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^9*c^9+256*x^10*c^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*c^6*x^6+120*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.55603, size = 1231, normalized size = 1.4

$$\frac{3969\left(7\,b^{2}c^{10}d^{2}x^{10}-26\,b^{2}c^{8}d^{2}x^{8}+34\,b^{2}c^{6}d^{2}x^{6}-16\,b^{2}c^{4}d^{2}x^{4}-b^{2}c^{2}d^{2}x^{2}+2\,b^{2}d^{2}\right)\sqrt{-c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)^{2}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] 1/250047*(3969*(7*b^2*c^10*d^2*x^10 - 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*x
^6 - 16*b^2*c^4*d^2*x^4 - b^2*c^2*d^2*x^2 + 2*b^2*d^2)*sqrt(-c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1))^2 - 126*(49*a*b*c^9*d^2*x^9 - 171*a*b*c^7*d^2
*x^7 + 189*a*b*c^5*d^2*x^5 - 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 126*((49*b^2*c^9*d^2*x^9 - 171*b^2*c^7*d^2
*x^7 + 189*b^2*c^5*d^2*x^5 - 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 63*(7*a*b*c^10*d^2*x^10 - 26*a*b*c^8*d^2*x
^8 + 34*a*b*c^6*d^2*x^6 - 16*a*b*c^4*d^2*x^4 - a*b*c^2*d^2*x^2 + 2*a*b*d^2)
*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (343*(81*a^2 + 2*b^2)
*c^10*d^2*x^10 - 2*(51597*a^2 + 1490*b^2)*c^8*d^2*x^8 + 2*(67473*a^2 + 2152
*b^2)*c^6*d^2*x^6 - 4*(15876*a^2 + 53*b^2)*c^4*d^2*x^4 - (3969*a^2 + 14078*
b^2)*c^2*d^2*x^2 + 2*(3969*a^2 + 6140*b^2)*d^2)*sqrt(-c^2*d*x^2 + d))/(c^6*
x^2 - c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.187 $\int x^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=841

$$-\frac{bc^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^8}{32\sqrt{cx-1}\sqrt{cx+1}}+\frac{b^2c^4d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}x^7}{256(1-cx)(cx+1)}+\frac{17bc^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^6}{144\sqrt{cx-1}\sqrt{cx+1}}-$$

```
[Out] (35*b^2*d^2*x*Sqrt[d - c^2*d*x^2])/(9216*c^2) + (215*b^2*d^2*x^3*Sqrt[d - c
^2*d*x^2])/13824 - (5*b^2*c^2*d^2*x^5*Sqrt[d - c^2*d*x^2])/864 + (73*b^2*d^
2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(12288*c^2*(1 - c*x)*(1 + c*x)) + (7
3*b^2*d^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(18432*(1 - c*x)*(1 + c*x)
) - (43*b^2*c^2*d^2*x^5*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(4608*(1 - c*x)*
(1 + c*x)) + (b^2*c^4*d^2*x^7*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(256*(1 -
c*x)*(1 + c*x)) + (35*b^2*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(9216*c^3*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/(128*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (59*b*c*d^2*x^4*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(384*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (17*b
*c^3*d^2*x^6*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(144*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (b*c^5*d^2*x^8*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(
32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])^2)/(128*c^2) + (5*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^2)/64 + (5*d*x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/48 + (x^3*
(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/8 - (5*d^2*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^3)/(384*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (73*b^
2*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^
2]])/(12288*c^3*(1 - c*x)*(1 + c*x))
```

Rubi [A] time = 2.12388, antiderivative size = 872, normalized size of antiderivative = 1.04, number of steps used = 30, number of rules used = 21, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.724, Rules used = {5798, 5745, 5743, 5759, 5676, 5662, 90, 52, 100, 12, 14, 5731, 460, 266, 43, 520, 1267, 459, 321, 217, 206}

$$-\frac{bc^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^8}{32\sqrt{cx-1}\sqrt{cx+1}}+\frac{b^2c^4d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}x^7}{256(1-cx)(cx+1)}+\frac{17bc^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^6}{144\sqrt{cx-1}\sqrt{cx+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```


```
[Out] (35*b^2*d^2*x*Sqrt[d - c^2*d*x^2])/(9216*c^2) + (215*b^2*d^2*x^3*Sqrt[d - c
^2*d*x^2])/13824 - (5*b^2*c^2*d^2*x^5*Sqrt[d - c^2*d*x^2])/864 + (73*b^2*d^
```

```
2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(12288*c^2*(1 - c*x)*(1 + c*x)) + (7
3*b^2*d^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(18432*(1 - c*x)*(1 + c*x)
) - (43*b^2*c^2*d^2*x^5*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(4608*(1 - c*x)*
(1 + c*x)) + (b^2*c^4*d^2*x^7*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(256*(1 -
c*x)*(1 + c*x)) + (35*b^2*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(9216*c^3*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/(128*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (59*b*c*d^2*x^4*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(384*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (17*b
*c^3*d^2*x^6*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(144*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (b*c^5*d^2*x^8*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(
32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])^2)/(128*c^2) + (5*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^2)/64 + (5*d^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])^2)/48 + (d^2*x^3*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^2)/8 - (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(38
4*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (73*b^2*d^2*Sqrt[-1 + c^2*x^2]*Sqrt
[d - c^2*d*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(12288*c^3*(1 - c*x)*(1
+ c*x))
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
```

```
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right) \int x^2(-1+cx)^{5/2}(1+cx)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{8}d^2x^3(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\left(5d^2\sqrt{d-c^2dx^2}\right)}{}$$

$$= -\frac{bcd^2x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d^2x^6\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{12\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{11bcd^2x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{96\sqrt{-1+cx}\sqrt{1+cx}} + \frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{144\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{59bcd^2x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{384\sqrt{-1+cx}\sqrt{1+cx}} + \frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{144\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{5}{512}b^2d^2x^3\sqrt{d-c^2dx^2} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d-c^2dx^2} + \frac{b^2c^4d^2x^7\left(1-c^2x^2\right)\sqrt{d}}{256(1-cx)(1+c}$$

$$= -\frac{5b^2d^2x\sqrt{d-c^2dx^2}}{256c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d-c^2dx^2} -$$

$$= -\frac{5b^2d^2x\sqrt{d-c^2dx^2}}{1024c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d-c^2dx^2} +$$

$$= \frac{35b^2d^2x\sqrt{d-c^2dx^2}}{9216c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d-c^2dx^2} +$$

$$= \frac{35b^2d^2x\sqrt{d-c^2dx^2}}{9216c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d-c^2dx^2} +$$

$$= \frac{35b^2d^2x\sqrt{d-c^2dx^2}}{9216c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d-c^2dx^2} +$$

Mathematica [A] time = 5.82776, size = 910, normalized size = 1.08

$$-\frac{d^2\left(-110592a^2c^8\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}x^8 - 110592a^2c^7\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}x^7 + 313344a^2c^6\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}x^6 + 313344a^2c^5\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -(d^2*(34560*a^2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 34560
*a^2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 271872*a^2*c^
3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 271872*a^2*c^4*x^4*S
qrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 313344*a^2*c^5*x^5*Sqrt[(-1
 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 313344*a^2*c^6*x^6*Sqrt[(-1 + c*x)
/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 110592*a^2*c^7*x^7*Sqrt[(-1 + c*x)/(1 + c
*x)]*Sqrt[d - c^2*d*x^2] - 110592*a^2*c^8*x^8*Sqrt[(-1 + c*x)/(1 + c*x)]*Sq
rt[d - c^2*d*x^2] + 11520*b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^3 + 34560*a^
2*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt
[d]*(-1 + c^2*x^2))] + 34560*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Arc
Tan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 13824*a*b*Sqrt[d
- c^2*d*x^2]*Cosh[2*ArcCosh[c*x]] + 3456*a*b*Sqrt[d - c^2*d*x^2]*Cosh[4*Arc
Cosh[c*x]] - 1536*a*b*Sqrt[d - c^2*d*x^2]*Cosh[6*ArcCosh[c*x]] + 216*a*b*Sq
rt[d - c^2*d*x^2]*Cosh[8*ArcCosh[c*x]] - 6912*b^2*Sqrt[d - c^2*d*x^2]*Sinh[
2*ArcCosh[c*x]] - 864*b^2*Sqrt[d - c^2*d*x^2]*Sinh[4*ArcCosh[c*x]] + 256*b^
2*Sqrt[d - c^2*d*x^2]*Sinh[6*ArcCosh[c*x]] - 27*b^2*Sqrt[d - c^2*d*x^2]*Sin
h[8*ArcCosh[c*x]] + 24*b*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*(576*b*Cosh[2*Arc
Cosh[c*x]] + 144*b*Cosh[4*ArcCosh[c*x]] - 64*b*Cosh[6*ArcCosh[c*x]] + 9*b*C
osh[8*ArcCosh[c*x]] - 1152*a*Sinh[2*ArcCosh[c*x]] - 576*a*Sinh[4*ArcCosh[c*
x]] + 384*a*Sinh[6*ArcCosh[c*x]] - 72*a*Sinh[8*ArcCosh[c*x]]) - 288*b*Sqrt[
d - c^2*d*x^2]*ArcCosh[c*x]^2*(-120*a + 48*b*Sinh[2*ArcCosh[c*x]] + 24*b*Si
nh[4*ArcCosh[c*x]] - 16*b*Sinh[6*ArcCosh[c*x]] + 3*b*Sinh[8*ArcCosh[c*x]]))
)/(884736*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.533, size = 1312, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/48*a^2/c^2*x*(-c^2*d*x^2+d)^(5/2)+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x
+1)*c^6/(c*x-1)*arccosh(c*x)*x^9-23/24*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+
1)*c^4/(c*x-1)*arccosh(c*x)*x^7+127/96*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+
1)*c^2/(c*x-1)*arccosh(c*x)*x^5+5/64*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)
/c^2/(c*x-1)*arccosh(c*x)*x-5/128*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*d^2-133/192*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2
/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+17/144*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*
x+1)^(1/2)*c^3/(c*x-1)^(1/2)*arccosh(c*x)*x^6+1/8*b^2*(-d*(c^2*x^2-1))^(1/2
```

```
)*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)^2*x^9-23/48*b^2*(-d*(c^2*x^2-1))^(1/
2)*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)^2*x^7+127/192*b^2*(-d*(c^2*x^2-1))^
(1/2)*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)^2*x^5+5/128*b^2*(-d*(c^2*x^2-1))
^(1/2)*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)^2*x-59/384*b^2*(-d*(c^2*x^2-1))
^(1/2)*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*arccosh(c*x)*x^4+5/128*b^2*(-d*(c^
2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*arccosh(c*x)*x^2-1/32*b^2
*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*arccosh(c*x)*x^
8-1/32*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^8+1
7/144*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-59
/384*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+5/128
*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+5/128*a^2
/c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/128*a^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2
*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1081/110592*b^2*(-d*(c^2*x^2-1))^(1/2)*d^
2/(c*x+1)/(c*x-1)*x^3+5/192*a^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)-1/8*a^2*x*(-c^
2*d*x^2+d)^(7/2)/c^2/d+359/36864*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/
2)/c^3/(c*x-1)^(1/2)*arccosh(c*x)+359/36864*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/
(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)-5/384*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^3*d^2+1/256*b^2*(-d*(c^2*x^2-1))^(1/2)*d
^2/(c*x+1)*c^6/(c*x-1)*x^9-263/13824*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)
*c^4/(c*x-1)*x^7+1915/55296*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^2/(c*x
-1)*x^5-359/36864*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/c^2/(c*x-1)*x-133/
384*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2c^4d^2x^6 - 2\,a^2c^2d^2x^4 + a^2d^2x^2 + \left(b^2c^4d^2x^6 - 2\,b^2c^2d^2x^4 + b^2d^2x^2\right)\mathrm{arcosh}\left(cx\right)^2 + 2\left(abc^4d^2x^6 - 2\,abc^2d^2x^4 +\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^6 - 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2*
x^6 - 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^6
- 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.188 $\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=470

$$-\frac{2bc^5d^2x^7\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{6bc^3d^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (-32*b^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(245*c^2*(1 - c*x)*(1 + c*x
)) - (16*b^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(735*c^2*(1 - c*x)*(1
 + c*x)) - (12*b^2*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(1225*c^2*(1 -
c*x)*(1 + c*x)) - (2*b^2*d^2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(343*c^2*
(1 - c*x)*(1 + c*x)) + (2*b*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*b*c^3*d^2*x^5*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(2*b*c^5*d^2*x^7*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(49*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x])^2)/(7*c^2*d
)
```

Rubi [A] time = 0.680231, antiderivative size = 485, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5718, 194, 5680, 12, 1610, 1799, 1850}

$$-\frac{2bc^5d^2x^7\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{6bc^3d^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-32*b^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(245*c^2*(1 - c*x)*(1 + c*x
)) - (16*b^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(735*c^2*(1 - c*x)*(1
 + c*x)) - (12*b^2*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(1225*c^2*(1 -
c*x)*(1 + c*x)) - (2*b^2*d^2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(343*c^2*
(1 - c*x)*(1 + c*x)) + (2*b*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*b*c^3*d^2*x^5*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(2*b*c^5*d^2*x^7*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(49*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - (d^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b
```

```
*ArcCosh[c*x])^2)/(7*c^2)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
```

```
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{7c^2}-\frac{\left(2bd^2\sqrt{d-c^2dx^2}\right)\int}{7c}$$

$$=\frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{7\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{7\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{7\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{7\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{7\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{32b^2d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{245c^2(1-cx)(1+cx)}-\frac{16b^2d^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}}{735c^2(1-cx)(1+cx)}-\frac{12b^2d^2\left(1\right.}{1225c}$$

Mathematica [A] time = 0.59218, size = 234, normalized size = 0.5

$$\frac{d^2\sqrt{d-c^2dx^2}\left(3675a^2\left(c^2x^2-1\right)^4-210abcx\sqrt{cx-1}\sqrt{cx+1}\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)+210b\cosh^{-1}(cx)\left(35a\left(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(3675*a^2*(-1 + c^2*x^2)^4 - 210*a*b*c*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + 2*b^2*(21
61 - 2918*c^2*x^2 + 1108*c^4*x^4 - 426*c^6*x^6 + 75*c^8*x^8) + 210*b*(35*a*
(-1 + c^2*x^2)^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(35 - 35*c^2*x^2 + 21
*c^4*x^4 - 5*c^6*x^6))*ArcCosh[c*x] + 3675*b^2*(-1 + c^2*x^2)^4*ArcCosh[c*x
]^2))/(25725*c^2*(-1 + c^2*x^2))
```

Maple [B] time = 0.468, size = 1958, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/7*a^2/c^2/d*(-c^2*d*x^2+d)^(7/2)+b^2*(1/43904*(-d*(c^2*x^2-1))^(1/2)*(64
*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(49*arccosh(c*x)^2-14*arcc
osh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)-1/3200*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x
^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*arccos
h(c*x)^2-10*arccosh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)+1/384*(-d*(c^2*x^2-1))^
(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)*d^2/(c*x+1)/
c^2/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c
^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)-5/128*(
-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh
(c*x)^2+2*arccosh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1
/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)*d^2/(c*x+1)/c
^2/(c*x-1)-1/3200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
```

```
^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c*x)^2+10*arccosh(c*x)
+2)*d^2/(c*x+1)/c^2/(c*x-1)+1/43904*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5
-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(49*arccosh(c*x)^2+14*arccosh(c*x)+2)*
d^2/(c*x+1)/c^2/(c*x-1))+2*a*b*(1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-1
44*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^
3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c^2/
(c*x-1)-1/640*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*
x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*
d^2/(c*x+1)/c^2/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-5/128*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^
2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-1/640*(-d*(c^2*x^2-1))^(1/2
)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1
)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/6272*(-d*(c^2*x^2-1))^(1/2)*
(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4
*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))*d^2
/(c*x+1)/c^2/(c*x-1))
```

Maxima [A] time = 1.3219, size = 455, normalized size = 0.97

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}b^2\operatorname{arcosh}(cx)^2}{7\,c^2d}-\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}ab\operatorname{arcosh}(cx)}{7\,c^2d}+\frac{2}{25725}\,b^2\left(\frac{75\,\sqrt{c^2x^2-1}c^4\sqrt{-d}d^3x^6-351\,\sqrt{c^2x^2-1}c^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -1/7*(-c^2*d*x^2 + d)^(7/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/7*(-c^2*d*x^2 +
d)^(7/2)*a*b*arccosh(c*x)/(c^2*d) + 2/25725*b^2*((75*sqrt(c^2*x^2 - 1)*c^4*
```

```
sqrt(-d)*d^3*x^6 - 351*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*d^3*x^4 + 757*sqrt(c^
2*x^2 - 1)*sqrt(-d)*d^3*x^2 - 2161*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^3/c^2)/d -
105*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5 + 35*c^2*sqrt(-d)*d^3
*x^3 - 35*sqrt(-d)*d^3*x)*arccosh(c*x)/(c*d)) - 1/7*(-c^2*d*x^2 + d)^(7/2)*
a^2/(c^2*d) - 2/245*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5 + 35*
c^2*sqrt(-d)*d^3*x^3 - 35*sqrt(-d)*d^3*x)*a*b/(c*d)
```

Fricas [A] time = 2.56769, size = 1031, normalized size = 2.19

$$\frac{3675\left(b^2c^8d^2x^8 - 4\,b^2c^6d^2x^6 + 6\,b^2c^4d^2x^4 - 4\,b^2c^2d^2x^2 + b^2d^2\right)\sqrt{-c^2dx^2 + d}\log\left(cx + \sqrt{c^2x^2 - 1}\right)^2 - 210\left(5\,abc^7d^2x^7 - 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/25725*(3675*(b^2*c^8*d^2*x^8 - 4*b^2*c^6*d^2*x^6 + 6*b^2*c^4*d^2*x^4 - 4*
b^2*c^2*d^2*x^2 + b^2*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)
)^2 - 210*(5*a*b*c^7*d^2*x^7 - 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3 - 35
*a*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 210*((5*b^2*c^7*d^2*
x^7 - 21*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^2*x^3 - 35*b^2*c*d^2*x)*sqrt(-c^2*d
*x^2 + d)*sqrt(c^2*x^2 - 1) - 35*(a*b*c^8*d^2*x^8 - 4*a*b*c^6*d^2*x^6 + 6*a
*b*c^4*d^2*x^4 - 4*a*b*c^2*d^2*x^2 + a*b*d^2)*sqrt(-c^2*d*x^2 + d))*log(c*x
 + sqrt(c^2*x^2 - 1)) + (75*(49*a^2 + 2*b^2)*c^8*d^2*x^8 - 12*(1225*a^2 + 7
1*b^2)*c^6*d^2*x^6 + 2*(11025*a^2 + 1108*b^2)*c^4*d^2*x^4 - 4*(3675*a^2 + 1
459*b^2)*c^2*d^2*x^2 + (3675*a^2 + 4322*b^2)*d^2)*sqrt(-c^2*d*x^2 + d))/(c^
4*x^2 - c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.189 $\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=486

$$\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18c\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48c\sqrt{cx-1}\sqrt{cx+1}}-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+\right.}{48bc\sqrt{cx-1}}$$

```
[Out] (245*b^2*d^2*x*Sqrt[d - c^2*d*x^2])/1152 + (65*b^2*d^2*x*(1 - c*x)*(1 + c*x
)*Sqrt[d - c^2*d*x^2])/1728 + (b^2*d^2*x*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c
^2*d*x^2])/108 + (115*b^2*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(1152*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*d^2*(1 - c^2*x^2)^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(48*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (b*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(18*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x])^2)/16 + (5*d*x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/24 + (x*(
d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/6 - (5*d^2*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x])^3)/(48*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.867642, antiderivative size = 517, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5713, 5685, 5683, 5676, 5662, 90, 52, 5716, 38}

$$\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18c\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48c\sqrt{cx-1}\sqrt{cx+1}}-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+\right.}{48bc\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (245*b^2*d^2*x*Sqrt[d - c^2*d*x^2])/1152 + (65*b^2*d^2*x*(1 - c*x)*(1 + c*x
)*Sqrt[d - c^2*d*x^2])/1728 + (b^2*d^2*x*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c
^2*d*x^2])/108 + (115*b^2*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(1152*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*d^2*(1 - c^2*x^2)^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(48*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (b*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(18*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x])^2)/16 + (5*d^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x])^2)/24 + (d^2*x*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*A
```

```
rcCosh[c*x])^2)/6 - (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(48*
b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\left(5d^2\sqrt{d-c^2dx^2}\right)\int(-$$

$$= \frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5}{24}d^2x(1-cx)(1+cx)\sqrt{d-c$$

$$= \frac{1}{108}b^2d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2} + \frac{5bd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\,c\right.}{48c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728} + \frac{1}{108}b^2d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2} -$$

$$= \frac{245b^2d^2x\sqrt{d-c^2dx^2}}{1152} + \frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728} + \frac{1}{108}b^2d^2x(1-c$$

$$= \frac{245b^2d^2x\sqrt{d-c^2dx^2}}{1152} + \frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728} + \frac{1}{108}b^2d^2x(1-c$$

Mathematica [A] time = 3.3673, size = 740, normalized size = 1.52

$$\frac{d^2\left(2304a^2c^6x^6\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2} + 2304a^2c^5x^5\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2} - 7488a^2c^4x^4\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2} - 7488a^2c^3x^3\sqrt{\frac{cx-1}{cx+1}}\sqrt{d\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*(9504*a^2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 9504*a^
2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 7488*a^2*c^3*x^3
*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 7488*a^2*c^4*x^4*Sqrt[(-1
 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 2304*a^2*c^5*x^5*Sqrt[(-1 + c*x)/(
1 + c*x)]*Sqrt[d - c^2*d*x^2] + 2304*a^2*c^6*x^6*Sqrt[(-1 + c*x)/(1 + c*x)]
*Sqrt[d - c^2*d*x^2] - 1440*b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^3 - 4320*a
^2*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqr
t[d]*(-1 + c^2*x^2))] - 4320*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Arc
Tan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 3240*a*b*Sqrt[d -
```

```
 c^2*d*x^2]*Cosh[2*ArcCosh[c*x]] + 324*a*b*Sqrt[d - c^2*d*x^2]*Cosh[4*ArcCo
sh[c*x]] - 24*a*b*Sqrt[d - c^2*d*x^2]*Cosh[6*ArcCosh[c*x]] + 1620*b^2*Sqrt[
d - c^2*d*x^2]*Sinh[2*ArcCosh[c*x]] - 81*b^2*Sqrt[d - c^2*d*x^2]*Sinh[4*Arc
Cosh[c*x]] + 4*b^2*Sqrt[d - c^2*d*x^2]*Sinh[6*ArcCosh[c*x]] - 12*b*Sqrt[d -
 c^2*d*x^2]*ArcCosh[c*x]*(270*b*Cosh[2*ArcCosh[c*x]] - 27*b*Cosh[4*ArcCosh[
c*x]] + 2*b*Cosh[6*ArcCosh[c*x]] - 540*a*Sinh[2*ArcCosh[c*x]] + 108*a*Sinh[
4*ArcCosh[c*x]] - 12*a*Sinh[6*ArcCosh[c*x]]) + 72*b*Sqrt[d - c^2*d*x^2]*Arc
Cosh[c*x]^2*(-60*a + 45*b*Sinh[2*ArcCosh[c*x]] - 9*b*Sinh[4*ArcCosh[c*x]] +
 b*Sinh[6*ArcCosh[c*x]])))/(13824*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] time = 0.339, size = 1053, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/6*x*(-c^2*d*x^2+d)^(5/2)*a^2+5/24*a^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a^2*d
^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-c^2*d*x^2+d)^(1/2))-299/1152*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x
-1)*x+59/48*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)
^2*x^3+1/6*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^6*arccosh(c*x)^
2*x^7-17/24*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)
^2*x^5-5/16*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccos
h(c*x)^2*d^2-11/8*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*
x)*x-1/18*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5*ar
ccosh(c*x)*x^6+13/48*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(
1/2)*c^3*arccosh(c*x)*x^4-11/16*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2
)/(c*x-1)^(1/2)*c*arccosh(c*x)*x^2-1/18*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x
+1)^(1/2)/(c*x-1)^(1/2)*c^5*x^6+13/48*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*c^3*x^4-11/16*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^
(1/2)/(c*x-1)^(1/2)*c*x^2+59/24*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x
-1)*c^2*arccosh(c*x)*x^3+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)
*c^6*arccosh(c*x)*x^7-17/12*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*
c^4*arccosh(c*x)*x^5+299/1152*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/
(c*x-1)^(1/2)/c+299/1152*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-
1)^(1/2)/c*arccosh(c*x)+1/108*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1
)*c^6*x^7-113/1728*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^4*x^5+1
091/3456*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^2*x^3-5/48*b^2*(-
d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^3*d^2-11/16
*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2c^4d^2x^4 - 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 - 2\,b^2c^2d^2x^2 + b^2d^2\right)\mathrm{arcosh}\,(cx)^2 + 2\left(abc^4d^2x^4 - 2\,abc^2d^2x^2 + abd\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.190 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=836

$$-\frac{2bc^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^5}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{22bc^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^3}{45\sqrt{cx-1}\sqrt{cx+1}}-\frac{2}{27}b^2c^2d^2\sqrt{d-c^2dx^2}x^2-\frac{2b^2cd^2}{}$$

```
[Out] (68*b^2*d^2*Sqrt[d - c^2*d*x^2])/27 - (2*b^2*c^2*d^2*x^2*Sqrt[d - c^2*d*x^2
])/27 - (2*a*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (16*b^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(75*(1 - c*x)*(1 + c*x)) +
 (8*b^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(225*(1 - c*x)*(1 + c*x))
+ (2*b^2*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(125*(1 - c*x)*(1 + c*x))
 - (2*b^2*c*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) - (16*b*c*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (22*b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(45*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^5*d^2*x^5*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2 + (d*(d - c^2*d*x^2)^(3/2)*(a + b*A
rcCosh[c*x])^2)/3 + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/5 - (2*d
^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((
2*I)*b*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[
c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b^2*d^2*Sqrt[d - c^2*d*x^2]*
PolyLog[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^
2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqr
t[1 + c*x])
```

Rubi [A] time = 1.77567, antiderivative size = 867, normalized size of antiderivative = 1.04, number of steps used = 25, number of rules used = 17, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.586, Rules used = {5798, 5745, 5743, 5761, 4180, 2531, 2282, 6589, 5654, 74, 5680, 12, 460, 194, 520, 1247, 698}

$$-\frac{2bc^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^5}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{22bc^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^3}{45\sqrt{cx-1}\sqrt{cx+1}}-\frac{2}{27}b^2c^2d^2\sqrt{d-c^2dx^2}x^2-\frac{2b^2cd^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] (68*b^2*d^2*Sqrt[d - c^2*d*x^2])/27 - (2*b^2*c^2*d^2*x^2*Sqrt[d - c^2*d*x^2
])/27 - (2*a*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (16*b^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(75*(1 - c*x)*(1 + c*x)) +
 (8*b^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(225*(1 - c*x)*(1 + c*x))
+ (2*b^2*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(125*(1 - c*x)*(1 + c*x))
 - (2*b^2*c*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) - (16*b*c*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (22*b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(45*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^5*d^2*x^5*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2 + (d^2*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/3 + (d^2*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/5 - (2*d^2*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
(2*I)*b*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcC
osh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b*d^2*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) - ((2*I)*b^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*E^ArcCosh[c*x
]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^2*d^2*Sqrt[d - c^2*d*x^2]*Pol
yLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
```

```
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{5}d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-}{}}{\sqrt{-1}}$$

$$= -\frac{2bcd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4bc^3d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{15\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{16bcd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15\sqrt{-1+cx}\sqrt{1+cx}} + \frac{22bc^3d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{45\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{16bcd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15\sqrt{-1+cx}\sqrt{1+cx}} + \frac{22bc^3d^2x^3}{}$$

$$= -\frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cd^2x\sqrt{d-c^2dx^2}\cosh^{-1}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{68}{27}b^2d^2\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cd^2}{}$$

$$= \frac{68}{27}b^2d^2\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{16b^2d^2}{7}$$

$$= \frac{68}{27}b^2d^2\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{16b^2d^2}{7}$$

Mathematica [A] time = 7.19022, size = 1031, normalized size = 1.23

$$a^2\log(cx)d^{5/2} - a^2\log\left(d+\sqrt{-d\left(c^2x^2-1\right)}\sqrt{d}\right)d^{5/2} + \frac{ab\sqrt{-d(cx-1)(cx+1)}\left(-12\left(\frac{cx-1}{cx+1}\right)^{3/2}\cosh^{-1}(cx)(cx+1)^3 - 9cx +\right.}{9\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((23*a^2*d^2)/15 - (11*a^2*c^2*d^2*x^2)/15 + (a^2
*c^4*d^2*x^4)/5) + (a*b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*((
```

```
-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]])
)/(9*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b^2*d^2*Sqrt[-(d*(-1 + c*x)*(
1 + c*x))]*(-26 + (27*c*x*ArcCosh[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)) - 9*ArcCosh[c*x]^2 + (2 + 9*ArcCosh[c*x]^2)*Cosh[2*ArcCosh[c*x]] - (3*A
rcCosh[c*x]*Cosh[3*ArcCosh[c*x]])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))))/
27 + a^2*d^(5/2)*Log[c*x] - a^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*
x^2))]] + 2*a*b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-((c*x)/(Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x))) + ArcCosh[c*x] + (I*ArcCosh[c*x]*(Log[1 - I/E^Arc
Cosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)) + (I*(PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]]))
/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) + b^2*d^2*Sqrt[-(d*(-1 + c*x)*(1 +
 c*x))]*(2 - (2*c*x*ArcCosh[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) +
ArcCosh[c*x]^2 + (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x
]^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*
x]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^Arc
Cosh[c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x))) - (a*b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-450*c*x + 450*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3*ArcCosh[c*x]] + 9*
Cosh[5*ArcCosh[c*x]] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 45*ArcCosh[c*
x]*Sinh[5*ArcCosh[c*x]]))/(1800*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b^
2*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(13500*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
+ c*x) - 13500*c*x*ArcCosh[c*x] + 6750*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*ArcCosh[c*x]^2 + 750*ArcCosh[c*x]*Cosh[3*ArcCosh[c*x]] + 270*ArcCosh[c*x]*
Cosh[5*ArcCosh[c*x]] - 250*Sinh[3*ArcCosh[c*x]] - 1125*ArcCosh[c*x]^2*Sinh[
3*ArcCosh[c*x]] - 54*Sinh[5*ArcCosh[c*x]] - 675*ArcCosh[c*x]^2*Sinh[5*ArcCo
sh[c*x]]))/(54000*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [F] time = 0.413, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{arccosh}(cx))^2}{x} \left(-c^2dx^2 + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4 - 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 - 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arcosh}\,(cx)^2 + 2\left(abc^4d^2x^4 - 2\,abc^2d^2x^2 + ab\right.\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="fricas"
)
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx) + a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2/x, x)
```

3.191 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=607

$$-\frac{b^2cd^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{15bc^3d^2x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\right.$$

```
[Out] (-31*b^2*c^2*d^2*x*Sqrt[d - c^2*d*x^2])/64 - (b^2*c^2*d^2*x*(1 - c*x)*(1 +
c*x)*Sqrt[d - c^2*d*x^2])/32 - (89*b^2*c*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*
x])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*b*c^3*d^2*x^2*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d^2*(1 - c
^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (b*c*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/
(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*c^2*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])^2)/8 + (c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d*x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcC
osh[c*x])^2)/4 - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x + (5*c*d^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(8*b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (2*b*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*A
rcCosh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b^2*c*d^2*Sqrt[d - c^2*d*x
^2]*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.29001, antiderivative size = 638, normalized size of antiderivative = 1.05, number of steps used = 24, number of rules used = 16, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.552, Rules used = {5798, 5740, 5685, 5683, 5676, 5662, 90, 52, 5716, 38, 5727, 5660, 3718, 2190, 2279, 2391}

$$\frac{b^2cd^2\sqrt{d-c^2dx^2}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{15bc^3d^2x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cos\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```


```
[Out] (-31*b^2*c^2*d^2*x*Sqrt[d - c^2*d*x^2])/64 - (b^2*c^2*d^2*x*(1 - c*x)*(1 +
c*x)*Sqrt[d - c^2*d*x^2])/32 - (89*b^2*c*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*
x])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*b*c^3*d^2*x^2*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d^2*(1 - c
^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 +
```

```
c*x]) - (b*c*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/
(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*c^2*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])^2)/8 - (c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x])^2)/4 - (d^2*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x + (5*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^3)/(8*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c*d^2*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + (b^2*c*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(2*ArcCosh[c
*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
```

```
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} + \frac{\left(2bcd^2\sqrt{d-c^2dx^2}\right)}{\sqrt{-1}}$$

$$= \frac{bcd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5}{4}c^2d^2x(1-cx)(1+cx)\sqrt{d}$$

$$= \frac{1}{8}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} + \frac{bcd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{11}{16}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} + \frac{15bc^3d^2x^2\sqrt{}}{8}$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} - \frac{11b^2cd^2\sqrt{}}{16\sqrt{}}$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} - \frac{89b^2cd^2\sqrt{}}{64\sqrt{}}$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} - \frac{89b^2cd^2\sqrt{}}{64\sqrt{}}$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} - \frac{89b^2cd^2\sqrt{}}{64\sqrt{}}$$

Mathematica [A] time = 5.71888, size = 554, normalized size = 0.91

$$d^2\left(-256b^2\sqrt{d-c^2dx^2}\left(3cx\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+\cosh^{-1}(cx)\left(3\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)-cx\left(\cosh^{-1}(cx)\left(\cos\right.\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*(96*a^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-8 -
 9*c^2*x^2 + 2*c^4*x^4) + 1440*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 768*a
*b*Sqrt[d - c^2*d*x^2]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]
 - c*x*(ArcCosh[c*x]^2 + 2*Log[c*x])) - 256*b^2*Sqrt[d - c^2*d*x^2]*(ArcCos
h[c*x]*(3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[
c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])) + 3*c*x*PolyLog[
2, -E^(-2*ArcCosh[c*x])]) + 384*a*b*c*x*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh
[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + 64*b^2*c*x
*Sqrt[d - c^2*d*x^2]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]
] - 3*(1 + 2*ArcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]) - 12*a*b*c*x*Sqrt[d - c^
2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*A
rcCosh[c*x]]) - b^2*c*x*Sqrt[d - c^2*d*x^2]*(32*ArcCosh[c*x]^3 + 12*ArcCosh
[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*Sinh[4*ArcCosh[c*x]])
))/(768*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] time = 0.411, size = 1227, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x)
```

```
[Out] -5/4*a^2*c^2*d*x*(-c^2*d*x^2+d)^(3/2)-a^2/d/x*(-c^2*d*x^2+d)^(7/2)-a^2*c^2*
x*(-c^2*d*x^2+d)^(5/2)+1/2*a*b*(-d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c*x+1)/(c*x-
1)*arccosh(c*x)*x^5-11/4*a*b*(-d*(c^2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)
*arccosh(c*x)*x^3+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)*c^2*d^2/(c*x+1)/(c*x-1)*ar
ccosh(c*x)*x-15/8*a^2*c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*
x^2+d)^(1/2))-1/8*a*b*(-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*x^4+9/8*a*b*(-d*(c^2*x^2-1))^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2
)*x^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)*c*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccos
h(c*x)+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)^
2*x^5-11/8*b^2*(-d*(c^2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)^
2*x^3+1/8*b^2*(-d*(c^2*x^2-1))^(1/2)*c^2*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)^2
*x+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c*d^2-1/8*b^2*(-d*(c^2*x^2-1))^(1/2
)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*x^4+9/8*b^2*(-d*(c^2*x^2
```

```
-1))^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*x^2+2*a*b*(-d*(
c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2+1)*c*d^2+15/8*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)*arccosh(c*x)^2*c*d^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*d^2/(c*x
+1)/(c*x-1)/x-15/8*a^2*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+b^2*(-d*(c^2*x^2-1))^
(1/2)*arccosh(c*x)^2*d^2/(c*x+1)/(c*x-1)/x+1/32*b^2*(-d*(c^2*x^2-1))^(1/2)*
c^6*d^2/(c*x+1)/(c*x-1)*x^5+5/8*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)*arccosh(c*x)^3*c*d^2+b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d^2-35/64*b^
2*(-d*(c^2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*x^3+33/64*b^2*(-d*(c^2*x^2
-1))^(1/2)*c^2*d^2/(c*x+1)/(c*x-1)*x-33/64*a*b*(-d*(c^2*x^2-1))^(1/2)*c*d^2
/(c*x+1)^(1/2)/(c*x-1)^(1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)*c*d^2/(c*x+1)^(1/2)
/(c*x-1)^(1/2)*arccosh(c*x)^2-33/64*b^2*(-d*(c^2*x^2-1))^(1/2)*c*d^2/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4 - 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 - 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arcosh}\,(cx)^2 + 2\left(abc^4d^2x^4 - 2\,abc^2d^2x^2 + ab\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2/x^2, x)
```

3.192 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=890

$$-\frac{2bd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)c^5}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{5}{27}b^2d^2x^2\sqrt{d-c^2dx^2}c^4+\frac{5b^2d^2x\sqrt{d-c^2dx^2}\cosh^{-1}(cx)c^3}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bd^2x\sqrt{d-c^2d}}{3\sqrt{c}}$$

```
[Out] (-170*b^2*c^2*d^2*Sqrt[d - c^2*d*x^2])/27 + (5*b^2*c^4*d^2*x^2*Sqrt[d - c^2
*d*x^2])/27 + (5*a*b*c^3*d^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (5*b^2*c^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(3*(1 - c*x)*(1
 + c*x)) + (b^2*c^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(9*(1 - c*x)*(
1 + c*x)) + (5*b^2*c^3*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (b*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(x*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^5*d^2*x^3*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (5*c^2*d*(d - c^2*d*x^2)^
(3/2)*(a + b*ArcCosh[c*x])^2)/6 - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x
])^2)/(2*x^2) + (5*c^2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTa
n[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b^2*c^2*d^2*Sqrt[-1 +
c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/((1 - c*x)*(1 + c*
x)) - ((5*I)*b*c^2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2,
(-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((5*I)*b*c^2*d^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + ((5*I)*b^2*c^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[3,
 (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((5*I)*b^2*c^2*d^2*
Sqrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rubi [A] time = 1.98538, antiderivative size = 921, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 21, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.724, Rules used = {5798, 5740, 5745, 5743, 5761, 4180, 2531, 2282, 6589, 5654, 74, 5680, 12, 460, 270, 5731, 520, 1251, 897, 1153, 205}

$$-\frac{2bd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)c^5}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{5}{27}b^2d^2x^2\sqrt{d-c^2dx^2}c^4+\frac{5b^2d^2x\sqrt{d-c^2dx^2}\cosh^{-1}(cx)c^3}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bd^2x\sqrt{d-c^2d}}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] (-170*b^2*c^2*d^2*Sqrt[d - c^2*d*x^2])/27 + (5*b^2*c^4*d^2*x^2*Sqrt[d - c^2
*d*x^2])/27 + (5*a*b*c^3*d^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (5*b^2*c^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(3*(1 - c*x)*(1
 + c*x)) + (b^2*c^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(9*(1 - c*x)*(
1 + c*x)) + (5*b^2*c^3*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (b*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(x*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^5*d^2*x^3*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (5*c^2*d^2*(1 - c*x)*(1 +
 c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/6 - (d^2*(1 - c*x)^2*(1 +
 c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*x^2) + (5*c^2*d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (b^2*c^2*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*
ArcTan[Sqrt[-1 + c^2*x^2]])/((1 - c*x)*(1 + c*x)) - ((5*I)*b*c^2*d^2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + ((5*I)*b*c^2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((5
*I)*b^2*c^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - ((5*I)*b^2*c^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[3
, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{\left(bcd^2\sqrt{d-c^2dx^2}\right)\int \cdots}{\sqrt{-1+\cdots}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bc^3d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\cdots\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^3d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\cdots}{3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^3d^2x\sqrt{d-c^2\cdots}}{3\sqrt{-1\cdots}}$$

$$= \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5b^2c^3d^2x\sqrt{d-c^2dx^2}\cosh^{\cdots}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \cdots$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \cdots$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \cdots$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \cdots$$

Mathematica [A] time = 95.217, size = 1384, normalized size = 1.56

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((-7*a^2*c^2*d^2)/3 - (a^2*d^2)/(2*x^2) + (a^2*c^
4*d^2*x^2)/3) - (a*b*c^2*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*(
(-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]]
))/(18*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (5*a^2*c^2*d^(5/2)*Log[x])/2
 + (5*a^2*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/2 - 4*a*b
*c^2*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-((c*x)/(Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x))) + ArcCosh[c*x] + (I*ArcCosh[c*x]*(Log[1 - I/E^ArcCosh[c*x]]
- Log[1 + I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (I*(
PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x))) + (I*a*b*c^2*d^3*(((-I)*Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x))/(c*x) - (I*(-1 + c*x)*(1 + c*x)*ArcCosh[c*x])/(c^2*x^2) +
Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]]
 - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*
x]] + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]]
- Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[
-(d*(-1 + c*x)*(1 + c*x))] + (b^2*d^2*Sqrt[d - c^2*d*x^2]*((244*c^2)/(-1 +
c*x) - (244*c^3*x)/(-1 + c*x) - (4*c^4*x^2)/(-1 + c*x) + (4*c^5*x^3)/(-1 +
c*x) - (54*c^2*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (54*c*ArcCo
sh[c*x])/(x*(-1 + c*x)^(3/2)*Sqrt[1 + c*x]) - (252*c^3*x*ArcCosh[c*x])/((-1
 + c*x)^(3/2)*Sqrt[1 + c*x]) + (252*c^4*x^2*ArcCosh[c*x])/((-1 + c*x)^(3/2)
*Sqrt[1 + c*x]) + (12*c^5*x^3*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]
) - (12*c^6*x^4*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (126*c^2*A
rcCosh[c*x]^2)/(-1 + c*x) + (27*ArcCosh[c*x]^2)/(x^2*(-1 + c*x)) - (126*c^3
*x*ArcCosh[c*x]^2)/(-1 + c*x) - (18*c^4*x^2*ArcCosh[c*x]^2)/(-1 + c*x) + (1
8*c^5*x^3*ArcCosh[c*x]^2)/(-1 + c*x) + (27*c*ArcCosh[c*x]^2)/(x - c*x^2) +
(54*c^2*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1 + c*x)*Sqrt[-1 + c^2*x^2]) - (54
*c^3*x*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1 + c*x)*Sqrt[-1 + c^2*x^2]) - ((13
5*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]
])/(-1 + c*x) + ((135*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]^2*Log[
1 + I/E^ArcCosh[c*x]])/(-1 + c*x) - ((270*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]
*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]])/(-1 + c*x) + ((270*I)*c^2*Sq
rt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]])/(-1 + c
*x) - ((270*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[3, (-I)/E^ArcCosh[c*x
]])/(-1 + c*x) + ((270*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[3, I/E^Arc
Cosh[c*x]])/(-1 + c*x)))/54
```

Maple [F] time = 0.444, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{arccosh}(cx))^2}{x^3} \left(-c^2dx^2 + d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4-2\,a^2c^2d^2x^2+a^2d^2+\left(b^2c^4d^2x^4-2\,b^2c^2d^2x^2+b^2d^2\right)\text{arcosh}\left(cx\right)^2+2\left(abc^4d^2x^4-2\,abc^2d^2x^2+ab\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2/x^3, x)
```

3.193 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=638

$$\frac{7b^2c^3d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{5bc^5d^2x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\left(a+b\,\text{co}\right.$$

```
[Out] (7*b^2*c^4*d^2*x*Sqrt[d - c^2*d*x^2])/12 + (b^2*c^2*d^2*(1 - c*x)*(1 + c*x)
*Sqrt[d - c^2*d*x^2])/(3*x) + (23*b^2*c^3*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c
*x])/(12*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c^5*d^2*x^2*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c^3*d^2*(1
 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (b*c*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^4*d^2*x*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])^2)/2 - (7*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^2*d*(d - c^2*d*x^2)^(3/2
)*(a + b*ArcCosh[c*x])^2)/(3*x) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x
])^2)/(3*x^3) - (5*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(6*b
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (14*b*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (7*b^2*c^3*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.61533, antiderivative size = 669, normalized size of antiderivative = 1.05, number of steps used = 29, number of rules used = 17, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.586, Rules used = {5798, 5740, 5683, 5676, 5662, 90, 52, 5727, 5660, 3718, 2190, 2279, 2391, 38, 5729, 97, 12}

$$-\frac{7b^2c^3d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{5bc^5d^2x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\left(a+b\,\text{c}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (7*b^2*c^4*d^2*x*Sqrt[d - c^2*d*x^2])/12 + (b^2*c^2*d^2*(1 - c*x)*(1 + c*x)
*Sqrt[d - c^2*d*x^2])/(3*x) + (23*b^2*c^3*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c
*x])/(12*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c^5*d^2*x^2*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c^3*d^2*(1
```

```
 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (b*c*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^4*d^2*x*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])^2)/2 + (7*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^2*d^2*(1 - c*x)*(1 + c*x
)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*x) - (d^2*(1 - c*x)^2*(1 +
 c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*x^3) - (5*c^3*d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(6*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (14*b*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*Arc
Cosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b^2*c^3*d^2*Sqrt[d - c^2
*d*x^2]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3} + \frac{\left(2bcd^2\sqrt{d-c^2dx^2}\right)}{3\sqrt{-1}}$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5c^2d^2(1-cx)(1+cx)\sqrt{d}}{}$$

$$= \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} - \frac{7bc^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{7}{6}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} - \frac{5bc^5d^2x^2\sqrt{d-}}{2\sqrt{-}}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} + \frac{7b^2c^3d^2\sqrt{d-c}}{6\sqrt{-1+c}}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} + \frac{23b^2c^3d^2\sqrt{d-}}{12\sqrt{-1+}}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} + \frac{23b^2c^3d^2\sqrt{d-}}{12\sqrt{-1+}}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} + \frac{23b^2c^3d^2\sqrt{d-}}{12\sqrt{-1+}}$$

Mathematica [A] time = 3.3581, size = 803, normalized size = 1.26

$$-12a^2c^6d^3\sqrt{\frac{cx-1}{cx+1}}x^6+6abc^4d^3\cosh\left(2\cosh^{-1}(cx)\right)x^4+112abc^4d^3\log(cx)x^4-3b^2c^4d^3\sinh\left(2\cosh^{-1}(cx)\right)x^4-44a^2c^4d^3$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (-8*a*b*c*d^3*x + 8*a*b*c^2*d^3*x^2 - 8*a^2*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]
+ 64*a^2*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 8*b^2*c^2*d^3*x^2*Sqrt[(-
1 + c*x)/(1 + c*x)] - 44*a^2*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 8*b^2
*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 12*a^2*c^6*d^3*x^6*Sqrt[(-1 + c*x
)/(1 + c*x)] + 20*b^2*c^3*d^3*x^3*(-1 + c*x)*ArcCosh[c*x]^3 - 60*a^2*c^3*d^
(5/2)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d
 - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 6*a*b*c^3*d^3*x^3*Cosh[2*ArcCosh
[c*x]] + 6*a*b*c^4*d^3*x^4*Cosh[2*ArcCosh[c*x]] - 112*a*b*c^3*d^3*x^3*Log[c
*x] + 112*a*b*c^4*d^3*x^4*Log[c*x] - 56*b^2*c^3*d^3*x^3*(-1 + c*x)*PolyLog[
2, -E^(-2*ArcCosh[c*x])] + 3*b^2*c^3*d^3*x^3*Sinh[2*ArcCosh[c*x]] - 3*b^2*c
^4*d^3*x^4*Sinh[2*ArcCosh[c*x]] + 2*b*d^3*(-1 + c*x)*ArcCosh[c*x]*(4*b*c*x
+ 8*a*Sqrt[(-1 + c*x)/(1 + c*x)] + 8*a*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - 56*
a*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] - 56*a*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*
x)] + 3*b*c^3*x^3*Cosh[2*ArcCosh[c*x]] + 56*b*c^3*x^3*Log[1 + E^(-2*ArcCosh
[c*x])] - 6*a*c^3*x^3*Sinh[2*ArcCosh[c*x]]) - 2*b*d^3*(-1 + c*x)*ArcCosh[c*
x]^2*(-30*a*c^3*x^3 + 4*b*(-Sqrt[(-1 + c*x)/(1 + c*x)] - c*x*Sqrt[(-1 + c*x
)/(1 + c*x)] + 7*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 7*c^3*x^3*(-1 + Sqrt[
(-1 + c*x)/(1 + c*x)])) + 3*b*c^3*x^3*Sinh[2*ArcCosh[c*x]]))/(24*x^3*Sqrt[(
-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.434, size = 3431, normalized size = 5.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x)
```

```
[Out] 4/3*a^2*c^2/d/x*(-c^2*d*x^2+d)^(7/2)-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63
*c^4*x^4-15*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)
*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^2-14/3*b^2*
```

(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln((c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c^3*d^2+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*c^6*d
^2/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x^3-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*c^4*d^
2/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x+56/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*
c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-71/3*b^2*(-d*(c^2*x^2-1))^(1/
2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+16/3*b^2*(-d*(c^2*
x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-1/2*a*b*(
-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2+5*b^2*(-d*(c^
2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/
2)*c^5-7/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)
^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^3+5*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(6
3*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3-5/2*a*
b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c^3*d^2
+28/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c
^3*d^2-14/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln((c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c^3*d^2+5*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/
(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/2*b^2*(-d*(c^2*
x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*x^2+7/3*b^2*
(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3*c^6-1/3*a^2/d/x^3*
(-c^2*d*x^2+d)^(7/2)+4/3*a^2*c^4*x*(-c^2*d*x^2+d)^(5/2)+294*a*b*(-d*(c^2*x^
2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*
c^8-406*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1
)/(c*x-1)*arccosh(c*x)*c^6+380/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4
-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-46/3*a*b*(-d*(c^2*x^2-1))
^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2+70*
a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*arccosh(c*x)*c^5-294*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x
^4-15*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7-21*b^2*(-
d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1
)^(1/2)*c^7+5/3*a^2*c^4*d*x*(-c^2*d*x^2+d)^(3/2)+5/2*a^2*c^4*d^2*x*(-c^2*d*
x^2+d)^(1/2)+5/2*a^2*c^4*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x
^2+d)^(1/2))+a*b*(-d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c*x+1)/(c*x-1)*arccosh(c*x
)*x^3-a*b*(-d*(c^2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x+49/
3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x
-1)*c^8-56/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(
c*x+1)/(c*x-1)*c^6+7/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^
2+1)*x/(c*x+1)/(c*x-1)*c^4+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-1
5*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)-21*a*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-14/3*a*b
(-d(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)
^(1/2)*arccosh(c*x)*c^3-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c
^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+35*b^2*(-d*(c^2*x^2-1))^(1/2)*d
^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2
*c^5-21*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-147*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(

```
63*c^4*x^4-15*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^7
-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x^2/(c*x+1)^(
1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c+147*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^
4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^8+49/3*b^2*(-d*(c^
2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c
*x)*c^8-203*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c
*x+1)/(c*x-1)*arccosh(c*x)^2*c^6-56/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^
4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+190/3*b^2*(-d*(c^2
*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)
^2*c^4+7/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+
1)/(c*x-1)*arccosh(c*x)*c^4-23/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4
-15*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^2+1/4*a*b*(-d*(c^2*x^2-1)
)^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)-49/3*a*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3*c^6+7/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(
63*c^4*x^4-15*c^2*x^2+1)*x*c^4-49/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*
x^4-15*c^2*x^2+1)*x^3*arccosh(c*x)*c^6+7/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(
63*c^4*x^4-15*c^2*x^2+1)*x*arccosh(c*x)*c^4-7/3*b^2*(-d*(c^2*x^2-1))^(1/2)/
(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
*c^3*d^2-5/6*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh
(c*x)^3*c^3*d^2+14/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
*arccosh(c*x)^2*c^3*d^2+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c*x+1)/(c*x
-1)*x^3-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*x+1/4*b^2*(-
d*(c^2*x^2-1))^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)-1/3*b
^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4 - 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 - 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arcosh}\,(cx)^2 + 2\left(abc^4d^2x^4 - 2\,abc^2d^2x^2 + abd\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="giac"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2/x^4, x)
```

3.194 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=421

$$-\frac{16abx\sqrt{cx-1}\sqrt{cx+1}}{15c^5\sqrt{d-c^2dx^2}}-\frac{2bx^5\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{25c\sqrt{d-c^2dx^2}}-\frac{x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2d}-\frac{8bx^3\sqrt{cx-1}\sqrt{\ldots}}{45c\ldots}$$

```
[Out] (-16*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (41
44*b^2*(1 - c*x)*(1 + c*x))/(3375*c^6*Sqrt[d - c^2*d*x^2]) - (272*b^2*x^2*(
1 - c*x)*(1 + c*x))/(3375*c^4*Sqrt[d - c^2*d*x^2]) - (2*b^2*x^4*(1 - c*x)*(
1 + c*x))/(125*c^2*Sqrt[d - c^2*d*x^2]) - (16*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*ArcCosh[c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (8*b*x^3*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(45*c^3*Sqrt[d - c^2*d*x^2]) - (2*b*x^5
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(25*c*Sqrt[d - c^2*d*x^
2]) - (8*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^6*d) - (4*x^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^4*d) - (x^4*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x])^2)/(5*c^2*d)
```

Rubi [A] time = 1.13135, antiderivative size = 445, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5798, 5759, 5718, 5654, 74, 5662, 100, 12}

$$-\frac{16abx\sqrt{cx-1}\sqrt{cx+1}}{15c^5\sqrt{d-c^2dx^2}}-\frac{2bx^5\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{25c\sqrt{d-c^2dx^2}}-\frac{x^4(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2\sqrt{d-c^2dx^2}}-\frac{8bx^3\sqrt{cx-\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-16*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (41
44*b^2*(1 - c*x)*(1 + c*x))/(3375*c^6*Sqrt[d - c^2*d*x^2]) - (272*b^2*x^2*(
1 - c*x)*(1 + c*x))/(3375*c^4*Sqrt[d - c^2*d*x^2]) - (2*b^2*x^4*(1 - c*x)*(
1 + c*x))/(125*c^2*Sqrt[d - c^2*d*x^2]) - (16*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*ArcCosh[c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (8*b*x^3*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(45*c^3*Sqrt[d - c^2*d*x^2]) - (2*b*x^5
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(25*c*Sqrt[d - c^2*d*x^
2]) - (8*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(15*c^6*Sqrt[d - c^2*d
*x^2]) - (4*x^2*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(15*c^4*Sqrt[d
- c^2*d*x^2]) - (x^4*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(5*c^2*Sqr
```

```
t[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^4(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2\sqrt{d-c^2dx^2}} + \frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{5c^2\sqrt{d-c^2dx^2}} -$$

$$= -\frac{2bx^5\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{25c\sqrt{d-c^2dx^2}} - \frac{4x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{15c^4\sqrt{d-c^2dx^2}} -$$

$$= -\frac{2b^2x^4(1-cx)(1+cx)}{125c^2\sqrt{d-c^2dx^2}} - \frac{8bx^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{45c^3\sqrt{d-c^2dx^2}} - \frac{2bx^5\sqrt{-1+cx}\sqrt{1}}{25c}$$

$$= -\frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{8b^2x^2(1-cx)(1+cx)}{135c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^4(1-cx)(1+cx)}{125c^2\sqrt{d-c^2dx^2}} - \frac{8bx^3\sqrt{-1+}}{}$$

$$= -\frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{272b^2x^2(1-cx)(1+cx)}{3375c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^4(1-cx)(1+cx)}{125c^2\sqrt{d-c^2dx^2}} - \frac{16b^2x\sqrt{}}{}$$

$$= -\frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{32b^2(1-cx)(1+cx)}{27c^6\sqrt{d-c^2dx^2}} - \frac{272b^2x^2(1-cx)(1+cx)}{3375c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^4(1-}{125c^2\sqrt{}}$$

$$= -\frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{4144b^2(1-cx)(1+cx)}{3375c^6\sqrt{d-c^2dx^2}} - \frac{272b^2x^2(1-cx)(1+cx)}{3375c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^4(}{125c}$$

Mathematica [A] time = 0.53885, size = 255, normalized size = 0.61

$$\frac{\sqrt{d-c^2dx^2}\left(-225a^2\left(3c^6x^6+c^4x^4+4c^2x^2-8\right)+30abcx\sqrt{cx-1}\sqrt{cx+1}\left(9c^4x^4+20c^2x^2+120\right)+30b\cosh^{-1}(cx)\left(bcx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(30*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(120 + 20*c^2
*x^2 + 9*c^4*x^4) - 225*a^2*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6) - 2*b^2*
(-2072 + 1936*c^2*x^2 + 109*c^4*x^4 + 27*c^6*x^6) + 30*b*(b*c*x*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(120 + 20*c^2*x^2 + 9*c^4*x^4) - 15*a*(-8 + 4*c^2*x^2 + c
^4*x^4 + 3*c^6*x^6))*ArcCosh[c*x] - 225*b^2*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c
^6*x^6)*ArcCosh[c*x]^2))/(3375*c^6*d*(-1 + c*x)*(1 + c*x))
```

Maple [B] time = 0.477, size = 1314, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(-1/5*x^4/c^2/d*(-c^2*d*x^2+d)^(1/2)+4/5/c^2*(-1/3*x^2/c^2/d*(-c^2*d*x^
2+d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2)))+b^2*(-1/4000*(-d*(c^2*x^2-1))^(
1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x
^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-1)*(25*arccosh(c*x)^2-10*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-5/864*(-d*(c^2*
x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)/c^6/
d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-5/16*(-d*(c^
2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^
2+2*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-5/864*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-1/4000*
(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+
20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c*x)^2+10*arccosh(c*x)+2)/c^6/d/(c^2*x^2
-1))+2*a*b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*
c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))/c^6/d/(c^2*x^2
-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/c
^6/d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/
2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/c^6/d/(c^2
*x^2-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^
3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x
))/c^6/d/(c^2*x^2-1)-1/800*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^
4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))/c^6/d/
(c^2*x^2-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17923, size = 771, normalized size = 1.83

$$-\frac{225\left(3\,b^{2}c^{6}x^{6}+b^{2}c^{4}x^{4}+4\,b^{2}c^{2}x^{2}-8\,b^{2}\right)\sqrt{-c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)^{2}-30\left(9\,abc^{5}x^{5}+20\,abc^{3}x^{3}+120\,abcx\right)\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/3375*(225*(3*b^2*c^6*x^6 + b^2*c^4*x^4 + 4*b^2*c^2*x^2 - 8*b^2)*sqrt(-c^
2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(9*a*b*c^5*x^5 + 20*a*b*c^
3*x^3 + 120*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30*((9*b^2*c^
5*x^5 + 20*b^2*c^3*x^3 + 120*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1
) - 15*(3*a*b*c^6*x^6 + a*b*c^4*x^4 + 4*a*b*c^2*x^2 - 8*a*b)*sqrt(-c^2*d*x^
2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (27*(25*a^2 + 2*b^2)*c^6*x^6 + (225*
a^2 + 218*b^2)*c^4*x^4 + 4*(225*a^2 + 968*b^2)*c^2*x^2 - 1800*a^2 - 4144*b^
2)*sqrt(-c^2*d*x^2 + d))/(c^8*d*x^2 - c^6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2 x^5}{\sqrt{-c^2 d x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^5/sqrt(-c^2*d*x^2 + d), x)
```

3.195 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=355

$$-\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2d}-\frac{3bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{8c^3\sqrt{d-c^2dx^2}}$$

```
[Out] (-15*b^2*x*(1 - c*x)*(1 + c*x))/(64*c^4*Sqrt[d - c^2*d*x^2]) - (b^2*x^3*(1
- c*x)*(1 + c*x))/(32*c^2*Sqrt[d - c^2*d*x^2]) + (15*b^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*ArcCosh[c*x])/(64*c^5*Sqrt[d - c^2*d*x^2]) - (3*b*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(8*c^3*Sqrt[d - c^2*d*x^2]) - (b*x
^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[d - c^2*d*x
^2]) - (3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(8*c^4*d) - (x^3*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*c^2*d) + (Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])^3)/(8*b*c^5*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.02443, antiderivative size = 371, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5798, 5759, 5676, 5662, 90, 52, 100, 12}

$$-\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{d-c^2dx^2}}-\frac{x^3(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2\sqrt{d-c^2dx^2}}-\frac{3bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(\right.}{8c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-15*b^2*x*(1 - c*x)*(1 + c*x))/(64*c^4*Sqrt[d - c^2*d*x^2]) - (b^2*x^3*(1
- c*x)*(1 + c*x))/(32*c^2*Sqrt[d - c^2*d*x^2]) + (15*b^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*ArcCosh[c*x])/(64*c^5*Sqrt[d - c^2*d*x^2]) - (3*b*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(8*c^3*Sqrt[d - c^2*d*x^2]) - (b*x
^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[d - c^2*d*x
^2]) - (3*x*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(8*c^4*Sqrt[d - c^2
*d*x^2]) - (x^3*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(4*c^2*Sqrt[d -
 c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(8*b*c
^5*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int\frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$=-\frac{x^3(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2\sqrt{d-c^2dx^2}}+\frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\int\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{4c^2\sqrt{d-c^2dx^2}}-$$

$$=-\frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{d-c^2dx^2}}-\frac{3x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{8c^4\sqrt{d-c^2dx^2}}-\frac{x^3}{}$$

$$=-\frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}}-\frac{3bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{8c^3\sqrt{d-c^2dx^2}}-\frac{bx^4\sqrt{-1+cx}\sqrt{1+}}{8c\sqrt{d}}$$

$$=-\frac{3b^2x(1-cx)(1+cx)}{16c^4\sqrt{d-c^2dx^2}}-\frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}}-\frac{3bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx\right.}{8c^3\sqrt{d-c^2dx^2}}$$

$$=-\frac{15b^2x(1-cx)(1+cx)}{64c^4\sqrt{d-c^2dx^2}}-\frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}}+\frac{3b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{16c^5\sqrt{d-c^2dx^2}}-\frac{3b}{}$$

$$=-\frac{15b^2x(1-cx)(1+cx)}{64c^4\sqrt{d-c^2dx^2}}-\frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}}+\frac{15b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{64c^5\sqrt{d-c^2dx^2}}-\frac{3b}{}$$

Mathematica [A] time = 1.52373, size = 295, normalized size = 0.83

$$32a^2c\sqrt{d}x\left(c^2x^2-1\right)\left(2c^2x^2+3\right)-96a^2\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-4ab\sqrt{d}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(16\cosh\left(2\cosh^{-1}(cx)\right)+\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (32*a^2*c*Sqrt[d]*x*(-1 + c^2*x^2)*(3 + 2*c^2*x^2) - 96*a^2*Sqrt[d - c^2*d*
x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b^2*Sqrt[
d]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(32*ArcCosh[c*x]^3 - 4*ArcCosh[c*x]
*(16*Cosh[2*ArcCosh[c*x]] + Cosh[4*ArcCosh[c*x]]) + 32*Sinh[2*ArcCosh[c*x]]
 + Sinh[4*ArcCosh[c*x]] + 8*ArcCosh[c*x]^2*(8*Sinh[2*ArcCosh[c*x]] + Sinh[4
*ArcCosh[c*x]])) - 4*a*b*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(16*C
osh[2*ArcCosh[c*x]] + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*(6*ArcCosh[c*x]
 + 8*Sinh[2*ArcCosh[c*x]] + Sinh[4*ArcCosh[c*x]])))/(256*c^5*Sqrt[d]*Sqrt[d
 - c^2*d*x^2])
```

Maple [B] time = 0.48, size = 887, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a^2*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(-c^2*d*x^2+d)^(1/2
)+3/8*a^2/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/
32*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*x^5-13/64*b^2*(-d*(c^2*x^2-1))^
(1/2)/d/c^2/(c^2*x^2-1)*x^3+15/64*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^4/(c^2*x^2
-1)*x+1/8*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c/(c^2*x^2-1)*arccosh(c*x)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*x^4+3/8*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*arc
cosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/d/
(c^2*x^2-1)*arccosh(c*x)^2*x^5-1/8*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^
2-1)*arccosh(c*x)^2*x^3+3/8*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^4/(c^2*x^2-1)*ar
ccosh(c*x)^2*x-1/8*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
/c^5/(c^2*x^2-1)*arccosh(c*x)^3-15/64*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^5/(c^2
*x^2-1)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/2*a*b*(-d*(c^2*x^2-1))^(
1/2)/d/(c^2*x^2-1)*arccosh(c*x)*x^5-1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c
```

```
^2*x^2-1)*arccosh(c*x)*x^3+3/4*a*b*(-d*(c^2*x^2-1))^(1/2)/d/c^4/(c^2*x^2-1)
*arccosh(c*x)*x-15/64*a*b*(-d*(c^2*x^2-1))^(1/2)/d/c^5/(c^2*x^2-1)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)-3/8*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/d/c^5/(c^2*x^2-1)*arccosh(c*x)^2+1/8*a*b*(-d*(c^2*x^2-1))^(1/2)/d/c/(c^
2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+3/8*a*b*(-d*(c^2*x^2-1))^(1/2)/d/c
^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2x^4\operatorname{arcosh}(cx)^2+2\,abx^4\operatorname{arcosh}(cx)+a^2x^4\right)\sqrt{-c^2dx^2+d}}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt(
-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\left(a+b\operatorname{acosh}(cx)\right)^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2 x^4}{\sqrt{-c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^4/sqrt(-c^2*d*x^2 + d), x)
```

3.196 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=292

$$-\frac{4abx\sqrt{cx-1}\sqrt{cx+1}}{3c^3\sqrt{d-c^2dx^2}}-\frac{2bx^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d}-\frac{2\sqrt{d-c^2dx^2}\,(}{3}$$

```
[Out] (-4*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (40*b
^2*(1 - c*x)*(1 + c*x))/(27*c^4*Sqrt[d - c^2*d*x^2]) - (2*b^2*x^2*(1 - c*x)
*(1 + c*x))/(27*c^2*Sqrt[d - c^2*d*x^2]) - (4*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*ArcCosh[c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (2*b*x^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c*Sqrt[d - c^2*d*x^2]) - (2*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*c^4*d) - (x^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])^2)/(3*c^2*d)
```

Rubi [A] time = 0.769446, antiderivative size = 308, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5798, 5759, 5718, 5654, 74, 5662, 100, 12}

$$-\frac{4abx\sqrt{cx-1}\sqrt{cx+1}}{3c^3\sqrt{d-c^2dx^2}}-\frac{2bx^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c\sqrt{d-c^2dx^2}}-\frac{x^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2\sqrt{d-c^2dx^2}}-\frac{2(1-cx)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-4*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (40*b
^2*(1 - c*x)*(1 + c*x))/(27*c^4*Sqrt[d - c^2*d*x^2]) - (2*b^2*x^2*(1 - c*x)
*(1 + c*x))/(27*c^2*Sqrt[d - c^2*d*x^2]) - (4*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*ArcCosh[c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (2*b*x^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c*Sqrt[d - c^2*d*x^2]) - (2*(1 - c*x)
*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*c^4*Sqrt[d - c^2*d*x^2]) - (x^2*(1 -
c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*c^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
```

```
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2\sqrt{d-c^2dx^2}} + \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3c^2\sqrt{d-c^2dx^2}} - \ldots$$

$$= -\frac{2bx^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{9c\sqrt{d-c^2dx^2}} - \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4\sqrt{d-c^2dx^2}} - \ldots$$

$$= -\frac{4abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{2b^2x^2(1-cx)(1+cx)}{27c^2\sqrt{d-c^2dx^2}} - \frac{2bx^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}\ldots\right)}{9c\sqrt{d-c^2dx^2}}$$

$$= -\frac{4abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{2b^2x^2(1-cx)(1+cx)}{27c^2\sqrt{d-c^2dx^2}} - \frac{4b^2x\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{3c^3\sqrt{d-c^2dx^2}} - \ldots$$

$$= -\frac{4abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{40b^2(1-cx)(1+cx)}{27c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^2(1-cx)(1+cx)}{27c^2\sqrt{d-c^2dx^2}} - \frac{4b^2x\sqrt{-1+c\ldots}}{3c^3\ldots}$$

Mathematica [A] time = 0.444763, size = 201, normalized size = 0.69

$$\frac{\sqrt{d-c^2dx^2}\left(-9a^2\left(c^4x^4+c^2x^2-2\right)+6abcx\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2+6\right)+6b\cosh^{-1}(cx)\left(bcx\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2+6\right)\ldots\right.\right.}{27c^4d(cx-1)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(6*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2)
- 9*a^2*(-2 + c^2*x^2 + c^4*x^4) - 2*b^2*(-20 + 19*c^2*x^2 + c^4*x^4) + 6*b
*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2) - 3*a*(-2 + c^2*x^2 + c^
4*x^4))*ArcCosh[c*x] - 9*b^2*(-2 + c^2*x^2 + c^4*x^4)*ArcCosh[c*x]^2))/(27*
c^4*d*(-1 + c*x)*(1 + c*x))
```

Maple [B] time = 0.375, size = 752, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(-1/3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2))+b^
2*(-1/216*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*a
rccosh(c*x)+2)/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)/c^4/d/(c^2*x
^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-
1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)/c^4/d/(c^2*x^2-1)-1/216*(-d*(c^2*x^2-1
))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)/c^4/d/(c
^2*x^2-1))+2*a*b*(-1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*a
rccosh(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2
*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x)
)/c^4/d/(c^2*x^2-1)-1/72*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*
arccosh(c*x))/c^4/d/(c^2*x^2-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20677, size = 598, normalized size = 2.05

$$-\frac{9\left(b^2c^4x^4+b^2c^2x^2-2\,b^2\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-6\left(abc^3x^3+6\,abcx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}-6\left(\left(b^2c^3x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/27*(9*(b^2*c^4*x^4 + b^2*c^2*x^2 - 2*b^2)*sqrt(-c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 - 1))^2 - 6*(a*b*c^3*x^3 + 6*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sq
rt(c^2*x^2 - 1) - 6*((b^2*c^3*x^3 + 6*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^
2*x^2 - 1) - 3*(a*b*c^4*x^4 + a*b*c^2*x^2 - 2*a*b)*sqrt(-c^2*d*x^2 + d))*lo
g(c*x + sqrt(c^2*x^2 - 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 + (9*a^2 + 38*b^2)*c^
2*x^2 - 18*a^2 - 40*b^2)*sqrt(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2 x^3}{\sqrt{-c^2 d x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^3/sqrt(-c^2*d*x^2 + d), x)
```

3.197 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{2c\sqrt{d-c^2dx^2}}-\frac{b^2x}{4}$$

```
[Out] -(b^2*x*(1 - c*x)*(1 + c*x))/(4*c^2*Sqrt[d - c^2*d*x^2]) + (b^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(4*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*c*Sqrt[d - c^2*d*x^2]) - (x
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*c^2*d) + (Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(6*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.626361, antiderivative size = 234, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5798, 5759, 5676, 5662, 90, 52}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{2c\sqrt{d-c^2dx^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -(b^2*x*(1 - c*x)*(1 + c*x))/(4*c^2*Sqrt[d - c^2*d*x^2]) + (b^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(4*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*c*Sqrt[d - c^2*d*x^2]) - (x
*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(2*c^2*Sqrt[d - c^2*d*x^2]) +
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(6*b*c^3*Sqrt[d - c^2
*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2c^2\sqrt{d-c^2dx^2}} - \frac{(b\sqrt{}}{}$$

$$= -\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{2c\sqrt{d-c^2dx^2}} - \frac{x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2\sqrt{d-c^2dx^2}} + \frac{\sqrt{-}}{}$$

$$= -\frac{b^2x(1-cx)(1+cx)}{4c^2\sqrt{d-c^2dx^2}} - \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{2c\sqrt{d-c^2dx^2}} - \frac{x(1-cx)(1+cx)\left(a+\right.}{2c^2\sqrt{d-c}}$$

$$= -\frac{b^2x(1-cx)(1+cx)}{4c^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{4c^3\sqrt{d-c^2dx^2}} - \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\mathrm{c}\right.}{2c\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.862718, size = 228, normalized size = 1.01

$$\frac{-\frac{12a^2cx\sqrt{d-c^2dx^2}}{d} - \frac{12a^2\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(c^2x^2-1)}\right)}{\sqrt{d}} + \frac{6ab\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+\sinh\left(2\cosh^{-1}(cx)\right)\right)-\cosh\left(2\cosh^{-1}(cx)\right)\right)}{\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{\frac{cx-1}{cx+1}}(cx+}{}}{24c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-12*a^2*c*x*Sqrt[d - c^2*d*x^2])/d - (12*a^2*ArcTan[(c*x*Sqrt[d - c^2*d*x
^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*(4*ArcCosh[c*x]^3 - 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] + (3 + 6*A
rcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2] + (6*a*b*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*(-Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCo
sh[c*x] + Sinh[2*ArcCosh[c*x]])))/Sqrt[d - c^2*d*x^2])/(24*c^3)
```

Maple [B] time = 0.309, size = 624, normalized size = 2.8

$$-\frac{a^2x}{2c^2d}\sqrt{-c^2dx^2+d} + \frac{a^2}{2c^2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}} - \frac{b^2x^3}{4d\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)} + \frac{b^2x}{4c^2d\left(c^2x^2-1\right)}\sqrt{-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a^2/c^2/(c^2*d)^(1/2)*arctan((c^2
*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2
-1)*x^3+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2-1)*x-1/6*b^2*(-d*(c^2
*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)^3
+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^2-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)
^2*x^3+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2-1)*arccosh(c*x)^2*x-1/
4*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(
c*x-1)^(1/2)-1/2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c
^3/(c^2*x^2-1)*arccosh(c*x)^2-a*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arcc
osh(c*x)*x^3+1/2*a*b*(-d*(c^2*x^2-1))^(1/2)/d/c/(c^2*x^2-1)*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^2+a*b*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2-1)*arccosh(c*x)*
x-1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2x^2\operatorname{arcosh}(cx)^2 + 2\,abx^2\operatorname{arcosh}(cx) + a^2x^2\right)\sqrt{-c^2dx^2 + d}}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(
-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{acosh}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2 x^2}{\sqrt{-c^2 d x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^2/sqrt(-c^2*d*x^2 + d), x)
```

3.198 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=155

$$-\frac{2abx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d}-\frac{2b^2(1-cx)(cx+1)}{c^2\sqrt{d-c^2dx^2}}-\frac{2b^2x\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{c\sqrt{d-c^2dx^2}}$$

```
[Out] (-2*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (2*b^2*(1
 - c*x)*(1 + c*x))/(c^2*Sqrt[d - c^2*d*x^2]) - (2*b^2*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*ArcCosh[c*x])/(c*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]*(a +
 b*ArcCosh[c*x])^2)/(c^2*d)
```

Rubi [A] time = 0.341503, antiderivative size = 163, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5718, 5654, 74}

$$-\frac{2abx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{c^2\sqrt{d-c^2dx^2}}-\frac{2b^2(1-cx)(cx+1)}{c^2\sqrt{d-c^2dx^2}}-\frac{2b^2x\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (2*b^2*(1
 - c*x)*(1 + c*x))/(c^2*Sqrt[d - c^2*d*x^2]) - (2*b^2*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*ArcCosh[c*x])/(c*Sqrt[d - c^2*d*x^2]) - ((1 - c*x)*(1 + c*x)*(a +
 b*ArcCosh[c*x])^2)/(c^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
```

```
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{c^2\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\sqrt{1+cx}\right)\int \left(a+b\cosh^{-1}(cx)\right)\,dx}{c\sqrt{d-c^2dx^2}}$$

$$= -\frac{2abx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{c^2\sqrt{d-c^2dx^2}} - \frac{\left(2b^2\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{d-c^2}}$$

$$= -\frac{2abx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{2b^2x\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\,\mathrm{co}\right.}{c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{2abx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^2\sqrt{d-c^2dx^2}} - \frac{2b^2x\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c\sqrt{d-c^2dx^2}} - \frac{(1}{}$$

Mathematica [A] time = 0.379791, size = 149, normalized size = 0.96

$$\frac{\sqrt{d-c^2dx^2}\left(a^2\left(1-c^2x^2\right)+2b\cosh^{-1}(cx)\left(-ac^2x^2+a+bcx\sqrt{cx-1}\sqrt{cx+1}\right)+2abcx\sqrt{cx-1}\sqrt{cx+1}-2b^2\left(c^2x^2-1\right)\right.}{c^2d(cx-1)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(2*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + a^2*(1 - c^2
*x^2) - 2*b^2*(-1 + c^2*x^2) + 2*b*(a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])*ArcCosh[c*x] + b^2*(1 - c^2*x^2)*ArcCosh[c*x]^2))/(c^2*d*(-1 +
 c*x)*(1 + c*x))
```

Maple [B] time = 0.236, size = 314, normalized size = 2.

$$-\frac{a^2}{c^2d}\sqrt{-c^2dx^2+d}+b^2\left(-\frac{(\operatorname{arccosh}(cx))^2-2\operatorname{arccosh}(cx)+2}{2c^2d\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\left(\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)-\frac{(\operatorname{arccos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a^2/c^2/d*(-c^2*d*x^2+d)^(1/2)+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)/c^2/d/(
c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2
*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)/c^2/d/(c^2*x^2-1))+2*a*b*(-1/2*(-
d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccos
h(c*x))/c^2/d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/c^2/d/(c^2*x^2-1))
```

Maxima [A] time = 1.14279, size = 196, normalized size = 1.26

$$2b^2\left(\frac{\sqrt{-d}x\operatorname{arcosh}(cx)}{cd}-\frac{\sqrt{c^2x^2-1}\sqrt{-d}}{c^2d}\right)+\frac{2ab\sqrt{-d}x}{cd}-\frac{\sqrt{-c^2dx^2+d}b^2\operatorname{arcosh}(cx)^2}{c^2d}-\frac{2\sqrt{-c^2dx^2+d}ab\operatorname{arcosh}(cx)}{c^2d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2*b^2*(sqrt(-d)*x*arccosh(c*x)/(c*d) - sqrt(c^2*x^2 - 1)*sqrt(-d)/(c^2*d))
+ 2*a*b*sqrt(-d)*x/(c*d) - sqrt(-c^2*d*x^2 + d)*b^2*arccosh(c*x)^2/(c^2*d)
```

```
- 2*sqrt(-c^2*d*x^2 + d)*a*b*arccosh(c*x)/(c^2*d) - sqrt(-c^2*d*x^2 + d)*a^
2/(c^2*d)
```

Fricas [A] time = 2.19751, size = 448, normalized size = 2.89

$$\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}abcx-\left(b^2c^2x^2-b^2\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2+2\left(\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}b^2cx-\left(ab\right.\right.}{c^4dx^2-c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a*b*c*x - (b^2*c^2*x^2 - b^2)*sqr
t(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 + 2*(sqrt(-c^2*d*x^2 + d)*
sqrt(c^2*x^2 - 1)*b^2*c*x - (a*b*c^2*x^2 - a*b)*sqrt(-c^2*d*x^2 + d))*log(c
*x + sqrt(c^2*x^2 - 1)) - ((a^2 + 2*b^2)*c^2*x^2 - a^2 - 2*b^2)*sqrt(-c^2*d
*x^2 + d))/(c^4*d*x^2 - c^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\operatorname{acosh}(cx)\right)^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arcosh}(cx)+a\right)^2x}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x/sqrt(-c^2*d*x^2 + d), x)
```

3.199 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}$$

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c*Sqrt[d - c^2*d
*x^2])
```

Rubi [A] time = 0.195441, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5713, 5676}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c*Sqrt[d - c^2*d
*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
```

]

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.0497908, size = 53, normalized size = 1.

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c*Sqrt[d - c^2*d
*x^2])
```

Maple [B] time = 0.066, size = 149, normalized size = 2.8

$$a^2\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}-\frac{b^2(\operatorname{arccosh}(cx))^3}{3\,cd\left(c^2x^2-1\right)}\sqrt{-(cx-1)(cx+1)d}\sqrt{cx-1}\sqrt{cx+1}-\frac{ab(\operatorname{arccosh}(cx))^2}{cd\left(c^2x^2-1\right)}\sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/3*b^2*(-(c
*x-1)*(c*x+1)*d)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(c^2*x^2-1)*arccosh(
c*x)^3-a*b*(-(c*x-1)*(c*x+1)*d)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(c^2*
x^2-1)*arccosh(c*x)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2\right)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\text{acosh}\,(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2}{\sqrt{-c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/sqrt(-c^2*d*x^2 + d), x)
```

3.200 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=273

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\right.}{\sqrt{d-c^2dx^2}}$$

```
[Out] (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x
]])/Sqrt[d - c^2*d*x^2] - ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + ((2*I)*b*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x
]])/Sqrt[d - c^2*d*x^2] + ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3
, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rubi [A] time = 0.52126, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5798, 5761, 4180, 2531, 2282, 6589}

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\right.}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```


```
[Out] (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x
]])/Sqrt[d - c^2*d*x^2] - ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + ((2*I)*b*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x
]])/Sqrt[d - c^2*d*x^2] + ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3
, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^2\textbf{sech}(x)\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{\left(2ib\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}}{}$$

$$= \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\text{co}\right.}{\sqrt{d-c}}$$

$$= \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\text{co}\right.}{\sqrt{d-c}}$$

$$= \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\text{co}\right.}{\sqrt{d-c}}$$

Mathematica [A] time = 0.628862, size = 315, normalized size = 1.15

$$-\frac{2iab\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+\cosh^{-1}(cx)\left(\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-\text{log}\right.\right.}{\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d]
 - ((2*I)*a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(Log[1 - I
/E^ArcCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]) + PolyLog[2, (-I)/E^ArcCosh[c
*x]] - PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2] + (I*b^2*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*(-(ArcCosh[c*x]^2*(Log[1 - I/E^ArcCosh[c*x]] -
 Log[1 + I/E^ArcCosh[c*x]])) - 2*ArcCosh[c*x]*(PolyLog[2, (-I)/E^ArcCosh[c*
x]] - PolyLog[2, I/E^ArcCosh[c*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c*x]] + 2
*PolyLog[3, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2]
```

Maple [F] time = 0.315, size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x}\frac{1}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^2dx^3-dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^3 - d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{acosh}(cx))^2}{x\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{\sqrt{-c^2dx^2+d}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x), x)
```

3.201 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=186

$$\frac{b^2c\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{dx}-\frac{c\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}$$

```
[Out] -((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^
2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(d*x) - (2*b*c*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/Sqrt
[d - c^2*d*x^2] + (b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^(-2*Arc
Cosh[c*x])])/Sqrt[d - c^2*d*x^2]
```

Rubi [A] time = 0.518662, antiderivative size = 194, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5798, 5724, 5660, 3718, 2190, 2279, 2391}

$$-\frac{b^2c\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}}+\frac{c\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(\right.}{\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2]
 - ((1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(x*Sqrt[d - c^2*d*x^2]) - (
2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCos
h[c*x])])/Sqrt[d - c^2*d*x^2] - (b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog
[2, -E^(2*ArcCosh[c*x])])/Sqrt[d - c^2*d*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx\right.}{\sqrt{d-c^2dx^2}}$$

$$= \frac{c\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{\left(4bc\sqrt{-1+\right.}{}$$

$$= \frac{c\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{2bc\sqrt{-1+c}}{}$$

$$= \frac{c\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{2bc\sqrt{-1+c}}{}$$

$$= \frac{c\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{2bc\sqrt{-1+c}}{}$$

Mathematica [A] time = 0.839799, size = 179, normalized size = 0.96

$$\frac{b^2c\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+\cosh^{-1}(cx)\left(\frac{\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)}{cx}-\cosh^{-1}(cx)-2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\right.\right.}{\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] ((a^2*(-1 + c^2*x^2))/x - (2*a*b*(-1 + c^2*x^2)*(-ArcCosh[c*x] + (c*x*Log[c
*x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/x + b^2*c*Sqrt[(-1 + c*x)/(1 + c*x)]*
(1 + c*x)*(ArcCosh[c*x]*(-ArcCosh[c*x] + (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)*ArcCosh[c*x])/(c*x) - 2*Log[1 + E^(-2*ArcCosh[c*x])]) + PolyLog[2, -E^(
-2*ArcCosh[c*x])]))/Sqrt[d - c^2*d*x^2]
```

Maple [B] time = 0.257, size = 513, normalized size = 2.8

$$-\frac{a^2}{dx}\sqrt{-c^2dx^2+d}-\frac{b^2\left(\operatorname{arccosh}\left(cx\right)\right)^2c}{d\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}-\frac{b^2\left(\operatorname{arccosh}\left(cx\right)\right)^2xc^2}{d\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}+\frac{b^2\left(a\right.}{xa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a^2/d/x*(-c^2*d*x^2+d)^(1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)^2*c-b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh
(c*x)^2*x/(c^2*x^2-1)/d*c^2+b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)^2/x/(c^
2*x^2-1)/d+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*
x^2-1)*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c+b^2*(-d*(c^
2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*polylog(2,-(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)*ar
ccosh(c*x)*x/(c^2*x^2-1)/d*c^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/x/
(c^2*x^2-1)/d+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c
^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}\left(cx\right)^2+2\,ab\operatorname{arcosh}\left(cx\right)+a^2\right)}{c^2dx^4-dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^4 - d*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x^2\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2}{\sqrt{-c^2dx^2 + d}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

3.202 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=430

$$-\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+\right.}{\sqrt{d-c^2dx^2}}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(x*Sqrt[d - c^2*d*x
^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*d*x^2) + (c^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/Sqrt[
d - c^2*d*x^2] - (b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[Sqrt[-1 + c*x
]*Sqrt[1 + c*x]])/Sqrt[d - c^2*d*x^2] - (I*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2
] + (I*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I
*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + (I*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] - (I*b^2*c^2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rubi [A] time = 0.880644, antiderivative size = 438, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5798, 5748, 5761, 4180, 2531, 2282, 6589, 5662, 92, 205}

$$-\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+\right.}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(x*Sqrt[d - c^2*d*x
^2]) - ((1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(2*x^2*Sqrt[d - c^2*d*x
^2]) + (c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^Ar
cCosh[c*x]])/Sqrt[d - c^2*d*x^2] - (b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Ar
cTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/Sqrt[d - c^2*d*x^2] - (I*b*c^2*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/
Sqrt[d - c^2*d*x^2] + (I*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[
c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + (I*b^2*c^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

```
 - (I*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])/Sq
rt[d - c^2*d*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(c^2\sqrt{-1+\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} + \frac{\left(c^2\sqrt{-1+c\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}}{}$$

Mathematica [A] time = 86.9073, size = 697, normalized size = 1.62

$$\frac{ab(cx+1)\left(-ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)+ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)-ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\cosh^{-1}(cx)\log\right.}{x^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[d - c^2*d*x^2])/(2*d*x^2) + (a^2*c^2*Log[x])/(2*Sqrt[d]) - (a^2*
c^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/(2*Sqrt[d]) - (b^2*ArcCosh[c*x]^2
*(Sqrt[d - c^2*d*x^2]/x^2 - c^2*Sqrt[d]*Log[x] + c^2*Sqrt[d]*Log[d + Sqrt[d
]*Sqrt[d - c^2*d*x^2]]))/(2*d) + (b^2*c*(-((Sqrt[d - c^2*d*x^2]*ArcCosh[c*x
])/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + c*Sqrt[d]*(-Log[x] + Log[Sqrt[d] + S
qrt[d - c^2*d*x^2]]) + (c*Sqrt[d]*ArcCosh[c*x]^2*(-Log[x] + Log[d + Sqrt[d]
*Sqrt[d - c^2*d*x^2]]))/2))/d + (a*b*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1 + c*
x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)
]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c
```

```
*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1
+ c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*
x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]) - ((I/2)*b^2*c
^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh
[c*x]] - ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[
2, (-I)/E^ArcCosh[c*x]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*P
olyLog[3, (-I)/E^ArcCosh[c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/Sqrt[d -
c^2*d*x^2]
```

Maple [F] time = 0.354, size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x^3}\frac{1}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^2dx^5-dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x^3\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2}{\sqrt{-c^2dx^2 + dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

3.203 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=328

$$\frac{2b^2c^3\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{3\sqrt{d-c^2dx^2}}-\frac{2c^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{3\sqrt{d-c^2dx^2}}-\frac{2c^2\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{3dx}$$

```
[Out] (b^2*c^2*(1 - c*x)*(1 + c*x))/(3*x*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[d - c^2*d*x^2]) - (2*c^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[d - c^2*d*x^2
]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*d*x^3) - (2*c^2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*d*x) - (4*b*c^3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(3*Sqrt[d - c
^2*d*x^2]) + (2*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^(-2*ArcC
osh[c*x])])/(3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.868904, antiderivative size = 344, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5798, 5748, 5724, 5660, 3718, 2190, 2279, 2391, 5662, 95}

$$-\frac{2b^2c^3\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3\sqrt{d-c^2dx^2}}+\frac{2c^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{3\sqrt{d-c^2dx^2}}-\frac{2c^2(1-cx)(cx+1)\left(a+\right.}{3x\sqrt{d-c^2d\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b^2*c^2*(1 - c*x)*(1 + c*x))/(3*x*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[d - c^2*d*x^2]) + (2*c^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[d - c^2*d*x^2
]) - ((1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*x^3*Sqrt[d - c^2*d*x^2
]) - (2*c^2*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*x*Sqrt[d - c^2*d
*x^2]) - (4*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 +
 E^(2*ArcCosh[c*x])])/(3*Sqrt[d - c^2*d*x^2]) - (2*b^2*c^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{3\sqrt{d-c^2dx^2}} + \frac{\left(2c^2\sqrt{-\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3\sqrt{d-c^2dx^2}} - \frac{2c^2(1-cx)(}{}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh\right.}{3x^3\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{3\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{3\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{3\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{3\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.57971, size = 346, normalized size = 1.05

$$\frac{\frac{2b^2c^3x^3(cx-1)^{3/2}\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{\sqrt{\frac{cx-1}{cx+1}}} + (cx-1)\sqrt{cx+1}\left(a^2\sqrt{cx-1}\sqrt{cx+1}\left(2c^2x^2+1\right) - 4abc^3x^3\log(cx) + abcx - b^2c^2x^2\sqrt{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (-(b^2*Sqrt[-1 + c*x]*(1 + c*x)*(1 - c*x + 2*c^2*x^2 + 2*c^3*x^3*(-1 + Sqrt
[(-1 + c*x)/(1 + c*x)]))*ArcCosh[c*x]^2) + b*Sqrt[-1 + c*x]*(1 + c*x)*ArcCo
sh[c*x]*(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] + 2*a*(-1 + c*x - 2*c^2*x^2 + 2*c
^3*x^3) - 4*b*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[1 + E^(-2*ArcCosh[c*x]
)]) + (-1 + c*x)*Sqrt[1 + c*x]*(a*b*c*x - b^2*c^2*x^2*Sqrt[-1 + c*x]*Sqrt[1
```

```
 + c*x] + a^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 + 2*c^2*x^2) - 4*a*b*c^3*x^3*
Log[c*x]) + (2*b^2*c^3*x^3*(-1 + c*x)^(3/2)*PolyLog[2, -E^(-2*ArcCosh[c*x])
])/Sqrt[(-1 + c*x)/(1 + c*x)])/(3*x^3*Sqrt[-1 + c*x]*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.338, size = 2198, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -2/3*a^2*c^2/d/x*(-c^2*d*x^2+d)^(1/2)+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c
^4*x^4-2*c^2*x^2-1)*x^5*c^8-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c
^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/
(3*c^4*x^4-2*c^2*x^2-1)/x*arccosh(c*x)^2*c^2+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)
/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*c^8-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*
x^4-2*c^2*x^2-1)*x^3*c^6-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*
x^2-1)*x*c^4+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x^3*a
rccosh(c*x)+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*ar
ccosh(c*x)*c^8-2*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*a
rccosh(c*x)^2*c^6-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*
x^3*arccosh(c*x)*c^6+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-
1)*x*arccosh(c*x)^2*c^4-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x
^2-1)*x*arccosh(c*x)*c^4-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(1/2)+4*a*b*(-d*(c^2*
x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*c^5-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*
arccosh(c*x)*(c*x+1)*(c*x-1)*c^6+2*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-
2*c^2*x^2-1)*x^2*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5-2/3*b^2*(-d
*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*arccosh(c*x)*(c*x+1)*(c*x-1
)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x^2*arccosh(
c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2+1)*c^3-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/d/(c^2*x^2-1)*arccosh(c*x)*c^3-4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4
-2*c^2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c
^4*x^4-2*c^2*x^2-1)/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+4/3*a*b*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2+1)*c^3-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^
2-1)*x*(c*x+1)*(c*x-1)*c^4+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^
2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-b^2*(-d*(c^2*x^2-1))^
(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5+2/3*b^2
```

```
*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*arccosh(c*x)^2*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*c^3-b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*
arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)^2*c^3+2/3*b^2*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*polylog(2,-(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^
4*x^4-2*c^2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6-4*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(
3*c^4*x^4-2*c^2*x^2-1)*x^3*arccosh(c*x)*c^6+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/
d/(3*c^4*x^4-2*c^2*x^2-1)*x*arccosh(c*x)*c^4-a*b*(-d*(c^2*x^2-1))^(1/2)/d/(
3*c^4*x^4-2*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+8/3*a*b*(-d*(c^2*x^2
-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x*arccosh(c*x)*c^2+1/3*b^2*(-d*(c^2*x^
2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*c^6-2/3*b^2*(-d*(c^2*x^2-1))^(1/2
)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x
^4-2*c^2*x^2-1)/x*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2
-1)/x^3*arccosh(c*x)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^2dx^6-dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^6 - d*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))^2}{x^4\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**4/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)^2}{\sqrt{-c^2dx^2 + d}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^4), x)
```

3.204 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=556

$$\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}}+\frac{4x^2\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{3c^4d^2}$$

```
[Out] (16*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^5*d*Sqrt[d - c^2*d*x^2]) + (94
*b^2*(1 - c*x)*(1 + c*x))/(27*c^6*d*Sqrt[d - c^2*d*x^2]) + (2*b^2*x^2*(1 -
c*x)*(1 + c*x))/(27*c^4*d*Sqrt[d - c^2*d*x^2]) + (16*b^2*x*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*ArcCosh[c*x])/(3*c^5*d*Sqrt[d - c^2*d*x^2]) - (2*b*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(c^5*d*Sqrt[d - c^2*d*x^2]) + (2*
b*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3*d*Sqrt[d -
c^2*d*x^2]) + (x^4*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (8
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*c^6*d^2) + (4*x^2*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*c^4*d^2) + (4*b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^6*d*Sqrt[d - c^2*d
*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(
c^6*d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2,
 E^ArcCosh[c*x]])/(c^6*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.36703, antiderivative size = 578, normalized size of antiderivative = 1.04, number of steps used = 23, number of rules used = 14, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.483, Rules used = {5798, 5752, 5759, 5718, 5654, 74, 5662, 100, 12, 5766, 5694, 4182, 2279, 2391}

$$\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}}+\frac{16abx\sqrt{cx-1}\sqrt{cx+1}}{3c^5d\sqrt{d-c^2dx^2}}+\frac{x^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```


```
[Out] (16*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^5*d*Sqrt[d - c^2*d*x^2]) + (94
*b^2*(1 - c*x)*(1 + c*x))/(27*c^6*d*Sqrt[d - c^2*d*x^2]) + (2*b^2*x^2*(1 -
c*x)*(1 + c*x))/(27*c^4*d*Sqrt[d - c^2*d*x^2]) + (16*b^2*x*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*ArcCosh[c*x])/(3*c^5*d*Sqrt[d - c^2*d*x^2]) - (2*b*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(c^5*d*Sqrt[d - c^2*d*x^2]) + (2*
```

```
b*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3*d*Sqrt[d -
c^2*d*x^2]) + (x^4*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (8
*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*c^6*d*Sqrt[d - c^2*d*x^2])
+ (4*x^2*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*c^4*d*Sqrt[d - c^2*
d*x^2]) + (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^
ArcCosh[c*x]])/(c^6*d*Sqrt[d - c^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(c^6*d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(c^6*d*Sqrt[d - c^2*d
*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^5 \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\sqrt{1+\right.}{cd\sqrt{}}$$

$$= -\frac{2bx^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4x^2(1-cx)(1+c}{3c^4d\sqrt{}}$$

$$= -\frac{2b^2x^2(1-cx)(1+cx)}{9c^4d\sqrt{d-c^2dx^2}} - \frac{2bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c^5d\sqrt{d-c^2dx^2}} + \frac{2bx^3\sqrt{-1+cx}\sqrt{1+}{9c^3d\sqrt{}}$$

$$= \frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}} - \frac{2bx\sqrt{-1+cx}\sqrt{1}{c^5d}$$

$$= \frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} - \frac{22b^2(1-cx)(1+cx)}{9c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}} + \frac{16b^2x\sqrt{-1+c}{3c^5d}$$

$$= \frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{94b^2(1-cx)(1+cx)}{27c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}} + \frac{16b^2x\sqrt{-1+c}{3c^5d}$$

$$= \frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{94b^2(1-cx)(1+cx)}{27c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}} + \frac{16b^2x\sqrt{-1+c}{3c^5d}$$

Mathematica [A] time = 3.78422, size = 358, normalized size = 0.64

$$\frac{-b^2\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(216\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)-216\text{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right)+378\sqrt{\frac{cx-1}{cx+1}}(cx+1)+189\sqrt{\frac{cx-1}{cx+1}}(cx+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-36*a^2*(-8 + 4*c^2*x^2 + c^4*x^4) + 3*a*b*(135*ArcCosh[c*x] - 60*ArcCosh[
c*x]*Cosh[2*ArcCosh[c*x]] - 3*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 72*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + 62*Sinh[2*ArcCos
h[c*x]] + Sinh[4*ArcCosh[c*x]]) - b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*
(378*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - 378*c*x*ArcCosh[c*x] + 189*Sqrt
```

```
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 - 6*ArcCosh[c*x]*Cosh[3*Arc
Cosh[c*x]] - 54*ArcCosh[c*x]^2*Coth[ArcCosh[c*x]/2] + 216*ArcCosh[c*x]*Log[
1 - E^(-ArcCosh[c*x])] - 216*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 216*
PolyLog[2, -E^(-ArcCosh[c*x])] - 216*PolyLog[2, E^(-ArcCosh[c*x])] + 2*Sinh
[3*ArcCosh[c*x]] + 9*ArcCosh[c*x]^2*Sinh[3*ArcCosh[c*x]] + 54*ArcCosh[c*x]^
2*Tanh[ArcCosh[c*x]/2]))/(108*c^6*d*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.486, size = 1099, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/3*a^2*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)-4/3*a^2/c^4*x^2/d/(-c^2*d*x^2+d)^(1
/2)+8/3*a^2/c^6/d/(-c^2*d*x^2+d)^(1/2)-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/c^6/d
^2/(c^2*x^2-1)*arccosh(c*x)^2-2/9*b^2*(-d*(c^2*x^2-1))^(1/2)/c^3/d^2/(c^2*x
^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3-10/3*b^2*(-d*(c^2*x^2-1)
)^(1/2)/c^5/d^2/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-2*b^
2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*ar
ccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c
*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/c^6/d^2/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2/2
7*b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*x^4+92/27*b^2*(-d*(c^2*x^2
-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*x^2-94/27*b^2*(-d*(c^2*x^2-1))^(1/2)/c^6/d^2
/(c^2*x^2-1)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x
)^2*x^4+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)^2*x
^2-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^
2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*a*b*(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/
d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-16/3*a*b*(-d*(c^2*x^2
-1))^(1/2)/c^6/d^2/(c^2*x^2-1)*arccosh(c*x)+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/
c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*x^4+8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/c^4/d^2
/(c^2*x^2-1)*arccosh(c*x)*x^2-2/9*a*b*(-d*(c^2*x^2-1))^(1/2)/c^3/d^2/(c^2*x
^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3-10/3*a*b*(-d*(c^2*x^2-1))^(1/2)/c^5/d
^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^5\operatorname{arcosh}(cx)^2+2\,abx^5\operatorname{arcosh}(cx)+a^2x^5\right)\sqrt{-c^2dx^2+d}}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^5*arccosh(c*x)^2 + 2*a*b*x^5*arccosh(c*x) + a^2*x^5)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2x^5}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^5/(-c^2*d*x^2 + d)^(3/2), x)
```

3.205 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=440

$$-\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{c^5d\sqrt{d-c^2dx^2}}+\frac{3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2c^4d^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-}}{}$$

```
[Out] (b^2*x*(1 - c*x)*(1 + c*x))/(4*c^4*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(4*c^5*d*Sqrt[d - c^2*d*x^2]) + (b*x^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x^2
]) + (x^3*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(c^5*d*Sqrt[d - c^2*d*x^2]) + (3
*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*c^4*d^2) - (Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(2*b*c^5*d*Sqrt[d - c^2*d*x^2]) - (
2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[
c*x])])/(c^5*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Pol
yLog[2, E^(2*ArcCosh[c*x])])/(c^5*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.20541, antiderivative size = 451, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5752, 5759, 5676, 5662, 90, 52, 5766, 5715, 3716, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{c^5d\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{3x(1-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (b^2*x*(1 - c*x)*(1 + c*x))/(4*c^4*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(4*c^5*d*Sqrt[d - c^2*d*x^2]) + (b*x^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x^2
]) + (x^3*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(c^5*d*Sqrt[d - c^2*d*x^2]) + (3
*x*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(2*c^4*d*Sqrt[d - c^2*d*x^2]
) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(2*b*c^5*d*Sqrt[d
 - c^2*d*x^2]) - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log
```

```
[1 - E^(2*ArcCosh[c*x])])/(c^5*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(c^5*d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\sqrt{1+\right.}{cd\sqrt{}}$$

$$= -\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)(1+cx)\,(}{2c^4d\sqrt{d}}$$

$$= -\frac{b^2x(1-cx)(1+cx)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{2c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.95431, size = 343, normalized size = 0.78

$$\frac{b^2\sqrt{d}\left(8\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)+8cx\cosh^{-1}(cx)^2-\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(4\cosh^{-1}(cx)^3-2\cosh^{-1}(cx)\left(\cosh\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-4*a^2*c*Sqrt[d]*x*(-3 + c^2*x^2) + 12*a^2*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x
*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*a*b*Sqrt[d]*(8*c*x*ArcC
osh[c*x] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(6*ArcCosh[c*x]^2 - Cosh[2*
ArcCosh[c*x]] + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + 2*ArcCosh[c*x
]*Sinh[2*ArcCosh[c*x]])) + b^2*Sqrt[d]*(8*c*x*ArcCosh[c*x]^2 + 8*Sqrt[(-1 +
```

```
c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])] - Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*(4*ArcCosh[c*x]^3 - 2*ArcCosh[c*x]*(Cosh[2*ArcCosh[c
*x]] - 8*Log[1 - E^(-2*ArcCosh[c*x])]) + Sinh[2*ArcCosh[c*x]] + 2*ArcCosh[c
*x]^2*(4 + Sinh[2*ArcCosh[c*x]]))))/(8*c^5*d^(3/2)*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.495, size = 1141, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/2*a^2*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(-c^2*d*x^2+d)^(1/2
)-3/2*a^2/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+
2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1
)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arccosh(c*x)^2-1/2*b^2*(-d*(c^2*x^2-1))^(1/2
)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/2*b^2*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arcc
osh(c*x)^3+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5
/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/4*b^2*(-d*(c^2*x^
2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+1/
4*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*x^3-1/4*b^2*(-d*(c^2*x^2-1
))^(1/2)/d^2/c^4/(c^2*x^2-1)*x+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2)
)+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccosh(c*x)^2*x^3-3/2
*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arccosh(c*x)^2*x+3/2*a*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arccos
h(c*x)^2+a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccosh(c*x)*x^3-1/
2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)
^(1/2)*(c*x-1)^(1/2)-3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arcco
sh(c*x)*x+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/
c^5/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 \operatorname{arcosh}(cx)^2 + 2\,abx^4 \operatorname{arcosh}(cx) + a^2x^4\right)\sqrt{-c^2dx^2 + d}}{c^4d^2x^4 - 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\,(a + b\operatorname{acosh}(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{2}x^{4}}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^4/(-c^2*d*x^2 + d)^(3/2), x)
```

3.206 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=413

$$\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{c^4d^2}$$

```
[Out] (4*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) + (2*b^2
*(1 - c*x)*(1 + c*x))/(c^4*d*Sqrt[d - c^2*d*x^2]) + (4*b^2*x*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*ArcCosh[c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) - (2*b*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(c^3*d*Sqrt[d - c^2*d*x^2]) + (x^
2*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x])^2)/(c^4*d^2) + (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^4*d*Sqrt[d - c^2*d*x^2]) +
(2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(c^4*d*Sqr
t[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCos
h[c*x]])/(c^4*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.921047, antiderivative size = 424, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5798, 5752, 5718, 5654, 74, 5766, 5694, 4182, 2279, 2391}

$$\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}+\frac{4abx\sqrt{cx-1}\sqrt{cx+1}}{c^3d\sqrt{d-c^2dx^2}}-\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (4*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) + (2*b^2
*(1 - c*x)*(1 + c*x))/(c^4*d*Sqrt[d - c^2*d*x^2]) + (4*b^2*x*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*ArcCosh[c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) - (2*b*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(c^3*d*Sqrt[d - c^2*d*x^2]) + (x^
2*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (2*(1 - c*x)*(1 + c
*x)*(a + b*ArcCosh[c*x])^2)/(c^4*d*Sqrt[d - c^2*d*x^2]) + (4*b*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^4*d*Sqrt[
d - c^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh
[c*x]])/(c^4*d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*P
```

```
olyLog[2, E^ArcCosh[c*x]])/(c^4*d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\sqrt{1-\right.}{cd}$$

$$= -\frac{2bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-cx)(1+cx)}{c^4d\sqrt{d}}$$

$$= \frac{4abx\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} - \frac{2bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c^3d\sqrt{d-c^2dx^2}}$$

$$= \frac{4abx\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{4b^2x\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c^3d\sqrt{d-c^2dx^2}} - \frac{2b}{}$$

$$= \frac{4abx\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{4b^2x\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c^3d\sqrt{d-c^2dx^2}} - \frac{2b}{}$$

$$= \frac{4abx\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{4b^2x\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c^3d\sqrt{d-c^2dx^2}} - \frac{2b}{}$$

Mathematica [A] time = 1.58449, size = 302, normalized size = 0.73

$$\frac{b^2\left(-4\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)+4\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right)-\cosh\left(2\cosh^{-1}(cx)\right)\cosh}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-2*a^2*(-2 + c^2*x^2) + 2*a*b*(-(ArcCosh[c*x]*(-3 + Cosh[2*ArcCosh[c*x]]))
 - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + Sinh[
2*ArcCosh[c*x]]) + b^2*(2 + 3*ArcCosh[c*x]^2 - 2*Cosh[2*ArcCosh[c*x]] - Arc
Cosh[c*x]^2*Cosh[2*ArcCosh[c*x]] - 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*A
rcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] - 4*Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)*PolyLog[2, E^(-ArcCosh[c*x])] + 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]
```

```
))/(2*c^4*d*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.394, size = 836, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a^2*x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*a^2/d/c^4/(-c^2*d*x^2+d)^(1/2)+b^2*(-
d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)^2*x^2-2*b^2*(-d*(c^2*
x^2-1))^(1/2)/c^3/d^2/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x+2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*x^2-2*b^2*(-d*(c^2*x^2-1
))^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)^2-2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^
4/d^2/(c^2*x^2-1)+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c^4/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^
2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*po
lylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^4/d^2/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*a*b*(-d*
(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*x^2-2*a*b*(-d*(c^2*x^2-
1))^(1/2)/c^3/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-4*a*b*(-d*(c^2*
x^2-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)+2*a*b*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)-1)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^
2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^3 \operatorname{arcosh}(cx)^2 + 2\,abx^3 \operatorname{arcosh}(cx) + a^2x^3\right)\sqrt{-c^2dx^2 + d}}{c^4d^2x^4 - 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b \operatorname{acosh}(cx))^2}{(-d (cx - 1) (cx + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2 x^3}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^3/(-c^2*d*x^2 + d)^(3/2), x)
```

3.207 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{c^3d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\sqrt{}}{}$$

```
[Out] (x*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(c^3*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c^3*d*Sqrt[d - c^2*d*x^2]
) - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*Arc
Cosh[c*x])])/(c^3*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*PolyLog[2, E^(2*ArcCosh[c*x])])/(c^3*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.795287, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5798, 5752, 5676, 5715, 3716, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{c^3d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(c^3*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c^3*d*Sqrt[d - c^2*d*x^2]
) - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*Arc
Cosh[c*x])])/(c^3*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*PolyLog[2, E^(2*ArcCosh[c*x])])/(c^3*d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\sqrt{1+cx}\right)}{cd\sqrt{d-}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{S}}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{3bc^3d\sqrt{d-c^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{3bc^3d\sqrt{d-c^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{3bc^3d\sqrt{d-c^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{3bc^3d\sqrt{d-c^2}}$$

Mathematica [A] time = 2.00724, size = 270, normalized size = 1.05

$$\frac{-b^2d\left(\cosh^{-1}(cx)\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+3\right)+6\log\left(1-e^{-2\cosh^{-1}(cx)}\right)\right)-3cx\cosh^{-1}(cx)\right)-3\sqrt{\frac{cx-1}{cx+1}}(c}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (3*a^2*c*d*x + 3*a^2*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d
*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 3*a*b*d*(2*c*x*ArcCosh[c*x] - Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]^2 + 2*Log[Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)])) - b^2*d*(ArcCosh[c*x]*(-3*c*x*ArcCosh[c*x] + Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 - E^(-2*A
rcCosh[c*x])])) - 3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*A
rcCosh[c*x])]))/(3*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.333, size = 738, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a^2/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/
2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^3-b^2*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-b^2*(-d*(c^2*x
^2-1))^(1/2)*arccosh(c*x)^2/d^2/c^2/(c^2*x^2-1)*x+2*b^2*(-d*(c^2*x^2-1))^(1
/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(
c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*
b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*
arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)
^(1/2)*(c*x+1)^(1/2))+a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)-2*a*b*(-d*(c^2*x^2-1))^
(1/2)*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2\,\text{arcosh}\,(cx)^2+2\,abx^2\,\text{arcosh}\,(cx)+a^2x^2\right)\sqrt{-c^2dx^2+d}}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,(a+b\,\text{acosh}\,(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^2x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

3.208 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=196

$$\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{4b\sqrt{}}{}$$

```
[Out] (a + b*ArcCosh[c*x])^2/(c^2*d*Sqrt[d - c^2*d*x^2]) + (4*b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^2*d*Sqrt[d - c
^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]
])/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLo
g[2, E^ArcCosh[c*x]])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.445352, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 5718, 5694, 4182, 2279, 2391}

$$\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{4b\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```


```
[Out] (a + b*ArcCosh[c*x])^2/(c^2*d*Sqrt[d - c^2*d*x^2]) + (4*b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^2*d*Sqrt[d - c
^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]
])/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLo
g[2, E^ArcCosh[c*x]])/(c^2*d*Sqrt[d - c^2*d*x^2])
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{-1+c^2x^2}\,dx}{cd\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int(a+bx)\mathrm{csch}(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{\left(2b^2\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{\left(2b^2\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2b^2\sqrt{}}{}$$

Mathematica [A] time = 0.965408, size = 210, normalized size = 1.07

$$\frac{-2b^2\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)+2b^2\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right)+a^2+2ab\cosh^{-1}(cx)-2ab}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (a^2 + 2*a*b*ArcCosh[c*x] + b^2*ArcCosh[c*x]*(ArcCosh[c*x] - 2*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*(Log[1 - E^(-ArcCosh[c*x])] - Log[1 + E^(-ArcCosh[
c*x])])) - 2*a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]
/2]] - 2*b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c
*x])] + 2*b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-ArcCosh[c
*x])])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.269, size = 542, normalized size = 2.8

$$\frac{a^2}{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}} - \frac{b^2\left(\operatorname{arccosh}(cx)\right)^2}{c^2d^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)} + 2\,\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}\operatorname{arccosh}(cx)\ln\left(1-cx-\sqrt{c}\right.}{c^2d^2\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a^2/c^2/d/(-c^2*d*x^2+d)^(1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-
1)*arccosh(c*x)^2+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^
2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*po
lylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^2/d^2/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*a*b*(-d*
(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)+2*a*b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2)-1)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
^2/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2}{\sqrt{-c^2dx^2+d}c^2d}+\int \frac{b^2x\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}+\frac{2\,abx\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] a^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2) + 2*a*b*x*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2x\,\text{arcosh}\,(cx)^2+2\,abx\,\text{arcosh}\,(cx)+a^2x\right)}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
+ a^2*x)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{2}x}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x/(-c^2*d*x^2 + d)^(3/2), x)
```

3.209 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=198

$$-\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-}}{}$$

```
[Out] (x*(a + b*ArcCosh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])^2)/(c*d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c*d*S
qrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*Arc
Cosh[c*x])])/(c*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.351189, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5713, 5688, 5715, 3716, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])^2)/(c*d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c*d*S
qrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*Arc
Cosh[c*x])])/(c*d*Sqrt[d - c^2*d*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} + \frac{\left(4b\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Su}}{cd}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{cd}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{cd}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{cd}$$

Mathematica [A] time = 0.438871, size = 126, normalized size = 0.64

$$\frac{\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-2b^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-2b^2\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)+\left(a+b\cosh^{-1}(cx)\right)\left(a+b\cosh^{-1}(cx)-2b\log\left(1-e^{\cosh^{-1}(cx)}\right)-2b\log\left(e^{\cosh^{-1}(cx)}+1\right)\right)\right)}{c}}{d\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x])^2 + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c
*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 - E^ArcCosh[c*x]] - 2*b*Log[1 + E^ArcC
osh[c*x]]) - 2*b^2*PolyLog[2, -E^ArcCosh[c*x]] - 2*b^2*PolyLog[2, E^ArcCosh
[c*x]]))/c)/(d*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.216, size = 578, normalized size = 2.9

$$\frac{a^2x}{d}\frac{1}{\sqrt{-c^2dx^2+d}}-\frac{b^2(\operatorname{arccosh}(cx))^2}{cd^2(c^2x^2-1)}\sqrt{cx-1}\sqrt{cx+1}\sqrt{-d(c^2x^2-1)}-\frac{b^2(\operatorname{arccosh}(cx))^2x}{d^2(c^2x^2-1)}\sqrt{-d(c^2x^2-1)}+2\frac{b^2\sqrt{cx+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a^2/d*x/(-c^2*d*x^2+d)^(1/2)-b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1
))^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)^2-b^2*(-d*(c^2*x^2-1))^(1/2)*arccos
h(c*x)^2/d^2/(c^2*x^2-1)*x+2*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1
))^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))+2*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c/(c^2*x^2-
1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))+2*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1
/2)/d^2/c/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*a*b*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c
*x)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x+2*a*b*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c/(c^2*x^2-1)*ln((c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{abc\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)}{d}+b^2\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx+\frac{2\,abx\operatorname{arcosh}(cx)}{\sqrt{-c^2dx^2+d}d}+\frac{a^2x}{\sqrt{-c^2dx^2+d}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*b*c*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b^2*integrate(log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2), x) + 2*a*b*x*arccosh(c*x)/
(sqrt(-c^2*d*x^2 + d)*d) + a^2*x/(sqrt(-c^2*d*x^2 + d)*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(-c^2*d*x^2 + d)^(3/2), x)
```

3.210 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=471

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\,\text{co}\right.}{d\sqrt{d-c^2dx^2}}$$

```
[Out] (a + b*ArcCosh[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]
) + (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCos
h[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Poly
Log[2, -E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt
[d - c^2*d*x^2]) + ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
])*PolyLog[2, I*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (
(2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d*
Sqrt[d - c^2*d*x^2]) - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I
*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.963503, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {5798, 5756, 5761, 4180, 2531, 2282, 6589, 5694, 4182, 2279, 2391}

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\,\text{co}\right.}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcCosh[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]
) + (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCos
h[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Poly
Log[2, -E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt
[d - c^2*d*x^2]) + ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
```

```
])*PolyLog[2, I*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (
(2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d*
Sqrt[d - c^2*d*x^2]) - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I
*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int}{d\sqrt{d-c^2d}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^2\operatorname{sech}(x)\,dx, x, \cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} -$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1}}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1}}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1}}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1}}{}$$

Mathematica [A] time = 3.52595, size = 577, normalized size = 1.23

$$-\frac{2iabd\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+i\cosh^{-1}(cx)+\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-\sqrt{\frac{c}{c}}}{\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(((a^2*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2) - a^2*Sqrt[d]*Log[c*x] + a^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + ((2*I)*a*b*d*(I*ArcCosh[c*x] +
 Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]
```

```
] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c
*x]] - I*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2
*d*x^2] + (b^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((Sqrt[(-1 + c*x)/(1
+ c*x)]*ArcCosh[c*x]^2)/(1 - c*x) + 2*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x]
)] + I*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]^2*Log[1 +
I/E^ArcCosh[c*x]] - 2*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 2*PolyLog[2
, -E^(-ArcCosh[c*x])] + (2*I)*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]]
- (2*I)*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] - 2*PolyLog[2, E^(-ArcCos
h[c*x])] + (2*I)*PolyLog[3, (-I)/E^ArcCosh[c*x]] - (2*I)*PolyLog[3, I/E^Arc
Cosh[c*x]]))/Sqrt[d - c^2*d*x^2])/d^2)
```

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x}\left(-c^2dx^2+d\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^4d^2x^5-2\,c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x\left(-d(cx-1)(cx+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

3.211 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=341

$$-\frac{b^2c\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}-\frac{b^2c\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}$$

```
[Out] -((a + b*ArcCosh[c*x])^2/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcCo
sh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + (2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
+ b*ArcCosh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (4*b*c*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d
*x^2]) - (4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E
^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d*x^2]) - (b^2*c*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d*x^2]) - (b^2*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*
d*x^2])
```

Rubi [A] time = 0.936362, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {5798, 5748, 5688, 5715, 3716, 2190, 2279, 2391, 5721, 5461, 4182}

$$-\frac{b^2c\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}-\frac{b^2c\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```



```
[Out] -((a + b*ArcCosh[c*x])^2/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcCo
sh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + (2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
+ b*ArcCosh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (4*b*c*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d
*x^2]) - (4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E
^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d*x^2]) - (b^2*c*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d*x^2]) - (b^2*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*
d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x\left(-1+c^2x^2\right)}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(2c^2\sqrt{-1+cx}\sqrt{1+cx}\right.}{d\sqrt{d-}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int(a+\right.}{d\sqrt{d-}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} - \frac{\left(4bc\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int(a+\right.}{d\sqrt{d-c^2d}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right.}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right.}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right.}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right.}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.63105, size = 315, normalized size = 0.92

$$\frac{b^2\left(cx\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+cx\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)+\cosh^{-1}(cx)\left(c^2x^2\cosh^{-}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a^2*(-1 + 2*c^2*x^2) + 2*a*b*(c^2*x^2*ArcCosh[c*x] + Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x)*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(L
og[c*x] + Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)]))) + b^2*(ArcCosh[c*x]*
(c^2*x^2*ArcCosh[c*x] + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - 2*c*x*(ArcCosh[c*x] + Log[1 - E^(-2*
ArcCosh[c*x])] + Log[1 + E^(-2*ArcCosh[c*x])]))) + c*x*Sqrt[(-1 + c*x)/(1 +
```

```
 c*x)]*(1 + c*x)*PolyLog[2, -E^(-2*ArcCosh[c*x])] + c*x*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(d*x*Sqrt[d - c^2*d*x^2
])
```

Maple [B] time = 0.286, size = 826, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a^2/d/x/(-c^2*d*x^2+d)^(1/2)+2*a^2*c^2/d*x/(-c^2*d*x^2+d)^(1/2)-2*b^2*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)
^2*c-2*b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)^2*x/(c^2*x^2-1)/d^2*c^2+b^2*
(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)^2/x/(c^2*x^2-1)/d^2+2*b^2*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)*ln(1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))*c+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c+
2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*ar
ccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c+2*b^2*(-d*(c^2*x^2-1))^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*polylog(2,-c*x-(c*x-1)^(1/
2)*(c*x+1)^(1/2))*c+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/(c^2*x^2-1)/d^2*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c+
b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*poly
log(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c-4*a*b*(-d*(c^2*x^2-1))^(1/2)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)*c-4*a*b*(-d*(c^2*x
^2-1))^(1/2)*arccosh(c*x)*x/(c^2*x^2-1)/d^2*c^2+2*a*b*(-d*(c^2*x^2-1))^(1/2
)*arccosh(c*x)/x/(c^2*x^2-1)/d^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^4d^2x^6-2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\operatorname{acosh}(cx))^2}{x^2\left(-d(cx-1)(cx+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

3.212 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=650

$$-\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a\right.}{d\sqrt{d-c^2dx^2}}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(d*x*Sqrt[d - c^2*d
*x^2]) + (3*c^2*(a + b*ArcCosh[c*x])^2)/(2*d*Sqrt[d - c^2*d*x^2]) - (a + b*
ArcCosh[c*x])^2/(2*d*x^2*Sqrt[d - c^2*d*x^2]) + (3*c^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x
^2]) - (b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (4*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
+ b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (2*b^2
*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(d*Sqrt[d -
c^2*d*x^2]) - ((3*I)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + ((3*I)*b*c^2*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x
]])/(d*Sqrt[d - c^2*d*x^2]) - (2*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyL
og[2, E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + ((3*I)*b^2*c^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
- ((3*I)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])
/(d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.48649, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {5798, 5748, 5756, 5761, 4180, 2531, 2282, 6589, 5694, 4182, 2279, 2391, 5746, 92, 205}

$$-\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a\right.}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```


```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(d*x*Sqrt[d - c^2*d
*x^2]) + (3*c^2*(a + b*ArcCosh[c*x])^2)/(2*d*Sqrt[d - c^2*d*x^2]) - (a + b*
```

```
ArcCosh[c*x])^2/(2*d*x^2*Sqrt[d - c^2*d*x^2]) + (3*c^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x
^2]) - (b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (4*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
+ b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (2*b^2
*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(d*Sqrt[d -
c^2*d*x^2]) - ((3*I)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + ((3*I)*b*c^2*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x
]])/(d*Sqrt[d - c^2*d*x^2]) - (2*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyL
og[2, E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + ((3*I)*b^2*c^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
- ((3*I)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])
/(d*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
```

```
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(-1+c^2x^2\right)}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(3c^2\sqrt{-1+cx}\sqrt{1+cx}\right)}{2d\sqrt{d-}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}} +$$

Mathematica [A] time = 91.3921, size = 979, normalized size = 1.51

$$\frac{b^2\sqrt{d-c^2dx^2}\left(-2\cosh^2\left(\frac{1}{2}\cosh^{-1}(cx)\right)\cosh^{-1}(cx)^2+2\sinh^2\left(\frac{1}{2}\cosh^{-1}(cx)\right)\cosh^{-1}(cx)^2+\left(\frac{1}{c^2x^2}-1\right)\cosh^{-1}(cx)^2+3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b^2*c^2*Sqrt[d - c^2*d*x^2]*((-2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcC
osh[c*x])/(c*x) + (-1 + 1/(c^2*x^2))*ArcCosh[c*x]^2 + 4*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*ArcTan[Tanh[ArcCosh[c*x]/2]] - 2*ArcCosh[c*x]^2*Cosh[ArcC
osh[c*x]/2]^2 + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 -
 E^(-ArcCosh[c*x])] + (3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*
x]^2*Log[1 - I/E^ArcCosh[c*x]] - (3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] - 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c*x])] + (6*I)*Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - (6*I)*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] - 4
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-ArcCosh[c*x])] + (6*I)
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[3, (-I)/E^ArcCosh[c*x]] - (6*
I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[3, I/E^ArcCosh[c*x]] + 2*Ar
cCosh[c*x]^2*Sinh[ArcCosh[c*x]/2]^2))/(2*d^2*(-1 + c^2*x^2)) + (a*(-((a*(-1
 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(x^2*(-1 + c^2*x^2))) + 3*a*c^2*Sqrt[d]*
Log[x] - 3*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (2*b*c^2*d*
(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x)) + (-1 + 1/(c^2*x^2))*ArcCo
sh[c*x] - 2*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 + (3*I)*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (3*I)*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + 2*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + (3*I)*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - (3*I)*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]
*Sinh[ArcCosh[c*x]/2]^2))/Sqrt[d - c^2*d*x^2]))/(2*d^2)
```

Maple [F] time = 0.372, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{arccosh}(cx))^2}{x^3} \left(-c^2dx^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^4d^2x^7-2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\operatorname{acosh}(cx))^2}{x^3\left(-d\,(cx-1)\,(cx+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^3), x)
```

3.213 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=496

$$-\frac{5b^2c^3\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{b^2c^3\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}\right.}{3d\sqrt{d-c^2dx^2}}$$

```
[Out] (b^2*c^2*(1 - c*x)*(1 + c*x))/(3*d*x*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d*x^2*Sqrt[d - c^2*d*x^2]) - (a
 + b*ArcCosh[c*x])^2/(3*d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcCosh[
c*x])^2)/(3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x])^2)/(3*
d*Sqrt[d - c^2*d*x^2]) + (8*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])^2)/(3*d*Sqrt[d - c^2*d*x^2]) - (20*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2
]) - (16*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^
(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2]) - (5*b^2*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2]) - (b
^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(d*Sqrt
[d - c^2*d*x^2])
```

Rubi [A] time = 1.46136, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5748, 5688, 5715, 3716, 2190, 2279, 2391, 5721, 5461, 4182, 5746, 95}

$$-\frac{5b^2c^3\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{b^2c^3\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}\right.}{3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```


```
[Out] (b^2*c^2*(1 - c*x)*(1 + c*x))/(3*d*x*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d*x^2*Sqrt[d - c^2*d*x^2]) - (a
 + b*ArcCosh[c*x])^2/(3*d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcCosh[
c*x])^2)/(3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x])^2)/(3*
d*Sqrt[d - c^2*d*x^2]) + (8*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])^2)/(3*d*Sqrt[d - c^2*d*x^2]) - (20*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
```

```
]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2
]) - (16*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^
(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2]) - (5*b^2*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2]) - (b
^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(d*Sqrt
[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(-1+c^2x^2\right)}\,dx}{3d\sqrt{d-c^2dx^2}} - \frac{\left(4c^2\sqrt{-1+cx}\sqrt{1+cx}\right.}{3d\sqrt{d}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)^2}{3dx\sqrt{d-c^2dx^2}} -$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2}{}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2}{}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2}{}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2}{}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2}{}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2}{}$$

Mathematica [A] time = 2.51462, size = 529, normalized size = 1.07

$$\frac{b^2\left(5c^3x^3\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+3c^3x^3\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)-c^4x^4+c^2x^2+3c^4\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a^2*(-1 - 4*c^2*x^2 + 8*c^4*x^4) + a*b*(6*c^4*x^4*ArcCosh[c*x] + Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*(c*x + 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*A
```

```
rcCosh[c*x]) + 2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(5*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(5*Log[c*x] + 3*Log[Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)]))) + b^2*(c^2*x^2 - c^4*x^4 + c*x*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 3*c^4*x^4*ArcCosh[c*x]^2 + (-1 + c*
x)*(1 + c*x)*ArcCosh[c*x]^2 + 5*c^2*x^2*(-1 + c*x)*(1 + c*x)*ArcCosh[c*x]^2
 - 8*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 - 6*c^3*x^
3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-2*ArcCosh[c
*x])] - 10*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1
+ E^(-2*ArcCosh[c*x])] + 5*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Pol
yLog[2, -E^(-2*ArcCosh[c*x])] + 3*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*d*x^3*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.385, size = 2868, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/3*a^2/d/x^3/(-c^2*d*x^2+d)^(1/2)+8/3*a^2*c^4/d*x/(-c^2*d*x^2+d)^(1/2)-7/
3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*c^4+32*a*b*(-d*(
c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*c^8-8*a*b*(-d*(c^2*x^2-1)
)^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*c^6-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/
d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*c^4+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4
*x^4-7*c^2*x^2-1)/x^3*arccosh(c*x)-128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*
c^4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)*c^6+16*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2
/(8*c^4*x^4-7*c^2*x^2-1)*x*arccosh(c*x)*c^4-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/
d^2/(8*c^4*x^4-7*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+8*a*b*(-d*(c^2*
x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x*arccosh(c*x)*c^2-16/3*b^2*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)^
2*c^3+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2
-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3+8/3*b^2*(-d*(c^2*x^2-1))
^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*c^3-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*arccosh(c
*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*
c^4*x^4-7*c^2*x^2-1)*x^5*(c*x+1)*(c*x-1)*c^8-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)
/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6-8/3*b^2*(-d*(c^2*x^2-1
))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5+5/
3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*po
lylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3+2*b^2*(-d*(c^2*x^2-1))^(1
/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2
```

```
)*(c*x+1)^(1/2))*c^3-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^
2-1)/x*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x^3*a
rccosh(c*x)^2-32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x
^7*c^10+40/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*c^8
-4/3*a^2*c^2/d/x/(-c^2*d*x^2+d)^(1/2)+128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/
(8*c^4*x^4-7*c^2*x^2-1)*x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5+2*
b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arcc
osh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3+2*b^2*(-d*(c^2*x^2-1))^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x
-1)^(1/2)*(c*x+1)^(1/2))*c^3+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2+1)*c^3+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x
^5*arccosh(c*x)*(c*x+1)*(c*x-1)*c^8-32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*
c^4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)*(c*x+1)*(c*x-1)*c^6+64/3*b^2*(-d*(c^2
*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^2*arccosh(c*x)^2*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*c^5-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-
1)*x*arccosh(c*x)*(c*x+1)*(c*x-1)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8
*c^4*x^4-7*c^2*x^2-1)/x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-32/3*a
*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arcco
sh(c*x)*c^3+64/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5
*(c*x+1)*(c*x-1)*c^8-32/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x
^2-1)*x^3*(c*x+1)*(c*x-1)*c^6-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4
-7*c^2*x^2-1)*x*(c*x+1)*(c*x-1)*c^4+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*
c^4*x^4-7*c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-1/3*a*b*(
-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x^2*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*c+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*
x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*c^3+10/3*a*b*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2+1)*c^3+32*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2
*x^2-1)*x^5*arccosh(c*x)*c^8-64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4
-7*c^2*x^2-1)*x^3*arccosh(c*x)^2*c^6-8*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^
4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)*c^6-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/
(8*c^4*x^4-7*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+8*b^2*(-d*(c^2*x^2-
1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*arccosh(c*x)^2*c^4-8/3*b^2*(-d*(c^2
*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*arccosh(c*x)*c^4+4*b^2*(-d*(c^
2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x*arccosh(c*x)^2*c^2-64/3*b^2*(
-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^7*arccosh(c*x)*c^10-64/
3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^7*c^10
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^4d^2x^8-2\,c^2d^2x^6+d^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**4/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

3.214 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=568

$$-\frac{11b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{3c^6d^2\sqrt{d-c^2dx^2}}+\frac{11b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{3c^6d^2\sqrt{d-c^2dx^2}}-\frac{16abx\sqrt{cx-1}\sqrt{cx+1}}{3c^5d^2\sqrt{d-c^2dx^2}}$$

```
[Out] -(b^2*x^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) - (16*a*b*x*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) - (7*b^2*(1 - c*x)*(1 + c*x))/(3*
c^6*d^2*Sqrt[d - c^2*d*x^2]) - (16*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCo
sh[c*x])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (11*b*x*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (b*x^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d
 - c^2*d*x^2]) + (x^4*(a + b*ArcCosh[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2
)) - (4*x^2*(a + b*ArcCosh[c*x])^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) - (8*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*c^6*d^3) - (22*b*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*c^6*d^2*Sq
rt[d - c^2*d*x^2]) - (11*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^Arc
Cosh[c*x]])/(3*c^6*d^2*Sqrt[d - c^2*d*x^2]) + (11*b^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*c^6*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.47623, antiderivative size = 594, normalized size of antiderivative = 1.05, number of steps used = 27, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5752, 5718, 5654, 74, 5766, 5694, 4182, 2279, 2391, 5750, 98, 21}

$$-\frac{11b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{3c^6d^2\sqrt{d-c^2dx^2}}+\frac{11b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{3c^6d^2\sqrt{d-c^2dx^2}}-\frac{16abx\sqrt{cx-1}\sqrt{cx+1}}{3c^5d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```


```
[Out] -(b^2*x^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) - (16*a*b*x*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) - (7*b^2*(1 - c*x)*(1 + c*x))/(3*
c^6*d^2*Sqrt[d - c^2*d*x^2]) - (16*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCo
sh[c*x])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (11*b*x*Sqrt[-1 + c*x]*Sqrt[1 +
```

```
c*x]*(a + b*ArcCosh[c*x]))/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (b*x^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d
 - c^2*d*x^2]) - (4*x^2*(a + b*ArcCosh[c*x])^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x
^2]) + (x^4*(a + b*ArcCosh[c*x])^2)/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2]) - (8*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*c^6*d^2*Sq
rt[d - c^2*d*x^2]) - (22*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)*ArcTanh[E^ArcCosh[c*x]])/(3*c^6*d^2*Sqrt[d - c^2*d*x^2]) - (11*b^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(3*c^6*d^2*Sqrt[d - c^2
*d*x^2]) + (11*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])
/(3*c^6*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\int \frac{x^5 \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{\left(2b\sqrt{-1\right.}{}$$

$$= \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{4x^2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh\right.}{3c^2d^2(1-cx)(1+c}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{11bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}\left(a\right.}{3c^3d^2\left(1-c^2x^2\right)}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{11b^2(1-cx)(1+cx)}{3c^6d^2\sqrt{d-c^2dx^2}} + \frac{11bx\sqrt{-1+cx}\sqrt{1}}{3c^5d^2}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{3b^2(1-cx)(1+cx)}{c^6d^2\sqrt{d-c^2dx^2}} - \frac{16b^2x\sqrt{-1+cx}\sqrt{1}}{3c^5d^2\sqrt{d}}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} - \frac{7b^2(1-cx)(1+cx)}{3c^6d^2\sqrt{d-c^2dx^2}} - \frac{16b^2x\sqrt{-1+cx}\sqrt{1}}{3c^5d^2\sqrt{d}}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} - \frac{7b^2(1-cx)(1+cx)}{3c^6d^2\sqrt{d-c^2dx^2}} - \frac{16b^2x\sqrt{-1+cx}\sqrt{1}}{3c^5d^2\sqrt{d}}$$

Mathematica [A] time = 5.56397, size = 437, normalized size = 0.77

$$-\frac{b^2\left(88\left(\frac{cx-1}{cx+1}\right)^{3/2}(cx+1)^3\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right) - 88\left(\frac{cx-1}{cx+1}\right)^{3/2}(cx+1)^3\text{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right) + 25\cosh^{-1}(cx)^2 -\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -(8*a^2*(8 - 12*c^2*x^2 + 3*c^4*x^4) + 2*a*b*(25*ArcCosh[c*x] - 36*ArcCosh[
c*x]*Cosh[2*ArcCosh[c*x]] + 3*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 33*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + 4*Sinh[2*ArcCosh
```

```
[c*x]] + 11*Log[Tanh[ArcCosh[c*x]/2]]*Sinh[3*ArcCosh[c*x]] - 3*Sinh[4*ArcCo
sh[c*x]]) + b^2*(22 + 25*ArcCosh[c*x]^2 - 4*(7 + 9*ArcCosh[c*x]^2)*Cosh[2*A
rcCosh[c*x]] + 3*(2 + ArcCosh[c*x]^2)*Cosh[4*ArcCosh[c*x]] - 66*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] + 66*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] +
88*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*x])]
- 88*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c*x])]
 + 8*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] + 22*ArcCosh[c*x]*Log[1 - E^(-ArcCos
h[c*x])]*Sinh[3*ArcCosh[c*x]] - 22*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])]*
Sinh[3*ArcCosh[c*x]] - 6*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/(24*c^6*d*(d -
 c^2*d*x^2)^(3/2))
```

Maple [B] time = 0.479, size = 1211, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^6+b^2*(-d*(c^2*x^2-1))^
(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)^2+11/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))-2*a*b*(-d*(c^2*x^2-1))^(1/2)/c^4/d^3/(c^2*x^2-1)*arccosh(c*x
)*x^2+4*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)*x^2-1
1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2
-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-5/3*b^2*(-d*(c^2*x^2-1))^(1/
2)/d^3/(c^2*x^2-1)^2/c^6*arccosh(c*x)^2-2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^4/d^
3/(c^2*x^2-1)*x^2-8/3*a^2/c^6/d/(-c^2*d*x^2+d)^(3/2)+2*b^2*(-d*(c^2*x^2-1))
^(1/2)/c^6/d^3/(c^2*x^2-1)-a^2*x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)+4*a^2/c^4*x^2
/d/(-c^2*d*x^2+d)^(3/2)-b^2*(-d*(c^2*x^2-1))^(1/2)/c^4/d^3/(c^2*x^2-1)*arcc
osh(c*x)^2*x^2+2*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c
*x)^2*x^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)-10/
3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^6*arccosh(c*x)+11/3*b^2*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*arccos
h(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-11/3*b^2*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*
x-1)^(1/2)*(c*x+1)^(1/2))+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/
c^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-11/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1
)+11/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*
x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)/c
```

```
^5/d^3/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/3*b^2*(-d*(
c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^5*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x+2*a*b*(-d*(c^2*x^2-1))^(1/2)/c^5/d^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2x^5\operatorname{arcosh}(cx)^2+2\,abx^5\operatorname{arcosh}(cx)+a^2x^5\right)\sqrt{-c^2dx^2+d}}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^5*arccosh(c*x)^2 + 2*a*b*x^5*arccosh(c*x) + a^2*x^5)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2 x^5}{(-c^2dx^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^5/(-c^2*d*x^2 + d)^(5/2), x)
```

3.215 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=482

$$\frac{4b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\ldots}{}$$

```
[Out] -b^2/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*(1 - c*x))/(3*c^5*d^2*Sqrt[d -
c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^5*d^2*Sq
rt[d - c^2*d*x^2]) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (x^3*(a + b*ArcCosh[c*x
])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) - (x*(a + b*ArcCosh[c*x])^2)/(c^4*d^2
*Sqrt[d - c^2*d*x^2]) - (4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)^2)/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])^3)/(3*b*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (8*b*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*c^5*d^2*S
qrt[d - c^2*d*x^2]) + (4*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*A
rcCosh[c*x])])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.33307, antiderivative size = 497, normalized size of antiderivative = 1.03, number of steps used = 19, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5752, 5676, 5715, 3716, 2190, 2279, 2391, 5750, 89, 12, 78, 52}

$$\frac{4b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -b^2/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*(1 - c*x))/(3*c^5*d^2*Sqrt[d -
c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^5*d^2*Sq
rt[d - c^2*d*x^2]) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (x*(a + b*ArcCosh[c*x])
^2)/(c^4*d^2*Sqrt[d - c^2*d*x^2]) + (x^3*(a + b*ArcCosh[c*x])^2)/(3*c^2*d^2
*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x])^2)/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*
```

```
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (
8*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[
c*x])])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (4*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^4 \left(a + b\cosh^{-1}(cx)\right)^2}{\left(d - c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{c^2d^2\sqrt{d-c^2dx^2}} + \frac{\left(2b\sqrt{-1\ldots\right.}{\ldots}$$

$$= \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}\ldots\right.}{3c^2d^2(1-cx)(1+cx)\ldots}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^4d^2\sqrt{d-c^2dx^2}} + \ldots$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^4d^2\sqrt{d-c^2dx^2}} + \ldots$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x\left(a\ldots\right.}{c^{\ldots}}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx\ldots}}{3c^3d^2\left(\ldots\right.}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx\ldots}}{3c^3d^2\left(\ldots\right.}$$

Mathematica [A] time = 2.56995, size = 382, normalized size = 0.79

$$\frac{b^2d\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(-4\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)-\frac{cx\left(c^2x^2+\left(4c^2x^2-3\right)\cosh^{-1}(cx)^2-1\right)}{\left(\frac{cx-1}{cx+1}\right)^{3/2}(cx+1)^3}+\cosh^{-1}(cx)\left(\frac{1}{1-c^2x^2}+\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+4\right)+8\log\left(1-e^{-2\cosh^{-1}(cx)}\right)\right)\right)}{\sqrt{d-c^2dx^2}} + \ldots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((a^2*c*x*(-3 + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2)^2 - 3*a^2*Sq
rt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (a*b*d*(
```

```
-8*c*x*ArcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh
[c*x])/(-1 + c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*ArcCosh[c*x
]^2 + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)])))/Sqrt[d - c^2*d*x^2] +
(b^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((c*x*(-1 + c^2*x^2 + (-3 + 4
*c^2*x^2)*ArcCosh[c*x]^2))/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3)) + Ar
cCosh[c*x]*((1 - c^2*x^2)^(-1) + ArcCosh[c*x]*(4 + ArcCosh[c*x]) + 8*Log[1
- E^(-2*ArcCosh[c*x])]) - 4*PolyLog[2, E^(-2*ArcCosh[c*x])]))/Sqrt[d - c^2*
d*x^2])/(3*c^5*d^3)
```

Maple [B] time = 0.492, size = 4074, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 32*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2
+16)*c^2/d^3*arccosh(c*x)^2*x^7-4*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87
*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*(c*x+1)*(c*x-1)*x+4/3*b^2*(-d*(
c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*(c*
x+1)*(c*x-1)*x^5-440/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+11
8*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^2-16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^
2*x^2+16)/d^3*(c*x+1)*(c*x-1)*x^5+64*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8
-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*arccosh(c*x)*x^7+28/3*a*b*(-
d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/
d^3*(c*x+1)*(c*x-1)*x^3+8*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6
+118*c^4*x^4-71*c^2*x^2+16)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-32*a*b*(-
d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/
d^3*arccosh(c*x)*x+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+1
18*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-8*b^2*(-d*(c^
2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c/d^3*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^6-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*
c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*(c*x+1)*(c*x-1)*x^3+21*b^2*(-d*(
c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c/d^3*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-8
7*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*(c*x+1)*(c*x-1)*x+8/3*b^2*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(
c*x)^2-16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-7
1*c^2*x^2+16)/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)*x^5-1/3*b^2*(-d*(c^2*x^2-1))
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c*x)^3-55/3*
```

b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16
)/c^3/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(
24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*arccosh(c*x)^2*(c*
x+1)^(1/2)*(c*x-1)^(1/2)+16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6
*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^
(1/2)-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c
^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-8/3*b^2*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*polylog(2,-c*x-(c*
x-1)^(1/2)*(c*x+1)^(1/2))-64*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*
x^6+118*c^4*x^4-71*c^2*x^2+16)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*x^6+168*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71
*c^2*x^2+16)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+16/3*a*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccos
h(c*x)-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(
c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-32*b^2*(-d*(c
^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c/d^3*ar
ccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6+84*b^2*(-d*(c^2*x^2-1))^(1/2)/
(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c/d^3*arccosh(c*x)^2*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^4+28/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*
c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)*x^3
+8*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2
+16)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-8/3*b^2*(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c*x)*ln
(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-220/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^
8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*arccosh(c*x)^2*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^2-4*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6
+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)*x-13*b^2*(
-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3
/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-a*b*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c*x)^2-76*b^2*(-d
*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*a
rccosh(c*x)^2*x^5-44/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+11
8*c^4*x^4-71*c^2*x^2+16)/d^3*arccosh(c*x)*x^5+20/3*b^2*(-d*(c^2*x^2-1))^(1/
2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*x^7+43/3*b^2*(
-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2
/d^3*x^3-4*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71
*c^2*x^2+16)/c^4/d^3*x-44/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x
^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*x^5+16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c
^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*arccosh(c*x)*x^7-a^2/c
^4/d^2*x/(-c^2*d*x^2+d)^(1/2)-13*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*
c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+
128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*
x^2+16)/c^5/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)-8/3*a*b*(-d*(c^2*x
^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*ln((c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))^2-1)+40/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*

```
c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*arccosh(c*x)*x^3+1/3*a^2*x^3/c^2
/d/(-c^2*d*x^2+d)^(3/2)-17*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^
6+118*c^4*x^4-71*c^2*x^2+16)/d^3*x^5+a^2/c^4/d^2/(c^2*d)^(1/2)*arctan((c^2*
d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-16*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-
87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*arccosh(c*x)^2*x-4*b^2*(-d*(c
^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*
arccosh(c*x)*x+16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c
^4*x^4-71*c^2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+16/3*a*b*(-d*(c^2
*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*x^
7-152*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*
x^2+16)/d^3*arccosh(c*x)*x^5+40/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87
*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*x^3-4*a*b*(-d*(c^2*x^2-1))^(1/2
)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*x+362/3*a*b*(-d
*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d
^3*arccosh(c*x)*x^3+181/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6
+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*arccosh(c*x)^2*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2x^4\operatorname{arcosh}(cx)^2+2\,abx^4\operatorname{arcosh}(cx)+a^2x^4\right)\sqrt{-c^2dx^2+d}}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^2x^4}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^4/(-c^2*d*x^2 + d)^(5/2), x)
```

3.216 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{5b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{5b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}\left(a+\right.}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d}}$$

```
[Out] -b^2/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (x^2*(a
 + b*ArcCosh[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) - (2*(a + b*ArcCosh[c
*x])^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) - (10*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*c^4*d^2*Sqrt[d - c^2*d*x
^2]) - (5*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(3*
c^4*d^2*Sqrt[d - c^2*d*x^2]) + (5*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[
2, E^ArcCosh[c*x]])/(3*c^4*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.968666, antiderivative size = 351, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {5798, 5752, 5718, 5694, 4182, 2279, 2391, 5750, 74}

$$-\frac{5b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{5b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}\left(a+\right.}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -b^2/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (2*(a +
 b*ArcCosh[c*x])^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (x^2*(a + b*ArcCosh[c
*x])^2)/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (10*b*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*c^4*
d^2*Sqrt[d - c^2*d*x^2]) - (5*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -
E^ArcCosh[c*x]])/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (5*b^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*c^4*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{\left(2b\sqrt{-1\right.}{}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^2\left(a+b\cosh^{-1}(\right.}{3c^2d^2(1-cx)(1+cx)\sqrt{}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{}{3}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{}{3}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{}{3}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{}{3}$$

Mathematica [A] time = 4.46191, size = 341, normalized size = 1.01

$$\frac{-b^2\left(20\left(\frac{cx-1}{cx+1}\right)^{3/2}(cx+1)^3\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)-20\left(\frac{cx-1}{cx+1}\right)^{3/2}(cx+1)^3\text{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right)+2\cosh^{-1}(cx)^2-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a^2*(-2 + 3*c^2*x^2) - b^2*(2 + 2*ArcCosh[c*x]^2 - 2*(1 + 3*ArcCosh[c*x]
^2)*Cosh[2*ArcCosh[c*x]] - 15*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[
c*x]*(Log[1 - E^(-ArcCosh[c*x])] - Log[1 + E^(-ArcCosh[c*x])]) + 20*((-1 +
c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*x])] - 20*((-1
+ c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c*x])] - 2*ArcCo
sh[c*x]*Sinh[2*ArcCosh[c*x]] + 5*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])]*Si
```

```
nh[3*ArcCosh[c*x]] - 5*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])]*Sinh[3*ArcCo
sh[c*x]]) - a*b*(ArcCosh[c*x]*(4 - 12*Cosh[2*ArcCosh[c*x]]) - 2*Sinh[2*ArcC
osh[c*x]] + 5*Log[Tanh[ArcCosh[c*x]/2]]*(-3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x) + Sinh[3*ArcCosh[c*x]])))/(12*c^4*d*(d - c^2*d*x^2)^(3/2))
```

Maple [B] time = 0.408, size = 835, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] a^2*x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3*a^2/d/c^4/(-c^2*d*x^2+d)^(3/2)+b^2*(
-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c*x)^2*x^2+1/3*b^2*(-d*
(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*x^2-2/3*b^2*(
-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)^2-1/3*b^2*(-d*(c^2
*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2
)*(c*x+1)^(1/2))-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/d^3/c^4/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/3*b^2*(-d
*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*arccosh
(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/3*b^2*(-d*(c^2*x^2-1))^(1/2)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2
)*(c*x+1)^(1/2))+2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh
(c*x)*x^2+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^3*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x-4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arcc
osh(c*x)-5/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4
/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)+5/3*a*b*(-d*(c^2*x^2-1))
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\,abc\left(\frac{2\sqrt{-d}x}{c^6d^3x^2-c^4d^3}+\frac{5\sqrt{-d}\log\left(cx+1\right)}{c^5d^3}-\frac{5\sqrt{-d}\log\left(cx-1\right)}{c^5d^3}\right)+\frac{2}{3}\,ab\left(\frac{3\,x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}-\frac{2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)\text{arcos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/6*a*b*c*(2*sqrt(-d)*x/(c^6*d^3*x^2 - c^4*d^3) + 5*sqrt(-d)*log(c*x + 1)/(
c^5*d^3) - 5*sqrt(-d)*log(c*x - 1)/(c^5*d^3)) + 2/3*a*b*(3*x^2/((-c^2*d*x^2
 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arccosh(c*x) + 1/3*a
^2*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d)
) + b^2*integrate(x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2
+ d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2x^3\operatorname{arcosh}(cx)^2+2\,abx^3\operatorname{arcosh}(cx)+a^2x^3\right)\sqrt{-c^2dx^2+d}}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\left(a+b\operatorname{acosh}(cx)\right)^2}{\left(-d\,(cx-1)\,(cx+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{2}x^{3}}{\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^3/(-c^2*d*x^2 + d)^(5/2), x)
```

3.217 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=389

$$\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx\right.}{3c^3d^2\sqrt{d-c^2dx^2}}$$

```
[Out] -b^2/(3*c^3*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*(1 - c*x))/(3*c^3*d^2*Sqrt[d -
c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^3*d^2*Sq
rt[d - c^2*d*x^2]) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (x^3*(a + b*ArcCosh[c*x])
^2)/(3*d*(d - c^2*d*x^2)^(3/2)) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x])^2)/(3*c^3*d^2*Sqrt[d - c^2*d*x^2]) + (2*b*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*c^3*d^2*Sqrt[d -
 c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*
x])])/(3*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.730808, antiderivative size = 404, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {5798, 5724, 5750, 89, 12, 78, 52, 5715, 3716, 2190, 2279, 2391}

$$\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -b^2/(3*c^3*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*(1 - c*x))/(3*c^3*d^2*Sqrt[d -
c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^3*d^2*Sq
rt[d - c^2*d*x^2]) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (x^3*(a + b*ArcCosh[c*x])
^2)/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*c^3*d^2*Sqrt[d - c^2*d*x^2]) + (2*b*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])
/(3*c^3*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLo
g[2, E^(2*ArcCosh[c*x])])/(3*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
```

```
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(-1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(b^2\sqrt{-1+\right.}{}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(c\right.}{3d^2(1-cx)(1+cx)\sqrt{d}}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(c\right.}{3d^2(1-cx)(1+cx)\sqrt{d}}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{}{3d}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+}}{3cd^2}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+}}{3cd^2}$$

Mathematica [A] time = 1.67831, size = 264, normalized size = 0.68

$$\frac{b^2\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(-\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)-\frac{cx\left(c^2x^2+c^2x^2\cosh^{-1}(cx)^2-1\right)}{\left(\frac{cx-1}{cx+1}\right)^{3/2}(cx+1)^3}+\cosh^{-1}(cx)\left(\frac{1}{1-c^2x^2}+\cosh^{-1}(cx)+2\log\left(1-\right.\right.\right.}{3c^3d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((a^2*c^3*x^3)/(1 - c^2*x^2) + a*b*((2*c^3*x^3*ArcCosh[c*x])/(1 - c^2*x^2)
+ (Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + 2*(-1 + c^2*x^2)*Log[Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)]))/(-1 + c*x)) + b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x
)*(-((c*x*(-1 + c^2*x^2 + c^2*x^2*ArcCosh[c*x]^2))/(((-1 + c*x)/(1 + c*x))^
(3/2)*(1 + c*x)^3)) + ArcCosh[c*x]*((1 - c^2*x^2)^(-1) + ArcCosh[c*x] + 2*L
og[1 - E^(-2*ArcCosh[c*x])]) - PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*c^3*d^2
*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.352, size = 3445, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)
/d^3*x^3+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c
^2*x^2+1)/d^3*arccosh(c*x)^2*x^3+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-
9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*arccosh(c*x)*x^3+2/3*b^2*(-d*(c^2*x^2
-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*x^7-b^2*(-d
*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*x^
5-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+
1)/d^3*(c*x+1)*(c*x-1)*x^3+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*
x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*arccosh(c*x)*x^7-b^2*(-d*(c^2*x^2-1))^(
1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*arccosh(c*x)^2*x^
5-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+
1)*c^2/d^3*arccosh(c*x)*x^5+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6
*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+b^2*(-d*(c
^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*arcco
sh(c*x)^2*x^7-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^
4-5*c^2*x^2+1)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-4/3*b^2*(
-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c
^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)
^(1/2)-2/3*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c^3/(
c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2/3*b^2*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c^3/(c^2*x^2-1)*arccosh(c*x)^2-2/
3*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c^3/(c^2*x^2-1
)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/3*b^2*(-d*(c^2*x^2-1))^(1/2
```

)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*arccosh(c*x)*(c*x+1)
^(1/2)*(c*x-1)^(1/2)+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c
^4*x^4-5*c^2*x^2+1)*c^4/d^3*arccosh(c*x)*x^7-2*a*b*(-d*(c^2*x^2-1))^(1/2)/(
3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*arccosh(c*x)*x^5+1/3*a*
b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*(
c*x+1)*(c*x-1)*x^3+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c
^4*x^4-5*c^2*x^2+1)/c^3/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/3*b^2*(-d*(c^2*x^
2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*arccosh(c*x)*(
c*x+1)*(c*x-1)*x^3+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c
^4*x^4-5*c^2*x^2+1)*c^2/d^3*(c*x+1)*(c*x-1)*x^5-b^2*(-d*(c^2*x^2-1))^(1/2)/
(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3/d^3*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^6+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^
2*x^2+1)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+1/3*b^2*(-d*(c^2*x^2-1))^(1/
2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^2/d^3*(c*x+1)*(c*x-1)*x-1
/3*a^2/c^2/d^2*x/(-c^2*d*x^2+d)^(1/2)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8
*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*x^3+1/3*a^2/c^2/d*x/(-c^2*d*x^2+
d)^(3/2)+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c
^2*x^2+1)*c^4/d^3*x^7-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+1
0*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*x^5+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^
8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*arccosh(c*x)*x^3-2*a*b*(-d*(c^2*x^2
-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3/d^3*arccosh(c*x
)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6+4*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9
*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^4+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x
^2+1)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-4/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*arccosh(c*x
)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9
*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^2-2/3*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c^3
/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2/3*b^2*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c^3/(c^2*x^2-1)*arccosh
(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c
^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3/d^3*arccosh(c*x)^2*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^6-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10
*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)*x^5+2*b^2*(-d*(c
^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c/d^3*arccosh
(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+4/3*a*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)
(-d(c^2*x^2-1))^(1/2)/d^3/c^3/(c^2*x^2-1)*arccosh(c*x)-1/3*a*b*(-d*(c^2*x
^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*(c*x+1)*(
c*x-1)*x^5+a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2
*x^2+1)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-a*b*(-d*(c^2*x^2-1))^(1/2)/(3
*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^2+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*
x^2+1)/c^3/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)-2/3*a*b*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c^3/(c^2*x^2-1)*ln((c*x+(c*x-1)

```
^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,abc\left(\frac{\sqrt{-d}}{c^6d^3x^2-c^4d^3}-\frac{\sqrt{-d}\log\left(cx+1\right)}{c^4d^3}-\frac{\sqrt{-d}\log\left(cx-1\right)}{c^4d^3}\right)-\frac{2}{3}\,ab\left(\frac{x}{\sqrt{-c^2dx^2+d}c^2d^2}-\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)\operatorname{arcosh}\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*a*b*c*(sqrt(-d)/(c^6*d^3*x^2 - c^4*d^3) - sqrt(-d)*log(c*x + 1)/(c^4*d^
3) - sqrt(-d)*log(c*x - 1)/(c^4*d^3)) - 2/3*a*b*(x/(sqrt(-c^2*d*x^2 + d)*c^
2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arccosh(c*x) - 1/3*a^2*(x/(sqrt(
-c^2*d*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d)) + b^2*integrat
e(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(b^2x^2\operatorname{arcosh}\left(cx\right)^2+2\,abx^2\operatorname{arcosh}\left(cx\right)+a^2x^2\right)\sqrt{-c^2dx^2+d}}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2 x^2}{(-c^2 d x^2 + d)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^2/(-c^2*d*x^2 + d)^(5/2), x)
```

3.218 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}-\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh\right.}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2d}}$$

```
[Out] -b^2/(3*c^2*d^2*Sqrt[d - c^2*d*x^2]) + (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (a + b*Ar
cCosh[c*x])^2/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (2*b*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*c^2*d^2*Sqrt[d - c^
2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/
(3*c^2*d^2*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog
[2, E^ArcCosh[c*x]])/(3*c^2*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.473191, antiderivative size = 313, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5718, 5689, 74, 5694, 4182, 2279, 2391}

$$\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}-\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh\right.}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -b^2/(3*c^2*d^2*Sqrt[d - c^2*d*x^2]) + (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (a + b*Ar
cCosh[c*x])^2/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (2*b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3
*c^2*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2
, -E^ArcCosh[c*x]])/(3*c^2*d^2*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*c^2*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(2b\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\left(-1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(b^2\sqrt{-1+cx}\right.}{}$$

$$= -\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-}}$$

$$= -\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-}}$$

$$= -\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-}}$$

$$= -\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-}}$$

Mathematica [A] time = 2.49435, size = 332, normalized size = 1.11

$$b^2\left(4\left(\frac{cx-1}{cx+1}\right)^{3/2}(cx+1)^3\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)-4\left(\frac{cx-1}{cx+1}\right)^{3/2}(cx+1)^3\text{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right)+4\cosh^{-1}(cx)^2+2\,c$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a^2 + b^2*(-2 + 4*ArcCosh[c*x]^2 + 2*Cosh[2*ArcCosh[c*x]] - 3*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] + 3*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] +
4*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*x])] -
 4*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c*x])] +
 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] + ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x
])]*Sinh[3*ArcCosh[c*x]] - ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])]*Sinh[3*A
rcCosh[c*x]]) + a*b*(8*ArcCosh[c*x] + 2*Sinh[2*ArcCosh[c*x]] + Log[Tanh[Arc
Cosh[c*x]/2]]*(-3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + Sinh[3*ArcCosh[c*x
]])))/(12*c^2*d*(d - c^2*d*x^2)^(3/2))
```

Maple [B] time = 0.33, size = 720, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a^2/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*
x^2-1)^2/c*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/3*b^2*(-d*(c^2*x^2-
1))^(1/2)/d^3/(c^2*x^2-1)^2*x^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2
-1)^2/c^2*arccosh(c*x)^2-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c
^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*
x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/3*b^2*(-d*(c^2*
x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*polylog(2,c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^
2/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/3*a*b*(-d*(c^2*
x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+2/3*a*b*(-d
```

```
*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c*x)+1/3*a*b*(-d*(c^2*x^2
-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)-1)-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d^3/c^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2}{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\int\frac{b^2x\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}+\frac{2\,abx\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*a^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2) + 2*a*b*x*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2x\operatorname{arcosh}\left(cx\right)^2+2\,abx\operatorname{arcosh}\left(cx\right)+a^2x\right)}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
 + a^2*x)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2 x}{\left(-c^2 d x^2 + d\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x/(-c^2*d*x^2 + d)^(5/2), x)
```

3.219 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx-}}{}$$

```
[Out] -(b^2*x)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (x*(a + b*A
rcCosh[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcCosh[c*x])^2)/
(3*d^2*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCos
h[c*x])^2)/(3*c*d^2*Sqrt[d - c^2*d*x^2]) - (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*c*d^2*Sqrt[d - c^2*d
*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])]
)/(3*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.56041, antiderivative size = 346, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5713, 5691, 5688, 5715, 3716, 2190, 2279, 2391, 5716, 39}

$$-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -(b^2*x)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (2*x*(a + b
*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (x*(a + b*ArcCosh[c*x])^2)/
(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(a + b*ArcCosh[c*x])^2)/(3*c*d^2*Sqrt[d - c^2*d*x^2]) - (4*b*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*
c*d^2*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2,
 E^(2*ArcCosh[c*x])])/(3*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+\right.}{}$$

$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{x\left(a+b\cosh^{-1}(cx\right.}{3d^2(1-cx)(1+cx)\sqrt{d-}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{}{3d^2}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{}{3d^2}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{}{3d^2}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{}{3d^2}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{}{3d^2}$$

Mathematica [A] time = 1.48929, size = 289, normalized size = 0.87

$$b^2\left(2\sqrt{\frac{cx-1}{cx+1}}(cx+1)\mathrm{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right) - \frac{\cosh^{-1}(cx)\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)+cx\cosh^{-1}(cx)\right)}{c^2x^2-1} + cx\left(2\cosh^{-1}(cx)^2-1\right) - 2\sqrt{\frac{cx-1}{cx+1}}(cx\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((a^2*c*x*(-3 + 2*c^2*x^2))/(-1 + c^2*x^2) + a*b*(2*c*x*(2 + (1 - c^2*x^2)^
(-1))*ArcCosh[c*x] + (Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + (4 - 4*c^2*x^2)*Log[
```

```
Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)]))/(-1 + c*x)) + b^2*(-((ArcCosh[c*x]*
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + c*x*ArcCosh[c*x]))/(-1 + c^2*x^2))
+ c*x*(-1 + 2*ArcCosh[c*x]^2) - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcC
osh[c*x]*(ArcCosh[c*x] + 2*Log[1 - E^(-2*ArcCosh[c*x])]) + 2*Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*c*d^2*Sqrt[d -
c^2*d*x^2])
```

Maple [B] time = 0.289, size = 3050, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 14/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3
*x^5-16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^
2/d^3*x^3-8*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/
d^3*arccosh(c*x)*x-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*
c^2*x^2-4)*c^6/d^3*arccosh(c*x)*x^7-2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6
-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)^2*x^5+14/3*b^2*(-d*(c^2*x^2-
1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)*x^5+17/3
*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arc
cosh(c*x)^2*x^3+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x
^2-4)/d^3*x+8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2
-4)/c/d^3*arccosh(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-4/3*b^2*(-d*(c^2*x^2-1
))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*arccosh(c*x)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)+4/3*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)
/d^3/c/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^
2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*arccosh(c*x)*(c*x-1
)*(c*x+1)*x-4/3*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/
c/(c^2*x^2-1)*arccosh(c*x)^2+7/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c
^4*x^4+11*c^2*x^2-4)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-4/3*b^2*(-d*(c^2
*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*(c*x-1)*(c*x+1)*
x^3-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^3/d^3*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4-4*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*
c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)*x^5+34/3*a*b*(-d*(c^2*x^2-1))^(1
/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)*x^3+2*a*b*(-d*
(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*(c*x-1)*(c*x+1)*
x-4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*
(c*x-1)^(1/2)*(c*x+1)^(1/2)+4/3*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^
2-1))^(1/2)/d^3/c/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2
```

```
/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*(
c*x-1)*(c*x+1)*x^5+1/3*a^2/d*x/(-c^2*d*x^2+d)^(3/2)+2/3*a^2/d^2*x/(-c^2*d*x
^2+d)^(1/2)+4*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4
)*c^3/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4-28/3*a*b*(-d*(c^2*x^
2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*arccosh(c*x)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*x^2+4/3*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1)
)^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2)
)+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^
3*arccosh(c*x)*(c*x-1)*(c*x+1)*x^5+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-
10*c^4*x^4+11*c^2*x^2-4)*c^3/d^3*arccosh(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*x^4-10/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^
2/d^3*arccosh(c*x)*(c*x-1)*(c*x+1)*x^3+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^
6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*(c*x-1)*(c*x+1)*x^5-10/3*a*b*(-d*(c^
2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*(c*x-1)*(c*x+1)
*x^3+a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-1
0*c^4*x^4+11*c^2*x^2-4)/c/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4/3*
a*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c/(c^2*x^2-1)*ln
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c
^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^6/d^3*x^7+3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3
*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*x^5-13/3*b^2*(-d*(c^2*x^2-1))^(1/
2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*x^3-4*b^2*(-d*(c^2*x^2-1))^(
1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*arccosh(c*x)^2*x+2*b^2*(-d*(c^
2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*arccosh(c*x)*x+2*a*
b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*x-16/3*b^2
*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh
(c*x)*x^3-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4
)/c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x
^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*(c*x-1)*(c*x+1)*x-4/3*a*b*(-d*(c^2*x^2-1))^
(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^6/d^3*x^7-14/3*b^2*(-d*(c^2*x^2
-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*arccosh(c*x)^2*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*x^2+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+1
1*c^2*x^2-4)*c/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-8/3*a*b*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(c
*x)+4/3*b^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c/(c^2*x
^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}abc\left(\frac{\sqrt{-d}}{c^4d^3x^2-c^2d^3}+\frac{2\sqrt{-d}\log(cx+1)}{c^2d^3}+\frac{2\sqrt{-d}\log(cx-1)}{c^2d^3}\right)+\frac{2}{3}ab\left(\frac{2x}{\sqrt{-c^2dx^2+d}\,d^2}+\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}\right)\operatorname{arcosh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*b*c*(sqrt(-d)/(c^4*d^3*x^2 - c^2*d^3) + 2*sqrt(-d)*log(c*x + 1)/(c^2*
d^3) + 2*sqrt(-d)*log(c*x - 1)/(c^2*d^3)) + 2/3*a*b*(2*x/(sqrt(-c^2*d*x^2 +
 d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arccosh(c*x) + 1/3*a^2*(2*x/(sqrt(
-c^2*d*x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d)) + b^2*integrate(log(c*
x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}\left(cx\right)^2+2\,ab\operatorname{arcosh}\left(cx\right)+a^2\right)}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(-c^2*d*x^2 + d)^(5/2), x)
```

3.220 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=597

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\,\text{co}\right.}{d^2\sqrt{d-c^2dx^2}}$$

```
[Out] -b^2/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (a + b*ArcCos
h[c*x])^2/(3*d*(d - c^2*d*x^2)^(3/2)) + (a + b*ArcCosh[c*x])^2/(d^2*Sqrt[d
- c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcT
an[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (14*b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d
*x^2]) + (7*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(
3*d^2*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*A
rcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (
(2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^Ar
cCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) - (7*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*PolyLog[2, E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) + ((2*I)*b^2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2
*d*x^2]) - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c
*x]])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.33004, antiderivative size = 612, normalized size of antiderivative = 1.03, number of steps used = 25, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5756, 5761, 4180, 2531, 2282, 6589, 5694, 4182, 2279, 2391, 5689, 74}

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\,\text{co}\right.}{d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -b^2/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (a + b*ArcCos
```

```
h[c*x])^2/(d^2*Sqrt[d - c^2*d*x^2]) + (a + b*ArcCosh[c*x])^2/(3*d^2*(1 - c*
x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*
ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (14*b*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(
3*d^2*Sqrt[d - c^2*d*x^2]) + (7*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2,
 -E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt
[d - c^2*d*x^2]) + ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
])*PolyLog[2, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) - (7*b^2*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2
]) + ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]
])/(d^2*Sqrt[d - c^2*d*x^2]) - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Poly
Log[3, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d^2\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\right.}{3}$$

$$= \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c}}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}} + \frac{(}{3d^2(1}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}} + \frac{(}{3d^2(1}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}} + \frac{(}{3d^2(1}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}} + \frac{(}{3d^2(1}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}} + \frac{(}{3d^2(1}$$

Mathematica [A] time = 10.8273, size = 806, normalized size = 1.35

$$\frac{\log(cx)a^2}{d^{5/2}} - \frac{\log\left(d+\sqrt{-d\left(c^2x^2-1\right)}\sqrt{d}\right)a^2}{d^{5/2}} + \frac{b\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(-\frac{1}{2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)\mathrm{csch}^4\left(\frac{1}{2}\cosh^{-1}(cx)\right)-\mathrm{csch}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(a^2/(3*d^3*(-1 + c^2*x^2)^2) - a^2/(d^3*(-1 + c^
2*x^2))) + (a^2*Log[c*x])/d^(5/2) - (a^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2
```

```
*x^2))]])/d^(5/2) + (a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(14*ArcCosh[c
*x]*Coth[ArcCosh[c*x]/2] - Csch[ArcCosh[c*x]/2]^2 - (Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*ArcCosh[c*x]*Csch[ArcCosh[c*x]/2]^4)/2 - (24*I)*ArcCosh[c*x]
*Log[1 - I/E^ArcCosh[c*x]] + (24*I)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]]
- 28*Log[Tanh[ArcCosh[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (
24*I)*PolyLog[2, I/E^ArcCosh[c*x]] - Sech[ArcCosh[c*x]/2]^2 - (8*ArcCosh[c*
x]*Sinh[ArcCosh[c*x]/2]^4)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3) - 14*
ArcCosh[c*x]*Tanh[ArcCosh[c*x]/2]))/(12*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]
) + (b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-4*Coth[ArcCosh[c*x]/2] + 14
*ArcCosh[c*x]^2*Coth[ArcCosh[c*x]/2] - 2*ArcCosh[c*x]*Csch[ArcCosh[c*x]/2]^
2 - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2*Csch[ArcCosh[c*x]/
2]^4)/2 - 56*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] - (24*I)*ArcCosh[c*x]^
2*Log[1 - I/E^ArcCosh[c*x]] + (24*I)*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x
]] + 56*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] - 56*PolyLog[2, -E^(-ArcCos
h[c*x])] - (48*I)*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (48*I)*Arc
Cosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 56*PolyLog[2, E^(-ArcCosh[c*x])] -
 (48*I)*PolyLog[3, (-I)/E^ArcCosh[c*x]] + (48*I)*PolyLog[3, I/E^ArcCosh[c*x
]] - 2*ArcCosh[c*x]*Sech[ArcCosh[c*x]/2]^2 - (8*ArcCosh[c*x]^2*Sinh[ArcCosh
[c*x]/2]^4)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3) + 4*Tanh[ArcCosh[c*x
]/2] - 14*ArcCosh[c*x]^2*Tanh[ArcCosh[c*x]/2]))/(24*d^2*Sqrt[-(d*(-1 + c*x)
*(1 + c*x))])
```

Maple [F] time = 0.361, size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x}\left(-c^2dx^2+d\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^6d^3x^7-3\,c^4d^3x^5+3\,c^2d^3x^3-d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

3.221 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=476

$$-\frac{b^2c\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}-\frac{5b^2c\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\,(a}{3d^2\left(1-c^2x^2\right)}$$

```
[Out] -(b^2*c^2*x)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (a + b*
ArcCosh[c*x])^2/(d*x*(d - c^2*d*x^2)^(3/2)) + (4*c^2*x*(a + b*ArcCosh[c*x])
^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (8*c^2*x*(a + b*ArcCosh[c*x])^2)/(3*d^2*S
qrt[d - c^2*d*x^2]) + (8*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) - (4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/(d^2*Sqrt[d - c^2*d*x^2]) - (
16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCo
sh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2]) - (b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(d^2*Sqrt[d - c^2*d*x^2]) - (5*b^2*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c
^2*d*x^2])
```

Rubi [A] time = 1.19143, antiderivative size = 506, normalized size of antiderivative = 1.06, number of steps used = 20, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {5798, 5748, 5691, 5688, 5715, 3716, 2190, 2279, 2391, 5716, 39, 5754, 5721, 5461, 4182}

$$-\frac{b^2c\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}-\frac{5b^2c\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\,(a}{3d^2\left(1-c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(b^2*c^2*x)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (8*c^2*
x*(a + b*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCosh[c*x]
)^2/(d^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (4*c^2*x*(a + b*ArcCo
sh[c*x])^2)/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (8*c*Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) -
(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCo
sh[c*x])])/(d^2*Sqrt[d - c^2*d*x^2]) - (16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^
2]) - (b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/
(d^2*Sqrt[d - c^2*d*x^2]) - (5*b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2
, E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
```

```
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
```

```
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
IntegerQ[p]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x\left(-1+c^2x^2\right)^2}\,dx}{d^2\sqrt{d-c^2dx^2}} + \frac{\left(4c^2\sqrt{-1+\right.}{}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+\right.}{3d^2(1-cx)(1}$$

$$= \frac{b^2c^2x}{d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{}{d^2x}$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{}{d}$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{}{d}$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{}{d}$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{}{d}$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{}{d}$$

Mathematica [A] time = 3.125, size = 457, normalized size = 0.96

$$\frac{c\left(b^2\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(3\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+5\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)+\frac{\cosh^{-1}(cx)}{1-c^2x^2}+\frac{cx\sqrt{\frac{cx-1}{cx+1}}}{1-cx}+\frac{3\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh}{cx}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (c*((a^2*(3 - 12*c^2*x^2 + 8*c^4*x^4))/(c*x*(-1 + c^2*x^2)) + a*b*(10*c*x*A
rcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh[c*x])/(
-1 + c^2*x^2) - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((-3*Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x])/(c*x) + 3*Log[c*x] + 5*Log[Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)])) + b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((c*
x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + ArcCosh[c*x]/(1 - c^2*x^2) - 8*Ar
cCosh[c*x]^2 - (c*x*ArcCosh[c*x]^2)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)
^3) + (5*c*x*ArcCosh[c*x]^2)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (3*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/(c*x) - 10*ArcCosh[c*x]*
Log[1 - E^(-2*ArcCosh[c*x])] - 6*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])]
+ 3*PolyLog[2, -E^(-2*ArcCosh[c*x])] + 5*PolyLog[2, E^(-2*ArcCosh[c*x])])))
/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] time = 0.352, size = 3798, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a^2/d/x/(-c^2*d*x^2+d)^(3/2)-32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^
6-25*c^4*x^4+26*c^2*x^2-9)*x^9*c^10+128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^4*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*c^5-272/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x
^2-9)*x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+8/3*a^2*c^2/d^2*x/(-
c^2*d*x^2+d)^(1/2)-8*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+2
6*c^2*x^2-9)*x*(c*x+1)*(c*x-1)*c^2+8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^
6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+48*a*b*(
-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*arccosh(c*x)*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+10/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*c+
2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1
/2)*(c*x+1)^(1/2))*c+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*
x^4+26*c^2*x^2-9)*x^7*arccosh(c*x)*(c*x+1)*(c*x-1)*c^8-160/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*arccosh(c*x)*(c*x+
1)*(c*x-1)*c^6+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26
*c^2*x^2-9)*x^4*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5+40*b^2*(-d*(
c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*arccosh(c*x)*
(c*x+1)*(c*x-1)*c^4-136/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*
```

x^4+26*c^2*x^2-9)*x^2*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+80/3*b
^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*(c*x+
1)*(c*x-1)*c^4-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*
c^2*x^2-9)*x^4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5-8*b^2*(-d*(c^2*x^2-1))^(1/2)
/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*(c*x+1)*(c*x-1)*c^2+17/3*b^2*(-d
*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*c^3+b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/d^3/(c^2*x^2-1)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c+10/3*b^2
(-d(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*polylog
(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))*c-16/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(
c^2*x^2-1)*arccosh(c*x)^2*c+24*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25
*c^4*x^4+26*c^2*x^2-9)*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-3*b^2*(
-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*arccosh(c*x)*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^
6-25*c^4*x^4+26*c^2*x^2-9)*x^7*(c*x+1)*(c*x-1)*c^8-88/3*b^2*(-d*(c^2*x^2-1)
)^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*(c*x+1)*(c*x-1)*c^6-128
/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*a
rccosh(c*x)*c^6+112*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26
*c^2*x^2-9)*x^3*arccosh(c*x)*c^4-88*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x
^6-25*c^4*x^4+26*c^2*x^2-9)*x*arccosh(c*x)*c^2-3*a*b*(-d*(c^2*x^2-1))^(1/2)
/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+2*b^
2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccos
h(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)*c+8/3*b^2*(-d*(c^2*x^2-1))
^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*arccosh(c*x)*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*c^3+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c-
8*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*arcc
osh(c*x)*(c*x+1)*(c*x-1)*c^2-32/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*c+64/3*a*b*(-d*(c^2*x^2-1))^(1/2
)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*(c*x+1)*(c*x-1)*c^8-160/3*a*b
(-d(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*(c*x+1)
*(c*x-1)*c^6+40*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2
*x^2-9)*x^3*(c*x+1)*(c*x-1)*c^4+4/3*a^2*c^2/d*x/(-c^2*d*x^2+d)^(3/2)+40*b^2
(-d(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*c^8-160
/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*c
^6+29*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^
3*c^4-5*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*
x*c^2+9*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/
x*arccosh(c*x)^2+224/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4
+26*c^2*x^2-9)*x^7*arccosh(c*x)*c^8-64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*
c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*arccosh(c*x)^2*c^6-280/3*b^2*(-d*(c^2*
x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*arccosh(c*x)*c^6+
56*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*a

```
rccosh(c*x)^2*c^4+48*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+2
6*c^2*x^2-9)*x^3*arccosh(c*x)*c^4-44*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*
x^6-25*c^4*x^4+26*c^2*x^2-9)*x*arccosh(c*x)^2*c^2-8*b^2*(-d*(c^2*x^2-1))^(1
/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*arccosh(c*x)*c^2-3*b^2*(-d*(c
^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*c-64/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^
2*x^2-9)*x^9*c^10-64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4
+26*c^2*x^2-9)*x^9*arccosh(c*x)*c^10+224/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(
8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*c^8-280/3*a*b*(-d*(c^2*x^2-1))^(1/2)
/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*c^6+48*a*b*(-d*(c^2*x^2-1))^(1
/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*c^4-8*a*b*(-d*(c^2*x^2-1))^
(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*c^2+18*a*b*(-d*(c^2*x^2-1))
^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/x*arccosh(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^6d^3x^8-3\,c^4d^3x^6+3\,c^2d^3x^4-d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

3.222 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=796

$$-\frac{2bx\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)c^3}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{5\left(a+b\cosh^{-1}(cx)\right)^2c^2}{2d^2\sqrt{d-c^2dx^2}}+\frac{5\left(a+b\cosh^{-1}(cx)\right)^2c^2}{6d\left(d-c^2dx^2\right)^{3/2}}+\frac{5\sqrt{cx-1}\sqrt{cx+1}\left(a\right.}{}$$

```
[Out] -(b^2*c^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x]))/(d^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (2*b*c^3*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*S
qrt[d - c^2*d*x^2]) + (5*c^2*(a + b*ArcCosh[c*x])^2)/(6*d*(d - c^2*d*x^2)^(
3/2)) - (a + b*ArcCosh[c*x])^2/(2*d*x^2*(d - c^2*d*x^2)^(3/2)) + (5*c^2*(a
+ b*ArcCosh[c*x])^2)/(2*d^2*Sqrt[d - c^2*d*x^2]) + (5*c^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^
2*d*x^2]) - (b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[Sqrt[-1 + c*x]*Sqr
t[1 + c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (26*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^
2]) + (13*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])
/(3*d^2*Sqrt[d - c^2*d*x^2]) - ((5*I)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2
]) + ((5*I)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog
[2, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) - (13*b^2*c^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) +
((5*I)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]]
)/(d^2*Sqrt[d - c^2*d*x^2]) - ((5*I)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*P
olyLog[3, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.93812, antiderivative size = 826, normalized size of antiderivative = 1.04, number of steps used = 39, number of rules used = 19, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.655, Rules used = {5798, 5748, 5756, 5761, 4180, 2531, 2282, 6589, 5694, 4182, 2279, 2391, 5689, 74, 5746, 104, 21, 92, 205}

$$-\frac{2bx\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)c^3}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{5\left(a+b\cosh^{-1}(cx)\right)^2c^2}{6d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{5\left(a+b\cosh^{-1}(cx)\right)^2c^2}{2d^2\sqrt{d-c^2dx^2}}+\frac{5\sqrt{cx-1}\sqrt{cx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(b^2*c^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x]))/(d^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (2*b*c^3*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*S
qrt[d - c^2*d*x^2]) + (5*c^2*(a + b*ArcCosh[c*x])^2)/(2*d^2*Sqrt[d - c^2*d*
x^2]) + (5*c^2*(a + b*ArcCosh[c*x])^2)/(6*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d -
c^2*d*x^2]) - (a + b*ArcCosh[c*x])^2/(2*d^2*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d
- c^2*d*x^2]) + (5*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*
ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) - (b^2*c^2*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(d^2*Sqrt[d - c^2*d*x^
2]) + (26*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E
^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) + (13*b^2*c^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) - ((5*
I)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*
E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + ((5*I)*b*c^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d
- c^2*d*x^2]) - (13*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCo
sh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) + ((5*I)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) - ((5*I
)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])/(d^2*S
qrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
```

```
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2d^2x^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(-1+c^2x^2\right)^2}\,dx}{d^2\sqrt{d-c^2dx^2}} + \frac{\left(5c^2\sqrt{-1+\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cosh^{-1}(cx)\right)^2}{6d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(a+b\,\mathrm{co}\right.}{2d^2x^2(1-cx)(}$$

$$= -\frac{b^2c^2}{d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2}}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{3d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{3d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{3d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{3d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{3d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

Mathematica [A] time = 97.9998, size = 1181, normalized size = 1.48

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(-a^2/(2*d^3*x^2) + (a^2*c^2)/(3*d^3*(-1 + c^2*x^
2)^2) - (2*a^2*c^2)/(d^3*(-1 + c^2*x^2))) + (5*a^2*c^2*Log[x])/(2*d^(5/2))
```

```
- (5*a^2*c^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/(2*d^(5/2)) + (a*b
*c^2*((6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x) + (6*(-1 + c*x)*(1 + c
*x)*ArcCosh[c*x])/(c^2*x^2) + 26*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 - Coth
[ArcCosh[c*x]/2] - ArcCosh[c*x]*Coth[ArcCosh[c*x]/2]^2 - (30*I)*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + (30*I)*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]]
- 26*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] - (30*I
)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (3
0*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] - 26
*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^2 - Tanh[ArcCosh[c*x]/2] - ArcCosh[c*x]*
Tanh[ArcCosh[c*x]/2]^2))/(6*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]) - (b^2*c^2
*Sqrt[d - c^2*d*x^2]*((12*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]
)/(c*x) + 6*(1 - 1/(c^2*x^2))*ArcCosh[c*x]^2 - 24*Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x)*ArcTan[Tanh[ArcCosh[c*x]/2]] - 4*Cosh[ArcCosh[c*x]/2]^2 + 26*Ar
cCosh[c*x]^2*Cosh[ArcCosh[c*x]/2]^2 - 2*ArcCosh[c*x]*Coth[ArcCosh[c*x]/2] -
 ArcCosh[c*x]^2*Coth[ArcCosh[c*x]/2]^2 - 52*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] - (30*I)*Sqrt[(-1 + c*x)/(1 +
 c*x)]*(1 + c*x)*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] + (30*I)*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] + 52*
Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])
] - 52*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c*x])]
- (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*PolyLog[2, (-I)/
E^ArcCosh[c*x]] + (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*
PolyLog[2, I/E^ArcCosh[c*x]] + 52*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Poly
Log[2, E^(-ArcCosh[c*x])] - (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Pol
yLog[3, (-I)/E^ArcCosh[c*x]] + (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*
PolyLog[3, I/E^ArcCosh[c*x]] + 4*Sinh[ArcCosh[c*x]/2]^2 - 26*ArcCosh[c*x]^2
*Sinh[ArcCosh[c*x]/2]^2 - 2*ArcCosh[c*x]*Tanh[ArcCosh[c*x]/2] - ArcCosh[c*x
]^2*Tanh[ArcCosh[c*x]/2]^2))/(12*d^3*(-1 + c^2*x^2))
```

Maple [F] time = 0.449, size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{arccosh}\,(cx))^2}{x^3} \left(-c^2dx^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^6d^3x^9-3\,c^4d^3x^7+3\,c^2d^3x^5-d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

3.223 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=562

$$-\frac{8b^2c^3\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{8b^2c^3\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{16c^4x\left(a+b\cosh\right.}{3d^2\sqrt{d-c^2\ldots}}$$

```
[Out] (b^2*c^2)/(3*d^2*x*Sqrt[d - c^2*d*x^2]) - (2*b^2*c^4*x)/(3*d^2*Sqrt[d - c^2
*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d^2*x
^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCosh[c*x])^2/(3*d*x^3*(d
- c^2*d*x^2)^(3/2)) - (2*c^2*(a + b*ArcCosh[c*x])^2)/(d*x*(d - c^2*d*x^2)^(
3/2)) + (8*c^4*x*(a + b*ArcCosh[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (16*
c^4*x*(a + b*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (16*c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) -
 (32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*A
rcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2]) - (32*b*c^3*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d -
c^2*d*x^2]) - (8*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^(2*ArcC
osh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2]) - (8*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.82139, antiderivative size = 607, normalized size of antiderivative = 1.08, number of steps used = 34, number of rules used = 18, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.621, Rules used = {5798, 5748, 5691, 5688, 5715, 3716, 2190, 2279, 2391, 5716, 39, 5754, 5721, 5461, 4182, 5746, 103, 12}

$$-\frac{8b^2c^3\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{8b^2c^3\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{16c^4x\left(a+b\cosh\right.}{3d^2\sqrt{d-c^2\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2)/(3*d^2*x*Sqrt[d - c^2*d*x^2]) - (2*b^2*c^4*x)/(3*d^2*Sqrt[d - c^2
*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d^2*x
^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (16*c^4*x*(a + b*ArcCosh[c*x])^2)/(
3*d^2*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCosh[c*x])^2/(3*d^2*x^3*(1 - c*x)*(1
```

```
 + c*x)*Sqrt[d - c^2*d*x^2]) - (2*c^2*(a + b*ArcCosh[c*x])^2)/(d^2*x*(1 - c
*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x])^2)/(3*d^
2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (16*c^3*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(a + b*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) - (32*b*c^3*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/
(3*d^2*Sqrt[d - c^2*d*x^2]) - (32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2]) - (
8*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*
d^2*Sqrt[d - c^2*d*x^2]) - (8*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[
2, E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
```

```
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
IntegerQ[p]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(-1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{\left(2c^2\sqrt{-1+\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\right.}{d^2x(1-cx)(1}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{8bc^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\mathrm{c}\right.}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} + \frac{8b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{16c^4x\left(a+b\right.}{3d^2\sqrt{d}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{16c^4x\left(a+b\right.}{3d^2\sqrt{d}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{16c^4x\left(a+b\right.}{3d^2\sqrt{d}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{16c^4x\left(a+b\right.}{3d^2\sqrt{d}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{16c^4x\left(a+b\right.}{3d^2\sqrt{d}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{16c^4x\left(a+b\right.}{3d^2\sqrt{d}}$$

Mathematica [A] time = 3.68602, size = 534, normalized size = 0.95

$$b^2c^3\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(8\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+8\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)+\frac{\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)^2}{c^3x^3}+\frac{\cosh^{-1}(cx)}{1-c^2x^2}+\frac{\cosh^{-1}}{c^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((a^2*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6))/(x^3*(-1 + c^2*x^2)) + a*b
*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(1/(c^2*x^2) + (1 - c^2*x^2)^(-1)
 + (2*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6
)*ArcCosh[c*x])/(c^3*x^3*(-1 + c*x)^3) - 16*Log[c*x] - 16*Log[Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)]) + b^2*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((
c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1
+ c*x))/(c*x) + ArcCosh[c*x]/(c^2*x^2) + ArcCosh[c*x]/(1 - c^2*x^2) - 16*Ar
cCosh[c*x]^2 - (c*x*ArcCosh[c*x]^2)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)
^3) + (8*c*x*ArcCosh[c*x]^2)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/(c^3*x^3) + (8*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/(c*x) - 16*ArcCosh[c*x]*Log[1 - E
^(-2*ArcCosh[c*x])] - 16*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] + 8*Poly
Log[2, -E^(-2*ArcCosh[c*x])] + 8*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*d^2*S
qrt[d - c^2*d*x^2])
```

Maple [B] time = 0.392, size = 5251, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^6d^3x^{10}-3\,c^4d^3x^8+3\,c^2d^3x^6-d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**4/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x^4), x)
```

3.224 $\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=429

$$-\frac{8\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{15ac^3\sqrt{c-a^2cx^2}}-\frac{x}{3c^3\sqrt{c-a^2cx^2}}-\frac{x}{30c^3(1-ax)(ax+1)\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)^2}{15c^3\sqrt{c-a^2cx^2}}+\frac{8}{}$$

```
[Out] -x/(3*c^3*Sqrt[c - a^2*c*x^2]) - x/(30*c^3*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2
*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(10*a*c^3*(1 - a^2*x
^2)^2*Sqrt[c - a^2*c*x^2]) + (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/
(15*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh[a*x]^2)/(5*c*(c -
 a^2*c*x^2)^(5/2)) + (4*x*ArcCosh[a*x]^2)/(15*c^2*(c - a^2*c*x^2)^(3/2)) +
(8*x*ArcCosh[a*x]^2)/(15*c^3*Sqrt[c - a^2*c*x^2]) + (8*Sqrt[-1 + a*x]*Sqrt[
1 + a*x]*ArcCosh[a*x]^2)/(15*a*c^3*Sqrt[c - a^2*c*x^2]) - (16*Sqrt[-1 + a*x
]*Sqrt[1 + a*x]*ArcCosh[a*x]*Log[1 - E^(2*ArcCosh[a*x])])/(15*a*c^3*Sqrt[c
- a^2*c*x^2]) - (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, E^(2*ArcCosh[a*x
])])/(15*a*c^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.671582, antiderivative size = 459, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5713, 5691, 5688, 5715, 3716, 2190, 2279, 2391, 5716, 39, 40}

$$-\frac{8\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{15ac^3\sqrt{c-a^2cx^2}}-\frac{x}{3c^3\sqrt{c-a^2cx^2}}-\frac{x}{30c^3(1-ax)(ax+1)\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)^2}{15c^3\sqrt{c-a^2cx^2}}+\frac{8}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -x/(3*c^3*Sqrt[c - a^2*c*x^2]) - x/(30*c^3*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2
*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(10*a*c^3*(1 - a^2*x
^2)^2*Sqrt[c - a^2*c*x^2]) + (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/
(15*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]) + (8*x*ArcCosh[a*x]^2)/(15*c^3
*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh[a*x]^2)/(5*c^3*(1 - a*x)^2*(1 + a*x)^2*S
qrt[c - a^2*c*x^2]) + (4*x*ArcCosh[a*x]^2)/(15*c^3*(1 - a*x)*(1 + a*x)*Sqrt
[c - a^2*c*x^2]) + (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(15*a*c^
3*Sqrt[c - a^2*c*x^2]) - (16*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*Log[
1 - E^(2*ArcCosh[a*x])])/(15*a*c^3*Sqrt[c - a^2*c*x^2]) - (8*Sqrt[-1 + a*x]
```

```
*Sqrt[1 + a*x]*PolyLog[2, E^(2*ArcCosh[a*x])])/(15*a*c^3*Sqrt[c - a^2*c*x^2
])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
```

```
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{(-1+ax)^{7/2}(1+ax)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^2}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{\left(4\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{5c^3\sqrt{c-a^2cx^2}} - \frac{\left(2a\sqrt{-1+ax}\sqrt{1\right.}{5c^3\sqrt{}}$$

$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^2}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{4x\cosh^{-1}(ax)^2}{15c^3(1-ax)(1+ax)\sqrt{c-a^2c}}$$

$$= -\frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(a}{15ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4\sqrt{-1+ax}}{15ac^3\left(1-\right.}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4\sqrt{-1+ax}}{15ac^3\left(1-\right.}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4\sqrt{-1+ax}}{15ac^3\left(1-\right.}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4\sqrt{-1+ax}}{15ac^3\left(1-\right.}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4\sqrt{-1+ax}}{15ac^3\left(1-\right.}$$

Mathematica [A] time = 1.41474, size = 220, normalized size = 0.51

$$-\frac{-16\sqrt{\frac{ax-1}{ax+1}}(ax+1)\text{PolyLog}\left(2,e^{-2\cosh^{-1}(ax)}\right)+ax\left(\frac{1}{1-a^2x^2}+10\right)+2\left(ax\left(\frac{4}{a^2x^2-1}-\frac{3}{\left(a^2x^2-1\right)^2}+8\sqrt{\frac{ax-1}{ax+1}}-8\right)+8\sqrt{\frac{ax-1}{ax+1}}\right)\text{co}}{30ac^3\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(a*x*(10 + (1 - a^2*x^2)^(-1)) + 2*(8*Sqrt[(-1 + a*x)/(1 + a*x)] + a*x*(-8
 + 8*Sqrt[(-1 + a*x)/(1 + a*x)] - 3/(-1 + a^2*x^2)^2 + 4/(-1 + a^2*x^2)))*A
rcCosh[a*x]^2 + (((-1 + a*x)/(1 + a*x))^(3/2)*ArcCosh[a*x]*(-11 + 8*a^2*x^2
 + 32*(-1 + a^2*x^2)^2*Log[1 - E^(-2*ArcCosh[a*x])]))/(-1 + a*x)^3 - 16*Sqr
t[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*PolyLog[2, E^(-2*ArcCosh[a*x])])/(30*a*c^
3*Sqrt[c - a^2*c*x^2])
```

Maple [B] time = 0.255, size = 794, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/30*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-20*x^3*a^3-8*(a*x+1)^(1/2)*(a*x-1)^
(1/2)*x^4*a^4+15*a*x+16*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-8*(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(-64*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)*x^7*a^7-64*ar
ccosh(a*x)*x^8*a^8-32*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^7*a^7-32*x^8*a^8+248*ar
ccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^5*x^5+280*arccosh(a*x)*x^6*a^6+126
*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^5*a^5+142*x^6*a^6+80*arccosh(a*x)^2*x^4*a^4-
340*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x^3-456*arccosh(a*x)*x^4*a
^4-156*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)-265*x^4*a^4-190*arccosh(a*x)^2*a
^2*x^2+165*arccosh(a*x)*a*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)+328*a^2*x^2*arccosh
(a*x)+62*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+235*a^2*x^2+128*arccosh(a*x)^2-88*
arccosh(a*x)-80)/(40*a^10*x^10-215*a^8*x^8+469*a^6*x^6-517*a^4*x^4+287*a^2*
x^2-64)/a/c^4-16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/
a/(a^2*x^2-1)*arccosh(a*x)^2+16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2
-1))^(1/2)/c^4/a/(a^2*x^2-1)*arccosh(a*x)*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1
/2))+16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^
2-1)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+16/15*(a*x+1)^(1/2)*(a*x-1)
^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^2-1)*arccosh(a*x)*ln(1+a*x+(a*x-
1)^(1/2)*(a*x+1)^(1/2))+16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^
(1/2)/c^4/a/(a^2*x^2-1)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^2}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^2/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\text{arcosh}\,(ax)^2}{a^8c^4x^8-4\,a^6c^4x^6+6\,a^4c^4x^4-4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^2/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 +
 6*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcosh}\,(ax)^2}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/(-a^2*c*x^2 + c)^(7/2), x)
```

3.225 $\int \frac{x^4 \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=243

$$-\frac{x^3\sqrt{1-ax}\sqrt{ax+1}}{32a^2}-\frac{x^3\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}{4a^2}-\frac{3x^2\sqrt{ax-1}\cosh^{-1}(ax)}{8a^3\sqrt{1-ax}}-\frac{3x\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}{8a^4}-\frac{15x\sqrt{1-ax}\sqrt{a}}{64a^4}$$

```
[Out] (-15*x*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(64*a^4) - (x^3*Sqrt[1 - a*x]*Sqrt[1 +
a*x])/(32*a^2) + (15*Sqrt[-1 + a*x]*ArcCosh[a*x])/(64*a^5*Sqrt[1 - a*x]) -
(3*x^2*Sqrt[-1 + a*x]*ArcCosh[a*x])/(8*a^3*Sqrt[1 - a*x]) - (x^4*Sqrt[-1 +
a*x]*ArcCosh[a*x])/(8*a*Sqrt[1 - a*x]) - (3*x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x
]^2)/(8*a^4) - (x^3*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/(4*a^2) + (Sqrt[-1 +
a*x]*ArcCosh[a*x]^3)/(8*a^5*Sqrt[1 - a*x])
```

Rubi [A] time = 0.792808, antiderivative size = 329, normalized size of antiderivative = 1.35, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5759, 5676, 5662, 90, 52, 100, 12}

$$-\frac{x^3(1-ax)(ax+1)}{32a^2\sqrt{1-a^2x^2}}-\frac{15x(1-ax)(ax+1)}{64a^4\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{8a\sqrt{1-a^2x^2}}-\frac{x^3(1-ax)(ax+1)\cosh^{-1}(ax)^2}{4a^2\sqrt{1-a^2x^2}}-3x^2\sqrt{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-15*x*(1 - a*x)*(1 + a*x))/(64*a^4*Sqrt[1 - a^2*x^2]) - (x^3*(1 - a*x)*(1
+ a*x))/(32*a^2*Sqrt[1 - a^2*x^2]) + (15*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCo
sh[a*x])/(64*a^5*Sqrt[1 - a^2*x^2]) - (3*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x])/(8*a^3*Sqrt[1 - a^2*x^2]) - (x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*
ArcCosh[a*x])/(8*a*Sqrt[1 - a^2*x^2]) - (3*x*(1 - a*x)*(1 + a*x)*ArcCosh[a*
x]^2)/(8*a^4*Sqrt[1 - a^2*x^2]) - (x^3*(1 - a*x)*(1 + a*x)*ArcCosh[a*x]^2)/
(4*a^2*Sqrt[1 - a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(
8*a^5*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{x^4\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^4\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3(1-ax)(1+ax)\cosh^{-1}(ax)^2}{4a^2\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+ax}\sqrt{1+ax\right.}}{2a\sqrt{1}}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{8a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)\cosh^{-1}}{4a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{8a^3\sqrt{1-a^2x^2}} - \frac{x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{8a\sqrt{1-a^2x^2}} - 3x$$

$$= -\frac{3x(1-ax)(1+ax)}{16a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{8a^3\sqrt{1-a^2x^2}} - \frac{x^4\sqrt{-1+ax}\sqrt{1}}{8a\sqrt{1}}$$

$$= -\frac{15x(1-ax)(1+ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} + \frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{16a^5\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\sqrt{1}}{8a^3\sqrt{1}}$$

$$= -\frac{15x(1-ax)(1+ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} + \frac{15\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{64a^5\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\sqrt{1}}{8a^3\sqrt{1}}$$

Mathematica [A] time = 0.266279, size = 116, normalized size = 0.48

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(32\cosh^{-1}(ax)^3-4\left(16\cosh\left(2\cosh^{-1}(ax)\right)+\cosh\left(4\cosh^{-1}(ax)\right)\right)\cosh^{-1}(ax)+8\cosh^{-1}(ax)^2\left(8\sinh\left(\right.\right.}{256a^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(32*ArcCosh[a*x]^3 - 4*ArcCosh[a*x]*(
16*Cosh[2*ArcCosh[a*x]] + Cosh[4*ArcCosh[a*x]]) + 32*Sinh[2*ArcCosh[a*x]] +
 Sinh[4*ArcCosh[a*x]] + 8*ArcCosh[a*x]^2*(8*Sinh[2*ArcCosh[a*x]] + Sinh[4*A
rcCosh[a*x]])))/(256*a^5*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.28, size = 488, normalized size = 2.

$$-\frac{(\operatorname{arccosh}(ax))^3}{8a^5(a^2x^2-1)}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}-\frac{8(\operatorname{arccosh}(ax))^2-4\operatorname{arccosh}(ax)+1}{512a^5(a^2x^2-1)}\sqrt{-a^2x^2+1}\left(8x^5a^5-12x^3a^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arccosh
(a*x)^3-1/512*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5-12*x^3*a^3+8*(a*x+1)^(1/2)*(a*x
-1)^(1/2)*x^4*a^4+4*a*x-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(8*arccosh(a*x)^2-4*arccosh(a*x)+1)/a^5/(a^2*x^2-1)-1/16*(-
a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-(a*
x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2-2*arccosh(a*x)+1)/a^5/(a^2*x^2-
1)-1/16*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x
^2*a^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2+2*arccosh(a*x)+1)/a^5
/(a^2*x^2-1)-1/512*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5-12*x^3*a^3-8*(a*x+1)^(1/2)
*(a*x-1)^(1/2)*x^4*a^4+4*a*x+8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-(a*x-1)^
(1/2)*(a*x+1)^(1/2))*(8*arccosh(a*x)^2+4*arccosh(a*x)+1)/a^5/(a^2*x^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^4\operatorname{arcosh}\left(ax\right)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\operatorname{acosh}^2\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\operatorname{arcosh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

3.226 $\int \frac{x^3 \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=177

$$-\frac{2x^2\sqrt{1-ax}\sqrt{ax+1}}{27a^2}-\frac{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}{3a^2}-\frac{2\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}{3a^4}-\frac{40\sqrt{1-ax}\sqrt{ax+1}}{27a^4}-\frac{4x\sqrt{ax-1}\cosh^{-1}}{3a^3\sqrt{1-ax}}$$

```
[Out] (-40*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(27*a^4) - (2*x^2*Sqrt[1 - a*x]*Sqrt[1 +
a*x])/(27*a^2) - (4*x*Sqrt[-1 + a*x]*ArcCosh[a*x])/(3*a^3*Sqrt[1 - a*x]) -
(2*x^3*Sqrt[-1 + a*x]*ArcCosh[a*x])/(9*a*Sqrt[1 - a*x]) - (2*Sqrt[1 - a^2*x
^2]*ArcCosh[a*x]^2)/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/(3*a^2
)
```

Rubi [A] time = 0.591863, antiderivative size = 237, normalized size of antiderivative = 1.34, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5759, 5718, 5654, 74, 5662, 100, 12}

$$-\frac{2x^2(1-ax)(ax+1)}{27a^2\sqrt{1-a^2x^2}}-\frac{40(1-ax)(ax+1)}{27a^4\sqrt{1-a^2x^2}}-\frac{2x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{9a\sqrt{1-a^2x^2}}-\frac{x^2(1-ax)(ax+1)\cosh^{-1}(ax)^2}{3a^2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-40*(1 - a*x)*(1 + a*x))/(27*a^4*Sqrt[1 - a^2*x^2]) - (2*x^2*(1 - a*x)*(1
+ a*x))/(27*a^2*Sqrt[1 - a^2*x^2]) - (4*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcC
osh[a*x])/(3*a^3*Sqrt[1 - a^2*x^2]) - (2*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x])/(9*a*Sqrt[1 - a^2*x^2]) - (2*(1 - a*x)*(1 + a*x)*ArcCosh[a*x]^
2)/(3*a^4*Sqrt[1 - a^2*x^2]) - (x^2*(1 - a*x)*(1 + a*x)*ArcCosh[a*x]^2)/(3*
a^2*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
```

```
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{x^3\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^3\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)^2}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{3a^2\sqrt{1-a^2x^2}} - \frac{\left(2\sqrt{-1+ax}\sqrt{1+\right.}{3a}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{9a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)^2}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-ax)(1+ax)\cosh}{3a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{2x^2(1-ax)(1+ax)}{27a^2\sqrt{1-a^2x^2}} - \frac{4x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3a^3\sqrt{1-a^2x^2}} - \frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{9a\sqrt{1-a^2x^2}} -$$

$$= -\frac{4(1-ax)(1+ax)}{3a^4\sqrt{1-a^2x^2}} - \frac{2x^2(1-ax)(1+ax)}{27a^2\sqrt{1-a^2x^2}} - \frac{4x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3a^3\sqrt{1-a^2x^2}} - \frac{2x^3\sqrt{-1+ax}\sqrt{}}{9a\sqrt{1}}$$

$$= -\frac{40(1-ax)(1+ax)}{27a^4\sqrt{1-a^2x^2}} - \frac{2x^2(1-ax)(1+ax)}{27a^2\sqrt{1-a^2x^2}} - \frac{4x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3a^3\sqrt{1-a^2x^2}} - \frac{2x^3\sqrt{-1+ax}}{9a}$$

Mathematica [A] time = 0.150591, size = 123, normalized size = 0.69

$$\left(-\frac{2x^2}{27a^2}-\frac{40}{27a^4}\right)\sqrt{1-a^2x^2} - \frac{\sqrt{1-a^2x^2}\left(a^2x^2+2\right)\cosh^{-1}(ax)^2}{3a^4} + \frac{2x\sqrt{1-a^2x^2}\left(a^2x^2+6\right)\cosh^{-1}(ax)}{9a^3\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-40/(27*a^4) - (2*x^2)/(27*a^2))*Sqrt[1 - a^2*x^2] + (2*x*Sqrt[1 - a^2*x^2
]*(6 + a^2*x^2)*ArcCosh[a*x])/(9*a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (Sqrt[
1 - a^2*x^2]*(2 + a^2*x^2)*ArcCosh[a*x]^2)/(3*a^4)
```

Maple [B] time = 0.192, size = 343, normalized size = 1.9

$$-\frac{9\left(\operatorname{arccosh}(ax)\right)^2-6\operatorname{arccosh}(ax)+2}{216\,a^4\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\left(4\,x^4a^4-5\,a^2x^2+4\,a^3x^3\sqrt{ax-1}\sqrt{ax+1}-3\,\sqrt{ax+1}\sqrt{ax-1}ax+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/216*(-a^2*x^2+1)^(1/2)*(4*x^4*a^4-5*a^2*x^2+4*a^3*x^3*(a*x-1)^(1/2)*(a*x
+1)^(1/2)-3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(9*arccosh(a*x)^2-6*arccosh(
a*x)+2)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)
*a*x+a^2*x^2-1)*(arccosh(a*x)^2-2*arccosh(a*x)+2)/a^4/(a^2*x^2-1)-3/8*(-a^2
*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*(arccosh(a*x)^2+2
*arccosh(a*x)+2)/a^4/(a^2*x^2-1)-1/216*(-a^2*x^2+1)^(1/2)*(4*x^4*a^4-5*a^2*
x^2-4*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x
+1)*(9*arccosh(a*x)^2+6*arccosh(a*x)+2)/a^4/(a^2*x^2-1)
```

Maxima [C] time = 1.74646, size = 142, normalized size = 0.8

$$-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\operatorname{arcosh}(ax)^2+\frac{2\left(-i\sqrt{a^2x^2-1}x^2-\frac{20i\sqrt{a^2x^2-1}}{a^2}\right)}{27\,a^2}+\frac{2\left(i\,a^2x^3+6i\,x\right)\operatorname{arcosh}(ax)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arccosh(a*x)^2
 + 2/27*(-I*sqrt(a^2*x^2 - 1)*x^2 - 20*I*sqrt(a^2*x^2 - 1)/a^2)/a^2 + 2/9*(
I*a^2*x^3 + 6*I*x)*arccosh(a*x)/a^3
```

Fricas [A] time = 2.20258, size = 324, normalized size = 1.83

$$-\frac{9\left(a^4x^4+a^2x^2-2\right)\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2-6\left(a^3x^3+6\,ax\right)\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)+}{27\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*(9*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
- 1))^2 - 6*(a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 - 1)) + 2*(a^4*x^4 + 19*a^2*x^2 - 20)*sqrt(-a^2*x^2 + 1))/(a
^6*x^2 - a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\operatorname{acosh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [C] time = 1.19231, size = 161, normalized size = 0.91

$$\frac{\left(\left(-a^2x^2+1\right)^{\frac{3}{2}}-3\sqrt{-a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{3\,a^4}+\frac{3\left(-2i\,a^2x^3-12i\,x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{-2i\left(a^2x^2-1\right)^{\frac{3}{2}}-42i\sqrt{a^2}}{a}}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 -
1))^2/a^4 + 1/27*(3*(-2*I*a^2*x^3 - 12*I*x)*log(a*x + sqrt(a^2*x^2 - 1)) -
(-2*I*(a^2*x^2 - 1)^(3/2) - 42*I*sqrt(a^2*x^2 - 1))/a)/a^3
```

3.227 $\int \frac{x^2 \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=151

$$-\frac{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}{2a^2}-\frac{x\sqrt{1-ax}\sqrt{ax+1}}{4a^2}+\frac{\sqrt{ax-1}\cosh^{-1}(ax)^3}{6a^3\sqrt{1-ax}}+\frac{\sqrt{ax-1}\cosh^{-1}(ax)}{4a^3\sqrt{1-ax}}-\frac{x^2\sqrt{ax-1}\cosh^{-1}(ax)}{2a\sqrt{1-ax}}$$

```
[Out] -(x*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(4*a^2) + (Sqrt[-1 + a*x]*ArcCosh[a*x])/(4
*a^3*Sqrt[1 - a*x]) - (x^2*Sqrt[-1 + a*x]*ArcCosh[a*x])/(2*a*Sqrt[1 - a*x])
 - (x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/(2*a^2) + (Sqrt[-1 + a*x]*ArcCosh[a
*x]^3)/(6*a^3*Sqrt[1 - a*x])
```

Rubi [A] time = 0.510342, antiderivative size = 207, normalized size of antiderivative = 1.37, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5798, 5759, 5676, 5662, 90, 52}

$$-\frac{x(1-ax)(ax+1)}{4a^2\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{6a^3\sqrt{1-a^2x^2}}-\frac{x(1-ax)(ax+1)\cosh^{-1}(ax)^2}{2a^2\sqrt{1-a^2x^2}}-\frac{x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{2a\sqrt{1-a^2x^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x*(1 - a*x)*(1 + a*x))/(4*a^2*Sqrt[1 - a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*ArcCosh[a*x])/(4*a^3*Sqrt[1 - a^2*x^2]) - (x^2*Sqrt[-1 + a*x]*Sqrt[
1 + a*x]*ArcCosh[a*x])/(2*a*Sqrt[1 - a^2*x^2]) - (x*(1 - a*x)*(1 + a*x)*Arc
Cosh[a*x]^2)/(2*a^2*Sqrt[1 - a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcC
osh[a*x]^3)/(6*a^3*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^2\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2a^2\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int}{a\sqrt{1-a}}$$

$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(a}{6a^3\sqrt{1-a^2x^2}}$$

$$= -\frac{x(1-ax)(1+ax)}{4a^2\sqrt{1-a^2x^2}} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+}}{}$$

$$= -\frac{x(1-ax)(1+ax)}{4a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{4a^3\sqrt{1-a^2x^2}} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a\sqrt{1-a^2x^2}} - \frac{x(1-a}{}$$

Mathematica [A] time = 0.170177, size = 87, normalized size = 0.58

$$-\frac{\sqrt{-(ax-1)(ax+1)}\left(4\cosh^{-1}(ax)^3-6\cosh\left(2\cosh^{-1}(ax)\right)\cosh^{-1}(ax)+\left(6\cosh^{-1}(ax)^2+3\right)\sinh\left(2\cosh^{-1}(ax)\right)\right)}{24a^3\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(Sqrt[-((-1 + a*x)*(1 + a*x))]*(4*ArcCosh[a*x]^3 - 6*ArcCosh[a*x]*Cosh[2*A
rcCosh[a*x]] + (3 + 6*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]]))/(24*a^3*Sqrt[(
-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [A] time = 0.162, size = 239, normalized size = 1.6

$$-\frac{(\mathrm{arccosh}\,(ax))^3}{6\,a^3\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}-\frac{2\,(\mathrm{arccosh}\,(ax))^2-2\,\mathrm{arccosh}\,(ax)+1}{16\,a^3\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\left(2\,x^3a^3-2\,ax+2\,\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/6*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccosh
(a*x)^3-1/16*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1
/2)*x^2*a^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2-2*arccosh(a*x)+1
)/a^3/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x-2*(a*x+1)^(1/2)*
(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2+2*arcc
osh(a*x)+1)/a^3/(a^2*x^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^2\,\text{arcosh}\,(ax)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\,\text{acosh}^2\,(ax)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

3.228 $\int \frac{x \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}{a^2}-\frac{2\sqrt{1-ax}\sqrt{ax+1}}{a^2}-\frac{2x\sqrt{ax-1}\cosh^{-1}(ax)}{a\sqrt{1-ax}}$$

```
[Out] (-2*Sqrt[1 - a*x]*Sqrt[1 + a*x])/a^2 - (2*x*Sqrt[-1 + a*x]*ArcCosh[a*x])/(a
*Sqrt[1 - a*x]) - (Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/a^2
```

Rubi [A] time = 0.271709, antiderivative size = 109, normalized size of antiderivative = 1.38, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5798, 5718, 5654, 74}

$$-\frac{2(1-ax)(ax+1)}{a^2\sqrt{1-a^2x^2}}-\frac{(1-ax)(ax+1)\cosh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*(1 - a*x)*(1 + a*x))/(a^2*Sqrt[1 - a^2*x^2]) - (2*x*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*ArcCosh[a*x])/(a*Sqrt[1 - a^2*x^2]) - ((1 - a*x)*(1 + a*x)*ArcCos
h[a*x]^2)/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
```

```
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{x\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{\left(2\sqrt{-1+ax}\sqrt{1+ax}\right)\int \cosh^{-1}(ax)\,dx}{a\sqrt{1-a^2x^2}}$$

$$= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cdots}{\sqrt{-1+\cdots}}}{\sqrt{1-a^2x^2}}$$

$$= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{1-a^2x^2}} - \frac{2x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0943196, size = 54, normalized size = 0.68

$$\frac{\sqrt{1-a^2x^2}\left(-\cosh^{-1}(ax)^2 + \frac{2ax\cosh^{-1}(ax)}{\sqrt{ax-1}\sqrt{ax+1}} - 2\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2 + (2*a*x*ArcCosh[a*x])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]
) - ArcCosh[a*x]^2))/a^2
```

Maple [A] time = 0.13, size = 139, normalized size = 1.8

$$-\frac{(\operatorname{arccosh}(ax))^2-2\operatorname{arccosh}(ax)+2}{2a^2(a^2x^2-1)}\sqrt{-a^2x^2+1}\left(\sqrt{ax+1}\sqrt{ax-1}ax+a^2x^2-1\right)-\frac{(\operatorname{arccosh}(ax))^2+2\operatorname{arccosh}(ax)}{2a^2(a^2x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(arccos
h(a*x)^2-2*arccosh(a*x)+2)/a^2/(a^2*x^2-1)-1/2*(-a^2*x^2+1)^(1/2)*(a^2*x^2-
(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*(arccosh(a*x)^2+2*arccosh(a*x)+2)/a^2/(a
^2*x^2-1)
```

Maxima [C] time = 1.10081, size = 68, normalized size = 0.86

$$\frac{2i\,x\operatorname{arcosh}(ax)}{a}-\frac{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)^2}{a^2}-\frac{2i\sqrt{a^2x^2-1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*x*arccosh(a*x)/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/a^2 - 2*I*sqrt(a^2
*x^2 - 1)/a^2
```

Fricas [A] time = 2.14564, size = 246, normalized size = 3.11

$$\frac{2\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2-1}\right)+\left(-a^2x^2+1\right)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)^2-2\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}}{a^4x^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1)) +
(-a^2*x^2 + 1)^(3/2)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 2*(a^2*x^2 - 1)*sqrt(
-a^2*x^2 + 1))/(a^4*x^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{acosh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [C] time = 1.1736, size = 103, normalized size = 1.3

$$-\frac{\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{a^2} - \frac{2i\left(x\log\left(ax+\sqrt{a^2x^2-1}\right) - \frac{\sqrt{a^2x^2-1}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))^2/a^2 - 2*I*(x*log(a*x + s
qrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1)/a)/a
```

3.229 $\int \frac{\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{ax-1}\cosh^{-1}(ax)^3}{3a\sqrt{1-ax}}$$

```
[Out] (Sqrt[-1 + a*x]*ArcCosh[a*x]^3)/(3*a*Sqrt[1 - a*x])
```

Rubi [A] time = 0.149932, antiderivative size = 45, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5713, 5676}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(3*a*Sqrt[1 - a^2*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0243579, size = 45, normalized size = 1.41

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(3*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.038, size = 51, normalized size = 1.6

$$-\frac{(\operatorname{arccosh}(ax))^3}{3\,a\left(a^2x^2-1\right)}\sqrt{-(ax-1)(ax+1)}\sqrt{ax-1}\sqrt{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/3*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*arc
cosh(a*x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acosh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

3.230 $\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=183

$$-\frac{2i\sqrt{ax-1}\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{2i\sqrt{ax-1}\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{2i\sqrt{ax-1}\mathrm{PolyLog}}{\sqrt{1-}}$$

```
[Out] (2*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a*x] - ((
2*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 -
a*x] + ((2*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]])/Sqr
t[1 - a*x] + ((2*I)*Sqrt[-1 + a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]])/Sqrt[1
- a*x] - ((2*I)*Sqrt[-1 + a*x]*PolyLog[3, I*E^ArcCosh[a*x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.420395, antiderivative size = 248, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5798, 5761, 4180, 2531, 2282, 6589}

$$-\frac{2i\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{2i\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{2i}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]])/Sqrt
[1 - a^2*x^2] - ((2*I)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2,
 (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((2*I)*Sqrt[-1 + a*x]*Sqrt[1 + a
*x]*ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((2*I)*S
qrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x
^2] - ((2*I)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, I*E^ArcCosh[a*x]])/Sqr
t[1 - a^2*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int x^2\mathbf{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(2i\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int x\log\left(1-ie^x\right)\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{2i\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\mathbf{Li}_2\left(-ie^{\cosh^{-1}(}\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{2i\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\mathbf{Li}_2\left(-ie^{\cosh^{-1}(}\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{2i\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\mathbf{Li}_2\left(-ie^{\cosh^{-1}(}\right.}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.18457, size = 151, normalized size = 0.83

$$\frac{i\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(-2\cosh^{-1}(ax)\left(\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(ax)}\right)\right)-2\text{PolyLog}\left(3,-ie^{-\cosh^{-1}(ax)}\right)\right.}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (I*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-(ArcCosh[a*x]^2*(Log[1 - I/E^ArcC
osh[a*x]] - Log[1 + I/E^ArcCosh[a*x]])) - 2*ArcCosh[a*x]*(PolyLog[2, (-I)/E
^ArcCosh[a*x]] - PolyLog[2, I/E^ArcCosh[a*x]]) - 2*PolyLog[3, (-I)/E^ArcCos
h[a*x]] + 2*PolyLog[3, I/E^ArcCosh[a*x]]))/Sqrt[1 - a^2*x^2]
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{(\text{arccosh}(ax))^2}{x}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)^2}{a^2x^3-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^2(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**2/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.231 $\int \frac{\cosh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=124

$$-\frac{a\sqrt{ax-1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}{x}+\frac{a\sqrt{ax-1}\cosh^{-1}(ax)^2}{\sqrt{1-ax}}-\frac{2a\sqrt{ax-1}\cosh^{-1}(ax)\log\left(\right.}{\sqrt{1-ax}}$$

```
[Out] (a*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/Sqrt[1 - a*x] - (Sqrt[1 - a^2*x^2]*ArcCos
h[a*x]^2)/x - (2*a*Sqrt[-1 + a*x]*ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[a*x])])
/Sqrt[1 - a*x] - (a*Sqrt[-1 + a*x]*PolyLog[2, -E^(2*ArcCosh[a*x])])/Sqrt[1
- a*x]
```

Rubi [A] time = 0.444493, antiderivative size = 174, normalized size of antiderivative = 1.4, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5798, 5724, 5660, 3718, 2190, 2279, 2391}

$$-\frac{a\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}-\frac{(1-ax)(ax+1)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}+\frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2a\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2] - ((1 - a
*x)*(1 + a*x)*ArcCosh[a*x]^2)/(x*Sqrt[1 - a^2*x^2]) - (2*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[a*x])])/Sqrt[1 - a^2*x^2] -
(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, -E^(2*ArcCosh[a*x])])/Sqrt[1 - a
^2*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{\left(2a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{\left(2a\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{\left(4a\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\Big(}{\sqrt{1-a^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.450159, size = 111, normalized size = 0.9

$$\frac{a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax)}\right)+\cosh^{-1}(ax)\left(\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\cosh^{-1}(ax)}{ax}-\cosh^{-1}(ax)-2\log\left(e^{-2\cosh^{-1}(ax)}+1\right.\right.\right.}{\sqrt{-(ax-1)(ax+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(ArcCosh[a*x]*(-ArcCosh[a*x] + (Sqr
t[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a*x) - 2*Log[1 + E^(-2*Arc
Cosh[a*x])]) + PolyLog[2, -E^(-2*ArcCosh[a*x])]))/Sqrt[-((-1 + a*x)*(1 + a*
x))]
```

Maple [A] time = 0.151, size = 241, normalized size = 1.9

$$-\frac{(\operatorname{arccosh}(ax))^2}{x(a^2x^2-1)}\sqrt{-a^2x^2+1}\left(a^2x^2-\sqrt{ax+1}\sqrt{ax-1}ax-1\right)-2\,\frac{\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}(\operatorname{arccosh}(ax))^2a}{a^2x^2-1}+2\,\frac{\sqrt{-a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*arccosh(a*x
)^2/x/(a^2*x^2-1)-2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2
-1)*arccosh(a*x)^2*a+2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*
x^2-1)*arccosh(a*x)*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a+(-a^2*x^2+1
)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*polylog(2,-(a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2))^2)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(a^2x^2-1)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{\sqrt{ax+1}\sqrt{-ax+1}x}-\int\frac{2\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)}{\left(\sqrt{ax+1}ax^2+(ax+1)\sqrt{ax-1}x\right)\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] (a^2*x^2 - 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(sqrt(a*x + 1)*sqrt(
-a*x + 1)*x) - integrate(2*(a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a
)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/((sqrt(a*x + 1)*a*x^2 + (a*x + 1)*
sqrt(a*x - 1)*x)*sqrt(-a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)^2}{a^2x^4-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^4 - x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^2(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**2/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.232 $\int \frac{\cosh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=296

$$-\frac{ia^2\sqrt{ax-1}\cosh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{ia^2\sqrt{ax-1}\cosh^{-1}(ax)\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{ia^2\sqrt{ax-1}\text{PolyL}}{\sqrt{1}}$$

```
[Out] (a*Sqrt[-1 + a*x]*ArcCosh[a*x])/(x*Sqrt[1 - a*x]) - (Sqrt[1 - a^2*x^2]*ArcC
osh[a*x]^2)/(2*x^2) + (a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a
*x]])/Sqrt[1 - a*x] - (a^2*Sqrt[-1 + a*x]*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*
x]])/Sqrt[1 - a*x] - (I*a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[2, (-I)*E^A
rcCosh[a*x]])/Sqrt[1 - a*x] + (I*a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[2,
 I*E^ArcCosh[a*x]])/Sqrt[1 - a*x] + (I*a^2*Sqrt[-1 + a*x]*PolyLog[3, (-I)*E
^ArcCosh[a*x]])/Sqrt[1 - a*x] - (I*a^2*Sqrt[-1 + a*x]*PolyLog[3, I*E^ArcCos
h[a*x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.722817, antiderivative size = 398, normalized size of antiderivative = 1.34, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5798, 5748, 5761, 4180, 2531, 2282, 6589, 5662, 92, 205}

$$-\frac{ia^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{ia^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(x*Sqrt[1 - a^2*x^2]) - ((1 -
 a*x)*(1 + a*x)*ArcCosh[a*x]^2)/(2*x^2*Sqrt[1 - a^2*x^2]) + (a^2*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2]
 - (a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*x]])/
Sqrt[1 - a^2*x^2] - (I*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLo
g[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + (I*a^2*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + (I*a
^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a
^2*x^2] - (I*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, I*E^ArcCosh[a*x]])
/Sqrt[1 - a^2*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x^3\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x^2}\,dx}{\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\sqrt{1+ax}\right)\int}{2\sqrt{1-a^2x}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int\right.}{2\sqrt{1-}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.981897, size = 233, normalized size = 0.79

$$\frac{ia^2\sqrt{-(ax-1)(ax+1)}\left(2\cosh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(ax)}\right)-2\cosh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\cosh^{-1}(ax)}\right)+2\text{PolyLog}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((I/2)*a^2*Sqrt[-((-1 + a*x)*(1 + a*x))]*(((2*I)*ArcCosh[a*x])/(a*x) + (I*S
qrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]^2)/(a^2*x^2) - (4*I)*ArcTa
n[Tanh[ArcCosh[a*x]/2]] + ArcCosh[a*x]^2*Log[1 - I/E^ArcCosh[a*x]] - ArcCos
h[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] + 2*ArcCosh[a*x]*PolyLog[2, (-I)/E^ArcCo
sh[a*x]] - 2*ArcCosh[a*x]*PolyLog[2, I/E^ArcCosh[a*x]] + 2*PolyLog[3, (-I)/
E^ArcCosh[a*x]] - 2*PolyLog[3, I/E^ArcCosh[a*x]]))/(Sqrt[(-1 + a*x)/(1 + a*
x)]*(1 + a*x))
```

Maple [F] time = 0.163, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{arccosh}\left(ax\right)\right)^{2}}{x^{3}}\frac{1}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}\left(ax\right)^{2}}{\sqrt{-a^{2}x^{2}+1}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^{2}x^{2}+1}\operatorname{arcosh}\left(ax\right)^{2}}{a^{2}x^{5}-x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^5 - x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^2(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**2/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.233 $\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=1153

result too large to display

```
[Out] (-10*b^2*c^2*d^2*(f*x)^(3 + m)*Sqrt[d - c^2*d*x^2])/(f^3*(4 + m)^3*(6 + m))
 - (2*b^2*c^2*d^2*(52 + 15*m + m^2)*(f*x)^(3 + m)*(1 - c^2*x^2)*Sqrt[d - c^
2*d*x^2])/(f^3*(4 + m)^2*(6 + m)^3*(1 - c*x)*(1 + c*x)) + (2*b^2*c^4*d^2*(f
*x)^(5 + m)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(f^5*(6 + m)^3*(1 - c*x)*(1
+ c*x)) - (2*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
)/(f^2*(2 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (30*b*c*d^2*(f*x)^(2
 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^2*(2 + m)^2*(4 + m)*(6 +
 m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (10*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(f^2*(2 + m)*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (10*b*c^3*d^2*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x]))/(f^4*(4 + m)^2*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*c^3*d
^2*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^4*(4 + m)*(6
+ m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^5*d^2*(f*x)^(6 + m)*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^6*(6 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (15*d^2*(f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(f*(6
+ m)*(8 + 6*m + m^2)) + (5*d*(f*x)^(1 + m)*(d - c^2*d*x^2)^(3/2)*(a + b*Arc
Cosh[c*x])^2)/(f*(4 + m)*(6 + m)) + ((f*x)^(1 + m)*(d - c^2*d*x^2)^(5/2)*(a
 + b*ArcCosh[c*x])^2)/(f*(6 + m)) - (30*b^2*c^2*d^2*(f*x)^(3 + m)*Sqrt[1 -
c^2*x^2]*Sqrt[d - c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c
^2*x^2])/(f^3*(2 + m)^2*(3 + m)*(4 + m)*(6 + m)*(1 - c*x)*(1 + c*x)) - (10*
b^2*c^2*d^2*(10 + 3*m)*(f*x)^(3 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*
Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/(f^3*(2 + m)*(3 + m)
*(4 + m)^3*(6 + m)*(1 - c*x)*(1 + c*x)) - (2*b^2*c^2*d^2*(264 + 130*m + 15*
m^2)*(f*x)^(3 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*Hypergeometric2F1[
1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/(f^3*(2 + m)*(3 + m)*(4 + m)^2*(6 + m)
^3*(1 - c*x)*(1 + c*x)) + (15*d^3*Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x]
)^2)/Sqrt[d - c^2*d*x^2], x])/((6 + m)*(8 + 6*m + m^2))
```

Rubi [A] time = 0.544834, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*Defer[Int][(f*x)^m*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2
)*(a + b*ArcCosh[c*x])^2, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (fx)^m(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2 \, dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.63206, size = 0, normalized size = 0.

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2, x]
```

Maple [A] time = 1.329, size = 0, normalized size = 0.

$$\int \left(fx\right)^m \left(-c^2dx^2+d\right)^{\frac{5}{2}} \left(a+b\operatorname{arccosh}(cx)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2+d\right)^{\frac{5}{2}} \left(b\operatorname{arccosh}(cx)+a\right)^2 \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="m
axima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2c^4d^2x^4 - 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 - 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arcosh}\,(cx)^2 + 2\left(abc^4d^2x^4 - 2\,abc^2d^2x^2 + abd\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="f
ricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.234 $\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=583

$$\frac{3d^2\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}},x\right)}{m^2+6m+8}-\frac{2b^2c^2d(3m+10)\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2}\right.}{f^3(m+2)(m+3)(m+4)^3(1-cx)(cx+1)}$$

```
[Out] (-2*b^2*c^2*d*(f*x)^(3 + m)*Sqrt[d - c^2*d*x^2])/(f^3*(4 + m)^3) - (6*b*c*d
*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^2*(2 + m)^2*(4
+ m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(f^2*(2 + m)*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (2*b*c^3*d*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^
4*(4 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*(f*x)^(1 + m)*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])^2)/(f*(8 + 6*m + m^2)) + ((f*x)^(1 + m)*(d - c
^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/(f*(4 + m)) - (6*b^2*c^2*d*(f*x)^(3
 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/
2, (5 + m)/2, c^2*x^2])/(f^3*(2 + m)^2*(3 + m)*(4 + m)*(1 - c*x)*(1 + c*x))
 - (2*b^2*c^2*d*(10 + 3*m)*(f*x)^(3 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x
^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/(f^3*(2 + m)*(3
+ m)*(4 + m)^3*(1 - c*x)*(1 + c*x)) + (3*d^2*Unintegrable[((f*x)^m*(a + b*A
rcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2], x])/(8 + 6*m + m^2)
```

Rubi [A] time = 0.523117, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[d - c^2*d*x^2]*Defer[Int][(f*x)^m*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2
)*(a + b*ArcCosh[c*x])^2, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right) \int (fx)^m(-1+cx)^{3/2}(1+cx)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.513126, size = 0, normalized size = 0.

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2, x]
```

Maple [A] time = 1.082, size = 0, normalized size = 0.

$$\int \left(fx\right)^m \left(-c^2dx^2+d\right)^{\frac{3}{2}} (a+b\mathrm{arccosh}\,(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2+d\right)^{\frac{3}{2}} (b\,\mathrm{arcosh}\,(cx)+a)^2 \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="m
axima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2c^2dx^2 - a^2d + \left(b^2c^2dx^2 - b^2d\right)\operatorname{arcosh}(cx)^2 + 2\left(abc^2dx^2 - abd\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2 + d}\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="f
ricas")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.235 $\int (fx)^m\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=239

$$\frac{d\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}},x\right)}{m+2}-\frac{2b^2c^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(fx)^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{f^3(m+2)^2(m+3)(1-cx)(cx+1)}$$

```
[Out] (-2*b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^2*(2 + m
)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a +
 b*ArcCosh[c*x])^2)/(f*(2 + m)) - (2*b^2*c^2*(f*x)^(3 + m)*Sqrt[1 - c^2*x^2
]*Sqrt[d - c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2]
)/(f^3*(2 + m)^2*(3 + m)*(1 - c*x)*(1 + c*x)) + (d*Unintegrable[((f*x)^m*(a
 + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2], x])/(2 + m)
```

Rubi [A] time = 0.448144, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (fx)^m\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*Defer[Int][(f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])^2, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int (fx)^m\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=\frac{\sqrt{d-c^2dx^2}\int (fx)^m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.33527, size = 0, normalized size = 0.

$$\int (fx)^m\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

Maple [A] time = 1.018, size = 0, normalized size = 0.

$$\int (fx)^{m} \sqrt{-c^2dx^2+d}\,(a+b\operatorname{arccosh}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^2 (fx)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)(fx)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.236 $\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2], x]
```

Rubi [A] time = 0.474311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^2)/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[d - c^2*d*x^2]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 3.43598, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2], x]
```

Maple [A] time = 0.406, size = 0, normalized size = 0.

$$\int (fx)^m (a + b\operatorname{arccosh}(cx))^2 \frac{1}{\sqrt{-c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2 (fx)^m}{\sqrt{-c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2 + d}\left(b^2\operatorname{arcosh}(cx)^2 + 2\,ab\operatorname{arcosh}(cx) + a^2\right)(fx)^m}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b\,\text{acosh}\,(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arcosh}\,(cx) + a)^2 (fx)^m}{\sqrt{-c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

3.237 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [A] time = 0.561244, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^2)
/((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 4.43293, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2), x]
```

Maple [A] time = 0.539, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(a + b\operatorname{arccosh}\left(cx\right)\right)^{2} \left(-c^{2}dx^{2} + d\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arcosh}\left(cx\right) + a\right)^{2} \left(fx\right)^{m}}{\left(-c^{2}dx^{2} + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^{2}dx^{2} + d}\left(b^{2}\operatorname{arcosh}\left(cx\right)^{2} + 2\,ab\operatorname{arcosh}\left(cx\right) + a^{2}\right)\left(fx\right)^{m}}{c^{4}d^{2}x^{4} - 2\,c^{2}d^{2}x^{2} + d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)*(f*x)^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2 (fx)^m}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

3.238 $\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]
```

Rubi [A] time = 0.556631, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^2)/(
(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)), x])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 4.62682, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]
```

Maple [A] time = 0.55, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(a + b\mathrm{arccosh}\left(cx\right)\right)^{2} \left(-c^{2}dx^{2} + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{arcosh}\left(cx\right) + a\right)^{2} \left(fx\right)^{m}}{\left(-c^{2}dx^{2} + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^{2}dx^{2} + d}\left(b^{2} \operatorname{arcosh}\left(cx\right)^{2} + 2\,ab \operatorname{arcosh}\left(cx\right) + a^{2}\right)\left(fx\right)^{m}}{c^{6}d^{3}x^{6} - 3\,c^{4}d^{3}x^{4} + 3\,c^{2}d^{3}x^{2} - d^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)*(f*x)^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2 (fx)^m}{(-c^2dx^2 + d)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

3.239 $\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\cosh^{-1}(cx)^2(fx)^m}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2], x]
```

Rubi [A] time = 0.369915, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*ArcCosh[c*x]^2)/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.738518, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2], x]
```

Maple [A] time = 0.341, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(\operatorname{arccosh}\left(cx\right)\right)^{2} \frac{1}{\sqrt{-c^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^{m} \operatorname{arcosh}\left(cx\right)^{2}}{\sqrt{-c^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x)^m*arccosh(c*x)^2/sqrt(-c^2*x^2 + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^{2}x^{2}+1} \left(fx\right)^{m} \operatorname{arcosh}\left(cx\right)^{2}}{c^{2}x^{2}-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m*arccosh(c*x)^2/(c^2*x^2 - 1), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m \operatorname{acosh}^2(cx)}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*acosh(c*x)**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((f*x)**m*acosh(c*x)**2/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m \operatorname{arcosh}(cx)^2}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*arccosh(c*x)^2/sqrt(-c^2*x^2 + 1), x)
```

3.240 $\int \left(c-a^2cx^2\right)^3 \cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=505

$$\frac{6c^3\left(1-a^2x^2\right)^4}{2401a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2664c^3\left(1-a^2x^2\right)^3}{214375a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1184c^3\left(1-a^2x^2\right)^2}{42875a\sqrt{ax-1}\sqrt{ax+1}}+\frac{7104c^3\left(1-a^2x^2\right)}{42875a\sqrt{ax-1}\sqrt{ax+1}}-\frac{6}{343}a^6c^3x^7\,\mathrm{co}$$

```
[Out] (-976*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(315*a) + (16*a*c^3*x^2*Sqrt[-1 + a
*x]*Sqrt[1 + a*x])/315 + (7104*c^3*(1 - a^2*x^2))/(42875*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (1184*c^3*(1 - a^2*x^2)^2)/(42875*a*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]) + (2664*c^3*(1 - a^2*x^2)^3)/(214375*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
 + (6*c^3*(1 - a^2*x^2)^4)/(2401*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (4322*c^
3*x*ArcCosh[a*x])/1225 - (1514*a^2*c^3*x^3*ArcCosh[a*x])/3675 + (702*a^4*c^
3*x^5*ArcCosh[a*x])/6125 - (6*a^6*c^3*x^7*ArcCosh[a*x])/343 - (48*c^3*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(35*a) + (8*c^3*(-1 + a*x)^(3/2)*(1
 + a*x)^(3/2)*ArcCosh[a*x]^2)/(35*a) - (18*c^3*(-1 + a*x)^(5/2)*(1 + a*x)^(
5/2)*ArcCosh[a*x]^2)/(175*a) + (3*c^3*(-1 + a*x)^(7/2)*(1 + a*x)^(7/2)*ArcC
osh[a*x]^2)/(49*a) + (16*c^3*x*ArcCosh[a*x]^3)/35 + (8*c^3*x*(1 - a^2*x^2)*
ArcCosh[a*x]^3)/35 + (6*c^3*x*(1 - a^2*x^2)^2*ArcCosh[a*x]^3)/35 + (c^3*x*(
1 - a^2*x^2)^3*ArcCosh[a*x]^3)/7
```

Rubi [A] time = 1.40818, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5681, 5718, 194, 5680, 12, 1610, 1799, 1850, 520, 1247, 698, 460, 74, 5654}

$$\frac{6c^3\left(1-a^2x^2\right)^4}{2401a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2664c^3\left(1-a^2x^2\right)^3}{214375a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1184c^3\left(1-a^2x^2\right)^2}{42875a\sqrt{ax-1}\sqrt{ax+1}}+\frac{7104c^3\left(1-a^2x^2\right)}{42875a\sqrt{ax-1}\sqrt{ax+1}}-\frac{6}{343}a^6c^3x^7\,\mathrm{co}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3*ArcCosh[a*x]^3,x]
```


```
[Out] (-976*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(315*a) + (16*a*c^3*x^2*Sqrt[-1 + a
*x]*Sqrt[1 + a*x])/315 + (7104*c^3*(1 - a^2*x^2))/(42875*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (1184*c^3*(1 - a^2*x^2)^2)/(42875*a*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]) + (2664*c^3*(1 - a^2*x^2)^3)/(214375*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
 + (6*c^3*(1 - a^2*x^2)^4)/(2401*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (4322*c^
3*x*ArcCosh[a*x])/1225 - (1514*a^2*c^3*x^3*ArcCosh[a*x])/3675 + (702*a^4*c^
3*x^5*ArcCosh[a*x])/6125 - (6*a^6*c^3*x^7*ArcCosh[a*x])/343 - (48*c^3*Sqrt[
```

```
-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(35*a) + (8*c^3*(-1 + a*x)^(3/2)*(1
 + a*x)^(3/2)*ArcCosh[a*x]^2)/(35*a) - (18*c^3*(-1 + a*x)^(5/2)*(1 + a*x)^(
5/2)*ArcCosh[a*x]^2)/(175*a) + (3*c^3*(-1 + a*x)^(7/2)*(1 + a*x)^(7/2)*ArcC
osh[a*x]^2)/(49*a) + (16*c^3*x*ArcCosh[a*x]^3)/35 + (8*c^3*x*(1 - a^2*x^2)*
ArcCosh[a*x]^3)/35 + (6*c^3*x*(1 - a^2*x^2)^2*ArcCosh[a*x]^3)/35 + (c^3*x*(
1 - a^2*x^2)^3*ArcCosh[a*x]^3)/7
```

Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^3 \cosh^{-1}(ax)^3\,dx = \frac{1}{7}c^3x\left(1-a^2x^2\right)^3\cosh^{-1}(ax)^3+\frac{1}{7}(6c)\int\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^3\,dx+\frac{1}{7}\left(3ac^3\right)\int x(-$$

$$= \frac{3c^3(-1+ax)^{7/2}(1+ax)^{7/2}\cosh^{-1}(ax)^2}{49a}+\frac{6}{35}c^3x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^3+\frac{1}{7}c^3x\left(1-a^2\right.$$

$$= \frac{6}{49}c^3x\cosh^{-1}(ax)-\frac{6}{49}a^2c^3x^3\cosh^{-1}(ax)+\frac{18}{245}a^4c^3x^5\cosh^{-1}(ax)-\frac{6}{343}a^6c^3x^7\cosh^{-1}$$

$$= \frac{402c^3x\cosh^{-1}(ax)}{1225}-\frac{318a^2c^3x^3\cosh^{-1}(ax)}{1225}+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-\frac{6}{343}a^6c^3x^7\cosh$$

$$= \frac{962c^3x\cosh^{-1}(ax)}{1225}-\frac{1514a^2c^3x^3\cosh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-\frac{6}{343}a^6c^3x^7\cos$$

$$= \frac{4322c^3x\cosh^{-1}(ax)}{1225}-\frac{1514a^2c^3x^3\cosh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-\frac{6}{343}a^6c^3x^7\cos$$

$$= -\frac{96c^3\sqrt{-1+ax}\sqrt{1+ax}}{35a}+\frac{16}{315}ac^3x^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{4322c^3x\cosh^{-1}(ax)}{1225}-\frac{1514a^2}{}$$

$$= -\frac{976c^3\sqrt{-1+ax}\sqrt{1+ax}}{315a}+\frac{16}{315}ac^3x^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{96c^3\left(1-a^2x^2\right)}{1715a\sqrt{-1+ax}\sqrt{1+ax}}+$$

$$= -\frac{976c^3\sqrt{-1+ax}\sqrt{1+ax}}{315a}+\frac{16}{315}ac^3x^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{7104c^3\left(1-a^2x^2\right)}{42875a\sqrt{-1+ax}\sqrt{1+ax}}+$$

Mathematica [A] time = 0.41332, size = 179, normalized size = 0.35

$$\frac{c^3\left(2\sqrt{ax-1}\sqrt{ax+1}\left(16875a^6x^6-134541a^4x^4+747937a^2x^2-22329151\right)-385875ax\left(5a^6x^6-21a^4x^4+35a^2x^2-35\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3*ArcCosh[a*x]^3,x]
```

```
[Out] (c^3*(2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-22329151 + 747937*a^2*x^2 - 134541*a
^4*x^4 + 16875*a^6*x^6) - 210*a*x*(-226905 + 26495*a^2*x^2 - 7371*a^4*x^4 +
 1125*a^6*x^6)*ArcCosh[a*x] + 11025*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-2161 + 7
57*a^2*x^2 - 351*a^4*x^4 + 75*a^6*x^6)*ArcCosh[a*x]^2 - 385875*a*x*(-35 + 3
5*a^2*x^2 - 21*a^4*x^4 + 5*a^6*x^6)*ArcCosh[a*x]^3))/(13505625*a)
```

Maple [A] time = 0.076, size = 294, normalized size = 0.6

$$-\frac{c^3}{13505625\,a}\left(1929375\,(\mathrm{arccosh}\,(ax))^3\,a^7x^7-826875\,(\mathrm{arccosh}\,(ax))^2\,\sqrt{ax-1}\sqrt{ax+1}a^6x^6-8103375\,(\mathrm{arccosh}\,(ax))\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x)
```

```
[Out] -1/13505625/a*c^3*(1929375*arccosh(a*x)^3*a^7*x^7-826875*arccosh(a*x)^2*(a*
x-1)^(1/2)*(a*x+1)^(1/2)*a^6*x^6-8103375*arccosh(a*x)^3*a^5*x^5+3869775*arc
cosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^4*x^4+236250*arccosh(a*x)*a^7*x^7
-33750*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^6*x^6+13505625*arccosh(a*x)^3*a^3*x^3-
8345925*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^2*x^2-1547910*a^5*x^5*
arccosh(a*x)+269082*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^4*a^4-13505625*arccosh(a*
x)^3*a*x+23825025*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+5563950*arccos
h(a*x)*a^3*x^3-1495874*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-47650050*a*x*arc
cosh(a*x)+44658302*(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [A] time = 1.15399, size = 373, normalized size = 0.74

$$\frac{1}{1225}\left(75\,\sqrt{a^2x^2-1}a^4c^3x^6-351\,\sqrt{a^2x^2-1}a^2c^3x^4+757\,\sqrt{a^2x^2-1}c^3x^2-\frac{2161\,\sqrt{a^2x^2-1}c^3}{a^2}\right)a\,\mathrm{arcosh}\,(ax)^2-\frac{1}{35}\left(5\,a^6c^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/1225*(75*sqrt(a^2*x^2 - 1)*a^4*c^3*x^6 - 351*sqrt(a^2*x^2 - 1)*a^2*c^3*x^
4 + 757*sqrt(a^2*x^2 - 1)*c^3*x^2 - 2161*sqrt(a^2*x^2 - 1)*c^3/a^2)*a*arcco
sh(a*x)^2 - 1/35*(5*a^6*c^3*x^7 - 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 - 35*c^3*
x)*arccosh(a*x)^3 + 2/13505625*(16875*sqrt(a^2*x^2 - 1)*a^4*c^3*x^6 - 13454
1*sqrt(a^2*x^2 - 1)*a^2*c^3*x^4 + 747937*sqrt(a^2*x^2 - 1)*c^3*x^2 - 223291
51*sqrt(a^2*x^2 - 1)*c^3/a^2 - 105*(1125*a^6*c^3*x^7 - 7371*a^4*c^3*x^5 + 2
6495*a^2*c^3*x^3 - 226905*c^3*x)*arccosh(a*x)/a)*a
```

Fricas [A] time = 2.17705, size = 605, normalized size = 1.2

$$-\frac{385875\left(5\,a^7c^3x^7-21\,a^5c^3x^5+35\,a^3c^3x^3-35\,ac^3x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3-11025\left(75\,a^6c^3x^6-351\,a^4c^3x^4+757\,a^2c^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/13505625*(385875*(5*a^7*c^3*x^7 - 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 - 35*a
*c^3*x)*log(a*x + sqrt(a^2*x^2 - 1))^3 - 11025*(75*a^6*c^3*x^6 - 351*a^4*c^
3*x^4 + 757*a^2*c^3*x^2 - 2161*c^3)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^
2 - 1))^2 + 210*(1125*a^7*c^3*x^7 - 7371*a^5*c^3*x^5 + 26495*a^3*c^3*x^3 -
226905*a*c^3*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(16875*a^6*c^3*x^6 - 13454
1*a^4*c^3*x^4 + 747937*a^2*c^3*x^2 - 22329151*c^3)*sqrt(a^2*x^2 - 1))/a
```

Sympy [A] time = 24.5367, size = 367, normalized size = 0.73

$$\begin{cases} -\frac{a^6c^3x^7\operatorname{acosh}^3(ax)}{7}-\frac{6a^6c^3x^7\operatorname{acosh}(ax)}{343}+\frac{3a^5c^3x^6\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{49}+\frac{6a^5c^3x^6\sqrt{a^2x^2-1}}{2401}+\frac{3a^4c^3x^5\operatorname{acosh}^3(ax)}{5}+\frac{702a^4c^3x^5\operatorname{acosh}(ax)}{6125}-\frac{351a}{} \\ -\frac{i\pi^3c^3x}{8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*acosh(a*x)**3,x)
```

```
[Out] Piecewise((-a**6*c**3*x**7*acosh(a*x)**3/7 - 6*a**6*c**3*x**7*acosh(a*x)/34
3 + 3*a**5*c**3*x**6*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/49 + 6*a**5*c**3*x**
6*sqrt(a**2*x**2 - 1)/2401 + 3*a**4*c**3*x**5*acosh(a*x)**3/5 + 702*a**4*c*
*3*x**5*acosh(a*x)/6125 - 351*a**3*c**3*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)
**2/1225 - 29898*a**3*c**3*x**4*sqrt(a**2*x**2 - 1)/1500625 - a**2*c**3*x**
3*acosh(a*x)**3 - 1514*a**2*c**3*x**3*acosh(a*x)/3675 + 757*a*c**3*x**2*sqr
t(a**2*x**2 - 1)*acosh(a*x)**2/1225 + 1495874*a*c**3*x**2*sqrt(a**2*x**2 -
1)/13505625 + c**3*x*acosh(a*x)**3 + 4322*c**3*x*acosh(a*x)/1225 - 2161*c**
3*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(1225*a) - 44658302*c**3*sqrt(a**2*x**2
 - 1)/(13505625*a), Ne(a, 0)), (-I*pi**3*c**3*x/8, True))
```

Giac [A] time = 1.34547, size = 333, normalized size = 0.66

$$-\frac{1}{13505625}\left(210\left(1125\,a^6x^7-7371\,a^4x^5+26495\,a^2x^3-226905\,x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{11025\left(75\left(a^2x^2-1\right)^{\frac{7}{2}}-126\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] -1/13505625*(210*(1125*a^6*x^7 - 7371*a^4*x^5 + 26495*a^2*x^3 - 226905*x)*l
og(a*x + sqrt(a^2*x^2 - 1)) - 11025*(75*(a^2*x^2 - 1)^(7/2) - 126*(a^2*x^2
- 1)^(5/2) + 280*(a^2*x^2 - 1)^(3/2) - 1680*sqrt(a^2*x^2 - 1))*log(a*x + sq
rt(a^2*x^2 - 1))^2/a - 2*(16875*(a^2*x^2 - 1)^(7/2) - 83916*(a^2*x^2 - 1)^(
5/2) + 529480*(a^2*x^2 - 1)^(3/2) - 21698880*sqrt(a^2*x^2 - 1))/a)*c^3 - 1/
35*(5*a^6*c^3*x^7 - 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 - 35*c^3*x)*log(a*x + s
qrt(a^2*x^2 - 1))^3
```

3.241 $\int \left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=388

$$\frac{6c^2\left(1-a^2x^2\right)^3}{625a\sqrt{ax-1}\sqrt{ax+1}}+\frac{8c^2\left(1-a^2x^2\right)^2}{375a\sqrt{ax-1}\sqrt{ax+1}}+\frac{16c^2\left(1-a^2x^2\right)}{125a\sqrt{ax-1}\sqrt{ax+1}}+\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)-\frac{76}{225}a^2c^2x^3\cosh^{-1}(ax)$$

```
[Out] (-488*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(135*a) + (8*a*c^2*x^2*Sqrt[-1 + a*
x]*Sqrt[1 + a*x])/135 + (16*c^2*(1 - a^2*x^2))/(125*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]) + (8*c^2*(1 - a^2*x^2)^2)/(375*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (
6*c^2*(1 - a^2*x^2)^3)/(625*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (298*c^2*x*Ar
cCosh[a*x])/75 - (76*a^2*c^2*x^3*ArcCosh[a*x])/225 + (6*a^4*c^2*x^5*ArcCosh
[a*x])/125 - (8*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(5*a) + (4
*c^2*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]^2)/(15*a) - (3*c^2*(-1 +
 a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x]^2)/(25*a) + (8*c^2*x*ArcCosh[a*x]^
3)/15 + (4*c^2*x*(1 - a^2*x^2)*ArcCosh[a*x]^3)/15 + (c^2*x*(1 - a^2*x^2)^2*
ArcCosh[a*x]^3)/5
```

Rubi [A] time = 0.844015, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {5681, 5718, 194, 5680, 12, 520, 1247, 698, 460, 74, 5654}

$$\frac{6c^2\left(1-a^2x^2\right)^3}{625a\sqrt{ax-1}\sqrt{ax+1}}+\frac{8c^2\left(1-a^2x^2\right)^2}{375a\sqrt{ax-1}\sqrt{ax+1}}+\frac{16c^2\left(1-a^2x^2\right)}{125a\sqrt{ax-1}\sqrt{ax+1}}+\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)-\frac{76}{225}a^2c^2x^3\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2*ArcCosh[a*x]^3,x]
```


```
[Out] (-488*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(135*a) + (8*a*c^2*x^2*Sqrt[-1 + a*
x]*Sqrt[1 + a*x])/135 + (16*c^2*(1 - a^2*x^2))/(125*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]) + (8*c^2*(1 - a^2*x^2)^2)/(375*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (
6*c^2*(1 - a^2*x^2)^3)/(625*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (298*c^2*x*Ar
cCosh[a*x])/75 - (76*a^2*c^2*x^3*ArcCosh[a*x])/225 + (6*a^4*c^2*x^5*ArcCosh
[a*x])/125 - (8*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(5*a) + (4
*c^2*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]^2)/(15*a) - (3*c^2*(-1 +
 a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x]^2)/(25*a) + (8*c^2*x*ArcCosh[a*x]^
3)/15 + (4*c^2*x*(1 - a^2*x^2)*ArcCosh[a*x]^3)/15 + (c^2*x*(1 - a^2*x^2)^2*
ArcCosh[a*x]^3)/5
```

Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
```

```
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^3\,dx = \frac{1}{5}c^2x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^3+\frac{1}{5}(4c)\int\left(c-a^2cx^2\right)\cosh^{-1}(ax)^3\,dx-\frac{1}{5}\left(3ac^2\right)\int x(-$$

$$= -\frac{3c^2(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^2}{25a}+\frac{4}{15}c^2x\left(1-a^2x^2\right)\cosh^{-1}(ax)^3+\frac{1}{5}c^2x\left(1-\right.$$

$$= \frac{6}{25}c^2x\cosh^{-1}(ax)-\frac{4}{25}a^2c^2x^3\cosh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)+\frac{4c^2(-1+ax)^{3/2}(}{}$$

$$= \frac{58}{75}c^2x\cosh^{-1}(ax)-\frac{76}{225}a^2c^2x^3\cosh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)-\frac{8c^2\sqrt{-1+ax}\sqrt{}}{}$$

$$= \frac{298}{75}c^2x\cosh^{-1}(ax)-\frac{76}{225}a^2c^2x^3\cosh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)-\frac{8c^2\sqrt{-1+ax}}{}$$

$$= -\frac{16c^2\sqrt{-1+ax}\sqrt{1+ax}}{5a}+\frac{8}{135}ac^2x^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{298}{75}c^2x\cosh^{-1}(ax)-\frac{76}{225}a^2$$

$$= -\frac{488c^2\sqrt{-1+ax}\sqrt{1+ax}}{135a}+\frac{8}{135}ac^2x^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{298}{75}c^2x\cosh^{-1}(ax)-\frac{76}{225}a$$

$$= -\frac{488c^2\sqrt{-1+ax}\sqrt{1+ax}}{135a}+\frac{8}{135}ac^2x^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{16c^2\left(1-a^2x^2\right)}{125a\sqrt{-1+ax}\sqrt{1+ax}}+$$

Mathematica [A] time = 0.209834, size = 147, normalized size = 0.38

$$\frac{c^2\left(-2\sqrt{ax-1}\sqrt{ax+1}\left(81a^4x^4-842a^2x^2+31841\right)+1125ax\left(3a^4x^4-10a^2x^2+15\right)\cosh^{-1}(ax)^3-225\sqrt{ax-1}\sqrt{ax+1}\right.}{16875a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2*ArcCosh[a*x]^3,x]
```

```
[Out] (c^2*(-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(31841 - 842*a^2*x^2 + 81*a^4*x^4) +
30*a*x*(2235 - 190*a^2*x^2 + 27*a^4*x^4)*ArcCosh[a*x] - 225*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]*(149 - 38*a^2*x^2 + 9*a^4*x^4)*ArcCosh[a*x]^2 + 1125*a*x*(15
- 10*a^2*x^2 + 3*a^4*x^4)*ArcCosh[a*x]^3))/(16875*a)
```

Maple [A] time = 0.056, size = 218, normalized size = 0.6

$$\frac{c^2}{16875\,a}\left(3375\,(\mathrm{arccosh}\,(ax))^3\,a^5x^5-2025\,(\mathrm{arccosh}\,(ax))^2\,\sqrt{ax-1}\sqrt{ax+1}a^4x^4-11250\,(\mathrm{arccosh}\,(ax))^3\,a^3x^3+8550\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x)
```

```
[Out] 1/16875/a*c^2*(3375*arccosh(a*x)^3*a^5*x^5-2025*arccosh(a*x)^2*(a*x-1)^(1/2
)*(a*x+1)^(1/2)*a^4*x^4-11250*arccosh(a*x)^3*a^3*x^3+8550*arccosh(a*x)^2*(a
*x-1)^(1/2)*(a*x+1)^(1/2)*a^2*x^2+810*a^5*x^5*arccosh(a*x)-162*(a*x+1)^(1/2
)*(a*x-1)^(1/2)*x^4*a^4+16875*arccosh(a*x)^3*a*x-33525*arccosh(a*x)^2*(a*x-
1)^(1/2)*(a*x+1)^(1/2)-5700*arccosh(a*x)*a^3*x^3+1684*(a*x+1)^(1/2)*(a*x-1)
^(1/2)*x^2*a^2+67050*a*x*arccosh(a*x)-63682*(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [A] time = 1.2006, size = 284, normalized size = 0.73

$$-\frac{1}{75}\left(9\sqrt{a^2x^2-1}a^2c^2x^4-38\sqrt{a^2x^2-1}c^2x^2+\frac{149\sqrt{a^2x^2-1}c^2}{a^2}\right)a\operatorname{arcosh}(ax)^2+\frac{1}{15}\left(3a^4c^2x^5-10a^2c^2x^3+15c^2x\right)\operatorname{arcc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/75*(9*sqrt(a^2*x^2 - 1)*a^2*c^2*x^4 - 38*sqrt(a^2*x^2 - 1)*c^2*x^2 + 149
*sqrt(a^2*x^2 - 1)*c^2/a^2)*a*arccosh(a*x)^2 + 1/15*(3*a^4*c^2*x^5 - 10*a^2
*c^2*x^3 + 15*c^2*x)*arccosh(a*x)^3 - 2/16875*(81*sqrt(a^2*x^2 - 1)*a^2*c^2
*x^4 - 842*sqrt(a^2*x^2 - 1)*c^2*x^2 - 15*(27*a^4*c^2*x^5 - 190*a^2*c^2*x^3
 + 2235*c^2*x)*arccosh(a*x)/a + 31841*sqrt(a^2*x^2 - 1)*c^2/a^2)*a
```

Fricas [A] time = 2.17825, size = 467, normalized size = 1.2

$$\frac{1125\left(3a^5c^2x^5-10a^3c^2x^3+15ac^2x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3-225\left(9a^4c^2x^4-38a^2c^2x^2+149c^2\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a\right.}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(1125*(3*a^5*c^2*x^5 - 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt(
a^2*x^2 - 1))^3 - 225*(9*a^4*c^2*x^4 - 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2*x
^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 30*(27*a^5*c^2*x^5 - 190*a^3*c^2*x
```

```
^3 + 2235*a*c^2*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(81*a^4*c^2*x^4 - 842*a
^2*c^2*x^2 + 31841*c^2)*sqrt(a^2*x^2 - 1))/a
```

Sympy [A] time = 8.76748, size = 274, normalized size = 0.71

$$\begin{cases} \frac{a^4c^2x^5\operatorname{acosh}^3(ax)}{5} + \frac{6a^4c^2x^5\operatorname{acosh}(ax)}{125} - \frac{3a^3c^2x^4\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{25} - \frac{6a^3c^2x^4\sqrt{a^2x^2-1}}{625} - \frac{2a^2c^2x^3\operatorname{acosh}^3(ax)}{3} - \frac{76a^2c^2x^3\operatorname{acosh}(ax)}{225} + \frac{38ac^2x \\ -\frac{i\pi^3c^2x}{8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*acosh(a*x)**3,x)
```

```
[Out] Piecewise((a**4*c**2*x**5*acosh(a*x)**3/5 + 6*a**4*c**2*x**5*acosh(a*x)/125
 - 3*a**3*c**2*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/25 - 6*a**3*c**2*x**4
*sqrt(a**2*x**2 - 1)/625 - 2*a**2*c**2*x**3*acosh(a*x)**3/3 - 76*a**2*c**2*
x**3*acosh(a*x)/225 + 38*a*c**2*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/75 +
 1684*a*c**2*x**2*sqrt(a**2*x**2 - 1)/16875 + c**2*x*acosh(a*x)**3 + 298*c*
*2*x*acosh(a*x)/75 - 149*c**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(75*a) - 63
682*c**2*sqrt(a**2*x**2 - 1)/(16875*a), Ne(a, 0)), (-I*pi**3*c**2*x/8, True
))
```

Giac [A] time = 1.33745, size = 273, normalized size = 0.7

$$\frac{1}{15}\left(3\,a^4c^2x^5 - 10\,a^2c^2x^3 + 15\,c^2x\right)\log\left(ax + \sqrt{a^2x^2-1}\right)^3 + \frac{1}{16875}\left(30\left(27\,a^4x^5 - 190\,a^2x^3 + 2235\,x\right)\log\left(ax + \sqrt{a^2x^2 -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^4*c^2*x^5 - 10*a^2*c^2*x^3 + 15*c^2*x)*log(a*x + sqrt(a^2*x^2 - 1
))^3 + 1/16875*(30*(27*a^4*x^5 - 190*a^2*x^3 + 2235*x)*log(a*x + sqrt(a^2*x
^2 - 1)) - 225*(9*(a^2*x^2 - 1)^(5/2) - 20*(a^2*x^2 - 1)^(3/2) + 120*sqrt(a
^2*x^2 - 1))*log(a*x + sqrt(a^2*x^2 - 1))^2/a - 2*(81*(a^2*x^2 - 1)^(5/2) -
 680*(a^2*x^2 - 1)^(3/2) + 31080*sqrt(a^2*x^2 - 1))/a)*c^2
```

3.242 $\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=175

$$-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^3+\frac{2}{27}acx^2\sqrt{ax-1}\sqrt{ax+1}-\frac{122c\sqrt{ax-1}\sqrt{ax+1}}{27a}+\frac{2}{3}cx\cosh^{-1}(ax)^3$$

```
[Out] (-122*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(27*a) + (2*a*c*x^2*Sqrt[-1 + a*x]*Sq
rt[1 + a*x])/27 + (14*c*x*ArcCosh[a*x])/3 - (2*a^2*c*x^3*ArcCosh[a*x])/9 -
(2*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/a + (c*(-1 + a*x)^(3/2)*(
1 + a*x)^(3/2)*ArcCosh[a*x]^2)/(3*a) + (2*c*x*ArcCosh[a*x]^3)/3 + (c*x*(1 -
 a^2*x^2)*ArcCosh[a*x]^3)/3
```

Rubi [A] time = 0.477469, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5681, 5718, 5680, 12, 460, 74, 5654}

$$-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^3+\frac{2}{27}acx^2\sqrt{ax-1}\sqrt{ax+1}-\frac{122c\sqrt{ax-1}\sqrt{ax+1}}{27a}+\frac{2}{3}cx\cosh^{-1}(ax)^3$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)*ArcCosh[a*x]^3,x]
```



```
[Out] (-122*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(27*a) + (2*a*c*x^2*Sqrt[-1 + a*x]*Sq
rt[1 + a*x])/27 + (14*c*x*ArcCosh[a*x])/3 - (2*a^2*c*x^3*ArcCosh[a*x])/9 -
(2*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/a + (c*(-1 + a*x)^(3/2)*(
1 + a*x)^(3/2)*ArcCosh[a*x]^2)/(3*a) + (2*c*x*ArcCosh[a*x]^3)/3 + (c*x*(1 -
 a^2*x^2)*ArcCosh[a*x]^3)/3
```


Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^3\,dx = \frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^3+\frac{1}{3}(2c)\int \cosh^{-1}(ax)^3\,dx+(ac)\int x\sqrt{-1+ax}\sqrt{1+ax}\cosh$$

$$= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^2}{3a}+\frac{2}{3}cx\cosh^{-1}(ax)^3+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^3-$$

$$= \frac{2}{3}cx\cosh^{-1}(ax)-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)-\frac{2c\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a}+\frac{c(-1+ax)^{3/2}}{}$$

$$= \frac{14}{3}cx\cosh^{-1}(ax)-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)-\frac{2c\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a}+\frac{c(-1+ax)^3}{}$$

$$= -\frac{4c\sqrt{-1+ax}\sqrt{1+ax}}{a}+\frac{2}{27}acx^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{14}{3}cx\cosh^{-1}(ax)-\frac{2}{9}a^2cx^3\cosh^{-1}($$

$$= -\frac{122c\sqrt{-1+ax}\sqrt{1+ax}}{27a}+\frac{2}{27}acx^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{14}{3}cx\cosh^{-1}(ax)-\frac{2}{9}a^2cx^3\cosh^{-}$$

Mathematica [A] time = 0.116233, size = 109, normalized size = 0.62

$$\frac{c\left(2\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2-61\right)-9ax\left(a^2x^2-3\right)\cosh^{-1}(ax)^3+9\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2-7\right)\cosh^{-1}(ax)^2-6ax\left(a^2x^2-2\right.\right.}{27a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)*ArcCosh[a*x]^3,x]
```

```
[Out] (c*(2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-61 + a^2*x^2) - 6*a*x*(-21 + a^2*x^2)*
ArcCosh[a*x] + 9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-7 + a^2*x^2)*ArcCosh[a*x]^2
 - 9*a*x*(-3 + a^2*x^2)*ArcCosh[a*x]^3))/(27*a)
```

Maple [A] time = 0.049, size = 140, normalized size = 0.8

$$-\frac{c}{27\,a}\left(9\,(\mathrm{arccosh}\,(ax))^3\,a^3x^3-9\,(\mathrm{arccosh}\,(ax))^2\,\sqrt{ax-1}\sqrt{ax+1}a^2x^2-27\,(\mathrm{arccosh}\,(ax))^3\,ax+63\,(\mathrm{arccosh}\,(ax))^2\,\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*arccosh(a*x)^3,x)
```

```
[Out] -1/27/a*c*(9*arccosh(a*x)^3*a^3*x^3-9*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^
(1/2)*a^2*x^2-27*arccosh(a*x)^3*a*x+63*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)
^(1/2)+6*arccosh(a*x)*a^3*x^3-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-126*a*x
*arccosh(a*x)+122*(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [A] time = 1.22694, size = 167, normalized size = 0.95

$$\frac{1}{3}\left(\sqrt{a^2x^2-1}cx^2 - \frac{7\sqrt{a^2x^2-1}c}{a^2}\right)a\,\text{arcosh}\,(ax)^2 - \frac{1}{3}\left(a^2cx^3-3\,cx\right)\text{arcosh}\,(ax)^3 + \frac{2}{27}\left(\sqrt{a^2x^2-1}cx^2 - \frac{3\left(a^2cx^3-21\,cx\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 - 1)*c*x^2 - 7*sqrt(a^2*x^2 - 1)*c/a^2)*a*arccosh(a*x)^2
- 1/3*(a^2*c*x^3 - 3*c*x)*arccosh(a*x)^3 + 2/27*(sqrt(a^2*x^2 - 1)*c*x^2 -
3*(a^2*c*x^3 - 21*c*x)*arccosh(a*x)/a - 61*sqrt(a^2*x^2 - 1)*c/a^2)*a
```

Fricas [A] time = 2.17679, size = 316, normalized size = 1.81

$$-\frac{9\left(a^3cx^3-3\,acx\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3 - 9\left(a^2cx^2-7\,c\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2 + 6\left(a^3cx^3-21\,acx\right)\log\left(ax\right.}{27\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/27*(9*(a^3*c*x^3 - 3*a*c*x)*log(a*x + sqrt(a^2*x^2 - 1))^3 - 9*(a^2*c*x^
2 - 7*c)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 6*(a^3*c*x^3 -
21*a*c*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(a^2*c*x^2 - 61*c)*sqrt(a^2*x^2
- 1))/a
```

Sympy [A] time = 2.48989, size = 160, normalized size = 0.91

$$\begin{cases} -\frac{a^2cx^3\,\text{acosh}^3(ax)}{3} - \frac{2a^2cx^3\,\text{acosh}(ax)}{9} + \frac{acx^2\sqrt{a^2x^2-1}\,\text{acosh}^2(ax)}{3} + \frac{2acx^2\sqrt{a^2x^2-1}}{27} + cx\,\text{acosh}^3(ax) + \frac{14cx\,\text{acosh}(ax)}{3} - \frac{7c\sqrt{a^2x^2-1}\,\text{acosh}}{3a} \\ -\frac{i\pi^3cx}{8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*acosh(a*x)**3,x)
```

```
[Out] Piecewise((-a**2*c*x**3*acosh(a*x)**3/3 - 2*a**2*c*x**3*acosh(a*x)/9 + a*c*
x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/3 + 2*a*c*x**2*sqrt(a**2*x**2 - 1)/2
7 + c*x*acosh(a*x)**3 + 14*c*x*acosh(a*x)/3 - 7*c*sqrt(a**2*x**2 - 1)*acosh
(a*x)**2/(3*a) - 122*c*sqrt(a**2*x**2 - 1)/(27*a), Ne(a, 0)), (-I*pi**3*c*x
/8, True))
```

Giac [A] time = 1.31812, size = 196, normalized size = 1.12

$$-\frac{1}{3}\left(a^2cx^3-3\,cx\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3-\frac{1}{27}\left(6\left(a^2x^3-21\,x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{9\left(\left(a^2x^2-1\right)^{\frac{3}{2}}-6\,\sqrt{a^2x^2-1}\right)\log}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] -1/3*(a^2*c*x^3 - 3*c*x)*log(a*x + sqrt(a^2*x^2 - 1))^3 - 1/27*(6*(a^2*x^3
- 21*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 9*((a^2*x^2 - 1)^(3/2) - 6*sqrt(a^2*
x^2 - 1))*log(a*x + sqrt(a^2*x^2 - 1))^2/a - 2*((a^2*x^2 - 1)^(3/2) - 60*sq
rt(a^2*x^2 - 1))/a)*c
```

3.243 $\int \frac{\cosh^{-1}(ax)^3}{c-a^2cx^2}\,dx$

Optimal. Leaf size=144

$$\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{6\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac}$$

```
[Out] (2*ArcCosh[a*x]^3*ArcTanh[E^ArcCosh[a*x]])/(a*c) + (3*ArcCosh[a*x]^2*PolyLo
g[2, -E^ArcCosh[a*x]])/(a*c) - (3*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]]
)/(a*c) - (6*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])/(a*c) + (6*ArcCosh[a
*x]*PolyLog[3, E^ArcCosh[a*x]])/(a*c) + (6*PolyLog[4, -E^ArcCosh[a*x]])/(a*
c) - (6*PolyLog[4, E^ArcCosh[a*x]])/(a*c)
```

Rubi [A] time = 0.12915, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5694, 4182, 2531, 6609, 2282, 6589}

$$\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{6\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2),x]
```


```
[Out] (2*ArcCosh[a*x]^3*ArcTanh[E^ArcCosh[a*x]])/(a*c) + (3*ArcCosh[a*x]^2*PolyLo
g[2, -E^ArcCosh[a*x]])/(a*c) - (3*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]]
)/(a*c) - (6*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])/(a*c) + (6*ArcCosh[a
*x]*PolyLog[3, E^ArcCosh[a*x]])/(a*c) + (6*PolyLog[4, -E^ArcCosh[a*x]])/(a*
c) - (6*PolyLog[4, E^ArcCosh[a*x]])/(a*c)
```


Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{c-a^2cx^2}\,dx = -\frac{\mathrm{Subst}\left(\int x^3\mathbf{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac}$$
$$= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{3\,\mathrm{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \cosh^{-1}(ax)\right)}{ac} - \frac{3\,\mathrm{Subst}\left(\int x\right.}{}$$
$$= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{3\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{3\cosh^{-1}(ax)^2\mathrm{Li}_2\left(e^{\cosh^{-1}(\right.}}{ac}$$
$$= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{3\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{3\cosh^{-1}(ax)^2\mathrm{Li}_2\left(e^{\cosh^{-1}(\right.}}{ac}$$
$$= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{3\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{3\cosh^{-1}(ax)^2\mathrm{Li}_2\left(e^{\cosh^{-1}(\right.}}{ac}$$
$$= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{3\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{3\cosh^{-1}(ax)^2\mathrm{Li}_2\left(e^{\cosh^{-1}(\right.}}{ac}$$

Mathematica [A] time = 0.0991638, size = 129, normalized size = 0.9

$$\frac{3\cosh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right) - 3\cosh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right) - 6\cosh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2),x]
```

```
[Out] (-(ArcCosh[a*x]^3*Log[1 - E^ArcCosh[a*x]]) + ArcCosh[a*x]^3*Log[1 + E^ArcCo
sh[a*x]] + 3*ArcCosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]] - 3*ArcCosh[a*x]^2*
PolyLog[2, E^ArcCosh[a*x]] - 6*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]] + 6
*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]] + 6*PolyLog[4, -E^ArcCosh[a*x]] -
6*PolyLog[4, E^ArcCosh[a*x]])/(a*c)
```

Maple [A] time = 0.043, size = 273, normalized size = 1.9

$$-\frac{\left(\mathrm{arccosh}\,(ax)\right)^3}{ac}\ln\left(1-ax-\sqrt{ax-1}\sqrt{ax+1}\right) - 3\,\frac{\left(\mathrm{arccosh}\,(ax)\right)^2\mathrm{polylog}\left(2, ax+\sqrt{ax-1}\sqrt{ax+1}\right)}{ac} + 6\,\frac{\mathrm{arccosh}\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c),x)
```

```
[Out] -1/a/c*arccosh(a*x)^3*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-3*arccosh(a*x)^
2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+6*arccosh(a*x)*polylog(3,a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-6*polylog(4,a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2))/a/c+1/a/c*arccosh(a*x)^3*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3*arcc
osh(a*x)^2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-6*arccosh(a*x)*p
olylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+6*polylog(4,-a*x-(a*x-1)^(1/
2)*(a*x+1)^(1/2))/a/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(\log(ax+1)-\log(ax-1))\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{2\,ac}-\int\frac{3\left((ax\log(ax+1)-ax\log(ax-1))\sqrt{ax+1}\sqrt{ax-1}+\left(a^2\right.\right.}{2\left(a^3cx^3-acx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1) - log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/
(a*c) - integrate(3/2*((a*x*log(a*x + 1) - a*x*log(a*x - 1))*sqrt(a*x + 1)*
sqrt(a*x - 1) + (a^2*x^2 - 1)*log(a*x + 1) - (a^2*x^2 - 1)*log(a*x - 1))*lo
g(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(a^3*c*x^3 - a*c*x + (a^2*c*x^2 - c)
*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\operatorname{arcosh}(ax)^3}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^3/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\operatorname{acosh}^3(ax)}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(acosh(a*x)**3/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arcosh}(ax)^3}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)^3/(a^2*c*x^2 - c), x)
```

3.244 $\int \frac{\cosh^{-1}(ax)^3}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=260

$$\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{3\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac^2}$$

```
[Out] (-3*ArcCosh[a*x]^2)/(2*a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x
]^3)/(2*c^2*(1 - a^2*x^2)) - (6*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^
2) + (ArcCosh[a*x]^3*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) - (3*PolyLog[2, -E^Ar
cCosh[a*x]])/(a*c^2) + (3*ArcCosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]])/(2*a*
c^2) + (3*PolyLog[2, E^ArcCosh[a*x]])/(a*c^2) - (3*ArcCosh[a*x]^2*PolyLog[2
, E^ArcCosh[a*x]])/(2*a*c^2) - (3*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])
/(a*c^2) + (3*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]])/(a*c^2) + (3*PolyLog
[4, -E^ArcCosh[a*x]])/(a*c^2) - (3*PolyLog[4, E^ArcCosh[a*x]])/(a*c^2)
```

Rubi [A] time = 0.441588, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5689, 5718, 5694, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{3\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^2,x]
```


```
[Out] (-3*ArcCosh[a*x]^2)/(2*a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x
]^3)/(2*c^2*(1 - a^2*x^2)) - (6*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^
2) + (ArcCosh[a*x]^3*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) - (3*PolyLog[2, -E^Ar
cCosh[a*x]])/(a*c^2) + (3*ArcCosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]])/(2*a*
c^2) + (3*PolyLog[2, E^ArcCosh[a*x]])/(a*c^2) - (3*ArcCosh[a*x]^2*PolyLog[2
, E^ArcCosh[a*x]])/(2*a*c^2) - (3*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])
/(a*c^2) + (3*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]])/(a*c^2) + (3*PolyLog
[4, -E^ArcCosh[a*x]])/(a*c^2) - (3*PolyLog[4, E^ArcCosh[a*x]])/(a*c^2)
```


Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^2}\,dx = \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} + \frac{(3a)\int \frac{x\cosh^{-1}(ax)^2}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{2c^2} + \frac{\int \frac{\cosh^{-1}(ax)^3}{c-a^2cx^2}\,dx}{2c}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} + \frac{3\int \frac{\cosh^{-1}(ax)}{-1+a^2x^2}\,dx}{c^2} - \frac{\text{Subst}\left(\int x^3\mathbf{csch}(x)\,dx, x, \cosh^{-1}(\right.}{2ac^2}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{3\,\text{Subst}\left(\int x^2\log\right.}{}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\cosh^{-1}(ax)^3\tanh}{ac^2}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\cosh^{-1}(ax)^3\tanh}{ac^2}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\cosh^{-1}(ax)^3\tanh}{ac^2}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\cosh^{-1}(ax)^3\tanh}{ac^2}$$

Mathematica [A] time = 2.25245, size = 276, normalized size = 1.06

$$\frac{-24\cosh^{-1}(ax)^2\text{PolyLog}\left(2, e^{\cosh^{-1}(ax)}\right) - 48\cosh^{-1}(ax)\text{PolyLog}\left(3, -e^{-\cosh^{-1}(ax)}\right) + 48\cosh^{-1}(ax)\text{PolyLog}\left(3, e^{\cosh^{-1}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-Pi^4 + 2*ArcCosh[a*x]^4 - 12*ArcCosh[a*x]^2*Coth[ArcCosh[a*x]/2] - 2*ArcC
osh[a*x]^3*Csch[ArcCosh[a*x]/2]^2 + 48*ArcCosh[a*x]*Log[1 - E^(-ArcCosh[a*x
])] - 48*ArcCosh[a*x]*Log[1 + E^(-ArcCosh[a*x])] + 8*ArcCosh[a*x]^3*Log[1 +
 E^(-ArcCosh[a*x])] - 8*ArcCosh[a*x]^3*Log[1 - E^ArcCosh[a*x]] - 24*(-2 + A
rcCosh[a*x]^2)*PolyLog[2, -E^(-ArcCosh[a*x])] - 48*PolyLog[2, E^(-ArcCosh[a
*x])] - 24*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]] - 48*ArcCosh[a*x]*Poly
Log[3, -E^(-ArcCosh[a*x])] + 48*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]] - 4
8*PolyLog[4, -E^(-ArcCosh[a*x])] - 48*PolyLog[4, E^ArcCosh[a*x]] - 2*ArcCos
```

```
h[a*x]^3*Sech[ArcCosh[a*x]/2]^2 + 12*ArcCosh[a*x]^2*Tanh[ArcCosh[a*x]/2])/(
16*a*c^2)
```

Maple [A] time = 0.095, size = 464, normalized size = 1.8

$$-\frac{x\left(\operatorname{arccosh}\left(ax\right)\right)^{3}}{\left(2\,a^{2}x^{2}-2\right)c^{2}}-\frac{3\,\left(\operatorname{arccosh}\left(ax\right)\right)^{2}}{2\,a\left(a^{2}x^{2}-1\right)c^{2}}\sqrt{ax-1}\sqrt{ax+1}-\frac{\left(\operatorname{arccosh}\left(ax\right)\right)^{3}}{2\,ac^{2}}\ln\left(1-ax-\sqrt{ax-1}\sqrt{ax+1}\right)-\frac{3\,\left(\operatorname{arccosh}\left(\right.\right.}{2\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/(a^2*x^2-1)*arccosh(a*x)^3/c^2*x-3/2/a/(a^2*x^2-1)*arccosh(a*x)^2/c^2*
(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/2/a/c^2*arccosh(a*x)^3*ln(1-a*x-(a*x-1)^(1/2)
*(a*x+1)^(1/2))-3/2*arccosh(a*x)^2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2
))/a/c^2+3*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3*
polylog(4,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+1/2/a/c^2*arccosh(a*x)^3*l
n(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/2*arccosh(a*x)^2*polylog(2,-a*x-(a*x
-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3*arccosh(a*x)*polylog(3,-a*x-(a*x-1)^(1/2)*
(a*x+1)^(1/2))/a/c^2+3*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+3/
a/c^2*arccosh(a*x)*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))+3*polylog(2,a*x+(a
*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3/a/c^2*arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2
)*(a*x+1)^(1/2))-3*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,ax-\left(a^{2}x^{2}-1\right)\log\left(ax+1\right)+\left(a^{2}x^{2}-1\right)\log\left(ax-1\right)\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^{3}}{4\left(a^{3}c^{2}x^{2}-ac^{2}\right)}-\int-\frac{3\left(2\,a^{3}x^{3}+\left(2\,a^{2}x^{2}-\left(a^{3}x^{3}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(a*x - 1))*log(
a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(a^3*c^2*x^2 - a*c^2) - integrate(-3/4
*(2*a^3*x^3 + (2*a^2*x^2 - (a^3*x^3 - a*x)*log(a*x + 1) + (a^3*x^3 - a*x)*l
og(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x - 1) - 2*a*x - (a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x + 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x
```

```
 + 1)*sqrt(a*x - 1))^2/(a^5*c^2*x^5 - 2*a^3*c^2*x^3 + a*c^2*x + (a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcosh}(ax)^3}{a^4c^2x^4 - 2a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^3/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{acosh}^3(ax)}{a^4x^4 - 2a^2x^2 + 1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(acosh(a*x)**3/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{(a^2cx^2 - c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(a^2*c*x^2 - c)^2, x)
```

3.245 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=387

$$\frac{9\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{9\cosh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{9\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{4ac^3}$$

```
[Out] 1/(4*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (x*ArcCosh[a*x])/(4*c^3*(1 - a^2
*x^2)) + ArcCosh[a*x]^2/(4*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)) - (9*Arc
Cosh[a*x]^2)/(8*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x]^3)/(4
*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x]^3)/(8*c^3*(1 - a^2*x^2)) - (5*Arc
Cosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^3) + (3*ArcCosh[a*x]^3*ArcTanh[E^Ar
cCosh[a*x]])/(4*a*c^3) - (5*PolyLog[2, -E^ArcCosh[a*x]])/(2*a*c^3) + (9*Arc
Cosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]])/(8*a*c^3) + (5*PolyLog[2, E^ArcCos
h[a*x]])/(2*a*c^3) - (9*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]])/(8*a*c^3
) - (9*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])/(4*a*c^3) + (9*ArcCosh[a*x
]*PolyLog[3, E^ArcCosh[a*x]])/(4*a*c^3) + (9*PolyLog[4, -E^ArcCosh[a*x]])/(
4*a*c^3) - (9*PolyLog[4, E^ArcCosh[a*x]])/(4*a*c^3)
```

Rubi [A] time = 0.814226, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {5689, 5718, 74, 5694, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{9\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{9\cosh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{9\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^3,x]
```


```
[Out] 1/(4*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (x*ArcCosh[a*x])/(4*c^3*(1 - a^2
*x^2)) + ArcCosh[a*x]^2/(4*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)) - (9*Arc
Cosh[a*x]^2)/(8*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x]^3)/(4
*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x]^3)/(8*c^3*(1 - a^2*x^2)) - (5*Arc
Cosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^3) + (3*ArcCosh[a*x]^3*ArcTanh[E^Ar
cCosh[a*x]])/(4*a*c^3) - (5*PolyLog[2, -E^ArcCosh[a*x]])/(2*a*c^3) + (9*Arc
Cosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]])/(8*a*c^3) + (5*PolyLog[2, E^ArcCos
h[a*x]])/(2*a*c^3) - (9*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]])/(8*a*c^3
```

```
) - (9*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])/(4*a*c^3) + (9*ArcCosh[a*x
]*PolyLog[3, E^ArcCosh[a*x]])/(4*a*c^3) + (9*PolyLog[4, -E^ArcCosh[a*x]])/(
4*a*c^3) - (9*PolyLog[4, E^ArcCosh[a*x]])/(4*a*c^3)
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^3}\,dx = \frac{x\cosh^{-1}(ax)^3}{4c^3\left(1-a^2x^2\right)^2} - \frac{(3a)\int \frac{x\cosh^{-1}(ax)^2}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{4c^3} + \frac{3\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} + \frac{x\cosh^{-1}(ax)^3}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)^3}{8c^3\left(1-a^2x^2\right)} - \frac{\int \frac{\cosh^{-1}(ax)}{\left(-1+a^2x^2\right)^2}\,dx}{2c^3} + \frac{(9a)\int \frac{x\cos}{(-1+ax)}}{8c}$$

$$= -\frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(ax)^2}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\,\mathrm{co}}{8c^3(}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(ax)^2}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} +$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(ax)^2}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} +$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(ax)^2}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} +$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(ax)^2}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} +$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(ax)^2}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} +$$

Mathematica [A] time = 8.253, size = 455, normalized size = 1.18

$$-\frac{72\cosh^{-1}(ax)^2\mathrm{PolyLog}\left(2, e^{\cosh^{-1}(ax)}\right) + 144\cosh^{-1}(ax)\mathrm{PolyLog}\left(3, -e^{-\cosh^{-1}(ax)}\right) - 144\cosh^{-1}(ax)\mathrm{PolyLog}\left(3, e^{\cos}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^3,x]
```

```
[Out] -(3*Pi^4 - 6*ArcCosh[a*x]^4 - 8*Coth[ArcCosh[a*x]/2] + 40*ArcCosh[a*x]^2*Co
th[ArcCosh[a*x]/2] - 4*ArcCosh[a*x]*Csch[ArcCosh[a*x]/2]^2 + 6*ArcCosh[a*x]
```

```
^3*Csch[ArcCosh[a*x]/2]^2 - Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*
x]^2*Csch[ArcCosh[a*x]/2]^4 - ArcCosh[a*x]^3*Csch[ArcCosh[a*x]/2]^4 - 160*A
rcCosh[a*x]*Log[1 - E^(-ArcCosh[a*x])] + 160*ArcCosh[a*x]*Log[1 + E^(-ArcCo
sh[a*x])] - 24*ArcCosh[a*x]^3*Log[1 + E^(-ArcCosh[a*x])] + 24*ArcCosh[a*x]^
3*Log[1 - E^ArcCosh[a*x]] + 8*(-20 + 9*ArcCosh[a*x]^2)*PolyLog[2, -E^(-ArcC
osh[a*x])] + 160*PolyLog[2, E^(-ArcCosh[a*x])] + 72*ArcCosh[a*x]^2*PolyLog[
2, E^ArcCosh[a*x]] + 144*ArcCosh[a*x]*PolyLog[3, -E^(-ArcCosh[a*x])] - 144*
ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]] + 144*PolyLog[4, -E^(-ArcCosh[a*x])
] + 144*PolyLog[4, E^ArcCosh[a*x]] - 4*ArcCosh[a*x]*Sech[ArcCosh[a*x]/2]^2
+ 6*ArcCosh[a*x]^3*Sech[ArcCosh[a*x]/2]^2 + ArcCosh[a*x]^3*Sech[ArcCosh[a*x
]/2]^4 - (16*ArcCosh[a*x]^2*Sinh[ArcCosh[a*x]/2]^4)/(((-1 + a*x)/(1 + a*x))
^(3/2)*(1 + a*x)^3) + 8*Tanh[ArcCosh[a*x]/2] - 40*ArcCosh[a*x]^2*Tanh[ArcCo
sh[a*x]/2])/(64*a*c^3)
```

Maple [A] time = 0.171, size = 710, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -3/8*a^2/(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)^3*x^3-9/8*a/(a^4*x^4-2*a^2*
x^2+1)/c^3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*x^2+1/4*a^2/(a^4*x^4-
2*a^2*x^2+1)/c^3*x^3*arccosh(a*x)+1/4*a/(a^4*x^4-2*a^2*x^2+1)/c^3*(a*x+1)^(
1/2)*(a*x-1)^(1/2)*x^2+5/8/(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)^3*x+11/8/
a/(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/4/
(a^4*x^4-2*a^2*x^2+1)/c^3*x*arccosh(a*x)-1/4/a/(a^4*x^4-2*a^2*x^2+1)/c^3*(a
*x-1)^(1/2)*(a*x+1)^(1/2)+5/2/a/c^3*arccosh(a*x)*ln(1-a*x-(a*x-1)^(1/2)*(a*
x+1)^(1/2))+5/2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-5/2/a/c^3*
arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-5/2*polylog(2,-a*x-(a*x-
1)^(1/2)*(a*x+1)^(1/2))/a/c^3-3/8/a/c^3*arccosh(a*x)^3*ln(1-a*x-(a*x-1)^(1/
2)*(a*x+1)^(1/2))-9/8*arccosh(a*x)^2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1
/2))/a/c^3+9/4*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^
3-9/4*polylog(4,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+3/8/a/c^3*arccosh(a*
x)^3*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+9/8*arccosh(a*x)^2*polylog(2,-a*
x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-9/4*arccosh(a*x)*polylog(3,-a*x-(a*x-1
)^(1/2)*(a*x+1)^(1/2))/a/c^3+9/4*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2)
)/a/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(6\,a^3x^3-10\,ax-3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+1\right)+3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax-1\right)\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{16\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)
)^3/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3) - integrate(-3/16*(6*a^5*x^5 - 16
*a^3*x^3 + (6*a^4*x^4 - 10*a^2*x^2 - 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x
+ 1) + 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x -
 1) + 10*a*x - 3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) + 3*(a^
6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sq
rt(a*x - 1))^2/(a^7*c^3*x^7 - 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 - a*c^3*x + (a^
6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(a*x + 1)*sqrt(a*x - 1
)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{arcosh}\left(ax\right)^3}{a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^3/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\operatorname{acosh}^3\left(ax\right)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(acosh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**
3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arcosh}(ax)^3}{\left(a^2cx^2 - c\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)^3/(a^2*c*x^2 - c)^3, x)
```

3.246 $\int \left(c-a^2cx^2\right)^{5/2} \cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=605

$$\frac{65a^3c^2x^4\sqrt{c-a^2cx^2}}{2304\sqrt{ax-1}\sqrt{ax+1}}-\frac{865ac^2x^2\sqrt{c-a^2cx^2}}{2304\sqrt{ax-1}\sqrt{ax+1}}+\frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15ac^2x^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{32\sqrt{ax-1}\sqrt{ax+1}}+\frac{5}{16}c^2x\sqrt{}$$

```
[Out] (-865*a*c^2*x^2*Sqrt[c - a^2*c*x^2])/(2304*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) +
(65*a^3*c^2*x^4*Sqrt[c - a^2*c*x^2])/(2304*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) +
(c^2*(1 - a^2*x^2)^3*Sqrt[c - a^2*c*x^2])/(216*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]) + (245*c^2*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/384 + (65*c^2*x*(1 - a*x
)*(1 + a*x)*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/576 + (c^2*x*(1 - a*x)^2*(1 +
 a*x)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/36 + (115*c^2*Sqrt[c - a^2*c*x^2]
*ArcCosh[a*x]^2)/(768*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*a*c^2*x^2*Sqrt[
c - a^2*c*x^2]*ArcCosh[a*x]^2)/(32*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (5*c^2*(
1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(32*a*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]) + (c^2*(1 - a^2*x^2)^3*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(12*
a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (5*c^2*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]
^3)/16 + (5*c*x*(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^3)/24 + (x*(c - a^2*c*x^
2)^(5/2)*ArcCosh[a*x]^3)/6 - (5*c^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(64
*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 1.49667, antiderivative size = 636, normalized size of antiderivative = 1.05, number of steps used = 25, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {5713, 5685, 5683, 5676, 5662, 5759, 30, 5716, 14, 261}

$$\frac{65a^3c^2x^4\sqrt{c-a^2cx^2}}{2304\sqrt{ax-1}\sqrt{ax+1}}-\frac{865ac^2x^2\sqrt{c-a^2cx^2}}{2304\sqrt{ax-1}\sqrt{ax+1}}+\frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15ac^2x^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{32\sqrt{ax-1}\sqrt{ax+1}}+\frac{5}{16}c^2x\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^3,x]
```


```
[Out] (-865*a*c^2*x^2*Sqrt[c - a^2*c*x^2])/(2304*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) +
(65*a^3*c^2*x^4*Sqrt[c - a^2*c*x^2])/(2304*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) +
(c^2*(1 - a^2*x^2)^3*Sqrt[c - a^2*c*x^2])/(216*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]) + (245*c^2*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/384 + (65*c^2*x*(1 - a*x
)*(1 + a*x)*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/576 + (c^2*x*(1 - a*x)^2*(1 +
 a*x)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/36 + (115*c^2*Sqrt[c - a^2*c*x^2]
*ArcCosh[a*x]^2)/(768*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*a*c^2*x^2*Sqrt[
```

```
c - a^2*c*x^2]*ArcCosh[a*x]^2)/(32*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (5*c^2*(
1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(32*a*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]) + (c^2*(1 - a^2*x^2)^3*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(12*
a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (5*c^2*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]
^3)/16 + (5*c^2*x*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^3)/2
4 + (c^2*x*(1 - a*x)^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^3)/6 -
(5*c^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
```

```
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^3\,dx=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^3\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$=\frac{1}{6}c^2x(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3-\frac{\left(5c^2\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+a}{6\sqrt{-1+ax}\sqrt{1+a}$$

$$=\frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{12a\sqrt{-1+ax}\sqrt{1+ax}}+\frac{5}{24}c^2x(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax)$$

$$=\frac{1}{36}c^2x(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)+\frac{5c^2\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\cosh^{-1}(a}{32a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$=\frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{-1+ax}\sqrt{1+ax}}+\frac{65}{576}c^2x(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax)+\frac{1}{36}c^2x(1$$

$$=\frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{-1+ax}\sqrt{1+ax}}+\frac{245}{384}c^2x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)+\frac{65}{576}c^2x(1-ax)(1+ax)$$

$$=-\frac{865ac^2x^2\sqrt{c-a^2cx^2}}{2304\sqrt{-1+ax}\sqrt{1+ax}}+\frac{65a^3c^2x^4\sqrt{c-a^2cx^2}}{2304\sqrt{-1+ax}\sqrt{1+ax}}+\frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{-1+ax}\sqrt{1+ax}}+\frac{245}{384}$$

Mathematica [A] time = 1.15302, size = 189, normalized size = 0.31

$$c^2\sqrt{c-a^2cx^2}\left(-4320\cosh^{-1}(ax)^4-72\left(270\cosh\left(2\cosh^{-1}(ax)\right)-27\cosh\left(4\cosh^{-1}(ax)\right)+2\cosh\left(6\cosh^{-1}(ax)\right)\right)\cosh$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^3,x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(-4320*ArcCosh[a*x]^4 - 9720*Cosh[2*ArcCosh[a*x]]
+ 243*Cosh[4*ArcCosh[a*x]] - 8*Cosh[6*ArcCosh[a*x]] - 72*ArcCosh[a*x]^2*(27
0*Cosh[2*ArcCosh[a*x]] - 27*Cosh[4*ArcCosh[a*x]] + 2*Cosh[6*ArcCosh[a*x]])
+ 288*ArcCosh[a*x]^3*(45*Sinh[2*ArcCosh[a*x]] - 9*Sinh[4*ArcCosh[a*x]] + Si
nh[6*ArcCosh[a*x]]) + 12*ArcCosh[a*x]*(1620*Sinh[2*ArcCosh[a*x]] - 81*Sinh[
```

```
4*ArcCosh[a*x]] + 4*Sinh[6*ArcCosh[a*x]])))/(55296*a*Sqrt[(-1 + a*x)/(1 + a
*x)]*(1 + a*x))
```

Maple [A] time = 0.259, size = 887, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x)
```

```
[Out] -5/64*(-c*(a^2*x^2-1))^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)/a*arccosh(a*x)^4*c
^2+1/13824*(-c*(a^2*x^2-1))^(1/2)*(32*a^7*x^7-64*x^5*a^5+32*(a*x-1)^(1/2)*(
a*x+1)^(1/2)*a^6*x^6+38*x^3*a^3-48*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^4*a^4-6*a*
x+18*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(36*a
rccosh(a*x)^3-18*arccosh(a*x)^2+6*arccosh(a*x)-1)*c^2/(a*x-1)/(a*x+1)/a-3/4
096*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-12*x^3*a^3+8*(a*x+1)^(1/2)*(a*x-1)^(1
/2)*x^4*a^4+4*a*x-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2)*(a*x+
1)^(1/2))*(32*arccosh(a*x)^3-24*arccosh(a*x)^2+12*arccosh(a*x)-3)*c^2/(a*x-
1)/(a*x+1)/a+15/512*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3-2*a*x+2*(a*x+1)^(1/2)
*(a*x-1)^(1/2)*x^2*a^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arc
cosh(a*x)^2+6*arccosh(a*x)-3)*c^2/(a*x-1)/(a*x+1)/a+15/512*(-c*(a^2*x^2-1))
^(1/2)*(2*x^3*a^3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)*c^2/(a
*x-1)/(a*x+1)/a-3/4096*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-12*x^3*a^3-8*(a*x+
1)^(1/2)*(a*x-1)^(1/2)*x^4*a^4+4*a*x+8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-
(a*x-1)^(1/2)*(a*x+1)^(1/2))*(32*arccosh(a*x)^3+24*arccosh(a*x)^2+12*arccos
h(a*x)+3)*c^2/(a*x-1)/(a*x+1)/a+1/13824*(-c*(a^2*x^2-1))^(1/2)*(-32*(a*x-1)
^(1/2)*(a*x+1)^(1/2)*a^6*x^6+32*a^7*x^7+48*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^4*
a^4-64*x^5*a^5-18*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+38*x^3*a^3+(a*x-1)^(1
/2)*(a*x+1)^(1/2)-6*a*x)*(36*arccosh(a*x)^3+18*arccosh(a*x)^2+6*arccosh(a*x
)+1)*c^2/(a*x-1)/(a*x+1)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^4 - 2\,a^2c^2x^2 + c^2\right)\sqrt{-a^2cx^2 + c}\,\text{arcosh}\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(-a^2*c*x^2 + c)*arccosh(a
*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*acosh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2 + c\right)^{\frac{5}{2}}\text{arcosh}\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^3, x)
```

3.247 $\int \left(c-a^2cx^2\right)^{3/2} \cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=402

$$\frac{3a^3cx^4\sqrt{c-a^2cx^2}}{128\sqrt{ax-1}\sqrt{ax+1}}-\frac{51acx^2\sqrt{c-a^2cx^2}}{128\sqrt{ax-1}\sqrt{ax+1}}-\frac{9acx^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{16\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^3+\frac{3}{8}cx\sqrt{c-$$

```
[Out] (-51*a*c*x^2*Sqrt[c - a^2*c*x^2])/(128*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*a
^3*c*x^4*Sqrt[c - a^2*c*x^2])/(128*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (45*c*x*
Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/64 + (3*c*x*(1 - a*x)*(1 + a*x)*Sqrt[c -
a^2*c*x^2]*ArcCosh[a*x])/32 + (27*c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(12
8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (9*a*c*x^2*Sqrt[c - a^2*c*x^2]*ArcCosh[
a*x]^2)/(16*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*(1 - a^2*x^2)^2*Sqrt[c - a
^2*c*x^2]*ArcCosh[a*x]^2)/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt
[c - a^2*c*x^2]*ArcCosh[a*x]^3)/8 + (x*(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^3
)/4 - (3*c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(32*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x])
```

Rubi [A] time = 0.945539, antiderivative size = 414, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5713, 5685, 5683, 5676, 5662, 5759, 30, 5716, 14}

$$\frac{3a^3cx^4\sqrt{c-a^2cx^2}}{128\sqrt{ax-1}\sqrt{ax+1}}-\frac{51acx^2\sqrt{c-a^2cx^2}}{128\sqrt{ax-1}\sqrt{ax+1}}-\frac{9acx^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{16\sqrt{ax-1}\sqrt{ax+1}}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3+\frac{1}{4}cx(1-a$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^3,x]
```

```
[Out] (-51*a*c*x^2*Sqrt[c - a^2*c*x^2])/(128*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*a
^3*c*x^4*Sqrt[c - a^2*c*x^2])/(128*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (45*c*x*
Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/64 + (3*c*x*(1 - a*x)*(1 + a*x)*Sqrt[c -
a^2*c*x^2]*ArcCosh[a*x])/32 + (27*c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(12
8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (9*a*c*x^2*Sqrt[c - a^2*c*x^2]*ArcCosh[
a*x]^2)/(16*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*(1 - a^2*x^2)^2*Sqrt[c - a
^2*c*x^2]*ArcCosh[a*x]^2)/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt
[c - a^2*c*x^2]*ArcCosh[a*x]^3)/8 + (c*x*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c
*x^2]*ArcCosh[a*x]^3)/4 - (3*c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(32*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^3\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^3\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3 + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int\sqrt{-1+ax}\sqrt{1+ax}\cos}{4\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{16a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3 + \frac{1}{4}cx(1-ax)(1$$

$$= \frac{3}{32}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax) - \frac{9acx^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{16\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-\right.}{}$$

$$= \frac{45}{64}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax) + \frac{3}{32}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax) - \frac{9acx^2\sqrt{c}}{16\sqrt{}}$$

$$= -\frac{51acx^2\sqrt{c-a^2cx^2}}{128\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3a^3cx^4\sqrt{c-a^2cx^2}}{128\sqrt{-1+ax}\sqrt{1+ax}} + \frac{45}{64}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax) + \frac{3}{32}c$$

Mathematica [A] time = 0.493394, size = 148, normalized size = 0.37

$$-\frac{c\sqrt{c-a^2cx^2}\left(96\cosh^{-1}(ax)^4 - 24\left(\cosh\left(4\cosh^{-1}(ax)\right) - 16\cosh\left(2\cosh^{-1}(ax)\right)\right)\cosh^{-1}(ax)^2 - 3\left(\cosh\left(4\cosh^{-1}(ax)\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^3,x]
```

```
[Out] -(c*Sqrt[c - a^2*c*x^2]*(96*ArcCosh[a*x]^4 - 3*(-64*Cosh[2*ArcCosh[a*x]] +
Cosh[4*ArcCosh[a*x]]) - 24*ArcCosh[a*x]^2*(-16*Cosh[2*ArcCosh[a*x]] + Cosh[
4*ArcCosh[a*x]]) + 12*ArcCosh[a*x]*(-32*Sinh[2*ArcCosh[a*x]] + Sinh[4*ArcCo
sh[a*x]]) + 32*ArcCosh[a*x]^3*(-8*Sinh[2*ArcCosh[a*x]] + Sinh[4*ArcCosh[a*x
]])))/(1024*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [A] time = 0.174, size = 536, normalized size = 1.3

$$-\frac{3\,(\mathrm{arccosh}\,(ax))^4\,c}{32\,a}\sqrt{-c\left(a^2x^2-1\right)}\frac{1}{\sqrt{ax-1}}\frac{1}{\sqrt{ax+1}} - \frac{\left(32\,(\mathrm{arccosh}\,(ax))^3 - 24\,(\mathrm{arccosh}\,(ax))^2 + 12\,\mathrm{arccosh}\,(ax) - 3\right.}{(2048\,ax-2048)\,(ax+1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x)
```

```
[Out] -3/32*(-c*(a^2*x^2-1))^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)/a*arccosh(a*x)^4*c
-1/2048*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-12*x^3*a^3+8*(a*x+1)^(1/2)*(a*x-1
)^(1/2)*x^4*a^4+4*a*x-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2)*(
a*x+1)^(1/2))*(32*arccosh(a*x)^3-24*arccosh(a*x)^2+12*arccosh(a*x)-3)*c/(a*
x-1)/(a*x+1)/a+1/32*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3-2*a*x+2*(a*x+1)^(1/2)
*(a*x-1)^(1/2)*x^2*a^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arc
cosh(a*x)^2+6*arccosh(a*x)-3)*c/(a*x-1)/(a*x+1)/a+1/32*(-c*(a^2*x^2-1))^(1/
2)*(2*x^3*a^3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2)*(a*
x+1)^(1/2))*(4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)*c/(a*x-1)/
(a*x+1)/a-1/2048*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-12*x^3*a^3-8*(a*x+1)^(1/
2)*(a*x-1)^(1/2)*x^4*a^4+4*a*x+8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-(a*x-1
)^(1/2)*(a*x+1)^(1/2))*(32*arccosh(a*x)^3+24*arccosh(a*x)^2+12*arccosh(a*x)
+3)*c/(a*x-1)/(a*x+1)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2cx^2-c\right)\sqrt{-a^2cx^2+c}\,\mathrm{arcosh}\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arcosh}\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^3, x)
```

3.248 $\int \sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=231

$$-\frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^4}{8a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3-\frac{3ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{4\sqrt{ax-1}\sqrt{ax+1}}+\frac{3\sqrt{c-a^2}}{8a\sqrt{ax}}$$

```
[Out] (-3*a*x^2*Sqrt[c - a^2*c*x^2])/(8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*x*Sqrt
[c - a^2*c*x^2]*ArcCosh[a*x])/4 + (3*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(8
*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (3*a*x^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x
]^2)/(4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]
^3)/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(8*a*Sqrt[-1 + a*x]*Sqrt[1 + a
*x])
```

Rubi [A] time = 0.536833, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5713, 5683, 5676, 5662, 5759, 30}

$$-\frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^4}{8a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3-\frac{3ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{4\sqrt{ax-1}\sqrt{ax+1}}+\frac{3\sqrt{c-a^2}}{8a\sqrt{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^3,x]
```


```
[Out] (-3*a*x^2*Sqrt[c - a^2*c*x^2])/(8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*x*Sqrt
[c - a^2*c*x^2]*ArcCosh[a*x])/4 + (3*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(8
*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (3*a*x^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x
]^2)/(4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]
^3)/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(8*a*Sqrt[-1 + a*x]*Sqrt[1 + a
*x])
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3\,dx = \frac{\sqrt{c-a^2cx^2}\int \sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3 - \frac{\sqrt{c-a^2cx^2}\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(3a\sqrt{c-a^2cx^2}\right)\int x\cosh^{-}}{2\sqrt{-1+ax}\sqrt{1+a}}$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{4\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3 - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^4}{8a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{3}{4}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax) - \frac{3ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{4\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3}{4}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax) + \frac{3\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{8a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{}}{4}$$

Mathematica [A] time = 0.200555, size = 98, normalized size = 0.42

$$-\frac{\sqrt{-c(ax-1)(ax+1)}\left(2\cosh^{-1}(ax)^4+\left(6\cosh^{-1}(ax)^2+3\right)\cosh\left(2\cosh^{-1}(ax)\right)-2\left(2\cosh^{-1}(ax)^2+3\right)\cosh^{-1}(ax)\sin}{16a\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^3,x]
```

```
[Out] -(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(2*ArcCosh[a*x]^4 + (3 + 6*ArcCosh[a*x]^2
)*Cosh[2*ArcCosh[a*x]] - 2*ArcCosh[a*x]*(3 + 2*ArcCosh[a*x]^2)*Sinh[2*ArcCo
sh[a*x]]))/(16*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [A] time = 0.204, size = 256, normalized size = 1.1

$$-\frac{(\mathrm{arccosh}\,(ax))^4}{8a}\sqrt{-c\left(a^2x^2-1\right)}\frac{1}{\sqrt{ax-1}}\frac{1}{\sqrt{ax+1}} + \frac{4\,(\mathrm{arccosh}\,(ax))^3-6\,(\mathrm{arccosh}\,(ax))^2+6\,\mathrm{arccosh}\,(ax)-3}{(32\,ax-32)\,(ax+1)\,a}\sqrt{-c\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/8*(-c*(a^2*x^2-1))^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)/a*arccosh(a*x)^4+1/
32*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^
2*a^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arccosh(a*x)^2+6*arc
cosh(a*x)-3)/(a*x-1)/(a*x+1)/a+1/32*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3-2*a*x
-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arcc
osh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/(a*x-1)/(a*x+1)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a^2cx^2+c}\,\text{arcosh}\,(ax)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c\,(ax-1)\,(ax+1)}\,\text{acosh}^3\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*acosh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\,\text{arcosh}\,(ax)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3, x)
```

3.249 $\int \frac{\cosh^{-1}(ax)^3}{\sqrt{c-a^2cx^2}} dx$

Optimal. Leaf size=46

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.156155, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5713, 5676}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[c - a^2*c*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}} = \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0314924, size = 46, normalized size = 1.

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^3/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.039, size = 55, normalized size = 1.2

$$-\frac{(\operatorname{arccosh}(ax))^4}{4\,ca\left(a^2x^2-1\right)}\sqrt{-(ax-1)(ax+1)c}\sqrt{ax-1}\sqrt{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/4*(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c/(a^2*x^2-1)
*arccosh(a*x)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}(ax)^3}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^3(ax)}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**3/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*c*x^2 + c), x)
```

3.250 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{3/2}} dx$

Optimal. Leaf size=241

$$-\frac{3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{ac\sqrt{c-a^2cx^2}}+\frac{3\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)}{2ac\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}}$$

```
[Out] (x*ArcCosh[a*x]^3)/(c*Sqrt[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*
ArcCosh[a*x]^3)/(a*c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*ArcCosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])])/(a*c*Sqrt[c - a^2*c*x^2]) - (3
*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])])/
(a*c*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, E^(2
*ArcCosh[a*x])])/(2*a*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.348938, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5713, 5688, 5715, 3716, 2190, 2531, 2282, 6589}

$$-\frac{3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{ac\sqrt{c-a^2cx^2}}+\frac{3\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)}{2ac\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] (x*ArcCosh[a*x]^3)/(c*Sqrt[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*
ArcCosh[a*x]^3)/(a*c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*ArcCosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])])/(a*c*Sqrt[c - a^2*c*x^2]) - (3
*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])])/
(a*c*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, E^(2
*ArcCosh[a*x])])/(2*a*c*Sqrt[c - a^2*c*x^2])
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
```

```
!IntegerQ[p]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\left(3a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^2}{1-a^2x^2}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} - \frac{\left(3\sqrt{-1+ax}\sqrt{1+ax}\right)\operatorname{Subst}\left(\int x^2\coth(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} + \frac{\left(6\sqrt{-1+ax}\sqrt{1+ax}\right)\operatorname{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx, x, \mathrm{co}\right.}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\log\left(1-e^{2\cos}\right.}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\log\left(1-e^{2\cos}\right.}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\log\left(1-e^{2\cos}\right.}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\log\left(1-e^{2\cos}\right.}{ac\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.239766, size = 145, normalized size = 0.6

$$\frac{\frac{\sqrt{ax-1}\sqrt{ax+1}\left(-6\cosh^{-1}(ax)\operatorname{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)-6\cosh^{-1}(ax)\operatorname{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)+6\operatorname{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)+6\operatorname{PolyLog}\left(3,e^{\cosh^{-1}(ax)}\right)+\cosh^{-1}\right.}{a}}{c\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^3 + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(ArcCosh[a*x]^3 - 3*ArcCo
sh[a*x]^2*Log[1 - E^ArcCosh[a*x]] - 3*ArcCosh[a*x]^2*Log[1 + E^ArcCosh[a*x]
] - 6*ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]] - 6*ArcCosh[a*x]*PolyLog[2,
E^ArcCosh[a*x]] + 6*PolyLog[3, -E^ArcCosh[a*x]] + 6*PolyLog[3, E^ArcCosh[a*
x]]))/a)/(c*Sqrt[c - a^2*c*x^2])
```

Maple [B] time = 0.21, size = 548, normalized size = 2.3

$$-\frac{(\operatorname{arccosh}(ax))^3}{ac^2\left(a^2x^2-1\right)}\sqrt{-c\left(a^2x^2-1\right)}\left(-\sqrt{ax-1}\sqrt{ax+1}+ax\right)-2\,\frac{\sqrt{ax-1}\sqrt{ax+1}\sqrt{-c\left(a^2x^2-1\right)}\,(\operatorname{arccosh}(ax))^3}{ac^2\left(a^2x^2-1\right)}+3\,\frac{\sqrt{ax}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-(a*x-1)^(1/2)*(a*x+1)^(1/2)+a*x)*arccosh(a*x)^3/c
^2/a/(a^2*x^2-1)-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^2/a
/(a^2*x^2-1)*arccosh(a*x)^3+3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^
(1/2)/c^2/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2)
)+6*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^2/a/(a^2*x^2-1)*ar
ccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-6*(a*x+1)^(1/2)*(a*x-
1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^2/a/(a^2*x^2-1)*polylog(3,a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2))+3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^2/
a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+6*(a*x+1
)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^2/a/(a^2*x^2-1)*arccosh(a*x)
*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-6*(a*x+1)^(1/2)*(a*x-1)^(1/2)*
(-c*(a^2*x^2-1))^(1/2)/c^2/a/(a^2*x^2-1)*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+
1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\text{arcosh}\,(ax)^3}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 +
 c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acosh}^3\,(ax)}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(acosh(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcosh}\,(ax)^3}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(3/2), x)
```

3.251 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=413

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}}{2ac^2\sqrt{c}}$$

```
[Out] -((x*ArcCosh[a*x])/(c^2*Sqrt[c - a^2*c*x^2])) + (Sqrt[-1 + a*x]*Sqrt[1 + a*
x]*ArcCosh[a*x]^2)/(2*a*c^2*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh
[a*x]^3)/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x*ArcCosh[a*x]^3)/(3*c^2*Sqrt[c -
 a^2*c*x^2]) + (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(3*a*c^2*Sqr
t[c - a^2*c*x^2]) - (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*Log[1 -
E^(2*ArcCosh[a*x])])/(a*c^2*Sqrt[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*Log[1 - a^2*x^2])/(2*a*c^2*Sqrt[c - a^2*c*x^2]) - (2*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])])/(a*c^2*Sqrt[c - a
^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, E^(2*ArcCosh[a*x])])/
(a*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.637823, antiderivative size = 428, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5713, 5691, 5688, 5715, 3716, 2190, 2531, 2282, 6589, 5716, 260}

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}}{2ac^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x*ArcCosh[a*x])/(c^2*Sqrt[c - a^2*c*x^2])) + (Sqrt[-1 + a*x]*Sqrt[1 + a*
x]*ArcCosh[a*x]^2)/(2*a*c^2*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]) + (2*x*ArcCo
sh[a*x]^3)/(3*c^2*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh[a*x]^3)/(3*c^2*(1 - a*x
)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[
a*x]^3)/(3*a*c^2*Sqrt[c - a^2*c*x^2]) - (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Arc
Cosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])])/(a*c^2*Sqrt[c - a^2*c*x^2]) + (Sqr
t[-1 + a*x]*Sqrt[1 + a*x]*Log[1 - a^2*x^2])/(2*a*c^2*Sqrt[c - a^2*c*x^2]) -
 (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])
])/(a*c^2*Sqrt[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, E
```

```
^(2*ArcCosh[a*x])])/(a*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}} - \frac{\left(2\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{3c^2\sqrt{c-a^2cx^2}} + \frac{\left(a\sqrt{-1+ax}\sqrt{1+a\ldots\right.}{c^2\sqrt{c-\ldots}}$$

$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\left(\sqrt{-1+ax}\ldots\right.}{\ldots}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2c\ldots}}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2c\ldots}}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2c\ldots}}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2c\ldots}}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2c\ldots}}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2c\ldots}}$$

Mathematica [C] time = 0.942065, size = 258, normalized size = 0.62

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(-24\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)+12\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)+\frac{6\cosh^{-1}(ax)^2}{1-a^2x^2}+12\log\left(\sqrt{\frac{ax-1}{ax+1}}(ax\ldots\right.\right.}{12\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*((-I)*Pi^3 - (12*a*x*Sqrt[(-1 + a*x)/
(1 + a*x)]*ArcCosh[a*x])/(-1 + a*x) + (6*ArcCosh[a*x]^2)/(1 - a^2*x^2) + 8*
ArcCosh[a*x]^3 + (8*a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3)/(-1 + a*
x) - (4*a*x*((-1 + a*x)/(1 + a*x))^(3/2)*ArcCosh[a*x]^3)/(-1 + a*x)^3 - 24*
ArcCosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])] + 12*Log[Sqrt[(-1 + a*x)/(1 + a*
x)]*(1 + a*x)] - 24*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])] + 12*PolyLo
g[3, E^(2*ArcCosh[a*x])]))/(12*a*c^2*Sqrt[c - a^2*c*x^2])
```

Maple [B] time = 0.315, size = 955, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/6*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3-3*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*
x^2*a^2+2*(a*x-1)^(1/2)*(a*x+1)^(1/2))*arccosh(a*x)*(6*arccosh(a*x)*(a*x-1)
^(1/2)*(a*x+1)^(1/2)*a^3*x^3+6*arccosh(a*x)*x^4*a^4+6*a^3*x^3*(a*x-1)^(1/2)
*(a*x+1)^(1/2)+6*x^4*a^4+6*arccosh(a*x)^2*a^2*x^2-9*arccosh(a*x)*a*x*(a*x-1
)^(1/2)*(a*x+1)^(1/2)-12*a^2*x^2*arccosh(a*x)-6*(a*x+1)^(1/2)*(a*x-1)^(1/2)
*a*x-18*a^2*x^2-8*arccosh(a*x)^2+6*arccosh(a*x)+12)/(3*a^6*x^6-10*a^4*x^4+1
1*a^2*x^2-4)/a/c^3-(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^3/a
/(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)-1)+2*(a*x+1)^(1/2)*(a*x-1)^
(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^3/a/(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/2)*(a*x+1)
^(1/2))-(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^3/a/(a^2*x^2-1
)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-4/3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c
*(a^2*x^2-1))^(1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)^3+2*(a*x+1)^(1/2)*(a*x-1
)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1-a*x-(a
*x-1)^(1/2)*(a*x+1)^(1/2))+4*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(
1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/
2))-4*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^3/a/(a^2*x^2-1)*
polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-
c*(a^2*x^2-1))^(1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2
)*(a*x+1)^(1/2))+4*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^3/a
/(a^2*x^2-1)*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-4*(a*
x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^3/a/(a^2*x^2-1)*polylog(3
,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}(ax)^3}{a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^6*c^3*x^6 - 3*a^4*c^3*x^4
+ 3*a^2*c^3*x^2 - c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^3(ax)}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(acosh(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(5/2), x)
```

3.252 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=607

$$-\frac{8\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}+\frac{4\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{ax-1}\sqrt{a}}{20ac^3\left(1-a^2x^2\right)}$$

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(20*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]
) - (x*ArcCosh[a*x])/(c^3*Sqrt[c - a^2*c*x^2]) - (x*ArcCosh[a*x])/(10*c^3*(
1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x]^2)/(20*a*c^3*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]) + (2*Sqrt[-1
+ a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(5*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*
x^2]) + (x*ArcCosh[a*x]^3)/(5*c*(c - a^2*c*x^2)^(5/2)) + (4*x*ArcCosh[a*x]^
3)/(15*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x*ArcCosh[a*x]^3)/(15*c^3*Sqrt[c - a
^2*c*x^2]) + (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(15*a*c^3*Sqrt
[c - a^2*c*x^2]) - (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*Log[1 - E
^(2*ArcCosh[a*x])])/(5*a*c^3*Sqrt[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*Log[1 - a^2*x^2])/(2*a*c^3*Sqrt[c - a^2*c*x^2]) - (8*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])])/(5*a*c^3*Sqrt[c
- a^2*c*x^2]) + (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, E^(2*ArcCosh[a*x
])])/(5*a*c^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 1.07689, antiderivative size = 637, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5713, 5691, 5688, 5715, 3716, 2190, 2531, 2282, 6589, 5716, 260, 261}

$$-\frac{8\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}+\frac{4\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{ax-1}\sqrt{a}}{20ac^3\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(20*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]
) - (x*ArcCosh[a*x])/(c^3*Sqrt[c - a^2*c*x^2]) - (x*ArcCosh[a*x])/(10*c^3*(
1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x]^2)/(20*a*c^3*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]) + (2*Sqrt[-1
+ a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(5*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*
```

```
x^2]) + (8*x*ArcCosh[a*x]^3)/(15*c^3*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh[a*x]
^3)/(5*c^3*(1 - a*x)^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]) + (4*x*ArcCosh[a*x]
^3)/(15*c^3*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (8*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]*ArcCosh[a*x]^3)/(15*a*c^3*Sqrt[c - a^2*c*x^2]) - (8*Sqrt[-1 + a
*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])])/(5*a*c^3*Sqrt
[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[1 - a^2*x^2])/(2*a*c^3
*Sqrt[c - a^2*c*x^2]) - (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLo
g[2, E^(2*ArcCosh[a*x])])/(5*a*c^3*Sqrt[c - a^2*c*x^2]) + (4*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]*PolyLog[3, E^(2*ArcCosh[a*x])])/(5*a*c^3*Sqrt[c - a^2*c*x^2]
)
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
```

```
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
```

```
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{7/2}(1+ax)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{\left(4\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{5c^3\sqrt{c-a^2cx^2}} - \frac{\left(3a\sqrt{-1+ax}\sqrt{1\right.}{5c^3\sqrt{}}$$

$$= \frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{4x\cosh^{-1}(ax)^3}{15c^3(1-ax)(1+ax)\sqrt{c-a^2}}$$

$$= -\frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}}{5ac^3\left(1-a^2x^2\right)\sqrt{c-a^2c}}$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1+ax}\sqrt{1}}{20ac^3\left(1-a^2\right.}$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1+ax}\sqrt{1}}{20ac^3\left(1-a^2\right.}$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1+ax}\sqrt{1}}{20ac^3\left(1-a^2\right.}$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1+ax}\sqrt{1}}{20ac^3\left(1-a^2\right.}$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1+ax}\sqrt{1}}{20ac^3\left(1-a^2\right.}$$

$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1+ax}\sqrt{1}}{20ac^3\left(1-a^2\right.}$$

Mathematica [C] time = 1.71108, size = 363, normalized size = 0.6

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(96\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)-48\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)+\frac{3}{1-a^2x^2}+\frac{24\cosh^{-1}(ax)^2}{a^2x^2-1}-\frac{9\cosh^{-1}(}{\left(a^2x^2-1\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*((4*I)*Pi^3 + 3/(1 - a^2*x^2) + (60*
a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x])/(-1 + a*x) - (6*a*x*((-1 + a*x
)/(1 + a*x))^(3/2)*ArcCosh[a*x])/(-1 + a*x)^3 - (9*ArcCosh[a*x]^2)/(-1 + a^
2*x^2)^2 + (24*ArcCosh[a*x]^2)/(-1 + a^2*x^2) - 32*ArcCosh[a*x]^3 - (32*a*x
*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3)/(-1 + a*x) + (16*a*x*((-1 + a*x
)/(1 + a*x))^(3/2)*ArcCosh[a*x]^3)/(-1 + a*x)^3 - (12*a*x*Sqrt[(-1 + a*x)/(
1 + a*x)]*ArcCosh[a*x]^3)/((-1 + a*x)^3*(1 + a*x)^2) + 96*ArcCosh[a*x]^2*Lo
g[1 - E^(2*ArcCosh[a*x])] - 60*Log[Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)] +
96*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])] - 48*PolyLog[3, E^(2*ArcCosh
[a*x])]))/(60*a*c^3*Sqrt[c - a^2*c*x^2])
```

Maple [B] time = 0.359, size = 1319, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/60*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-20*x^3*a^3-8*(a*x+1)^(1/2)*(a*x-1)^
(1/2)*x^4*a^4+15*a*x+16*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-8*(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(24-192*arccosh(a*x)^2*x^8*a^8+840*arccosh(a*x)^2*x^6*a^6+1
60*arccosh(a*x)^3*x^4*a^4-96*a^2*x^2-192*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccos
h(a*x)*x^7*a^7-192*arccosh(a*x)^2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^7*a^7+852*a
rccosh(a*x)*x^6*a^6-1368*arccosh(a*x)^2*x^4*a^4+256*arccosh(a*x)^3-264*arcc
osh(a*x)^2-936*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x^3+372*arccosh
(a*x)*a*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)+1410*a^2*x^2*arccosh(a*x)+105*a^3*x^3
*(a*x-1)^(1/2)*(a*x+1)^(1/2)-45*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+984*arccosh
(a*x)^2*a^2*x^2-380*arccosh(a*x)^3*a^2*x^2-192*arccosh(a*x)*x^8*a^8+24*x^8*
a^8-96*x^6*a^6+144*x^4*a^4+756*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^5
*x^5-480*arccosh(a*x)-1020*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x
^3+495*arccosh(a*x)^2*a*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)+24*(a*x+1)^(1/2)*(a*x
-1)^(1/2)*x^7*a^7-84*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^5*a^5-1590*arccosh(a*x)*
x^4*a^4+744*arccosh(a*x)^2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^5*a^5)/(40*a^10*x^
10-215*a^8*x^8+469*a^6*x^6-517*a^4*x^4+287*a^2*x^2-64)/a/c^4-(a*x+1)^(1/2)*
(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/2)
*(a*x+1)^(1/2)-1)+2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/
a/(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-(a*x+1)^(1/2)*(a*x-1)^(1/
```

```
2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^2-1)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^
(1/2))-16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*
x^2-1)*arccosh(a*x)^3+8/5*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2
)/c^4/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))+16
/5*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^2-1)*arc
cosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-16/5*(a*x+1)^(1/2)*(a*
x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^2-1)*polylog(3,a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2))+8/5*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/
c^4/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+16/5
*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^2-1)*arcco
sh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-16/5*(a*x+1)^(1/2)*(a*x
-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^2-1)*polylog(3,-a*x-(a*x-1)^(
1/2)*(a*x+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}(ax)^3}{a^8c^4x^8-4\,a^6c^4x^6+6\,a^4c^4x^4-4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 +
 6*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(7/2), x)
```

3.253 $\int \frac{x^4 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=315

$$-\frac{45x^2\sqrt{ax-1}}{128a^3\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}{4a^2}-\frac{3x^3\sqrt{1-ax}\sqrt{ax+1}\cosh^{-1}(ax)}{32a^2}-\frac{9x^2\sqrt{ax-1}\cosh^{-1}(ax)^2}{16a^3\sqrt{1-ax}}-\frac{3x\sqrt{1-a^2x^2}}{8}$$

```
[Out] (-45*x^2*Sqrt[-1 + a*x])/(128*a^3*Sqrt[1 - a*x]) - (3*x^4*Sqrt[-1 + a*x])/(
128*a*Sqrt[1 - a*x]) - (45*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(64*
a^4) - (3*x^3*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(32*a^2) + (45*Sqrt
[-1 + a*x]*ArcCosh[a*x]^2)/(128*a^5*Sqrt[1 - a*x]) - (9*x^2*Sqrt[-1 + a*x]*
ArcCosh[a*x]^2)/(16*a^3*Sqrt[1 - a*x]) - (3*x^4*Sqrt[-1 + a*x]*ArcCosh[a*x]
^2)/(16*a*Sqrt[1 - a*x]) - (3*x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/(8*a^4) -
 (x^3*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/(4*a^2) + (3*Sqrt[-1 + a*x]*ArcCosh
[a*x]^4)/(32*a^5*Sqrt[1 - a*x])
```

Rubi [A] time = 1.44163, antiderivative size = 427, normalized size of antiderivative = 1.36, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5798, 5759, 5676, 5662, 30}

$$-\frac{3x^4\sqrt{ax-1}\sqrt{ax+1}}{128a\sqrt{1-a^2x^2}}-\frac{45x^2\sqrt{ax-1}\sqrt{ax+1}}{128a^3\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{16a\sqrt{1-a^2x^2}}-\frac{x^3(1-ax)(ax+1)\cosh^{-1}(ax)^3}{4a^2\sqrt{1-a^2x^2}}-3x$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-45*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(128*a^3*Sqrt[1 - a^2*x^2]) - (3*x^4
*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(128*a*Sqrt[1 - a^2*x^2]) - (45*x*(1 - a*x)*
(1 + a*x)*ArcCosh[a*x])/(64*a^4*Sqrt[1 - a^2*x^2]) - (3*x^3*(1 - a*x)*(1 +
a*x)*ArcCosh[a*x])/(32*a^2*Sqrt[1 - a^2*x^2]) + (45*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*ArcCosh[a*x]^2)/(128*a^5*Sqrt[1 - a^2*x^2]) - (9*x^2*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]^2)/(16*a^3*Sqrt[1 - a^2*x^2]) - (3*x^4*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(16*a*Sqrt[1 - a^2*x^2]) - (3*x*(1 - a*x
)*(1 + a*x)*ArcCosh[a*x]^3)/(8*a^4*Sqrt[1 - a^2*x^2]) - (x^3*(1 - a*x)*(1 +
 a*x)*ArcCosh[a*x]^3)/(4*a^2*Sqrt[1 - a^2*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*ArcCosh[a*x]^4)/(32*a^5*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^4\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3(1-ax)(1+ax)\cosh^{-1}(ax)^3}{4a^2\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{-1+ax}\sqrt{1+a\right.}{4a\sqrt{}}$$

$$= -\frac{3x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{16a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)^3}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)\cosh}{4a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{3x^3(1-ax)(1+ax)\cosh^{-1}(ax)}{32a^2\sqrt{1-a^2x^2}} - \frac{9x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{16a^3\sqrt{1-a^2x^2}} - \frac{3x^4\sqrt{-1+ax}\sqrt{1+ax}\,c}{16a\sqrt{1-a^2x^2}}$$

$$= -\frac{3x^4\sqrt{-1+ax}\sqrt{1+ax}}{128a\sqrt{1-a^2x^2}} - \frac{45x(1-ax)(1+ax)\cosh^{-1}(ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{3x^3(1-ax)(1+ax)\cosh^{-1}(ax)}{32a^2\sqrt{1-a^2x^2}} - \frac{9x}{}$$

$$= -\frac{45x^2\sqrt{-1+ax}\sqrt{1+ax}}{128a^3\sqrt{1-a^2x^2}} - \frac{3x^4\sqrt{-1+ax}\sqrt{1+ax}}{128a\sqrt{1-a^2x^2}} - \frac{45x(1-ax)(1+ax)\cosh^{-1}(ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{3x^3(1-ax}{32}$$

Mathematica [A] time = 0.4389, size = 136, normalized size = 0.43

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(-192\left(2\cosh^{-1}(ax)^2+1\right)\cosh\left(2\cosh^{-1}(ax)\right)-3\left(8\cosh^{-1}(ax)^2+1\right)\cosh\left(4\cosh^{-1}(ax)\right)+4\cosh^{-1}(a\right.}{1024a^5\sqrt{-(ax-1)(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-192*(1 + 2*ArcCosh[a*x]^2)*Cosh[2*A
rcCosh[a*x]] - 3*(1 + 8*ArcCosh[a*x]^2)*Cosh[4*ArcCosh[a*x]] + 4*ArcCosh[a*
x]*(24*ArcCosh[a*x]^3 + 32*(3 + 2*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]] + (3
 + 8*ArcCosh[a*x]^2)*Sinh[4*ArcCosh[a*x]])))/(1024*a^5*Sqrt[-((-1 + a*x)*(1
 + a*x))])
```

Maple [B] time = 0.28, size = 520, normalized size = 1.7

$$-\frac{3\,(\mathrm{arccosh}\,(ax))^4}{32\,a^5\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1} - \frac{32\,(\mathrm{arccosh}\,(ax))^3-24\,(\mathrm{arccosh}\,(ax))^2+12\,\mathrm{arccosh}\,(ax)-3}{2048\,a^5\left(a^2x^2-1\right)}\sqrt{-a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -3/32*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arccos
h(a*x)^4-1/2048*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5-12*x^3*a^3+8*(a*x+1)^(1/2)*(a
*x-1)^(1/2)*x^4*a^4+4*a*x-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/
2)*(a*x+1)^(1/2))*(32*arccosh(a*x)^3-24*arccosh(a*x)^2+12*arccosh(a*x)-3)/a
^5/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x+2*(a*x+1)^(1/2)*(a*
x-1)^(1/2)*x^2*a^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arccosh
(a*x)^2+6*arccosh(a*x)-3)/a^5/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(2*x^3*a^
3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*
(4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/a^5/(a^2*x^2-1)-1/2048
*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5-12*x^3*a^3-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^4
*a^4+4*a*x+8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2-(a*x-1)^(1/2)*(a*x+1)^(1/2
))*(32*arccosh(a*x)^3+24*arccosh(a*x)^2+12*arccosh(a*x)+3)/a^5/(a^2*x^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^4\operatorname{arcosh}(ax)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

3.254 $\int \frac{x^3 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=243

$$-\frac{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}{3a^2}-\frac{2x^2\sqrt{1-ax}\sqrt{ax+1}\cosh^{-1}(ax)}{9a^2}-\frac{2\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}{3a^4}-\frac{40x\sqrt{ax-1}}{9a^3\sqrt{1-ax}}-\frac{2x\sqrt{ax-1}\,\mathrm{co}}{a^3\sqrt{1-}}$$

```
[Out] (-40*x*Sqrt[-1 + a*x])/(9*a^3*Sqrt[1 - a*x]) - (2*x^3*Sqrt[-1 + a*x])/(27*a
*Sqrt[1 - a*x]) - (40*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(9*a^4) - (
2*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(9*a^2) - (2*x*Sqrt[-1 + a*
x]*ArcCosh[a*x]^2)/(a^3*Sqrt[1 - a*x]) - (x^3*Sqrt[-1 + a*x]*ArcCosh[a*x]^2
)/(3*a*Sqrt[1 - a*x]) - (2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/(3*a^4) - (x^2
*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/(3*a^2)
```

Rubi [A] time = 1.02626, antiderivative size = 329, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5798, 5759, 5718, 5654, 8, 5662, 30}

$$-\frac{2x^3\sqrt{ax-1}\sqrt{ax+1}}{27a\sqrt{1-a^2x^2}}-\frac{40x\sqrt{ax-1}\sqrt{ax+1}}{9a^3\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{3a\sqrt{1-a^2x^2}}-\frac{x^2(1-ax)(ax+1)\cosh^{-1}(ax)^3}{3a^2\sqrt{1-a^2x^2}}-2x^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-40*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(9*a^3*Sqrt[1 - a^2*x^2]) - (2*x^3*Sqr
t[-1 + a*x]*Sqrt[1 + a*x])/(27*a*Sqrt[1 - a^2*x^2]) - (40*(1 - a*x)*(1 + a*
x)*ArcCosh[a*x])/(9*a^4*Sqrt[1 - a^2*x^2]) - (2*x^2*(1 - a*x)*(1 + a*x)*Arc
Cosh[a*x])/(9*a^2*Sqrt[1 - a^2*x^2]) - (2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Ar
cCosh[a*x]^2)/(a^3*Sqrt[1 - a^2*x^2]) - (x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x]^2)/(3*a*Sqrt[1 - a^2*x^2]) - (2*(1 - a*x)*(1 + a*x)*ArcCosh[a*x
]^3)/(3*a^4*Sqrt[1 - a^2*x^2]) - (x^2*(1 - a*x)*(1 + a*x)*ArcCosh[a*x]^3)/(
3*a^2*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^3\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)^3}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{3a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+ax}\sqrt{1+a\right.}{a\sqrt{}}$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{3a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)^3}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-ax)(1+ax)\cosh^{-}}{3a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{2x^2(1-ax)(1+ax)\cosh^{-1}(ax)}{9a^2\sqrt{1-a^2x^2}} - \frac{2x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{x^3\sqrt{-1+ax}\sqrt{1+ax}\,\mathrm{co}}{3a\sqrt{1-a^2x^2}}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{27a\sqrt{1-a^2x^2}} - \frac{40(1-ax)(1+ax)\cosh^{-1}(ax)}{9a^4\sqrt{1-a^2x^2}} - \frac{2x^2(1-ax)(1+ax)\cosh^{-1}(ax)}{9a^2\sqrt{1-a^2x^2}} - 2$$

$$= -\frac{40x\sqrt{-1+ax}\sqrt{1+ax}}{9a^3\sqrt{1-a^2x^2}} - \frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{27a\sqrt{1-a^2x^2}} - \frac{40(1-ax)(1+ax)\cosh^{-1}(ax)}{9a^4\sqrt{1-a^2x^2}} - \frac{2x^2(1-ax)}{9a}$$

Mathematica [A] time = 0.155989, size = 140, normalized size = 0.58

$$\frac{\sqrt{1-a^2x^2}\left(2ax\left(a^2x^2+60\right)-9\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2+2\right)\cosh^{-1}(ax)^3+9ax\left(a^2x^2+6\right)\cosh^{-1}(ax)^2-6\sqrt{ax-1}\sqrt{ax+}\right.}{27a^4\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*a*x*(60 + a^2*x^2) - 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(
20 + a^2*x^2)*ArcCosh[a*x] + 9*a*x*(6 + a^2*x^2)*ArcCosh[a*x]^2 - 9*Sqrt[-1
 + a*x]*Sqrt[1 + a*x]*(2 + a^2*x^2)*ArcCosh[a*x]^3))/(27*a^4*Sqrt[-1 + a*x]
*Sqrt[1 + a*x])
```

Maple [A] time = 0.198, size = 375, normalized size = 1.5

$$-\frac{9\left(\operatorname{arccosh}(ax)\right)^3-9\left(\operatorname{arccosh}(ax)\right)^2+6\operatorname{arccosh}(ax)-2}{216\,a^4\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\left(4\,x^4a^4-5\,a^2x^2+4\,a^3x^3\sqrt{ax-1}\sqrt{ax+1}-3\,\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/216*(-a^2*x^2+1)^(1/2)*(4*x^4*a^4-5*a^2*x^2+4*a^3*x^3*(a*x-1)^(1/2)*(a*x
+1)^(1/2)-3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(9*arccosh(a*x)^3-9*arccosh(
a*x)^2+6*arccosh(a*x)-2)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1
/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(arccosh(a*x)^3-3*arccosh(a*x)^2+6*arccosh
(a*x)-6)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x
-1)^(1/2)*a*x-1)*(arccosh(a*x)^3+3*arccosh(a*x)^2+6*arccosh(a*x)+6)/a^4/(a^
2*x^2-1)-1/216*(-a^2*x^2+1)^(1/2)*(4*x^4*a^4-5*a^2*x^2-4*a^3*x^3*(a*x-1)^(1
/2)*(a*x+1)^(1/2)+3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(9*arccosh(a*x)^3+9*
arccosh(a*x)^2+6*arccosh(a*x)+2)/a^4/(a^2*x^2-1)
```

Maxima [C] time = 1.80281, size = 177, normalized size = 0.73

$$-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\operatorname{arcosh}(ax)^3+\frac{2}{27}a\left(\frac{3\left(-i\sqrt{a^2x^2-1}x^2-\frac{20i\sqrt{a^2x^2-1}}{a^2}\right)\operatorname{arcosh}(ax)}{a^3}+\frac{i\,a^2x^3+60i\,x}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arccosh(a*x)^3
 + 2/27*a*(3*(-I*sqrt(a^2*x^2 - 1)*x^2 - 20*I*sqrt(a^2*x^2 - 1)/a^2)*arccos
h(a*x)/a^3 + (I*a^2*x^3 + 60*I*x)/a^4) + 1/3*(I*a^2*x^3 + 6*I*x)*arccosh(a*
x)^2/a^3
```

Fricas [A] time = 2.20353, size = 447, normalized size = 1.84

$$-\frac{9\left(a^4x^4+a^2x^2-2\right)\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^3-9\left(a^3x^3+6\,ax\right)\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2+6}{27\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*(9*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
- 1))^3 - 9*(a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 - 1))^2 + 6*(a^4*x^4 + 19*a^2*x^2 - 20)*sqrt(-a^2*x^2 + 1)*l
og(a*x + sqrt(a^2*x^2 - 1)) - 2*(a^3*x^3 + 60*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-
a^2*x^2 + 1))/(a^6*x^2 - a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [C] time = 1.33952, size = 200, normalized size = 0.82

$$\frac{\left(\left(-a^2x^2+1\right)^{\frac{3}{2}}-3\sqrt{-a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3}{3a^4}+\frac{-2i\,a^2x^3+9\left(-i\,a^2x^3-6i\,x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2-120i\,x-}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 -
1))^3/a^4 + 1/27*(-2*I*a^2*x^3 + 9*(-I*a^2*x^3 - 6*I*x)*log(a*x + sqrt(a^2*
x^2 - 1))^2 - 120*I*x - 3*(-2*I*(a^2*x^2 - 1)^(3/2) - 42*I*sqrt(a^2*x^2 - 1
))*log(a*x + sqrt(a^2*x^2 - 1))/a)/a^3
```

3.255 $\int \frac{x^2 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=188

$$-\frac{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}{2a^2}+\frac{\sqrt{ax-1}\cosh^{-1}(ax)^4}{8a^3\sqrt{1-ax}}+\frac{3\sqrt{ax-1}\cosh^{-1}(ax)^2}{8a^3\sqrt{1-ax}}-\frac{3x\sqrt{1-ax}\sqrt{ax+1}\cosh^{-1}(ax)}{4a^2}-\frac{3x^2\sqrt{ax-}}{8a\sqrt{1-a}}$$

```
[Out] (-3*x^2*Sqrt[-1 + a*x])/(8*a*Sqrt[1 - a*x]) - (3*x*Sqrt[1 - a*x]*Sqrt[1 + a
*x]*ArcCosh[a*x])/(4*a^2) + (3*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(8*a^3*Sqrt[1
 - a*x]) - (3*x^2*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(4*a*Sqrt[1 - a*x]) - (x*S
qrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/(2*a^2) + (Sqrt[-1 + a*x]*ArcCosh[a*x]^4)/
(8*a^3*Sqrt[1 - a*x])
```

Rubi [A] time = 0.765304, antiderivative size = 257, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5798, 5759, 5676, 5662, 30}

$$-\frac{3x^2\sqrt{ax-1}\sqrt{ax+1}}{8a\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^4}{8a^3\sqrt{1-a^2x^2}}-\frac{x(1-ax)(ax+1)\cosh^{-1}(ax)^3}{2a^2\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-3*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(8*a*Sqrt[1 - a^2*x^2]) - (3*x*(1 - a
*x)*(1 + a*x)*ArcCosh[a*x])/(4*a^2*Sqrt[1 - a^2*x^2]) + (3*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]^2)/(8*a^3*Sqrt[1 - a^2*x^2]) - (3*x^2*Sqrt[-1 + a
*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(4*a*Sqrt[1 - a^2*x^2]) - (x*(1 - a*x)*(1
 + a*x)*ArcCosh[a*x]^3)/(2*a^2*Sqrt[1 - a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*ArcCosh[a*x]^4)/(8*a^3*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2a^2\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2a^2\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{-1+ax}\sqrt{1+ax}\right)}{2a\sqrt{1-}}$$

$$= -\frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-}}{8a^3\sqrt{1-a^2x^2}}$$

$$= -\frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}}{2a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}}{8a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}} + \frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{8a^3\sqrt{1-a^2x^2}} - \frac{3x}{}$$

Mathematica [A] time = 0.22204, size = 98, normalized size = 0.52

$$-\frac{\sqrt{-(ax-1)(ax+1)}\left(2\cosh^{-1}(ax)\left(\cosh^{-1}(ax)^3+\left(2\cosh^{-1}(ax)^2+3\right)\sinh\left(2\cosh^{-1}(ax)\right)\right)-3\left(2\cosh^{-1}(ax)^2+1\right)\cos\right.}{16a^3\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(Sqrt[-((-1 + a*x)*(1 + a*x))]*(-3*(1 + 2*ArcCosh[a*x]^2)*Cosh[2*ArcCosh[a
*x]] + 2*ArcCosh[a*x]*(ArcCosh[a*x]^3 + (3 + 2*ArcCosh[a*x]^2)*Sinh[2*ArcCo
sh[a*x]])))/(16*a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [A] time = 0.165, size = 255, normalized size = 1.4

$$-\frac{(\text{arccosh}(ax))^4}{8a^3\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1} - \frac{4(\text{arccosh}(ax))^3-6(\text{arccosh}(ax))^2+6\,\text{arccosh}(ax)-3}{32a^3\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccosh
(a*x)^4-1/32*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1
/2)*x^2*a^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arccosh(a*x)^2
+6*arccosh(a*x)-3)/a^3/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x
-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arcc
osh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/a^3/(a^2*x^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^2\,\text{arcosh}\,(ax)^3}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,\text{acosh}^3\,(ax)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

3.256 $\int \frac{x \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}{a^2}-\frac{6\sqrt{1-ax}\sqrt{ax+1}\cosh^{-1}(ax)}{a^2}-\frac{6x\sqrt{ax-1}}{a\sqrt{1-ax}}-\frac{3x\sqrt{ax-1}\cosh^{-1}(ax)^2}{a\sqrt{1-ax}}$$

```
[Out] (-6*x*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x]) - (6*Sqrt[1 - a*x]*Sqrt[1 + a*x]*Ar
cCosh[a*x])/a^2 - (3*x*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(a*Sqrt[1 - a*x]) - (
Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/a^2
```

Rubi [A] time = 0.392537, antiderivative size = 153, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5798, 5718, 5654, 8}

$$-\frac{6x\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{1-a^2x^2}}-\frac{(1-ax)(ax+1)\cosh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}-\frac{6(1-ax)(ax+1)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-6*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[1 - a^2*x^2]) - (6*(1 - a*x)*(1
 + a*x)*ArcCosh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) - (3*x*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*ArcCosh[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) - ((1 - a*x)*(1 + a*x)*ArcCosh
[a*x]^3)/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
```

```
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{-1+ax}\sqrt{1+ax}\right)\int \cosh^{-1}(ax)^2\,dx}{a\sqrt{1-a^2x^2}}$$
$$= -\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} + \frac{\left(6\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\,\mathrm{co}}{\sqrt{-1}}}{\sqrt{1-a^2x^2}}$$
$$= -\frac{6(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}$$
$$= -\frac{6x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{1-a^2x^2}} - \frac{6(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} - \frac{(1-}{}$$

Mathematica [A] time = 0.0992229, size = 101, normalized size = 0.92

$$\frac{\sqrt{1-a^2x^2}\left(6ax-\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3+3ax\cosh^{-1}(ax)^2-6\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\right)}{a^2\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(6*a*x - 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + 3
*a*x*ArcCosh[a*x]^2 - Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3))/(a^2*Sq
rt[-1 + a*x]*Sqrt[1 + a*x])
```

Maple [A] time = 0.121, size = 155, normalized size = 1.4

$$-\frac{(\operatorname{arccosh}(ax))^3 - 3(\operatorname{arccosh}(ax))^2 + 6\operatorname{arccosh}(ax) - 6}{2a^2(a^2x^2-1)}\sqrt{-a^2x^2+1}\left(\sqrt{ax+1}\sqrt{ax-1}ax + a^2x^2 - 1\right) - \frac{(\operatorname{arccosh}(ax)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(arccos
h(a*x)^3-3*arccosh(a*x)^2+6*arccosh(a*x)-6)/a^2/(a^2*x^2-1)-1/2*(-a^2*x^2+1
)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*(arccosh(a*x)^3+3*arcco
sh(a*x)^2+6*arccosh(a*x)+6)/a^2/(a^2*x^2-1)
```

Maxima [C] time = 1.14622, size = 88, normalized size = 0.8

$$\frac{3i\,x\operatorname{arcosh}(ax)^2}{a} - \frac{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)^3}{a^2} - \frac{3\left(-2i\,x + \frac{2i\sqrt{a^2x^2-1}\operatorname{arcosh}(ax)}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 3*I*x*arccosh(a*x)^2/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/a^2 - 3*(-2*I*x
+ 2*I*sqrt(a^2*x^2 - 1)*arccosh(a*x)/a)/a
```

Fricas [A] time = 2.17433, size = 348, normalized size = 3.16

$$\frac{3\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2-1}\right)^2 + \left(-a^2x^2+1\right)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)^3 + 6\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}ax - 6}{a^4x^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (3*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1))^2
+ (-a^2*x^2 + 1)^(3/2)*log(a*x + sqrt(a^2*x^2 - 1))^3 + 6*sqrt(a^2*x^2 - 1)
*sqrt(-a^2*x^2 + 1)*a*x - 6*(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt
(a^2*x^2 - 1)))/(a^4*x^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [C] time = 1.28001, size = 139, normalized size = 1.26

$$-\frac{\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^3}{a^2}-\frac{3i\left(x\log\left(ax+\sqrt{a^2x^2-1}\right)^2+2a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))^3/a^2 - 3*I*(x*log(a*x + s
qrt(a^2*x^2 - 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 -
 1))/a^2))/a
```

3.257 $\int \frac{\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=32

$$\frac{\sqrt{ax-1}\cosh^{-1}(ax)^4}{4a\sqrt{1-ax}}$$

```
[Out] (Sqrt[-1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[1 - a*x])
```

Rubi [A] time = 0.161407, antiderivative size = 45, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5713, 5676}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^4}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[1 - a^2*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^4}{4a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0182408, size = 45, normalized size = 1.41

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^4}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.035, size = 51, normalized size = 1.6

$$-\frac{(\operatorname{arccosh}(ax))^4}{4\,a\left(a^2x^2-1\right)}\sqrt{-(ax-1)(ax+1)}\sqrt{ax-1}\sqrt{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*arc
cosh(a*x)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)^3}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

3.258 $\int \frac{\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=265

$$-\frac{3i\sqrt{ax-1}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{3i\sqrt{ax-1}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{6i\sqrt{ax-1}\cosh^{-1}}{}$$

```
[Out] (2*Sqrt[-1 + a*x]*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a*x] - ((
3*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1
- a*x] + ((3*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*PolyLog[2, I*E^ArcCosh[a*x]])
/Sqrt[1 - a*x] + ((6*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCo
sh[a*x]])/Sqrt[1 - a*x] - ((6*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[3, I*E
^ArcCosh[a*x]])/Sqrt[1 - a*x] - ((6*I)*Sqrt[-1 + a*x]*PolyLog[4, (-I)*E^Arc
Cosh[a*x]])/Sqrt[1 - a*x] + ((6*I)*Sqrt[-1 + a*x]*PolyLog[4, I*E^ArcCosh[a*
x]])/Sqrt[1 - a*x]
```

Rubi [A] time = 0.478743, antiderivative size = 356, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5798, 5761, 4180, 2531, 6609, 2282, 6589}

$$-\frac{3i\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{3i\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]])/Sqrt
[1 - a^2*x^2] - ((3*I)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*PolyLog[
2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((3*I)*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*ArcCosh[a*x]^2*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((6*
I)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]
])/Sqrt[1 - a^2*x^2] - ((6*I)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*Pol
yLog[3, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] - ((6*I)*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*PolyLog[4, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((6*I)*Sqrt[-1
 + a*x]*Sqrt[1 + a*x]*PolyLog[4, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int x^3\,\mathbf{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}\\
&= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(3i\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int x^2\log\left(1-ie^{x}\right.\right.}{\sqrt{1-a^2x^2}}\\
&= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\mathbf{Li}_2\left(-ie^{\cosh^{-1}}\right.}{\sqrt{1-a^2x^2}}\\
&= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\mathbf{Li}_2\left(-ie^{\cosh^{-1}}\right.}{\sqrt{1-a^2x^2}}\\
&= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\mathbf{Li}_2\left(-ie^{\cosh^{-1}}\right.}{\sqrt{1-a^2x^2}}\\
&= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2\mathbf{Li}_2\left(-ie^{\cosh^{-1}}\right.}{\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.647126, size = 488, normalized size = 1.84

$$\frac{i\sqrt{-(ax-1)(ax+1)}\left(192\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)+192i\pi\cosh^{-1}(ax)\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)+384\cosh^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((I/64)*Sqrt[-((-1 + a*x)*(1 + a*x))]*(7*Pi^4 + (8*I)*Pi^3*ArcCosh[a*x] + 2
4*Pi^2*ArcCosh[a*x]^2 - (32*I)*Pi*ArcCosh[a*x]^3 - 16*ArcCosh[a*x]^4 + (8*I
)*Pi^3*Log[1 + I/E^ArcCosh[a*x]] + 48*Pi^2*ArcCosh[a*x]*Log[1 + I/E^ArcCosh
[a*x]] - (96*I)*Pi*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] - 64*ArcCosh[a*
x]^3*Log[1 + I/E^ArcCosh[a*x]] - 48*Pi^2*ArcCosh[a*x]*Log[1 - I*E^ArcCosh[a
*x]] + (96*I)*Pi*ArcCosh[a*x]^2*Log[1 - I*E^ArcCosh[a*x]] - (8*I)*Pi^3*Log[
1 + I*E^ArcCosh[a*x]] + 64*ArcCosh[a*x]^3*Log[1 + I*E^ArcCosh[a*x]] + (8*I)
*Pi^3*Log[Tan[(Pi + (2*I)*ArcCosh[a*x])/4]] - 48*(Pi - (2*I)*ArcCosh[a*x])^
2*PolyLog[2, (-I)/E^ArcCosh[a*x]] + 192*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^Ar
cCosh[a*x]] - 48*Pi^2*PolyLog[2, I*E^ArcCosh[a*x]] + (192*I)*Pi*ArcCosh[a*x
]*PolyLog[2, I*E^ArcCosh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcCosh[a*x]]
 + 384*ArcCosh[a*x]*PolyLog[3, (-I)/E^ArcCosh[a*x]] - 384*ArcCosh[a*x]*Poly
Log[3, (-I)*E^ArcCosh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcCosh[a*x]] + 384
*PolyLog[4, (-I)/E^ArcCosh[a*x]] + 384*PolyLog[4, (-I)*E^ArcCosh[a*x]]))/(S
qrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [F] time = 0.149, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{arccosh}\left(ax\right)\right)^{3}}{x}\frac{1}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}\left(ax\right)^{3}}{\sqrt{-a^{2}x^{2}+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)^3}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{acosh}^3\,(ax)}{x\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**3/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{arcosh}\,(ax)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.259 $\int \frac{\cosh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=166

$$-\frac{3a\sqrt{ax-1}\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{3a\sqrt{ax-1}\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}{x}+$$

```
[Out] (a*Sqrt[-1 + a*x]*ArcCosh[a*x]^3)/Sqrt[1 - a*x] - (Sqrt[1 - a^2*x^2]*ArcCos
h[a*x]^3)/x - (3*a*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*Log[1 + E^(2*ArcCosh[a*x])
])/Sqrt[1 - a*x] - (3*a*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[2, -E^(2*ArcCos
h[a*x])])/Sqrt[1 - a*x] + (3*a*Sqrt[-1 + a*x]*PolyLog[3, -E^(2*ArcCosh[a*x]
)])/(2*Sqrt[1 - a*x])
```

Rubi [A] time = 0.492011, antiderivative size = 229, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5724, 5660, 3718, 2190, 2531, 2282, 6589}

$$-\frac{3a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{3a\sqrt{ax-1}\sqrt{ax+1}\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)}{2\sqrt{1-a^2x^2}}-\frac{(1-ax)(}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2] - ((1 - a
*x)*(1 + a*x)*ArcCosh[a*x]^3)/(x*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]^2*Log[1 + E^(2*ArcCosh[a*x])])/Sqrt[1 - a^2*x^2]
- (3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, -E^(2*ArcCosh[a
*x])])/Sqrt[1 - a^2*x^2] + (3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, -E^
(2*ArcCosh[a*x])])/(2*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{\left(3a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{\left(3a\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{\left(6a\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\right.}{\sqrt{1-a^2}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax}{\sqrt{1-a^2x}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax}{\sqrt{1-a^2x}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax}{\sqrt{1-a^2x}}$$

$$= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax}{\sqrt{1-a^2x}}$$

Mathematica [A] time = 0.512007, size = 137, normalized size = 0.83

$$\frac{a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(6\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax)}\right)+3\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(ax)}\right)+2\cosh^{-1}(ax)^2\left(\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\,\mathrm{c}}{ax}\right.\right.}{2\sqrt{-(ax-1)(ax+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(2*ArcCosh[a*x]^2*(-ArcCosh[a*x] +
(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a*x) - 3*Log[1 + E^(-2
*ArcCosh[a*x])]) + 6*ArcCosh[a*x]*PolyLog[2, -E^(-2*ArcCosh[a*x])] + 3*Poly
Log[3, -E^(-2*ArcCosh[a*x])]))/(2*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

Maple [A] time = 0.165, size = 313, normalized size = 1.9

$$-\frac{(\operatorname{arccosh}(ax))^3}{x\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\left(a^2x^2-\sqrt{ax+1}\sqrt{ax-1}ax-1\right)-2\,\frac{\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}\,(\operatorname{arccosh}(ax))^3\,a}{a^2x^2-1}+3\,\frac{\sqrt{-a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*arccosh(a*x
)^3/x/(a^2*x^2-1)-2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2
-1)*arccosh(a*x)^3*a+3*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*
x^2-1)*arccosh(a*x)^2*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a+3*(-a^2*x
^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*polylog(2,
-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a-3/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2
)*(a*x+1)^(1/2)/(a^2*x^2-1)*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a^2x^2-1\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^3}{\sqrt{ax+1}\sqrt{-ax+1}x}-\int\frac{3\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)^2}{\left(\sqrt{ax+1}ax^2+(ax+1)\sqrt{ax-1}x\right)\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] (a^2*x^2 - 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(sqrt(a*x + 1)*sqrt(
-a*x + 1)*x) - integrate(3*(a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a
```

```
)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/((sqrt(a*x + 1)*a*x^2 + (a*x + 1
)*sqrt(a*x - 1)*x)*sqrt(-a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)^3}{a^2x^4-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^4 - x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acosh}^3\,(ax)}{x^2\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**3/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcosh}\,(ax)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.260 $\int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=460

$$-\frac{3ia^2\sqrt{ax-1}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{3ia^2\sqrt{ax-1}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{3ia^2\sqrt{ax-1}$$

```
[Out] (3*a*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(2*x*Sqrt[1 - a*x]) - (Sqrt[1 - a^2*x^2
]*ArcCosh[a*x]^3)/(2*x^2) - (6*a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]*ArcTan[E^Arc
Cosh[a*x]])/Sqrt[1 - a*x] + (a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]^3*ArcTan[E^Arc
Cosh[a*x]])/Sqrt[1 - a*x] + ((3*I)*a^2*Sqrt[-1 + a*x]*PolyLog[2, (-I)*E^Arc
Cosh[a*x]])/Sqrt[1 - a*x] - (((3*I)/2)*a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*Po
lyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a*x] - ((3*I)*a^2*Sqrt[-1 + a*x]*Po
lyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a*x] + (((3*I)/2)*a^2*Sqrt[-1 + a*x]*A
rcCosh[a*x]^2*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a*x] + ((3*I)*a^2*Sqrt
[-1 + a*x]*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a*x] - ((
3*I)*a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[3, I*E^ArcCosh[a*x]])/Sqrt[1 -
 a*x] - ((3*I)*a^2*Sqrt[-1 + a*x]*PolyLog[4, (-I)*E^ArcCosh[a*x]])/Sqrt[1 -
 a*x] + ((3*I)*a^2*Sqrt[-1 + a*x]*PolyLog[4, I*E^ArcCosh[a*x]])/Sqrt[1 - a*
x]
```

Rubi [A] time = 1.01337, antiderivative size = 614, normalized size of antiderivative = 1.33, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5798, 5748, 5761, 4180, 2531, 6609, 2282, 6589, 5662, 2279, 2391}

$$-\frac{3ia^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-a^2x^2}}+\frac{3ia^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*x*Sqrt[1 - a^2*x^2]) -
 ((1 - a*x)*(1 + a*x)*ArcCosh[a*x]^3)/(2*x^2*Sqrt[1 - a^2*x^2]) - (6*a^2*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a^
2*x^2] + (a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[
a*x]])/Sqrt[1 - a^2*x^2] + ((3*I)*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[
2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] - (((3*I)/2)*a^2*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*
```

```
x^2] - ((3*I)*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, I*E^ArcCosh[a*x]]
)/Sqrt[1 - a^2*x^2] + (((3*I)/2)*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a
*x]^2*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((3*I)*a^2*Sqrt[-1
+ a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]])/Sqrt[1 -
 a^2*x^2] - ((3*I)*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[3,
 I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] - ((3*I)*a^2*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]*PolyLog[4, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((3*I)*a^2*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]*PolyLog[4, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{\left(3a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x^2}\,dx}{2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\sqrt{1+ax}\right)}{2\sqrt{1-a^2}}$$

$$= \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}}{2\sqrt{1}}$$

$$= \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(a}{\sqrt{1-a^2x}}$$

$$= \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(}{\sqrt{1-a^2x}}$$

$$= \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(}{\sqrt{1-a^2x}}$$

$$= \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(}{\sqrt{1-a^2x}}$$

$$= \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(}{\sqrt{1-a^2x}}$$

Mathematica [B] time = 6.07934, size = 1051, normalized size = 2.28

$$-\frac{ia^2(ax+1)\left(-16\sqrt{\frac{ax-1}{ax+1}}\cosh^{-1}(ax)^4 + \frac{64i(ax-1)\cosh^{-1}(ax)^3}{a^2x^2} - 64\sqrt{\frac{ax-1}{ax+1}}\log\left(1+ie^{-\cosh^{-1}(ax)}\right)\cosh^{-1}(ax)^3 + 64\sqrt{\frac{ax-1}{ax+1}}\log\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((-I/128)*a^2*(1 + a*x)*(7*Pi^4*Sqrt[(-1 + a*x)/(1 + a*x)] + (8*I)*Pi^3*Sqr
t[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + 24*Pi^2*Sqrt[(-1 + a*x)/(1 + a*x)]*A
rcCosh[a*x]^2 + ((192*I)*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2)/(a*x) +
 ((64*I)*(-1 + a*x)*ArcCosh[a*x]^3)/(a^2*x^2) - (32*I)*Pi*Sqrt[(-1 + a*x)/(
1 + a*x)]*ArcCosh[a*x]^3 - 16*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^4 - 3
84*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]] + (8*I
)*Pi^3*Sqrt[(-1 + a*x)/(1 + a*x)]*Log[1 + I/E^ArcCosh[a*x]] + 384*Sqrt[(-1
+ a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] + 48*Pi^2*Sqrt[(-1
 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] - (96*I)*Pi*Sqrt[
(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] - 64*Sqrt[(-
1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3*Log[1 + I/E^ArcCosh[a*x]] - 48*Pi^2*Sqrt
[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I*E^ArcCosh[a*x]] + (96*I)*Pi*S
qrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*Log[1 - I*E^ArcCosh[a*x]] - (8*I)*
Pi^3*Sqrt[(-1 + a*x)/(1 + a*x)]*Log[1 + I*E^ArcCosh[a*x]] + 64*Sqrt[(-1 + a
*x)/(1 + a*x)]*ArcCosh[a*x]^3*Log[1 + I*E^ArcCosh[a*x]] + (8*I)*Pi^3*Sqrt[(
-1 + a*x)/(1 + a*x)]*Log[Tan[(Pi + (2*I)*ArcCosh[a*x])/4]] - 48*Sqrt[(-1 +
a*x)/(1 + a*x)]*(8 + Pi^2 - (4*I)*Pi*ArcCosh[a*x] - 4*ArcCosh[a*x]^2)*PolyL
og[2, (-I)/E^ArcCosh[a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, I/E^
ArcCosh[a*x]] + 192*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*PolyLog[2, (-
I)*E^ArcCosh[a*x]] - 48*Pi^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, I*E^ArcC
osh[a*x]] + (192*I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*PolyLog[2, I
*E^ArcCosh[a*x]] + (192*I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[3, (-I)/E^
ArcCosh[a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*PolyLog[3, (-I)
/E^ArcCosh[a*x]] - 384*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*PolyLog[3, (
-I)*E^ArcCosh[a*x]] - (192*I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[3, I*E^
ArcCosh[a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[4, (-I)/E^ArcCosh[a*
x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[4, (-I)*E^ArcCosh[a*x]]))/Sqrt
[1 - a^2*x^2]
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{arccosh}\left(ax\right)\right)^{3}}{x^{3}}\frac{1}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)^3}{a^2x^5-x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^3(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**3/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.261 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=32

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2], x]
```

Rubi [A] time = 0.458437, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^3)/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 3.49909, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2], x]
```

Maple [A] time = 0.379, size = 0, normalized size = 0.

$$\int (fx)^{m} (a + b\text{arccosh}(cx))^{3} \frac{1}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arcosh}(cx) + a)^{3} (fx)^{m}}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^3*(f*x)^m/sqrt(-c^2*x^2 + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^{3}\,\text{arcosh}(cx)^{3} + 3\,ab^{2}\,\text{arcosh}(cx)^{2} + 3\,a^{2}b\,\text{arcosh}(cx) + a^{3}\right)\sqrt{-c^{2}x^{2}+1}\,(fx)^{m}}{c^{2}x^{2}-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-(b^3*arccosh(c*x)^3 + 3*a*b^2*arccosh(c*x)^2 + 3*a^2*b*arccosh(c*
x) + a^3)*sqrt(-c^2*x^2 + 1)*(f*x)^m/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**3/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^3 (fx)^m}{\sqrt{-c^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)^3*(f*x)^m/sqrt(-c^2*x^2 + 1), x)
```

3.262 $\int \frac{(c-a^2cx^2)^3}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=67

$$\frac{35c^3\text{Shi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{21c^3\text{Shi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{c^3\text{Shi}\left(7\cosh^{-1}(ax)\right)}{64a}$$

```
[Out] (35*c^3*SinhIntegral[ArcCosh[a*x]])/(64*a) - (21*c^3*SinhIntegral[3*ArcCosh
[a*x]])/(64*a) + (7*c^3*SinhIntegral[5*ArcCosh[a*x]])/(64*a) - (c^3*SinhInt
egral[7*ArcCosh[a*x]])/(64*a)
```

Rubi [A] time = 0.140096, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5700, 3312, 3298}

$$\frac{35c^3\text{Shi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{21c^3\text{Shi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{c^3\text{Shi}\left(7\cosh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/ArcCosh[a*x],x]
```

```
[Out] (35*c^3*SinhIntegral[ArcCosh[a*x]])/(64*a) - (21*c^3*SinhIntegral[3*ArcCosh
[a*x]])/(64*a) + (7*c^3*SinhIntegral[5*ArcCosh[a*x]])/(64*a) - (c^3*SinhInt
egral[7*ArcCosh[a*x]])/(64*a)
```

Rule 5700

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(-d)^p/c, Subst[Int[(a + b*x)^n*Sinh[x]^(2*p + 1), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] &&
IGtQ[p, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^3}{\cosh^{-1}(ax)}\,dx = -\frac{c^3\,\text{Subst}\left(\int \frac{\sinh^7(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= -\frac{\left(ic^3\right)\text{Subst}\left(\int \left(\frac{35i\sinh(x)}{64x}-\frac{21i\sinh(3x)}{64x}+\frac{7i\sinh(5x)}{64x}-\frac{i\sinh(7x)}{64x}\right)dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= -\frac{c^3\,\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a}+\frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a}-\frac{\left(21c^3\right)\text{Sul}}{}$$
$$= \frac{35c^3\textbf{Shi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{21c^3\textbf{Shi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{7c^3\textbf{Shi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{c^3\textbf{Shi}\left(7\cosh^{-1}(ax)\right)}{64a}$$

Mathematica [A] time = 0.26443, size = 45, normalized size = 0.67

$$\frac{c^3\left(35\textbf{Shi}\left(\cosh^{-1}(ax)\right)-21\textbf{Shi}\left(3\cosh^{-1}(ax)\right)+7\textbf{Shi}\left(5\cosh^{-1}(ax)\right)-\textbf{Shi}\left(7\cosh^{-1}(ax)\right)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3/ArcCosh[a*x],x]
```

```
[Out] (c^3*(35*SinhIntegral[ArcCosh[a*x]] - 21*SinhIntegral[3*ArcCosh[a*x]] + 7*S
inhIntegral[5*ArcCosh[a*x]] - SinhIntegral[7*ArcCosh[a*x]]))/(64*a)
```

Maple [A] time = 0.039, size = 44, normalized size = 0.7

$$\frac{c^3\left(35\,\text{Shi}\left(\text{arccosh}\left(ax\right)\right)-21\,\text{Shi}\left(3\,\text{arccosh}\left(ax\right)\right)+7\,\text{Shi}\left(5\,\text{arccosh}\left(ax\right)\right)-\text{Shi}\left(7\,\text{arccosh}\left(ax\right)\right)\right)}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/arccosh(a*x),x)
```

```
[Out] 1/64/a*c^3*(35*Shi(arccosh(a*x))-21*Shi(3*arccosh(a*x))+7*Shi(5*arccosh(a*x
))-Shi(7*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a^2cx^2-c\right)^3}{\operatorname{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)^3/arccosh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3}{\operatorname{arcosh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)/arccosh(a*x),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int \frac{3a^2x^2}{\operatorname{acosh}\left(ax\right)}\,dx+\int -\frac{3a^4x^4}{\operatorname{acosh}\left(ax\right)}\,dx+\int \frac{a^6x^6}{\operatorname{acosh}\left(ax\right)}\,dx+\int -\frac{1}{\operatorname{acosh}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/acosh(a*x),x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/acosh(a*x), x) + Integral(-3*a**4*x**4/acosh(a*
x), x) + Integral(a**6*x**6/acosh(a*x), x) + Integral(-1/acosh(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^2cx^2-c\right)^3}{\operatorname{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3/arccosh(a*x), x)
```

3.263 $\int \frac{(c-a^2cx^2)^2}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=50

$$\frac{5c^2\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{5c^2\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a}$$

```
[Out] (5*c^2*SinhIntegral[ArcCosh[a*x]])/(8*a) - (5*c^2*SinhIntegral[3*ArcCosh[a*
x]])/(16*a) + (c^2*SinhIntegral[5*ArcCosh[a*x]])/(16*a)
```

Rubi [A] time = 0.113328, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5700, 3312, 3298}

$$\frac{5c^2\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{5c^2\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/ArcCosh[a*x],x]
```



```
[Out] (5*c^2*SinhIntegral[ArcCosh[a*x]])/(8*a) - (5*c^2*SinhIntegral[3*ArcCosh[a*
x]])/(16*a) + (c^2*SinhIntegral[5*ArcCosh[a*x]])/(16*a)
```


Rule 5700

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(-d)^p/c, Subst[Int[(a + b*x)^n*Sinh[x]^(2*p + 1), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] &&
IGtQ[p, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^2}{\cosh^{-1}(ax)}\,dx = \frac{c^2\,\text{Subst}\left(\int \frac{\sinh^5(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= -\frac{\left(ic^2\right)\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8x}-\frac{5i\sinh(3x)}{16x}+\frac{i\sinh(5x)}{16x}\right)dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= \frac{c^2\,\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{16a} - \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{16a} + \frac{\left(5c^2\right)\text{Subst}}{}$$
$$= \frac{5c^2\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a} - \frac{5c^2\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a} + \frac{c^2\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a}$$

Mathematica [A] time = 0.164153, size = 34, normalized size = 0.68

$$\frac{c^2\left(10\text{Shi}\left(\cosh^{-1}(ax)\right)-5\text{Shi}\left(3\cosh^{-1}(ax)\right)+\text{Shi}\left(5\cosh^{-1}(ax)\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2/ArcCosh[a*x],x]
```

```
[Out] (c^2*(10*SinhIntegral[ArcCosh[a*x]] - 5*SinhIntegral[3*ArcCosh[a*x]] + Sinh
Integral[5*ArcCosh[a*x]]))/(16*a)
```

Maple [A] time = 0.032, size = 33, normalized size = 0.7

$$\frac{c^2\left(10\,\text{Shi}\left(\text{arccosh}\left(ax\right)\right)-5\,\text{Shi}\left(3\,\text{arccosh}\left(ax\right)\right)+\text{Shi}\left(5\,\text{arccosh}\left(ax\right)\right)\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/arccosh(a*x),x)
```

```
[Out] 1/16/a*c^2*(10*Shi(arccosh(a*x))-5*Shi(3*arccosh(a*x))+Shi(5*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 - c\right)^2}{\operatorname{arcosh}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 - c)^2/arccosh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^4c^2x^4 - 2\,a^2c^2x^2 + c^2}{\operatorname{arcosh}\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)/arccosh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -\frac{2a^2x^2}{\operatorname{acosh}\left(ax\right)}\, dx + \int \frac{a^4x^4}{\operatorname{acosh}\left(ax\right)}\, dx + \int \frac{1}{\operatorname{acosh}\left(ax\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/acosh(a*x),x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/acosh(a*x), x) + Integral(a**4*x**4/acosh(a*x),
 x) + Integral(1/acosh(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 - c\right)^2}{\operatorname{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2/arccosh(a*x), x)
```

3.264 $\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{3c\text{Shi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{c\text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a}$$

```
[Out] (3*c*SinhIntegral[ArcCosh[a*x]])/(4*a) - (c*SinhIntegral[3*ArcCosh[a*x]])/(
4*a)
```

Rubi [A] time = 0.080804, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5700, 3312, 3298}

$$\frac{3c\text{Shi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{c\text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/ArcCosh[a*x],x]
```

```
[Out] (3*c*SinhIntegral[ArcCosh[a*x]])/(4*a) - (c*SinhIntegral[3*ArcCosh[a*x]])/(
4*a)
```

Rule 5700

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(-d)^p/c, Subst[Int[(a + b*x)^n*Sinh[x]^(2*p + 1), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] &&
IGtQ[p, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)}\,dx &= -\frac{c\,\text{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}\\
&= -\frac{(ic)\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x}-\frac{i\sinh(3x)}{4x}\right)dx, x, \cosh^{-1}(ax)\right)}{a}\\
&= -\frac{c\,\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a}+\frac{(3c)\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a}\\
&= \frac{3c\text{Shi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{c\text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a}
\end{aligned}
$$

Mathematica [A] time = 0.115985, size = 25, normalized size = 0.86

$$\frac{c\left(3\text{Shi}\left(\cosh^{-1}(ax)\right)-\text{Shi}\left(3\cosh^{-1}(ax)\right)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)/ArcCosh[a*x],x]

[Out] (c*(3*SinhIntegral[ArcCosh[a*x]] - SinhIntegral[3*ArcCosh[a*x]]))/(4*a)
```

Maple [A] time = 0.03, size = 24, normalized size = 0.8

$$\frac{c\left(3\,\text{Shi}\left(\text{arccosh}\left(ax\right)\right)-\text{Shi}\left(3\,\text{arccosh}\left(ax\right)\right)\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/arccosh(a*x),x)

[Out] 1/4/a*c*(3*Shi(arccosh(a*x))-Shi(3*arccosh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2cx^2 - c}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)/arccosh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{a^2cx^2 - c}{\operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)/arccosh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int \frac{a^2x^2}{\operatorname{acosh}(ax)}\,dx + \int -\frac{1}{\operatorname{acosh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/acosh(a*x),x)
```

```
[Out] -c*(Integral(a**2*x**2/acosh(a*x), x) + Integral(-1/acosh(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^2cx^2 - c}{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)/arccosh(a*x), x)
```

3.265 $\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/((c - a^2*c*x^2)*ArcCosh[a*x]), x]
```

Rubi [A] time = 0.0332269, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)*ArcCosh[a*x]),x]
```

```
[Out] Defer[Int][1/((c - a^2*c*x^2)*ArcCosh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx = \int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.4834, size = 0, normalized size = 0.

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]), x]
```

Maple [A] time = 0.136, size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right) \operatorname{arccosh}\left(a x \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)/arccosh(a*x),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)/arccosh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{{\left(a^{2} c x^{2} - c\right)} \operatorname{arcosh}\left(a x\right)}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a^2*c*x^2 - c)*arccosh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(-\frac{1}{{\left(a^{2} c x^{2} - c\right)} \operatorname{arcosh}\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-1/((a^2*c*x^2 - c)*arccosh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{a^2x^2 \operatorname{acosh}(ax) - \operatorname{acosh}(ax)}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)/acosh(a*x),x)
```

```
[Out] -Integral(1/(a**2*x**2*acosh(a*x) - acosh(a*x)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(a^2cx^2 - c) \operatorname{arcosh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*c*x^2 - c)*arccosh(a*x)), x)
```

3.266 $\int \frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]), x]
```

Rubi [A] time = 0.0316224, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]),x]
```

```
[Out] Defer[Int][1/((c - a^2*c*x^2)^2*ArcCosh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)} dx = \int \frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)} dx$$

Mathematica [A] time = 5.88946, size = 0, normalized size = 0.

$$\int \frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]), x]
```

Maple [A] time = 0.17, size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2 c x^2 + c\right)^2 \operatorname{arccosh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 c x^2 - c\right)^2 \operatorname{arcosh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^2*arccosh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\left(a^4 c^2 x^4 - 2\, a^2 c^2 x^2 + c^2\right) \operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*arccosh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^4 \operatorname{acosh}(ax) - 2a^2x^2 \operatorname{acosh}(ax) + \operatorname{acosh}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**2/acosh(a*x),x)
```

```
[Out] Integral(1/(a**4*x**4*acosh(a*x) - 2*a**2*x**2*acosh(a*x) + acosh(a*x)), x)
/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2 - c\right)^2 \operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^2*arccosh(a*x)), x)
```

3.267 $\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=339

$$-\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^5\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^5\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\cosh\right.}{b}\right.}{32bc^5\sqrt{cx-1}}$$

```
[Out] -(Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32
*b*c^5*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a +
b*ArcCosh[c*x]))/b])/(16*b*c^5*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(6*a)/
b]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^5*Sqrt[-1 + c*x]) - (S
qrt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(16*b*c^5*Sqrt[-1 + c*x]) + (Sqrt[1 -
 c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32*b*c^5*Sqr
t[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[
c*x]))/b])/(16*b*c^5*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(6*a)/b]*SinhInt
egral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^5*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.87942, antiderivative size = 430, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^5\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{16bc^5\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{6a}{b}\right)\text{C}}{32bc^5\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(
32*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*C
oshIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(16*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*CoshIntegral[(6*a)/b + 6*ArcCosh[c*x
]])/(32*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Log[a + b*
ArcCosh[c*x]])/(16*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]
*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(32*b*c^5*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b
 + 4*ArcCosh[c*x]])/(16*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2
*x^2]*Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c*x]])/(32*b*c^5*Sqrt[
```

```
-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^5\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{1}{16(a+bx)} - \frac{\cosh(2x)}{32(a+bx)} + \frac{\cosh(4x)}{16(a+bx)} + \frac{\cosh(6x)}{32(a+bx)}\right) dx, x, \cosh^{-1}(cx)\right)}{c^5\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{32c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2}}{}$$
$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-1}(\right.}{32c^5\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{16bc^5\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.501245, size = 188, normalized size = 0.55

$$\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\cosh^{-}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])]) +
 2*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]*CoshI
ntegral[6*(a/b + ArcCosh[c*x])] - 2*Log[a + b*ArcCosh[c*x]] + Sinh[(2*a)/b]
*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b
 + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/(3
2*c^5*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [A] time = 0.355, size = 591, normalized size = 1.7

$$\frac{1}{(64\,cx+64)\,c^{5}\,(cx-1)\,b}\sqrt{-c^{2}x^{2}+1}\left(-\sqrt{cx+1}\sqrt{cx-1}xc+c^{2}x^{2}-1\right)\operatorname{Ei}\left(1,6\operatorname{arccosh}\left(cx\right)+6\,\frac{a}{b}\right)\mathrm{e}^{\frac{b\operatorname{arccosh}(cx)+6\,a}{b}}+\frac{}{(64\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6
*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^5/(c*x-1)/b+1/64
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*ar
ccosh(c*x)-6*a/b)*exp((b*arccosh(c*x)-6*a)/b)/(c*x+1)/c^5/(c*x-1)/b-1/16*(-
c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^5*ln(a+b*arccosh(c*x))/b+1/3
2*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*ar
ccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^5/(c*x-1)/b-1/64*(-
c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccos
h(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^5/(c*x-1)/b-1/64*(-c^2*
x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c
*x)-2*a/b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/c^5/(c*x-1)/b+1/32*(-c^2*x^2
+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c*x)
-4*a/b)*exp((b*arccosh(c*x)-4*a)/b)/(c*x+1)/c^5/(c*x-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^{2}x^{2}+1}x^{4}}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^{2}x^{2}+1}x^{4}}{b\operatorname{arcosh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**4*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}x^4}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

3.268 $\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=297

$$-\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}\right.}{b}\right.}{16bc^4\sqrt{cx-1}}$$

```
[Out] -(Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(8*b*c^4*Sq
rt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh
[c*x]))/b])/(16*b*c^4*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(5*a)/b]*CoshIn
tegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b*c^4*Sqrt[-1 + c*x]) + (Sqrt[1 - c
*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b*c^4*Sqrt[-1 + c*x]
) - (Sqrt[1 - c*x]*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/
(16*b*c^4*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(5*a)/b]*SinhIntegral[(5*(a
 + b*ArcCosh[c*x]))/b])/(16*b*c^4*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.860935, antiderivative size = 371, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16bc^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{5a}{b}\right)\text{Chi}}{16bc^4\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(8*b*c^4*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshIntegral
[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt
[1 - c^2*x^2]*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b*c
^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegra
l[a/b + ArcCosh[c*x]])/(8*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c
^2*x^2]*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*c^4*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(5*a)/b]*SinhIntegral[
(5*a)/b + 5*ArcCosh[c*x]])/(16*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{8(a+bx)}+\frac{\cosh(3x)}{16(a+bx)}+\frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{16c^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{16c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{8c^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}}{16c^4\sqrt{-1}}$$

$$= -\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16bc^4\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 0.434021, size = 171, normalized size = 0.58

$$\frac{\sqrt{1-c^2x^2}\left(-2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx\right.\right.\right.}{16c^4\sqrt{\frac{cx-1}{cx+1}}(b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + Cosh[(3
*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Cosh[(5*a)/b]*CoshIntegral[5*
(a/b + ArcCosh[c*x])] + 2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - Sinh
[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - Sinh[(5*a)/b]*SinhIntegral
[5*(a/b + ArcCosh[c*x])]))/(16*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] time = 0.227, size = 543, normalized size = 1.8

$$\frac{1}{(32\,cx+32)\,c^4\,(cx-1)\,b}\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)\text{Ei}\left(1,5\,\text{arccosh}\,(cx)+5\,\frac{a}{b}\right)e^{\frac{b\,\text{arccosh}(cx)+5\,a}{b}} + \frac{}{(32\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5
*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/32
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arc
cosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/32*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccos
h(c*x)-3*a/b)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/32*(-c^2*
x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*arccosh(c
*x)-5*a/b)*exp((b*arccosh(c*x)-5*a)/b)/(c*x+1)/c^4/(c*x-1)/b-1/16*(-c^2*x^2
+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a
/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^4/(c*x-1)/b-1/16*(-c^2*x^2+1)^(1/2)
*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((a
+b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}x^3}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^3/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^2x^2+1}x^3}{b\,\mathrm{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^3/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{a+b\,\mathrm{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}\,x^3}{b\,\mathrm{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^3/(b*arccosh(c*x) + a), x)
```

3.269 $\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{cx-1}}$$

```
[Out] (Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(8*b
*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(8*b*c^3*Sqr
t[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[
c*x]))/b])/(8*b*c^3*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.671674, antiderivative size = 178, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh\right.}{8bc^3\sqrt{cx-1}\sqrt{cx+}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8
*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh
[c*x]])/(8*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(4
*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*c^3*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(-\frac{1}{8(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{8c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx, x, \cosh^{-1}(\right.}{8c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\mathrm{s}}{}$$

Mathematica [A] time = 0.297433, size = 103, normalized size = 0.74

$$-\frac{\sqrt{-(cx-1)(cx+1)}\left(-\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\log\left(a+b\cosh^{-1}(cx\right.\right.}{8bc^3\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[-((-1 + c*x)*(1 + c*x))]*(-(Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcC
osh[c*x])]) + Log[a + b*ArcCosh[c*x]] + Sinh[(4*a)/b]*SinhIntegral[4*(a/b +
 ArcCosh[c*x])]))/(8*b*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.167, size = 227, normalized size = 1.6

$$\frac{1}{(16\,cx+16)\,c^3\,(cx-1)\,b}\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)\mathrm{Ei}\left(1,4\,\mathrm{arccosh}\,(cx)+4\,\frac{a}{b}\right)\mathrm{e}^{\frac{b\mathrm{arccosh}(cx)+4\,a}{b}} + \frac{}{(16\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4
*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/16
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*ar
ccosh(c*x)-4*a/b)*exp((b*arccosh(c*x)-4*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/8*(-c
^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*ln(a+b*arccosh(c*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}x^2}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^2x^2+1}x^2}{b\operatorname{arcosh}(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}\,x^2}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)
```

3.270 $\int \frac{x\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=197

$$-\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4bc^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^2\sqrt{cx-1}}$$

```
[Out] -(Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(4*b*c^2*Sq
rt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh
[c*x]))/b])/(4*b*c^2*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Sinh[a/b]*SinhIntegra
l[(a + b*ArcCosh[c*x])/b])/(4*b*c^2*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(
3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b*c^2*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.571637, antiderivative size = 245, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\ldots\right)}{4bc^2\sqrt{cx-1}\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(4*b*c^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshIntegral
[(3*a)/b + 3*ArcCosh[c*x]])/(4*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[
1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b*c^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(3*a)/b]*SinhIntegral[(3*a)/
b + 3*ArcCosh[c*x]])/(4*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{4c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\right)\mathrm{Subst}}{4c^2\sqrt{-1}}$$

$$= -\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^2\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 0.310069, size = 127, normalized size = 0.64

$$\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\mathrm{s}\right.}{4c^2\sqrt{\frac{cx-1}{cx+1}}(bcx+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]]) + Cosh[(3
*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Sinh[a/b]*SinhIntegral[a/b +
ArcCosh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*c^2
*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] time = 0.156, size = 361, normalized size = 1.8

$$\frac{1}{(8cx+8)c^2(cx-1)b}\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)\mathrm{Ei}\left(1,3\,\mathrm{arccosh}\,(cx)+3\,\frac{a}{b}\right)\mathrm{e}^{\frac{b\,\mathrm{arccosh}(cx)+3\,a}{b}}+\frac{}{(8cx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*
arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b+1/8*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arcc
osh(c*x)-3*a/b)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/8*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*
x)-a/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/8*(-c^2*x^2+1)^(1
/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp
((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}x}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^2x^2+1}x}{b\operatorname{arcosh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}x}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

3.271 $\int \frac{\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{cx-1}}$$

```
[Out] (Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(2*b
*c*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(2*b*c*Sqrt[-1
 + c*x]) - (Sqrt[1 - c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]
))/b])/(2*b*c*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.343371, antiderivative size = 178, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {5713, 5701, 3312, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh\right.}{2bc\sqrt{cx-1}\sqrt{cx+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(2
*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh[c
*x]])/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(2*a)/
b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{1}{2(a+bx)} - \frac{\cosh(2x)}{2(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{2c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-1}(\right.}{2c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\text{s}}{}$$

Mathematica [A] time = 0.201807, size = 105, normalized size = 0.76

$$\frac{\sqrt{-(cx-1)(cx+1)}\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) - \sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) - \log\left(a+b\cosh^{-1}(cx)\right)\right)}{2bc\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[-((-1 + c*x)*(1 + c*x))]*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh
[c*x])] - Log[a + b*ArcCosh[c*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + Arc
Cosh[c*x])]))/(2*b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.106, size = 227, normalized size = 1.6

$$\frac{1}{(4cx+4)(cx-1)cb}\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)\text{Ei}\left(1, 2\,\text{arccosh}(cx)+2\,\frac{a}{b}\right)e^{\frac{b\,\text{arccosh}(cx)+2\,a}{b}} + \frac{1}{(4cx+4)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/4*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*
arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b+1/4*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccos
h(c*x)-2*a/b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/(c*x-1)/c/b-1/2*(-c^2*x^2
+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ln(a+b*arccosh(c*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b\operatorname{arcosh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{b \operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

3.272 $\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=116

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)-\frac{\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b\sqrt{1-cx}}+\frac{\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b\sqrt{1-cx}}$$

```
[Out] -((Sqrt[-1 + c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(b*Sqrt[1
 - c*x])) + (Sqrt[-1 + c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])
/(b*Sqrt[1 - c*x]) + Unintegrable[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x
])), x]
```

Rubi [A] time = 1.07703, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(b*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcC
osh[c*x]])/(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Defer[Int]
[1/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(-\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{c^2x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{1-c^2x^2}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1}}{}$$

Mathematica [A] time = 1.20046, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.234, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{arccosh}(cx)\right)}\sqrt{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\,\mathrm{arcosh}\,(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^2x^2+1}}{bx\,\mathrm{arcosh}\,(cx)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x*arccosh(c*x) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{x(a+b\,\mathrm{acosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\,\mathrm{arcosh}\,(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x), x)
```

3.273 $\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=65

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)-\frac{c\sqrt{cx-1}\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{1-cx}}$$

```
[Out] -((c*Sqrt[-1 + c*x]*Log[a + b*ArcCosh[c*x]])/(b*Sqrt[1 - c*x])) + Unintegra
ble[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.930562, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (c*Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh[c*x]])/(b*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - (Sqrt[1 - c^2*x^2]*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
+ b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(\frac{c^2}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,d}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{c\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.04271, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.139, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\,\mathrm{arccosh}\,(cx)\right)}\sqrt{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^2x^2+1}}{bx^2\operatorname{arcosh}(cx)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x^2*arccosh(c*x) + a*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{x^2(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^2), x)
```

3.274 $\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.447323, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][(Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(x^3*(a + b*Ar
cCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.39863, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.296, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(a + b \operatorname{arccosh}\left(c x\right)\right)} \sqrt{-c^{2} x^{2} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^{2} x^{2} + 1}}{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^{2} x^{2} + 1}}{b x^{3} \operatorname{arcosh}\left(c x\right) + a x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x^3*arccosh(c*x) + a*x^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{x^3 (a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**3/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**3*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^3), x)
```

3.275 $\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.437965, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][(Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(x^4*(a + b*Ar
cCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.858984, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.401, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4}\left(a+b\operatorname{arccosh}\left(cx\right)\right)}\sqrt{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^{2}x^{2}+1}}{(b\operatorname{arcosh}\left(cx\right)+a)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^{2}x^{2}+1}}{bx^{4}\operatorname{arcosh}\left(cx\right)+ax^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x^4*arccosh(c*x) + a*x^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{x^4(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**4/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**4*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^4), x)
```

3.276 $\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=397

$$-\frac{3\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64bc^4\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh\right.}{b}\right.}{64bc^4\sqrt{cx-1}}$$

```
[Out] (-3*Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(64*b*c^4
*Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Ar
cCosh[c*x]))/b])/(64*b*c^4*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(5*a)/b]*C
oshIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(64*b*c^4*Sqrt[-1 + c*x]) - (Sqrt[
1 - c*x]*Cosh[(7*a)/b]*CoshIntegral[(7*(a + b*ArcCosh[c*x]))/b])/(64*b*c^4*
Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*
x])/b])/(64*b*c^4*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Sinh[(3*a)/b]*SinhInte
gral[(3*(a + b*ArcCosh[c*x]))/b])/(64*b*c^4*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x
]*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(64*b*c^4*Sqrt[-1
 + c*x]) + (Sqrt[1 - c*x]*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcCosh[c*x]
))/b])/(64*b*c^4*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.945604, antiderivative size = 497, normalized size of antiderivative = 1.25, number of steps used = 16, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64bc^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{64bc^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{5a}{b}\right)\text{C}}{64bc^4\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(64*b*c^4
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshInt
egral[(3*a)/b + 3*ArcCosh[c*x]])/(64*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(Sqrt[1 - c^2*x^2]*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(6
4*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Cosh[(7*a)/b]*Co
shIntegral[(7*a)/b + 7*ArcCosh[c*x]])/(64*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (3*Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(64*b
*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*Sinh[(3*a)/b]*Sin
```

```
hIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(64*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (Sqrt[1 - c^2*x^2]*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]]
)/(64*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[(7*a)/b
]*SinhIntegral[(7*a)/b + 7*ArcCosh[c*x]])/(64*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^3(x)\sinh^4(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{3\cosh(x)}{64(a+bx)} - \frac{3\cosh(3x)}{64(a+bx)} - \frac{\cosh(5x)}{64(a+bx)} + \frac{\cosh(7x)}{64(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{64c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\right)\mathrm{Sub}}{64c^4\sqrt{-}}$$

$$= -\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64bc^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{64bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.923653, size = 215, normalized size = 0.54

$$\frac{\sqrt{1-c^2x^2}\left(-3\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3\cosh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\mathbf{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 3*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Cosh[(5*a)/b]*CoshIntegral[
5*(a/b + ArcCosh[c*x])] - Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcCosh[c*x])
] + 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Sinh[(3*a)/b]*SinhInte
gral[3*(a/b + ArcCosh[c*x])] - Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[
```

```
c*x])] + Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCosh[c*x])]))/(64*c^4*Sqrt[
(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] time = 0.263, size = 725, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] -1/128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1
,7*arccosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^4/(c*x-1)/b-1/
128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-7
*arccosh(c*x)-7*a/b)*exp((b*arccosh(c*x)-7*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/12
8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*ar
ccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b+3/128*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arcco
sh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/128*(-c^
2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c
*x)-a/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^4/(c*x-1)/b+3/128*(-c^2*x^2+1)
^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*
a/b)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/128*(-c^2*x^2+1)^(
1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*arccosh(c*x)-5*a/
b)*exp((b*arccosh(c*x)-5*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/128*(-c^2*x^2+1)^(1/
2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp(
(a+b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x^3}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^3/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(c^2x^5-x^3\right)\sqrt{-c^2x^2+1}}{b\,\mathrm{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^5 - x^3)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x^3}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^3/(b*arccosh(c*x) + a), x)
```

3.277 $\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=339

$$\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6(a+b\cosh^{-1}}{b}\right.}{32bc^3\sqrt{cx-1}}$$

```
[Out] (Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32*
b*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b
*ArcCosh[c*x]))/b])/(16*b*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Cosh[(6*a)/b
]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x]) - (Sq
rt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(16*b*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 -
c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt
[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c
*x]))/b])/(16*b*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Sinh[(6*a)/b]*SinhInte
gral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.881394, antiderivative size = 430, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{16bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}}{32bc^3\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(3
2*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*Co
shIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(16*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*CoshIntegral[(6*a)/b + 6*ArcCosh[c*x]
])/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Log[a + b*A
rcCosh[c*x]])/(16*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*
Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(32*b*c^3*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b
+ 4*ArcCosh[c*x]])/(16*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*
x^2]*Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c*x]])/(32*b*c^3*Sqrt[-
```

```
1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{1}{16(a+bx)}-\frac{\cosh(2x)}{32(a+bx)}-\frac{\cosh(4x)}{16(a+bx)}+\frac{\cosh(6x)}{32(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{1-c^2}}{}$$

$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-1}\right.}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}-$$

Mathematica [A] time = 0.716213, size = 188, normalized size = 0.55

$$-\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\cosh\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*(-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])])
- 2*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]*Cosh
Integral[6*(a/b + ArcCosh[c*x])] + 2*Log[a + b*ArcCosh[c*x]] + Sinh[(2*a)/b
]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/
b + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/(
32*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [A] time = 0.225, size = 591, normalized size = 1.7

$$-\frac{1}{(64\,cx+64)\,c^{3}\,(cx-1)\,b}\sqrt{-c^{2}x^{2}+1}\left(-\sqrt{cx+1}\sqrt{cx-1}xc+c^{2}x^{2}-1\right)\mathrm{Ei}\left(1,6\,\mathrm{arccosh}\,(cx)+6\,\frac{a}{b}\right)\mathrm{e}^{\frac{b\mathrm{arccosh}(cx)+6\,a}{b}}-\frac{}{(64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] -1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/6
4*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*a
rccosh(c*x)-6*a/b)*exp((b*arccosh(c*x)-6*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/16*(
-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*ln(a+b*arccosh(c*x))/b+1/
32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*a
rccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arcco
sh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(
c*x)-2*a/b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/32*(-c^2*x^
2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c*x
)-4*a/b)*exp((b*arccosh(c*x)-4*a)/b)/(c*x+1)/c^3/(c*x-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}x^{2}}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(c^{2}x^{4}-x^{2}\right)\sqrt{-c^{2}x^{2}+1}}{b\,\mathrm{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(-(cx-1)(cx+1)\right)^{\frac{3}{2}}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x^2}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a), x)
```

3.278 $\int \frac{x\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=297

$$-\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^2\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^2\sqrt{cx-1}}-\frac{\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}\right.}{b}\right.}{16bc^2\sqrt{cx-1}}$$

```
[Out] -(Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(8*b*c^2*Sq
rt[-1 + c*x]) + (3*Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCo
sh[c*x]))/b])/(16*b*c^2*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Cosh[(5*a)/b]*Cosh
Integral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b*c^2*Sqrt[-1 + c*x]) + (Sqrt[1 -
 c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b*c^2*Sqrt[-1 + c*
x]) - (3*Sqrt[1 - c*x]*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/
b])/(16*b*c^2*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Sinh[(5*a)/b]*SinhIntegral[(
5*(a + b*ArcCosh[c*x]))/b])/(16*b*c^2*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.675342, antiderivative size = 371, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16bc^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{5a}{b}\right)\text{Ch}}{16bc^2\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(8*b*c^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshIntegr
al[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sq
rt[1 - c^2*x^2]*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b
*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhInteg
ral[a/b + ArcCosh[c*x]])/(8*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1
 - c^2*x^2]*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*c^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[(5*a)/b]*SinhInteg
ral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)} - \frac{3\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{16c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{8c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{8c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\right)\mathrm{Subs}}{16c^2\sqrt{-1}}$$

$$= -\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.663466, size = 172, normalized size = 0.58

$$\frac{\sqrt{1-c^2x^2}\left(-2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(\right.\right.\right.}{16c^2\sqrt{\frac{cx-1}{cx+1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 3*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - Cosh[(5*a)/b]*CoshIntegral[
5*(a/b + ArcCosh[c*x])] + 2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*
Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + Sinh[(5*a)/b]*SinhInte
gral[5*(a/b + ArcCosh[c*x])]))/(16*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*
x))
```

Maple [B] time = 0.185, size = 543, normalized size = 1.8

$$-\frac{1}{(32cx+32)c^2(cx-1)b}\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)\mathrm{Ei}\left(1,5\,\mathrm{arccosh}(cx)+5\frac{a}{b}\right)\mathrm{e}^{\frac{b\,\mathrm{arccosh}(cx)+5a}{b}} - \frac{}{(32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] -1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
5*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/3
2*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*a
rccosh(c*x)-5*a/b)*exp((b*arccosh(c*x)-5*a)/b)/(c*x+1)/c^2/(c*x-1)/b+3/32*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arcco
sh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/16*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*
x)-a/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^2/(c*x-1)/b+3/32*(-c^2*x^2+1)^(
1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*a/
b)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/16*(-c^2*x^2+1)^(1/2
)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((
a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(c^2x^3-x\right)\sqrt{-c^2x^2+1}}{b\,\mathrm{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a), x)
```

3.279 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=239

$$\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc\sqrt{cx-1}}$$

```
[Out] (Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(2*b
*c*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*Ar
cCosh[c*x]))/b])/(8*b*c*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Log[a + b*ArcCos
h[c*x]])/(8*b*c*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(2*a)/b]*SinhIntegral
[(2*(a + b*ArcCosh[c*x]))/b])/(2*b*c*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Sinh[
(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(8*b*c*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.458781, antiderivative size = 304, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {5713, 5701, 3312, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{2a}{b}\right)\text{Shi}}{2bc\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(2
*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*CoshI
ntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
3*Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh[c*x]])/(8*b*c*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[
c*x]])/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[(4*a)
/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
```

```
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\sinh^4(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{3}{8(a+bx)}-\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{3\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{8bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{8c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c}}{}$$
$$= -\frac{3\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{8bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-}\right.}{2c\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc\sqrt{-1+cx}\sqrt{1+cx}} -$$

Mathematica [A] time = 0.441693, size = 147, normalized size = 0.62

$$-\frac{\sqrt{1-c^2x^2}\left(-4\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+4\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\cosh\right.\right.\right.}{8bc\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*(-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])]
+ Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + 3*Log[a + b*ArcCosh[
c*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - Sinh[(4*a)/b
]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(8*b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*
(1 + c*x))
```

Maple [A] time = 0.131, size = 409, normalized size = 1.7

$$-\frac{1}{(16\,cx+16)\,(cx-1)\,cb}\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)\mathrm{Ei}\left(1,4\,\mathrm{arccosh}\,(cx)+4\,\frac{a}{b}\right)\mathrm{e}^{\frac{b\,\mathrm{arccosh}(cx)+4\,a}{b}} - \frac{}{(16\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] -1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b-1/16*
(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arc
cosh(c*x)-4*a/b)*exp((b*arccosh(c*x)-4*a)/b)/(c*x+1)/(c*x-1)/c/b-3/8*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ln(a+b*arccosh(c*x))/b+1/4*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c
*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b+1/4*(-c^2*x^2+1)
^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c*x)-2*
a/b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/(c*x-1)/c/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b\operatorname{arcosh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

3.280 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=215

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)-\frac{5\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b\sqrt{1-cx}}+\frac{\sqrt{cx-1}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3}{}\right.}{4b\sqrt{1-cx}}$$

```
[Out] (-5*Sqrt[-1 + c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(4*b*Sqr
t[1 - c*x]) + (Sqrt[-1 + c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[
c*x]))/b])/(4*b*Sqrt[1 - c*x]) + (5*Sqrt[-1 + c*x]*Sinh[a/b]*SinhIntegral[(
a + b*ArcCosh[c*x])/b])/(4*b*Sqrt[1 - c*x]) - (Sqrt[-1 + c*x]*Sinh[(3*a)/b]
*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b*Sqrt[1 - c*x]) + Unintegrab
le[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 1.75167, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (5*Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(4*b*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshIntegral[(3
*a)/b + 3*ArcCosh[c*x]])/(4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*Sqrt[1 - c
^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*Arc
Cosh[c*x]])/(4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Defer[I
nt][1/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \left(\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{2c^2x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{c^4x^3}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(2c^2\sqrt{1-c^2x^2}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,d}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)dx,\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{2\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \cdots$$

$$= \frac{2\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \cdots$$

$$= \frac{5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4b\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.23866, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.247, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}}{bx\,\mathrm{arcosh}\left(cx\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x*arccosh(c*x) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{(b\operatorname{arcosh}(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x), x)
```

3.281 $\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=163

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)+\frac{c\sqrt{cx-1}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2b\sqrt{1-cx}}-\frac{c\sqrt{cx-1}\sinh\left(\frac{2a}{b}\right)\text{Sh}}{2b\sqrt{1-}}$$

```
[Out] (c*Sqrt[-1 + c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(
2*b*Sqrt[1 - c*x]) - (3*c*Sqrt[-1 + c*x]*Log[a + b*ArcCosh[c*x]])/(2*b*Sqrt
[1 - c*x]) - (c*Sqrt[-1 + c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh
[c*x]))/b])/(2*b*Sqrt[1 - c*x]) + Unintegrable[1/(x^2*Sqrt[1 - c^2*x^2]*(a
+ b*ArcCosh[c*x])), x]
```

Rubi [A] time = 1.52505, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -(c*Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])
/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*c*Sqrt[1 - c^2*x^2]*Log[a + b*ArcC
osh[c*x]])/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*Sqrt[1 - c^2*x^2]*Sinh[(
2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(2*b*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) - (Sqrt[1 - c^2*x^2]*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\cosh^{-1}(cx))}dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^2(a+b\cosh^{-1}(cx))}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \left(-\frac{2c^2}{\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))} + \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))} + \frac{c^4x^2}{\sqrt{-1+cx}\sqrt{1+cx}(a+b}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))}dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(2c^2\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{2c\sqrt{1-c^2x^2}\log(a+b\cosh^{-1}(cx))}{b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))}dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(c\sqrt{1-}}{}$$

$$= \frac{2c\sqrt{1-c^2x^2}\log(a+b\cosh^{-1}(cx))}{b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))}dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(c\sqrt{1-}}{}$$

$$= \frac{3c\sqrt{1-c^2x^2}\log(a+b\cosh^{-1}(cx))}{2b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))}dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(c\sqrt{1-}}{}$$

$$= \frac{3c\sqrt{1-c^2x^2}\log(a+b\cosh^{-1}(cx))}{2b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))}dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(c\sqrt{1-}}{}$$

$$= -\frac{c\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2b\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c\sqrt{1-c^2x^2}\log(a+b\cosh^{-1}(cx))}{2b\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c}{}$$

Mathematica [A] time = 1.37218, size = 0, normalized size = 0.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\cosh^{-1}(cx))}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.253, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b\operatorname{arccosh}(cx))} \left(-c^2x^2 + 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2 + 1\right)^{\frac{3}{2}}}{(b \operatorname{arcosh}(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(-c^2x^2 + 1\right)^{\frac{3}{2}}}{bx^2 \operatorname{arcosh}(cx) + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x^2*arccosh(c*x) + a*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x^{2}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**2*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^2), x)
```

3.282 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.525398, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(x^3*(a
+ b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx=-\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.37209, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.309, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(a+b\operatorname{arccosh}\left(cx\right)\right)}\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}}{bx^{3}\operatorname{arcosh}\left(cx\right)+ax^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x^3*arccosh(c*x) + a*x^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x^3(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**3/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**3*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{(b\operatorname{arcosh}(cx)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^3), x)
```

3.283 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.5394, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(x^4*(a
+ b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx=-\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.877244, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.382, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(a+b \,\mathrm{arccosh}\left(c x\right)\right)} \left(-c^{2} x^{2}+1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}}}{(b\,\mathrm{arcosh}\left(c x\right)+a)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}}}{b x^{4} \,\mathrm{arcosh}\left(c x\right)+a x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x^4*arccosh(c*x) + a*x^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x^4(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**4/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**4*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{(b\operatorname{arcosh}(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^4), x)
```

3.284 $\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=397

$$-\frac{3\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{128bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^4\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7\left(a+b\cosh\right.}{b}\right.}{256bc^4\sqrt{cx-1}}$$

```
[Out] (-3*Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(128*b*c^
4*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Arc
Cosh[c*x]))/b])/(32*b*c^4*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Cosh[(7*a)/b]*
CoshIntegral[(7*(a + b*ArcCosh[c*x]))/b])/(256*b*c^4*Sqrt[-1 + c*x]) + (Sqr
t[1 - c*x]*Cosh[(9*a)/b]*CoshIntegral[(9*(a + b*ArcCosh[c*x]))/b])/(256*b*c
^4*Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh
[c*x])/b])/(128*b*c^4*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(3*a)/b]*SinhIn
tegral[(3*(a + b*ArcCosh[c*x]))/b])/(32*b*c^4*Sqrt[-1 + c*x]) + (3*Sqrt[1 -
 c*x]*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcCosh[c*x]))/b])/(256*b*c^4*Sq
rt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(9*a)/b]*SinhIntegral[(9*(a + b*ArcCosh
[c*x]))/b])/(256*b*c^4*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.996848, antiderivative size = 497, normalized size of antiderivative = 1.25, number of steps used = 16, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{128bc^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{32bc^4\sqrt{cx-1}\sqrt{cx+1}}-\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{7a}{b}\right)\text{C}}{256bc^4\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(128*b*c^
4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshInte
gral[(3*a)/b + 3*ArcCosh[c*x]])/(32*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
3*Sqrt[1 - c^2*x^2]*Cosh[(7*a)/b]*CoshIntegral[(7*a)/b + 7*ArcCosh[c*x]])/(
256*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(9*a)/b]*
CoshIntegral[(9*a)/b + 9*ArcCosh[c*x]])/(256*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (3*Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(1
28*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(3*a)/b]*S
```

```
inhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(32*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (3*Sqrt[1 - c^2*x^2]*Sinh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcCosh[c
*x]])/(256*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(9
*a)/b]*SinhIntegral[(9*a)/b + 9*ArcCosh[c*x]])/(256*b*c^4*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^3(x)\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(-\frac{3\cosh(x)}{128(a+bx)}+\frac{\cosh(3x)}{32(a+bx)}-\frac{3\cosh(7x)}{256(a+bx)}+\frac{\cosh(9x)}{256(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(9x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{256c^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(3\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x, \cosh^{-1}\right.}{256c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{128c^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\right)\mathrm{Subs}}{32c^4\sqrt{-}}$$

$$= -\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{128bc^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{32bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.2813, size = 216, normalized size = 0.54

$$\frac{\sqrt{1-c^2x^2}\left(-6\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+8\cosh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-3\cosh\left(\frac{7a}{b}\right)\mathbf{Chi}\left(7\left(\frac{a}{b}+\cosh^{-1}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-6*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 8*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*Cosh[(7*a)/b]*CoshIntegra
l[7*(a/b + ArcCosh[c*x])] + Cosh[(9*a)/b]*CoshIntegral[9*(a/b + ArcCosh[c*x
])] + 6*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 8*Sinh[(3*a)/b]*SinhIn
tegral[3*(a/b + ArcCosh[c*x])] + 3*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcC
```

```
osh[c*x])] - Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcCosh[c*x])]))/(256*c^4*
Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] time = 0.254, size = 725, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/512*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
9*arccosh(c*x)+9*a/b)*exp((b*arccosh(c*x)+9*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/5
12*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-9*
arccosh(c*x)-9*a/b)*exp((b*arccosh(c*x)-9*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/512
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,7*arc
cosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/64*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arccosh
(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/256*(-c^2*
x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x
)-a/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/64*(-c^2*x^2+1)^(1
/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*a/b
)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/512*(-c^2*x^2+1)^(1/2
)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-7*arccosh(c*x)-7*a/b)*
exp((b*arccosh(c*x)-7*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/256*(-c^2*x^2+1)^(1/2)*
(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((a+
b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}x^3}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^3/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^{4}x^{7}-2\,c^{2}x^{5}+x^{3}\right)\sqrt{-c^{2}x^{2}+1}}{b\operatorname{arcosh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^7 - 2*c^2*x^5 + x^3)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{5}{2}}x^{3}}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^3/(b*arccosh(c*x) + a), x)
```

3.285 $\int \frac{x^2\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=439

$$\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\cosh^{-1}\right.}{b}\right.}{32bc^3\sqrt{cx-1}}$$

```
[Out] (Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32*
b*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b
*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Cosh[(6*a)/b
]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x]) + (Sq
rt[1 - c*x]*Cosh[(8*a)/b]*CoshIntegral[(8*(a + b*ArcCosh[c*x]))/b])/(128*b*
c^3*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(128*b*c^3*
Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCo
sh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(4*a)/b]*Sinh
Integral[(4*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 -
 c*x]*Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqr
t[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(8*a)/b]*SinhIntegral[(8*(a + b*ArcCosh[
c*x]))/b])/(128*b*c^3*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.979051, antiderivative size = 556, normalized size of antiderivative = 1.27, number of steps used = 16, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{32bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{6a}{b}\right)\text{Chi}}{32bc^3\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(3
2*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*Co
shIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*CoshIntegral[(6*a)/b + 6*ArcCosh[c*x]
])/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(8*a)/
b]*CoshIntegral[(8*a)/b + 8*ArcCosh[c*x]])/(128*b*c^3*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) - (5*Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh[c*x]])/(128*b*c^3*Sqrt[-1
```

```
 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(2*a)/b]*SinhIntegral[(2*a
)/b + 2*ArcCosh[c*x]])/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 -
c^2*x^2]*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(32*b*c^3*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[(6*a)/b]*SinhIntegral
[(6*a)/b + 6*ArcCosh[c*x]])/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt
[1 - c^2*x^2]*Sinh[(8*a)/b]*SinhIntegral[(8*a)/b + 8*ArcCosh[c*x]])/(128*b*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{5}{128(a+bx)}+\frac{\cosh(2x)}{32(a+bx)}+\frac{\cosh(4x)}{32(a+bx)}-\frac{\cosh(6x)}{32(a+bx)}+\frac{\cosh(8x)}{128(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{5\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{128bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(8x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{128c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c}}{}$$

$$= -\frac{5\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{128bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-}\right.}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}} -$$

Mathematica [A] time = 1.16922, size = 233, normalized size = 0.53

$$\frac{\sqrt{1-c^2x^2}\left(4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+4\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-4\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\cosh\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] +
4*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] - 4*Cosh[(6*a)/b]*Cosh
Integral[6*(a/b + ArcCosh[c*x])] + Cosh[(8*a)/b]*CoshIntegral[8*(a/b + ArcC
osh[c*x])] - 5*Log[a + b*ArcCosh[c*x]] - 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/
```

```
b + ArcCosh[c*x])] - 4*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])] +
 4*Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - Sinh[(8*a)/b]*SinhI
ntegral[8*(a/b + ArcCosh[c*x])]))/(128*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b +
b*c*x))
```

Maple [B] time = 0.262, size = 773, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/256*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
8*arccosh(c*x)+8*a/b)*exp((b*arccosh(c*x)+8*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/2
56*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-8*
arccosh(c*x)-8*a/b)*exp((b*arccosh(c*x)-8*a)/b)/(c*x+1)/c^3/(c*x-1)/b-5/128
*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*ln(a+b*arccosh(c*x))/b-
1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6
*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arc
cosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh
(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c^2*x
^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c*
x)-2*a/b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c^2*x^2+
1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c*x)-
4*a/b)*exp((b*arccosh(c*x)-4*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/64*(-c^2*x^2+1)^
(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*arccosh(c*x)-6*a
/b)*exp((b*arccosh(c*x)-6*a)/b)/(c*x+1)/c^3/(c*x-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}x^2}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^6 - 2\,c^2x^4 + x^2\right)\sqrt{-c^2x^2 + 1}}{b\,\text{arcosh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2 + 1\right)^{\frac{5}{2}}x^2}{b\,\text{arcosh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a), x)
```

3.286 $\int \frac{x(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=397

$$-\frac{5\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64bc^2\sqrt{cx-1}}+\frac{9\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^2\sqrt{cx-1}}-\frac{5\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^2\sqrt{cx-1}}$$

```
[Out] (-5*Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(64*b*c^2
*Sqrt[-1 + c*x]) + (9*Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Ar
cCosh[c*x]))/b])/(64*b*c^2*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*Cosh[(5*a)/b]
*CoshIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(64*b*c^2*Sqrt[-1 + c*x]) + (Sqr
t[1 - c*x]*Cosh[(7*a)/b]*CoshIntegral[(7*(a + b*ArcCosh[c*x]))/b])/(64*b*c^
2*Sqrt[-1 + c*x]) + (5*Sqrt[1 - c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[
c*x])/b])/(64*b*c^2*Sqrt[-1 + c*x]) - (9*Sqrt[1 - c*x]*Sinh[(3*a)/b]*SinhIn
tegral[(3*(a + b*ArcCosh[c*x]))/b])/(64*b*c^2*Sqrt[-1 + c*x]) + (5*Sqrt[1 -
 c*x]*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(64*b*c^2*Sqr
t[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcCosh[
c*x]))/b])/(64*b*c^2*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.791021, antiderivative size = 497, normalized size of antiderivative = 1.25, number of steps used = 16, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64bc^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{9\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{64bc^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{5\sqrt{1-c^2x^2}\cosh\left(\frac{5a}{b}\right)}{64bc^2\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5*Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(64*b*c^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (9*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshInt
egral[(3*a)/b + 3*ArcCosh[c*x]])/(64*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(5*Sqrt[1 - c^2*x^2]*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]])/
(64*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(7*a)/b]*
CoshIntegral[(7*a)/b + 7*ArcCosh[c*x]])/(64*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (5*Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(64
*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (9*Sqrt[1 - c^2*x^2]*Sinh[(3*a)/b]*S
```

```
inhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(64*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (5*Sqrt[1 - c^2*x^2]*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c
*x]])/(64*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(7*
a)/b]*SinhIntegral[(7*a)/b + 7*ArcCosh[c*x]])/(64*b*c^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{5\cosh(x)}{64(a+bx)}+\frac{9\cosh(3x)}{64(a+bx)}-\frac{5\cosh(5x)}{64(a+bx)}+\frac{\cosh(7x)}{64(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(5\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(\right.}{64c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(9\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\right)\text{Sub}}{64c^2\sqrt{-}}$$

$$= -\frac{5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{9\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{64bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.05884, size = 216, normalized size = 0.54

$$\frac{\sqrt{1-c^2x^2}\left(-5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-5*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 9*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 5*Cosh[(5*a)/b]*CoshIntegra
l[5*(a/b + ArcCosh[c*x])] + Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcCosh[c*x
])] + 5*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 9*Sinh[(3*a)/b]*SinhIn
tegral[3*(a/b + ArcCosh[c*x])] + 5*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcC
```

```
osh[c*x])] - Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCosh[c*x])]))/(64*c^2*S
qrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] time = 0.213, size = 725, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
7*arccosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^2/(c*x-1)/b+1/1
28*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-7*
arccosh(c*x)-7*a/b)*exp((b*arccosh(c*x)-7*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/128
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arc
cosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b+9/128*(-
c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arccos
h(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/128*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*
x)-a/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^2/(c*x-1)/b+9/128*(-c^2*x^2+1)^
(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*a
/b)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/128*(-c^2*x^2+1)^(1
/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*arccosh(c*x)-5*a/b
)*exp((b*arccosh(c*x)-5*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/128*(-c^2*x^2+1)^(1/2
)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((
a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}x}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^5 - 2\,c^2x^3 + x\right)\sqrt{-c^2x^2 + 1}}{b\,\text{arcosh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2 + 1\right)^{\frac{5}{2}}x}{b\,\text{arcosh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a), x)
```

3.287 $\int \frac{(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=339

$$\frac{15\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc\sqrt{cx-1}} - \frac{3\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc\sqrt{cx-1}} + \frac{\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{32bc\sqrt{cx-1}}$$

```
[Out] (15*Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(
32*b*c*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a
+ b*ArcCosh[c*x]))/b])/(16*b*c*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(6*a)/
b]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c*Sqrt[-1 + c*x]) - (5*S
qrt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(16*b*c*Sqrt[-1 + c*x]) - (15*Sqrt[1
- c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32*b*c*Sqrt
[-1 + c*x]) + (3*Sqrt[1 - c*x]*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh
[c*x]))/b])/(16*b*c*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(6*a)/b]*SinhInte
gral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.568075, antiderivative size = 430, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {5713, 5701, 3312, 3303, 3298, 3301}

$$\frac{15\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc\sqrt{cx-1}\sqrt{cx+1}} - \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{16bc\sqrt{cx-1}\sqrt{cx+1}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{6a}{b}\right)}{32bc\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (15*Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])
/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]
*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*CoshIntegral[(6*a)/b + 6*ArcCosh[c*x
]])/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*Sqrt[1 - c^2*x^2]*Log[a + b*
ArcCosh[c*x]])/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*Sqrt[1 - c^2*x^2
]*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(32*b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*Sinh[(4*a)/b]*SinhIntegral[(4*a)/
b + 4*ArcCosh[c*x]])/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*
x^2]*Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c*x]])/(32*b*c*Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{5}{16(a+bx)} - \frac{15\cosh(2x)}{32(a+bx)} + \frac{3\cosh(4x)}{16(a+bx)} - \frac{\cosh(6x)}{32(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{5\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{32c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(3\sqrt{1-\right.}{}$$

$$= -\frac{5\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(15\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cos\right.}{32c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{15\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx\right.}{16bc\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.787367, size = 191, normalized size = 0.56

$$\frac{\sqrt{1-c^2x^2}\left(15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) - 6\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + \cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\cosh^{-1}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(15*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] -
 6*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]*CoshI
ntegral[6*(a/b + ArcCosh[c*x])] - 10*Log[a + b*ArcCosh[c*x]] - 15*Sinh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 6*Sinh[(4*a)/b]*SinhIntegral[4*
(a/b + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])])
)/(32*b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.167, size = 591, normalized size = 1.7

$$\frac{1}{(64cx+64)(cx-1)cb}\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)\text{Ei}\left(1, 6\,\text{arccosh}(cx)+6\,\frac{a}{b}\right)e^{\frac{b\,\text{arccosh}(cx)+6a}{b}} + \frac{}{(64cx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6
*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/(c*x-1)/c/b+1/64*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*arcc
osh(c*x)-6*a/b)*exp((b*arccosh(c*x)-6*a)/b)/(c*x+1)/(c*x-1)/c/b-5/16*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ln(a+b*arccosh(c*x))/b-3/32*(-c^
2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(
c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b+15/64*(-c^2*x^2
+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+
2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b+15/64*(-c^2*x^2+1)^(
1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c*x)-2*a/
b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/(c*x-1)/c/b-3/32*(-c^2*x^2+1)^(1/2)*
(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c*x)-4*a/b)*ex
p((b*arccosh(c*x)-4*a)/b)/(c*x+1)/(c*x-1)/c/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{b\operatorname{arcosh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{b\,\mathrm{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a), x)
```

3.288 $\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=309

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))},x\right)-\frac{11\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b\sqrt{1-cx}}+\frac{7\sqrt{cx-1}\cosh\left(\frac{3a}{b}\right)\text{Chi}}{16b\sqrt{1-c}}$$

```
[Out] (-11*Sqrt[-1 + c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(8*b*Sq
rt[1 - c*x]) + (7*Sqrt[-1 + c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCo
sh[c*x]))/b])/(16*b*Sqrt[1 - c*x]) - (Sqrt[-1 + c*x]*Cosh[(5*a)/b]*CoshInte
gral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b*Sqrt[1 - c*x]) + (11*Sqrt[-1 + c*x]
*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b*Sqrt[1 - c*x]) - (7*S
qrt[-1 + c*x]*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(16*b
*Sqrt[1 - c*x]) + (Sqrt[-1 + c*x]*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcC
osh[c*x]))/b])/(16*b*Sqrt[1 - c*x]) + Unintegrable[1/(x*Sqrt[1 - c^2*x^2]*(
a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 2.37614, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x\,(a+b\cosh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (11*Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(8*b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (7*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshIntegral
[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 -
 c^2*x^2]*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (11*Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b
 + ArcCosh[c*x]])/(8*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*Sqrt[1 - c^2*x^2]
*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5
*ArcCosh[c*x]])/(16*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*De
fer[Int][1/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])), x])/(Sqrt
```

```
[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(-\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{3c^2x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{3c^4x^3}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(3c^2\sqrt{1-c^2x^2}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,d}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^5(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8(a+bx)}+\frac{5\cosh(3x)}{16(a+bx)}+\right.\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}}+$$

$$= \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}}-$$

$$= \frac{11\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b\sqrt{-1+cx}\sqrt{1+cx}}-\frac{7\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\textbf{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx\right.}{16b\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.26989, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.268, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a + b\operatorname{arccosh}\left(cx\right)\right)}\left(-c^2x^2+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{bx\operatorname{arcosh}\left(cx\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x*arccosh(c*x) + a
*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\,\mathrm{arcosh}\,(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x), x)
```

3.289 $\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=254

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)+\frac{c\sqrt{cx-1}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b\sqrt{1-cx}}-\frac{c\sqrt{cx-1}\cosh\left(\frac{4a}{b}\right)\text{C}}{8b\sqrt{1}}$$

```
[Out] (c*Sqrt[-1 + c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(
b*Sqrt[1 - c*x]) - (c*Sqrt[-1 + c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*A
rcCosh[c*x]))/b])/(8*b*Sqrt[1 - c*x]) - (15*c*Sqrt[-1 + c*x]*Log[a + b*ArcC
osh[c*x]])/(8*b*Sqrt[1 - c*x]) - (c*Sqrt[-1 + c*x]*Sinh[(2*a)/b]*SinhIntegr
al[(2*(a + b*ArcCosh[c*x]))/b])/(b*Sqrt[1 - c*x]) + (c*Sqrt[-1 + c*x]*Sinh[
(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(8*b*Sqrt[1 - c*x]) + Un
integrable[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 2.09303, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((c*Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]]
)/(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (c*Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*Co
shIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(15*c*Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh[c*x]])/(8*b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (c*Sqrt[1 - c^2*x^2]*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcC
osh[c*x]])/(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c*Sqrt[1 - c^2*x^2]*Sinh[(4*
a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (Sqrt[1 - c^2*x^2]*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(\frac{3c^2}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{3c^4x^2}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(3c^2\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{3c\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c\sqrt{1-\right.}{}$$

$$= \frac{3c\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c\sqrt{1-\right.}{}$$

$$= \frac{15c\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{8b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c\sqrt{1\right.}{}$$

$$= \frac{15c\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{8b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c\sqrt{1\right.}{}$$

$$= -\frac{c\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh\right.}{8b\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.26887, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.286, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(a+b \operatorname{arccosh}\left(c x\right)\right)} \left(-c^{2} x^{2}+1\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{5}{2}}}{(b \operatorname{arcosh}\left(c x\right)+a) x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^{4} x^{4}-2 \, c^{2} x^{2}+1\right) \sqrt{-c^{2} x^{2}+1}}{b x^{2} \operatorname{arcosh}\left(c x\right)+a x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^2*arccosh(c*x) +
 a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^2), x)
```

3.290 $\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.546165, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(x^3*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.42476, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.342, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b\operatorname{arccosh}(cx))} \left(-c^2x^2+1\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{(b\operatorname{arcosh}(cx)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{bx^3\operatorname{arcosh}(cx)+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^3*arccosh(c*x) +
 a*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**3/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{(b\,\mathrm{arcosh}\,(cx)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^3), x)
```

3.291 $\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.551468, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(x^4*(a +
b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.966696, size = 0, normalized size = 0.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.458, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(a+b \,\mathrm{arccosh}\left(c x\right)\right)} \left(-c^{2} x^{2}+1\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(-c^{2} x^{2} + 1\right)}^{\frac{5}{2}}}{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)} x^{4}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(c^{4} x^{4} - 2 \, c^{2} x^{2} + 1\right)} \sqrt{-c^{2} x^{2} + 1}}{b x^{4} \operatorname{arcosh}\left(c x\right) + a x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^4*arccosh(c*x) +
 a*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**4/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{(b\,\mathrm{arcosh}\,(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^4), x)
```

3.292 $\int \frac{x^4}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{ax-1}\,\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^5\sqrt{1-ax}}+\frac{\sqrt{ax-1}\,\text{Chi}\left(4\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-ax}}+\frac{3\sqrt{ax-1}\log\left(\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-ax}}$$

```
[Out] (Sqrt[-1 + a*x]*CoshIntegral[2*ArcCosh[a*x]])/(2*a^5*Sqrt[1 - a*x]) + (Sqrt
[-1 + a*x]*CoshIntegral[4*ArcCosh[a*x]])/(8*a^5*Sqrt[1 - a*x]) + (3*Sqrt[-1
 + a*x]*Log[ArcCosh[a*x]])/(8*a^5*Sqrt[1 - a*x])
```

Rubi [A] time = 0.465873, antiderivative size = 137, normalized size of antiderivative = 1.4, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5798, 5781, 3312, 3301}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\,\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^5\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\,\text{Chi}\left(4\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-a^2x^2}}+\frac{3\sqrt{ax-1}\sqrt{ax+1}\log\left(\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[2*ArcCosh[a*x]])/(2*a^5*Sqrt[1 -
 a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[4*ArcCosh[a*x]])/(8
*a^5*Sqrt[1 - a^2*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[ArcCosh[a*x]]
)/(8*a^5*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
```

```
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^4}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\operatorname{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^5\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\operatorname{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^5\sqrt{1-a^2x^2}}\\
&= \frac{3\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-a^2x^2}}+\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\operatorname{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \cosh^{-1}\right.}{8a^5\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{-1+ax}\sqrt{1+ax}\operatorname{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^5\sqrt{1-a^2x^2}}+\frac{\sqrt{-1+ax}\sqrt{1+ax}\operatorname{Chi}\left(4\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-a^2x^2}}+\frac{3\sqrt{-1+}}{}\end{aligned}$$

Mathematica [A] time = 0.112654, size = 69, normalized size = 0.7

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(4\operatorname{Chi}\left(2\cosh^{-1}(ax)\right)+\operatorname{Chi}\left(4\cosh^{-1}(ax)\right)+3\log\left(\cosh^{-1}(ax)\right)\right)}{8a^5\sqrt{-(ax-1)(ax+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(4*CoshIntegral[2*ArcCosh[a*x]] + Cos
hIntegral[4*ArcCosh[a*x]] + 3*Log[ArcCosh[a*x]]))/(8*a^5*Sqrt[-((-1 + a*x)*
(1 + a*x))])
```

Maple [B] time = 0.263, size = 249, normalized size = 2.5

$$\frac{\mathrm{Ei}\left(1,4\,\mathrm{arccosh}\left(ax\right)\right)}{16\,a^{5}\left(a^{2}x^{2}-1\right)}\sqrt{-a^{2}x^{2}+1}\sqrt{ax-1}\sqrt{ax+1}+\frac{\mathrm{Ei}\left(1,-4\,\mathrm{arccosh}\left(ax\right)\right)}{16\,a^{5}\left(a^{2}x^{2}-1\right)}\sqrt{-a^{2}x^{2}+1}\sqrt{ax-1}\sqrt{ax+1}-\frac{3\,\ln\left(\mathrm{arccos}\right.}{8\,a^{5}\left(a^{2}x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*Ei(1,4*
arccosh(a*x))+1/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*
x^2-1)*Ei(1,-4*arccosh(a*x))-3/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)/a^5/(a^2*x^2-1)*ln(arccosh(a*x))+1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*
(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*Ei(1,2*arccosh(a*x))+1/4*(-a^2*x^2+1)^(1/2)*(
a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*Ei(1,-2*arccosh(a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\sqrt{-a^{2}x^{2}+1}\,\mathrm{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^{2}x^{2}+1}\,x^{4}}{\left(a^{2}x^{2}-1\right)\mathrm{arcosh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-(ax-1)(ax+1)}\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

3.293 $\int \frac{x^3}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$\frac{3\sqrt{ax-1}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-ax}}+\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-ax}}$$

```
[Out] (3*Sqrt[-1 + a*x]*CoshIntegral[ArcCosh[a*x]])/(4*a^4*Sqrt[1 - a*x]) + (Sqrt
[-1 + a*x]*CoshIntegral[3*ArcCosh[a*x]])/(4*a^4*Sqrt[1 - a*x])
```

Rubi [A] time = 0.450474, antiderivative size = 91, normalized size of antiderivative = 1.4, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5798, 5781, 3312, 3301}

$$\frac{3\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[ArcCosh[a*x]])/(4*a^4*Sqrt[1 -
 a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[3*ArcCosh[a*x]])/(4
*a^4*Sqrt[1 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
```

```
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^4\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^4\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int\right.}{4a^4\sqrt{1-}}$$

$$= \frac{3\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0873484, size = 60, normalized size = 0.92

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(3\text{Chi}\left(\cosh^{-1}(ax)\right)+\text{Chi}\left(3\cosh^{-1}(ax)\right)\right)}{4a^4\sqrt{-(ax-1)(ax+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(3*CoshIntegral[ArcCosh[a*x]] + CoshI
ntegral[3*ArcCosh[a*x]]))/(4*a^4*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

Maple [B] time = 0.21, size = 200, normalized size = 3.1

$$\frac{\mathrm{Ei}\left(1,3\,\mathrm{arccosh}\left(ax\right)\right)}{8\,a^{4}\left(a^{2}x^{2}-1\right)}\sqrt{-a^{2}x^{2}+1}\sqrt{ax-1}\sqrt{ax+1}+\frac{\mathrm{Ei}\left(1,-3\,\mathrm{arccosh}\left(ax\right)\right)}{8\,a^{4}\left(a^{2}x^{2}-1\right)}\sqrt{-a^{2}x^{2}+1}\sqrt{ax-1}\sqrt{ax+1}+\frac{3\,\mathrm{Ei}\left(1,\mathrm{arcc}\right.}{8\,a^{4}\left(a^{2}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^4/(a^2*x^2-1)*Ei(1,3*a
rccosh(a*x))+1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^4/(a^2*x^
2-1)*Ei(1,-3*arccosh(a*x))+3/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)/a^4/(a^2*x^2-1)*Ei(1,arccosh(a*x))+3/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*
(a*x+1)^(1/2)/a^4/(a^2*x^2-1)*Ei(1,-arccosh(a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\sqrt{-a^{2}x^{2}+1}\,\mathrm{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^{2}x^{2}+1}x^{3}}{\left(a^{2}x^{2}-1\right)\mathrm{arcosh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^3/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-(ax-1)(ax+1)}\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

3.294 $\int \frac{x^2}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-ax}}+\frac{\sqrt{ax-1}\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-ax}}$$

```
[Out] (Sqrt[-1 + a*x]*CoshIntegral[2*ArcCosh[a*x]])/(2*a^3*Sqrt[1 - a*x]) + (Sqrt
[-1 + a*x]*Log[ArcCosh[a*x]])/(2*a^3*Sqrt[1 - a*x])
```

Rubi [A] time = 0.433989, antiderivative size = 91, normalized size of antiderivative = 1.4, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5798, 5781, 3312, 3301}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[2*ArcCosh[a*x]])/(2*a^3*Sqrt[1 -
 a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[ArcCosh[a*x]])/(2*a^3*Sqrt[1
 - a^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
```

```
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^3\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^3\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \cosh^{-1}\right.}{2a^3\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0990738, size = 60, normalized size = 0.92

$$-\frac{\sqrt{-(ax-1)(ax+1)}\left(\text{Chi}\left(2\cosh^{-1}(ax)\right)+\log\left(\cosh^{-1}(ax)\right)\right)}{2a^3\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] -(Sqrt[-((-1 + a*x)*(1 + a*x))]*(CoshIntegral[2*ArcCosh[a*x]] + Log[ArcCosh
[a*x]]))/(2*a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [B] time = 0.159, size = 149, normalized size = 2.3

$$\frac{\mathrm{Ei}\left(1,2\,\mathrm{arccosh}\left(ax\right)\right)}{4\,a^{3}\left(a^{2}x^{2}-1\right)}\sqrt{-a^{2}x^{2}+1}\sqrt{ax-1}\sqrt{ax+1}+\frac{\mathrm{Ei}\left(1,-2\,\mathrm{arccosh}\left(ax\right)\right)}{4\,a^{3}\left(a^{2}x^{2}-1\right)}\sqrt{-a^{2}x^{2}+1}\sqrt{ax-1}\sqrt{ax+1}-\frac{\ln\left(\mathrm{arccosh}\right.}{2\,a^{3}\left(a^{2}x^{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*Ei(1,2*a
rccosh(a*x))+1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^
2-1)*Ei(1,-2*arccosh(a*x))-1/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)/a^3/(a^2*x^2-1)*ln(arccosh(a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{-a^{2}x^{2}+1}\,\mathrm{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^{2}x^{2}+1}x^{2}}{\left(a^{2}x^{2}-1\right)\mathrm{arcosh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(ax-1)(ax+1)}\,\mathrm{acosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-a^2x^2+1}\,\mathrm{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

3.295 $\int \frac{x}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{1-ax}}$$

```
[Out] (Sqrt[-1 + a*x]*CoshIntegral[ArcCosh[a*x]])/(a^2*Sqrt[1 - a*x])
```

Rubi [A] time = 0.300595, antiderivative size = 41, normalized size of antiderivative = 1.46, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5798, 5781, 3301}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[ArcCosh[a*x]])/(a^2*Sqrt[1 - a^2
*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\operatorname{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^2\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0796933, size = 50, normalized size = 1.79

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] -((Sqrt[-((-1 + a*x)*(1 + a*x))]*CoshIntegral[ArcCosh[a*x]])/(a^2*Sqrt[(-1
+ a*x)/(1 + a*x)]*(1 + a*x)))
```

Maple [B] time = 0.132, size = 100, normalized size = 3.6

$$\frac{\mathrm{Ei}\left(1,\operatorname{arccosh}(ax)\right)}{2\,a^2\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}+\frac{\mathrm{Ei}\left(1,-\operatorname{arccosh}(ax)\right)}{2\,a^2\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^2/(a^2*x^2-1)*Ei(1,arc
cosh(a*x))+1/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^2/(a^2*x^2-
1)*Ei(1,-arccosh(a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a^2x^2+1}\,\mathrm{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}x}{\left(a^2x^2-1\right)\mathrm{arcosh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(ax-1)(ax+1)}\,\mathrm{acosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a^2x^2+1}\,\mathrm{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

3.296 $\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{ax-1}\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{1-ax}}$$

```
[Out] (Sqrt[-1 + a*x]*Log[ArcCosh[a*x]])/(a*Sqrt[1 - a*x])
```

Rubi [A] time = 0.162425, antiderivative size = 41, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5713, 5674}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[ArcCosh[a*x]])/(a*Sqrt[1 - a^2*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5674

```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[Log[a + b*ArcCosh[c*x]]/(b*c*Sqrt[-
(d1*d2)]), x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && Eq
Q[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0591602, size = 47, normalized size = 1.68

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{-(ax-1)(ax+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Log[ArcCosh[a*x]])/(a*Sqrt[-((-1 + a*
x)*(1 + a*x))])
```

Maple [A] time = 0.079, size = 48, normalized size = 1.7

$$-\frac{\ln\left(\operatorname{arccosh}(ax)\right)}{a\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*ln(arccosh(a*
x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Fricas [B] time = 2.02138, size = 117, normalized size = 4.18

$$-\frac{\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}\log\left(\log\left(ax+\sqrt{a^2x^2-1}\right)\right)}{a^3x^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(log(a*x + sqrt(a^2*x^2 - 1)))/(a^
3*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-(ax-1)(ax+1)}\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

3.297 $\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

Rubi [A] time = 0.37818, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][1/(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*
ArcCosh[a*x]), x])/Sqrt[1 - a^2*x^2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx=\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.550111, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

Maple [A] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{x\mathrm{arccosh}\,(ax)}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}x\,\mathrm{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x*arccosh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^2x^3-x\right)\mathrm{arcosh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^3 - x)*arccosh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-\left(ax-1\right)\left(ax+1\right)}\operatorname{acosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}x\operatorname{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x*arccosh(a*x)), x)
```

3.298 $\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

Rubi [A] time = 0.377071, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][1/(x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x
]*ArcCosh[a*x]), x])/Sqrt[1 - a^2*x^2]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.684712, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

Maple [A] time = 0.141, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arccosh}(ax)} \frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}x^2 \operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^2x^4-x^2\right)\operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^4 - x^2)*arccosh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{-(ax-1)(ax+1)}\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}x^2\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)), x)
```

3.299 $\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=197

$$\frac{3\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^4\sqrt{1-cx}}+\frac{\sqrt{cx-1}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4bc^4\sqrt{1-cx}}-\frac{3\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^4\sqrt{1-cx}}$$

```
[Out] (3*Sqrt[-1 + c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(4*b*c^4*
Sqrt[1 - c*x]) + (Sqrt[-1 + c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCo
sh[c*x]))/b])/(4*b*c^4*Sqrt[1 - c*x]) - (3*Sqrt[-1 + c*x]*Sinh[a/b]*SinhInt
egral[(a + b*ArcCosh[c*x])/b])/(4*b*c^4*Sqrt[1 - c*x]) - (Sqrt[-1 + c*x]*Si
nh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b*c^4*Sqrt[1 - c*x
])
```

Rubi [A] time = 0.689944, antiderivative size = 245, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 3312, 3303, 3298, 3301}

$$\frac{3\sqrt{cx-1}\sqrt{cx+1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^4\sqrt{1-c^2x^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^4\sqrt{1-c^2x^2}}-3\sqrt{cx-1}\sqrt{cx+}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])
/(4*b*c^4*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[(3*a)/b]*
CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b*c^4*Sqrt[1 - c^2*x^2]) - (3*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b
*c^4*Sqrt[1 - c^2*x^2]) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sinh[(3*a)/b]*SinhI
ntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b*c^4*Sqrt[1 - c^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4c^4\sqrt{1-c^2x^2}} + \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Su}}{4c}$$

$$= \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4c^4\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\right.}{}$$

$$= \frac{3\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^4\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{3a}{b}\right)\textbf{C}}{4bc^4\sqrt{1-}}$$

Mathematica [A] time = 0.279478, size = 130, normalized size = 0.66

$$\frac{\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(3\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\textbf{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-3\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right.}{4bc^4\sqrt{-(cx-1)(cx+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCo
sh[c*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*Sinh[a/b]
*SinhIntegral[a/b + ArcCosh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Cosh[c*x])]))/(4*b*c^4*Sqrt[-((-1 + c*x)*(1 + c*x))])
```

Maple [B] time = 0.192, size = 349, normalized size = 1.8

$$\frac{1}{8c^4\left(c^2x^2-1\right)b}\sqrt{-c^2x^2+1}\left(\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)\text{Ei}\left(1,3\,\text{arccosh}(cx)+3\frac{a}{b}\right)e^{-\frac{b\,\text{arccosh}(cx)-3a}{b}} + \frac{1}{8c^4\left(c^2x^2-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/8*(-c^2*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*a
rccosh(c*x)+3*a/b)*exp(-(b*arccosh(c*x)-3*a)/b)/c^4/(c^2*x^2-1)/b+1/8*(-c^2
*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c
*x)-3*a/b)*exp(-(b*arccosh(c*x)+3*a)/b)/c^4/(c^2*x^2-1)/b+3/8*(-c^2*x^2+1)^
(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a/b)*e
xp(-(a+b*arccosh(c*x))/b)/c^4/(c^2*x^2-1)/b+3/8*(-c^2*x^2+1)^(1/2)*((c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp(-(b*arccosh(
c*x)-a)/b)/c^4/(c^2*x^2-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}x^3}{ac^2x^2+\left(bc^2x^2-b\right)\mathrm{arcosh}\,(cx)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
- a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}\left(cx\right)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

3.300 $\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{cx-1}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc^3\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc^3\sqrt{1-cx}}+\frac{\sqrt{cx-1}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-cx}}$$

```
[Out] (Sqrt[-1 + c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(2*
b*c^3*Sqrt[1 - c*x]) + (Sqrt[-1 + c*x]*Log[a + b*ArcCosh[c*x]])/(2*b*c^3*Sq
rt[1 - c*x]) - (Sqrt[-1 + c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh
[c*x]))/b])/(2*b*c^3*Sqrt[1 - c*x])
```

Rubi [A] time = 0.631826, antiderivative size = 178, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 3312, 3303, 3298, 3301}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}}+\frac{\sqrt{cx-1}\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCos
h[c*x]])/(2*b*c^3*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[a
+ b*ArcCosh[c*x]])/(2*b*c^3*Sqrt[1 - c^2*x^2]) - (Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(2*b*c^3*Sqrt[1 -
 c^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}+\frac{\cosh(2x)}{2(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\right.}{2c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}}{2c^3\sqrt{1-}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{2a}{b}\right)\textbf{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\log\left(a\right.}{2bc^3\sqrt{1-c^2}}$$

Mathematica [A] time = 0.261181, size = 99, normalized size = 0.71

$$-\frac{\sqrt{1-c^2x^2}\left(\cosh\left(\frac{2a}{b}\right)\textbf{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\textbf{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\log\left(a+b\cosh^{-1}(cx)\right)\right)}{2c^3\sqrt{\frac{cx-1}{cx+1}}(bcx+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] + L
og[a + b*ArcCosh[c*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])]
))/(2*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [A] time = 0.166, size = 232, normalized size = 1.7

$$\frac{1}{4c^3\left(c^2x^2-1\right)b}\sqrt{-c^2x^2+1}\left(\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)\text{Ei}\left(1, 2\operatorname{arccosh}(cx)+2\frac{a}{b}\right)e^{-\frac{b\operatorname{arccosh}(cx)-2a}{b}} + \frac{1}{4c^3\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/4*(-c^2*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*a
rccosh(c*x)+2*a/b)*exp(-(b*arccosh(c*x)-2*a)/b)/c^3/(c^2*x^2-1)/b+1/4*(-c^2
*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c
*x)-2*a/b)*exp(-(b*arccosh(c*x)+2*a)/b)/c^3/(c^2*x^2-1)/b-1/2*(-c^2*x^2+1)^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(c^2*x^2-1)*ln(a+b*arccosh(c*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}x^2}{ac^2x^2+\left(bc^2x^2-b\right)\mathrm{arcosh}\,(cx)-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
- a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

3.301 $\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc^2\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc^2\sqrt{1-cx}}$$

```
[Out] (Sqrt[-1 + c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(b*c^2*Sqrt
[1 - c*x]) - (Sqrt[-1 + c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b]
)/(b*c^2*Sqrt[1 - c*x])
```

Rubi [A] time = 0.425927, antiderivative size = 114, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5798, 5781, 3303, 3298, 3301}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc^2\sqrt{1-c^2x^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc^2\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(
b*c^2*Sqrt[1 - c^2*x^2]) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sinh[a/b]*SinhInte
gral[a/b + ArcCosh[c*x]])/(b*c^2*Sqrt[1 - c^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
```

```
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}} - \frac{\left(\sqrt{-1+cx}\right.}{}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc^2\sqrt{1-c^2x^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\sinh\left(\frac{a}{b}\right)\text{Sh}}{bc^2\sqrt{1-c^2}}$$

Mathematica [A] time = 0.202297, size = 81, normalized size = 0.88

$$\frac{\sqrt{1-c^2x^2}\left(\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{c^2\sqrt{\frac{cx-1}{cx+1}}(bcx+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]]) + Sinh[a/
b]*SinhIntegral[a/b + ArcCosh[c*x]]))/(c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(b +
b*c*x))
```

Maple [B] time = 0.119, size = 173, normalized size = 1.9

$$\frac{1}{2\,c^2\left(c^2x^2-1\right)b}\sqrt{-c^2x^2+1}\left(\sqrt{cx+1}\sqrt{cx-1}xc+c^2x^2-1\right)\mathrm{Ei}\left(1,-\operatorname{arccosh}(cx)-\frac{a}{b}\right)\mathrm{e}^{-\frac{a+b\operatorname{arccosh}(cx)}{b}}+\frac{1}{2\,c^2\left(c^2x^2-1\right)b}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-c^2*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-ar
ccosh(c*x)-a/b)*exp(-(a+b*arccosh(c*x))/b)/c^2/(c^2*x^2-1)/b+1/2*(-c^2*x^2+
1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)
*exp(-(b*arccosh(c*x)-a)/b)/c^2/(c^2*x^2-1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}x}{ac^2x^2+\left(bc^2x^2-b\right)\operatorname{arcosh}(cx)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x) -
a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

3.302 $\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{cx-1}\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{1-cx}}$$

```
[Out] (Sqrt[-1 + c*x]*Log[a + b*ArcCosh[c*x]])/(b*c*Sqrt[1 - c*x])
```

Rubi [A] time = 0.220579, antiderivative size = 48, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {5713, 5674}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[a + b*ArcCosh[c*x]])/(b*c*Sqrt[1 - c^2*x^
2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5674

```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[Log[a + b*ArcCosh[c*x]]/(b*c*Sqrt[-
(d1*d2)]), x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && Eq
Q[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$
$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.105507, size = 54, normalized size = 1.54

$$\frac{\sqrt{\frac{cx-1}{cx+1}}(cx+1)\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{-(cx-1)(cx+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[a + b*ArcCosh[c*x]])/(b*c*Sqrt[-(
(-1 + c*x)*(1 + c*x))])
```

Maple [A] time = 0.079, size = 55, normalized size = 1.6

$$-\frac{\ln(a+b\,\mathrm{arccosh}(cx))}{c\left(c^2x^2-1\right)b}\sqrt{-c^2x^2+1}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] -(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(c^2*x^2-1)*ln(a+b*arccos
h(c*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [B] time = 1.90049, size = 136, normalized size = 3.89

$$-\frac{\sqrt{c^2x^2-1}\sqrt{-c^2x^2+1}\log\left(\frac{b\log\left(cx+\sqrt{c^2x^2-1}\right)+a}{b}\right)}{bc^3x^2-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*log((b*log(c*x + sqrt(c^2*x^2 - 1)) +
 a)/b)/(b*c^3*x^2 - b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

3.303 $\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.49831, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.707379, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.221, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{arccosh}\left(cx\right)\right)}\frac{1}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c^{2}x^{2}+1}(b\operatorname{arcosh}\left(cx\right)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^{2}x^{2}+1}}{ac^{2}x^{3}-ax+\left(bc^{2}x^{3}-bx\right)\operatorname{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^3 - a*x + (b*c^2*x^3 - b*x)*arccosh(c
*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x), x)
```

3.304 $\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.507394, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.2568, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b\operatorname{arccosh}(cx))} \frac{1}{\sqrt{-c^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c^2x^2+1}(b \operatorname{arcosh}(cx) + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{ac^2x^4 - ax^2 + \left(bc^2x^4 - bx^2\right)\operatorname{arcosh}(cx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^4 - a*x^2 + (b*c^2*x^4 - b*x^2)*arcco
sh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x^2), x)
```

3.305 $\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.57538, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^2/((-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 4.30466, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.165, size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b\operatorname{arccosh}(cx)} \left(-c^2x^2 + 1\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-c^2x^2 + 1\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^2x^2 + 1}x^2}{ac^4x^4 - 2\,ac^2x^2 + \left(bc^4x^4 - 2\,bc^2x^2 + b\right)\operatorname{arcosh}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b
*c^2*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**2/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

3.306 $\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.401472, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]

[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c*x)^(3/2)*(1 + c*x)^(3
/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 6.7297, size = 0, normalized size = 0.

$$\int \frac{x}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.223, size = 0, normalized size = 0.

$$\int \frac{x}{a+b\operatorname{arccosh}(cx)}\left(-c^2x^2+1\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x}{ac^4x^4-2\,ac^2x^2+\left(bc^4x^4-2\,bc^2x^2+b\right)\operatorname{arcosh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c
^2*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

3.307 $\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.249598, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/((-1 + c*x)^(3/2)*(1 + c*x)^(3
/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.142004, size = 0, normalized size = 0.

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.19, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\operatorname{arccosh}(cx)}\left(-c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}(cx)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{ac^4x^4-2\,ac^2x^2+\left(bc^4x^4-2\,bc^2x^2+b\right)\operatorname{arcosh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2
*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

3.308 $\int \frac{1}{x(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.567504, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x*(-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 3.43012, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.281, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{arccosh}\left(cx\right)\right)}\left(-c^{2}x^{2}+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^{2}x^{2}+1}}{ac^{4}x^{5}-2\,ac^{2}x^{3}+ax+\left(bc^{4}x^{5}-2\,bc^{2}x^{3}+bx\right)\operatorname{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^5 - 2*a*c^2*x^3 + a*x + (b*c^4*x^5 - 2
*b*c^2*x^3 + b*x)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x), x)
```

3.309 $\int \frac{1}{x^2(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.576767, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^2*(-1 + c*x)^(3/2)*(1 + c*x
)^(3/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 2.17069, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.218, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b\operatorname{arccosh}(cx))} \left(-c^2x^2 + 1\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2 + 1\right)^{\frac{3}{2}} (b \operatorname{arcosh}(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^2x^2 + 1}}{ac^4x^6 - 2\,ac^2x^4 + ax^2 + \left(bc^4x^6 - 2\,bc^2x^4 + bx^2\right) \operatorname{arcosh}(cx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^6 - 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6 -
 2*b*c^2*x^4 + b*x^2)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x^2), x)
```

3.310 $\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\left(1-c^2x^2\right)^{3/2}x^m}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]), x]
```

Rubi [A] time = 0.539941, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*Defer[Int][(x^m*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(a +
 b*ArcCosh[c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int \frac{x^m\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx=-\frac{\sqrt{1-c^2x^2}\int \frac{x^m(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.03556, size = 0, normalized size = 0.

$$\int \frac{x^m\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]), x]
```

Maple [A] time = 0.753, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{a+b\operatorname{arccosh}\left(cx\right)}\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}x^{m}}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^m/(b*arccosh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(c^{2}x^{2}-1\right)\sqrt{-c^{2}x^{2}+1}x^{m}}{b\operatorname{arcosh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x^m}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^m/(b*arccosh(c*x) + a), x)
```

3.311 $\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sqrt{1-c^2x^2}x^m}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]), x]
```

Rubi [A] time = 0.450933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(a + b*Arc
Cosh[c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{x^m\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.171441, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]), x]
```

Maple [A] time = 0.764, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{a+b\,\mathrm{arccosh}\left(cx\right)}\sqrt{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^{2}x^{2}+1}\,x^{m}}{b\,\mathrm{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^{2}x^{2}+1}\,x^{m}}{b\,\mathrm{arcosh}\left(cx\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sqrt{-(cx-1)(cx+1)}}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}\,x^m}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

3.312 $\int \frac{x^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{x^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.482797, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^m/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^m}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.617247, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.315, size = 0, normalized size = 0.

$$\int \frac{x^m}{a+b\,\mathrm{arccosh}\,(cx)}\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}x^m}{ac^2x^2+\left(bc^2x^2-b\right)\mathrm{arcosh}\,(cx)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
- a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

3.313 $\int \frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.565406, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^m/((-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^m}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.15695, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.46, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{a+b\operatorname{arccosh}\left(cx\right)}\left(-c^{2}x^{2}+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^{2}x^{2}+1}\,x^{m}}{ac^{4}x^{4}-2\,ac^{2}x^{2}+\left(bc^{4}x^{4}-2\,bc^{2}x^{2}+b\right)\operatorname{arcosh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^m/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b
*c^2*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

3.314 $\int \frac{x^m}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{x^m}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.579929, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^m/((-1 + c*x)^(5/2)*(1 + c*x)^(5
/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^m}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.66138, size = 0, normalized size = 0.

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.458, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{a+b\operatorname{arccosh}\left(cx\right)}\left(-c^{2}x^{2}+1\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(-c^{2}x^{2}+1\right)^{\frac{5}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^{2}x^{2}+1}\,x^{m}}{ac^{6}x^{6}-3\,ac^{4}x^{4}+3\,ac^{2}x^{2}+\left(bc^{6}x^{6}-3\,bc^{4}x^{4}+3\,bc^{2}x^{2}-b\right)\operatorname{arcosh}\left(cx\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 + (
b*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arccosh(c*x) - a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(-c^2x^2+1\right)^{\frac{5}{2}}(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)), x)
```

3.315 $\int \frac{(c-a^2cx^2)^3}{\cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=98

$$\frac{35c^3\text{Chi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{63c^3\text{Chi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{35c^3\text{Chi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{7c^3\text{Chi}\left(7\cosh^{-1}(ax)\right)}{64a}+\frac{c^3(ax-1)^{7/2}(ax}{a\cosh^{-1}(a}$$

```
[Out] (c^3*(-1 + a*x)^(7/2)*(1 + a*x)^(7/2))/(a*ArcCosh[a*x]) + (35*c^3*CoshInteg
ral[ArcCosh[a*x]])/(64*a) - (63*c^3*CoshIntegral[3*ArcCosh[a*x]])/(64*a) +
(35*c^3*CoshIntegral[5*ArcCosh[a*x]])/(64*a) - (7*c^3*CoshIntegral[7*ArcCos
h[a*x]])/(64*a)
```

Rubi [A] time = 0.325086, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5695, 5781, 5448, 3301}

$$\frac{35c^3\text{Chi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{63c^3\text{Chi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{35c^3\text{Chi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{7c^3\text{Chi}\left(7\cosh^{-1}(ax)\right)}{64a}+\frac{c^3(ax-1)^{7/2}(ax}{a\cosh^{-1}(a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/ArcCosh[a*x]^2,x]
```



```
[Out] (c^3*(-1 + a*x)^(7/2)*(1 + a*x)^(7/2))/(a*ArcCosh[a*x]) + (35*c^3*CoshInteg
ral[ArcCosh[a*x]])/(64*a) - (63*c^3*CoshIntegral[3*ArcCosh[a*x]])/(64*a) +
(35*c^3*CoshIntegral[5*ArcCosh[a*x]])/(64*a) - (7*c^3*CoshIntegral[7*ArcCos
h[a*x]])/(64*a)
```


Rule 5695

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((-d)^p*(-1 + c*x)^(p + 1/2)*(1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(-d)^p*(2*p + 1))/(b*(n +
1)), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n
, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^3}{\cosh^{-1}(ax)^2}\,dx = \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \left(7ac^3\right)\int \frac{x(-1+ax)^{5/2}(1+ax)^{5/2}}{\cosh^{-1}(ax)}\,dx$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^6(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \frac{\left(7c^3\right)\text{Subst}\left(\int \left(-\frac{5\cosh(x)}{64x} + \frac{9\cosh(3x)}{64x} - \frac{5\cosh(5x)}{64x} + \frac{\cosh(7x)}{64x}\right) dx, x, \cos\right.}{a}$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a} + \frac{\left(35c^3\right)\text{Subst}\left(\int \frac{\cosh(x)}{x}\,d\right.}{64a}$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} + \frac{35c^3\text{Chi}\left(\cosh^{-1}(ax)\right)}{64a} - \frac{63c^3\text{Chi}\left(3\cosh^{-1}(ax)\right)}{64a} + \frac{35c^3\text{Chi}\left(5\cosh^{-1}\right.}{64a}$$

Mathematica [A] time = 0.474794, size = 128, normalized size = 1.31

$$\frac{c^3\left(112\left(\text{Chi}\left(\cosh^{-1}(ax)\right) - \text{Chi}\left(3\cosh^{-1}(ax)\right)\right) + 56\left(-2\text{Chi}\left(\cosh^{-1}(ax)\right) + \text{Chi}\left(3\cosh^{-1}(ax)\right) + \text{Chi}\left(5\cosh^{-1}(ax\right.\right.}{64}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/ArcCosh[a*x]^2,x]
```

```
[Out] (c^3*((64*(-1 + a*x)^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)^4)/ArcCosh[a*x]
 + 112*(CoshIntegral[ArcCosh[a*x]] - CoshIntegral[3*ArcCosh[a*x]]) + 56*(-2
*CoshIntegral[ArcCosh[a*x]] + CoshIntegral[3*ArcCosh[a*x]] + CoshIntegral[5
*ArcCosh[a*x]]) + 7*(5*CoshIntegral[ArcCosh[a*x]] - CoshIntegral[3*ArcCosh[
a*x]] - 3*CoshIntegral[5*ArcCosh[a*x]] - CoshIntegral[7*ArcCosh[a*x]])))/(6
4*a)
```

Maple [A] time = 0.049, size = 107, normalized size = 1.1

$$\frac{c^3}{64\,a\mathrm{arccosh}\,(ax)}\Big(35\,\mathrm{Chi}\,(\mathrm{arccosh}\,(ax))\,\mathrm{arccosh}\,(ax) - 63\,\mathrm{Chi}\,(3\,\mathrm{arccosh}\,(ax))\,\mathrm{arccosh}\,(ax) + 35\,\mathrm{Chi}\,(5\,\mathrm{arccosh}\,(ax))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x)
```

```
[Out] 1/64/a*c^3*(35*Chi(arccosh(a*x))*arccosh(a*x)-63*Chi(3*arccosh(a*x))*arccos
h(a*x)+35*Chi(5*arccosh(a*x))*arccosh(a*x)-7*Chi(7*arccosh(a*x))*arccosh(a*
x)-35*(a*x-1)^(1/2)*(a*x+1)^(1/2)+21*sinh(3*arccosh(a*x))-7*sinh(5*arccosh(
a*x))+sinh(7*arccosh(a*x)))/arccosh(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^9c^3x^9 - 4\,a^7c^3x^7 + 6\,a^5c^3x^5 - 4\,a^3c^3x^3 + ac^3x + \left(a^8c^3x^8 - 4\,a^6c^3x^6 + 6\,a^4c^3x^4 - 4\,a^2c^3x^2 + c^3\right)\sqrt{ax+1}\sqrt{ax-1}}{\left(a^3x^2 + \sqrt{ax+1}\sqrt{ax-1}a^2x - a\right)\log\left(ax + \sqrt{ax+1}\sqrt{ax-1}\right)} - \int \frac{7\,a^1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] (a^9*c^3*x^9 - 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 4*a^3*c^3*x^3 + a*c^3*x + (a
^8*c^3*x^8 - 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 - 4*a^2*c^3*x^2 + c^3)*sqrt(a*x
+ 1)*sqrt(a*x - 1))/((a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(
a*x + sqrt(a*x + 1)*sqrt(a*x - 1))) - integrate((7*a^10*c^3*x^10 - 29*a^8*c
^3*x^8 + 46*a^6*c^3*x^6 - 34*a^4*c^3*x^4 + 11*a^2*c^3*x^2 + (7*a^8*c^3*x^8
```

```
- 20*a^6*c^3*x^6 + 18*a^4*c^3*x^4 - 4*a^2*c^3*x^2 - c^3)*(a*x + 1)*(a*x - 1
) - c^3 + 7*(2*a^9*c^3*x^9 - 7*a^7*c^3*x^7 + 9*a^5*c^3*x^5 - 5*a^3*c^3*x^3
+ a*c^3*x)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^4*x^4 + (a*x + 1)*(a*x - 1)*a^2
*x^2 - 2*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + 1)*log(a
*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{a^{6}c^{3}x^{6} - 3\,a^{4}c^{3}x^{4} + 3\,a^{2}c^{3}x^{2} - c^{3}}{\operatorname{arcosh}\left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)/arccosh(a*x)^
2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^{3}\left(\int \frac{3a^{2}x^{2}}{\operatorname{acosh}^{2}(ax)}\,dx + \int -\frac{3a^{4}x^{4}}{\operatorname{acosh}^{2}(ax)}\,dx + \int \frac{a^{6}x^{6}}{\operatorname{acosh}^{2}(ax)}\,dx + \int -\frac{1}{\operatorname{acosh}^{2}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/acosh(a*x)**2,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/acosh(a*x)**2, x) + Integral(-3*a**4*x**4/acosh
(a*x)**2, x) + Integral(a**6*x**6/acosh(a*x)**2, x) + Integral(-1/acosh(a*x
)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^{2}cx^{2} - c\right)^{3}}{\operatorname{arcosh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3/arccosh(a*x)^2, x)
```

3.316 $\int \frac{(c-a^2cx^2)^2}{\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=82

$$\frac{5c^2\text{Chi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{15c^2\text{Chi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{5c^2\text{Chi}\left(5\cosh^{-1}(ax)\right)}{16a}-\frac{c^2(ax-1)^{5/2}(ax+1)^{5/2}}{a\cosh^{-1}(ax)}$$

```
[Out] -((c^2*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2))/(a*ArcCosh[a*x])) + (5*c^2*CoshInt
egral[ArcCosh[a*x]])/(8*a) - (15*c^2*CoshIntegral[3*ArcCosh[a*x]])/(16*a) +
 (5*c^2*CoshIntegral[5*ArcCosh[a*x]])/(16*a)
```

Rubi [A] time = 0.305047, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5695, 5781, 5448, 3301}

$$\frac{5c^2\text{Chi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{15c^2\text{Chi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{5c^2\text{Chi}\left(5\cosh^{-1}(ax)\right)}{16a}-\frac{c^2(ax-1)^{5/2}(ax+1)^{5/2}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/ArcCosh[a*x]^2,x]
```

```
[Out] -((c^2*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2))/(a*ArcCosh[a*x])) + (5*c^2*CoshInt
egral[ArcCosh[a*x]])/(8*a) - (15*c^2*CoshIntegral[3*ArcCosh[a*x]])/(16*a) +
 (5*c^2*CoshIntegral[5*ArcCosh[a*x]])/(16*a)
```

Rule 5695

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((-d)^p*(-1 + c*x)^(p + 1/2)*(1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(-d)^p*(2*p + 1))/(b*(n +
1)), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n
, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
```

```
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c-a^2cx^2\right)^2}{\cosh^{-1}(ax)^2}\,dx &= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\left(5ac^2\right)\int \frac{x(-1+ax)^{3/2}(1+ax)^{3/2}}{\cosh^{-1}(ax)}\,dx\\
&= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{a}\\
&= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\frac{\left(5c^2\right)\text{Subst}\left(\int \left(\frac{\cosh(x)}{8x}-\frac{3\cosh(3x)}{16x}+\frac{\cosh(5x)}{16x}\right)dx,x,\cosh^{-1}(ax)\right)}{a}\\
&= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{16a}+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx\right.}{8a}\\
&= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\frac{5c^2\text{Chi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{15c^2\text{Chi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{5c^2\text{Chi}\left(5\cosh^{-1}(\right.}{16a}
\end{aligned}$$

Mathematica [A] time = 0.45385, size = 84, normalized size = 1.02

$$\frac{c^2\left(20\left(\text{Chi}\left(\cosh^{-1}(ax)\right)-\text{Chi}\left(3\cosh^{-1}(ax)\right)\right)+5\left(-2\text{Chi}\left(\cosh^{-1}(ax)\right)+\text{Chi}\left(3\cosh^{-1}(ax)\right)+\text{Chi}\left(5\cosh^{-1}(ax)\right)\right)-\right.}{16a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/ArcCosh[a*x]^2,x]
```

```
[Out] (c^2*((-16*((-1 + a*x)/(1 + a*x))^(5/2)*(1 + a*x)^5)/ArcCosh[a*x] + 20*(Cos
hIntegral[ArcCosh[a*x]] - CoshIntegral[3*ArcCosh[a*x]]) + 5*(-2*CoshIntegra
l[ArcCosh[a*x]] + CoshIntegral[3*ArcCosh[a*x]] + CoshIntegral[5*ArcCosh[a*x
]])))/(16*a)
```

Maple [A] time = 0.04, size = 87, normalized size = 1.1

$$\frac{c^2}{16\,a\mathrm{arccosh}\,(ax)}\left(10\,\mathrm{Chi}\,(\mathrm{arccosh}\,(ax))\,\mathrm{arccosh}\,(ax) - 15\,\mathrm{Chi}\,(3\,\mathrm{arccosh}\,(ax))\,\mathrm{arccosh}\,(ax) + 5\,\mathrm{Chi}\,(5\,\mathrm{arccosh}\,(ax))\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x)
```

```
[Out] 1/16/a*c^2*(10*Chi(arccosh(a*x))*arccosh(a*x)-15*Chi(3*arccosh(a*x))*arccos
h(a*x)+5*Chi(5*arccosh(a*x))*arccosh(a*x)-10*(a*x-1)^(1/2)*(a*x+1)^(1/2)+5*
sinh(3*arccosh(a*x))-sinh(5*arccosh(a*x)))/arccosh(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^7c^2x^7 - 3\,a^5c^2x^5 + 3\,a^3c^2x^3 - ac^2x + \left(a^6c^2x^6 - 3\,a^4c^2x^4 + 3\,a^2c^2x^2 - c^2\right)\sqrt{ax+1}\sqrt{ax-1}}{\left(a^3x^2 + \sqrt{ax+1}\sqrt{ax-1}a^2x - a\right)\log\left(ax + \sqrt{ax+1}\sqrt{ax-1}\right)} + \int \frac{5\,a^8c^2x^8 - 16\,a^6c^2x^6 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^7*c^2*x^7 - 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - a*c^2*x + (a^6*c^2*x^6 - 3*
a^4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*x^2 +
 sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
1))) + integrate((5*a^8*c^2*x^8 - 16*a^6*c^2*x^6 + 18*a^4*c^2*x^4 - 8*a^2*c
^2*x^2 + (5*a^6*c^2*x^6 - 9*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*(a*x + 1)*(a
*x - 1) + 5*(2*a^7*c^2*x^7 - 5*a^5*c^2*x^5 + 4*a^3*c^2*x^3 - a*c^2*x)*sqrt(
a*x + 1)*sqrt(a*x - 1) + c^2)/((a^4*x^4 + (a*x + 1)*(a*x - 1)*a^2*x^2 - 2*a
^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + 1)*log(a*x + sqrt(
a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4c^2x^4 - 2\,a^2c^2x^2 + c^2}{\operatorname{arcosh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)/arccosh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -\frac{2a^2x^2}{\operatorname{acosh}^2\left(ax\right)}\,dx + \int \frac{a^4x^4}{\operatorname{acosh}^2\left(ax\right)}\,dx + \int \frac{1}{\operatorname{acosh}^2\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/acosh(a*x)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/acosh(a*x)**2, x) + Integral(a**4*x**4/acosh(a*
x)**2, x) + Integral(acosh(a*x)**(-2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 - c\right)^2}{\operatorname{arcosh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2/arccosh(a*x)^2, x)
```

3.317 $\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=58

$$\frac{3c\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{3c\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{4a}+\frac{c(ax-1)^{3/2}(ax+1)^{3/2}}{a\cosh^{-1}(ax)}$$

```
[Out] (c*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2))/(a*ArcCosh[a*x]) + (3*c*CoshIntegral[A
rcCosh[a*x]])/(4*a) - (3*c*CoshIntegral[3*ArcCosh[a*x]])/(4*a)
```

Rubi [A] time = 0.235204, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5695, 5781, 5448, 3301}

$$\frac{3c\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{3c\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{4a}+\frac{c(ax-1)^{3/2}(ax+1)^{3/2}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/ArcCosh[a*x]^2,x]
```

```
[Out] (c*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2))/(a*ArcCosh[a*x]) + (3*c*CoshIntegral[A
rcCosh[a*x]])/(4*a) - (3*c*CoshIntegral[3*ArcCosh[a*x]])/(4*a)
```

Rule 5695

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((-d)^p*(-1 + c*x)^(p + 1/2)*(1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(-d)^p*(2*p + 1))/(b*(n +
1)), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n
, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
```

```
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)^2}\,dx = \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} - (3ac)\int \frac{x\sqrt{-1+ax}\sqrt{1+ax}}{\cosh^{-1}(ax)}\,dx$$

$$= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} - \frac{(3c)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} - \frac{(3c)\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} + \frac{(3c)\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a} - \frac{(3c)\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{co}\right.}{4a}$$

$$= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} + \frac{3c\,\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a} - \frac{3c\,\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a}$$

Mathematica [B] time = 0.897519, size = 140, normalized size = 2.41

$$\frac{c\sqrt{ax-1}\left(\left(4\sqrt{ax-1}-\sqrt{\frac{ax-1}{ax+1}}\sqrt{ax+1}\right)\cosh^{-1}(ax)\text{Chi}\left(\cosh^{-1}(ax)\right)+\sqrt{ax+1}\left(4(ax-1)^2(ax+1)-3\sqrt{\frac{ax-1}{ax+1}}\cosh^{-1}(ax\right.\right.}{8a\cosh^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/ArcCosh[a*x]^2,x]
```

```
[Out] (c*Sqrt[-1 + a*x]*((4*Sqrt[-1 + a*x] - Sqrt[(-1 + a*x)/(1 + a*x)]*Sqrt[1 +
a*x])*ArcCosh[a*x]*CoshIntegral[ArcCosh[a*x]] + Sqrt[1 + a*x]*(4*(-1 + a*x)
^2*(1 + a*x) - 3*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*CoshIntegral[3*Arc
Cosh[a*x]]))*Csch[ArcCosh[a*x]/2]^2)/(8*a*ArcCosh[a*x])
```

Maple [A] time = 0.036, size = 61, normalized size = 1.1

$$\frac{c}{4\,a\mathrm{arccosh}\left(ax\right)}\left(3\,\mathrm{Chi}\left(\mathrm{arccosh}\left(ax\right)\right)\mathrm{arccosh}\left(ax\right)-3\,\mathrm{Chi}\left(3\,\mathrm{arccosh}\left(ax\right)\right)\mathrm{arccosh}\left(ax\right)-3\,\sqrt{ax-1}\sqrt{ax+1}+\sinh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/arccosh(a*x)^2,x)
```

```
[Out] 1/4/a*c*(3*Chi(arccosh(a*x))*arccosh(a*x)-3*Chi(3*arccosh(a*x))*arccosh(a*x
)-3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+sinh(3*arccosh(a*x)))/arccosh(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^5cx^5-2\,a^3cx^3+acx+\left(a^4cx^4-2\,a^2cx^2+c\right)\sqrt{ax+1}\sqrt{ax-1}}{\left(a^3x^2+\sqrt{ax+1}\sqrt{ax-1}a^2x-a\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)}-\int\frac{3\,a^6cx^6-7\,a^4cx^4+5\,a^2cx^2+\left(3\,a^4cx^4-2\,a^2cx^2-c\right.}{\left(a^4x^4+(ax+1)(ax-1)a^2x^2-2\,a^2x^2+2\left(a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] (a^5*c*x^5 - 2*a^3*c*x^3 + a*c*x + (a^4*c*x^4 - 2*a^2*c*x^2 + c)*sqrt(a*x +
 1)*sqrt(a*x - 1))/((a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a
*x + sqrt(a*x + 1)*sqrt(a*x - 1))) - integrate((3*a^6*c*x^6 - 7*a^4*c*x^4 +
 5*a^2*c*x^2 + (3*a^4*c*x^4 - 2*a^2*c*x^2 - c)*(a*x + 1)*(a*x - 1) + 3*(2*a
^5*c*x^5 - 3*a^3*c*x^3 + a*c*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - c)/((a^4*x^4
+ (a*x + 1)*(a*x - 1)*a^2*x^2 - 2*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt(a*x + 1)
*sqrt(a*x - 1) + 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{a^2cx^2-c}{\mathrm{arcosh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)/arccosh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int \frac{a^2x^2}{\operatorname{acosh}^2(ax)}\,dx + \int -\frac{1}{\operatorname{acosh}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/acosh(a*x)**2,x)
```

```
[Out] -c*(Integral(a**2*x**2/acosh(a*x)**2, x) + Integral(-1/acosh(a*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^2cx^2 - c}{\operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)/arccosh(a*x)^2, x)
```

3.318 $\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=65

$$\frac{a\text{Unintegrable}\left(\frac{x}{(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)},x\right)}{c}+\frac{1}{ac\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}$$

```
[Out] 1/(a*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]) + (a*Unintegrable[x/((-1
+ a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]), x])/c
```

Rubi [A] time = 0.245898, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)*ArcCosh[a*x]^2),x]
```

```
[Out] 1/(a*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]) + (a*Defer[Int][x/((-1 +
a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]), x])/c
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx=\frac{1}{ac\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}+\frac{a\int \frac{x}{(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}\,dx}{c}$$

Mathematica [A] time = 2.97771, size = 0, normalized size = 0.

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]^2),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]^2), x]
```

Maple [A] time = 0.135, size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)\left(\operatorname{arccosh}\left(ax\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x)
```

```
[Out] int(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{ax+\sqrt{ax+1}\sqrt{ax-1}}{\left(a^3cx^2+\sqrt{ax+1}\sqrt{ax-1}a^2cx-ac\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)}+\int\frac{a^4x^4+\left(a^2x^2-1\right)}{\left(a^6cx^6-3\,a^4cx^4+3\,a^2cx^2+\left(a^4cx^4-a^2cx^2\right)(ax+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] (a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*c*x^2 + sqrt(a*x + 1)*sqrt(a*x -
1)*a^2*c*x - a*c)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))) + integrate((a^4*
x^4 + (a^2*x^2 - 1)*(a*x + 1)*(a*x - 1) + (2*a^3*x^3 - a*x)*sqrt(a*x + 1)*s
qrt(a*x - 1) - 1)/((a^6*c*x^6 - 3*a^4*c*x^4 + 3*a^2*c*x^2 + (a^4*c*x^4 - a^
2*c*x^2)*(a*x + 1)*(a*x - 1) + 2*(a^5*c*x^5 - 2*a^3*c*x^3 + a*c*x)*sqrt(a*x
 + 1)*sqrt(a*x - 1) - c)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{1}{\left(a^2cx^2-c\right)\operatorname{arcosh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^2*c*x^2 - c)*arccosh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{a^2x^2\operatorname{acosh}^2(ax)-\operatorname{acosh}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)/acosh(a*x)**2,x)
```

```
[Out] -Integral(1/(a**2*x**2*acosh(a*x)**2 - acosh(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2cx^2-c\right)\operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*c*x^2 - c)*arccosh(a*x)^2), x)
```

3.319 $\int \frac{1}{\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=67

$$-\frac{3a\text{Unintegrable}\left(\frac{x}{(ax-1)^{5/2}(ax+1)^{5/2}\cosh^{-1}(ax)},x\right)}{c^2}-\frac{1}{ac^2(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)}$$

```
[Out] -(1/(a*c^2*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])) - (3*a*Unintegra
ble[x/((-1 + a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x]), x])/c^2
```

Rubi [A] time = 0.253828, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])) - (3*a*Defer[Int
][x/((-1 + a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x]), x])/c^2
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^2}\,dx=-\frac{1}{ac^2(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}-\frac{(3a)\int \frac{x}{(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)}\,dx}{c^2}$$

Mathematica [A] time = 12.5104, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]^2),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]^2), x]
```

Maple [A] time = 0.172, size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^2 \left(\operatorname{arccosh}\left(ax\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ax+\sqrt{ax+1}\sqrt{ax-1}}{\left(a^5c^2x^4-2\,a^3c^2x^2+ac^2+\left(a^4c^2x^3-a^2c^2x\right)\sqrt{ax+1}\sqrt{ax-1}\right)\log\left(ax+\sqrt{ax+1}\sqrt{ax-1}\right)} - \int \frac{}{\left(a^8c^2x^8-4\,a^6c^2x^6+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/((a^5*c^2*x^4 - 2*a^3*c^2*x^2 + a*c^2
+ (a^4*c^2*x^3 - a^2*c^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x
 + 1)*sqrt(a*x - 1))) - integrate((3*a^4*x^4 - 2*a^2*x^2 + (3*a^2*x^2 - 1)*
(a*x + 1)*(a*x - 1) + 3*(2*a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - 1)/
((a^8*c^2*x^8 - 4*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 4*a^2*c^2*x^2 + (a^6*c^2*x^
6 - 2*a^4*c^2*x^4 + a^2*c^2*x^2)*(a*x + 1)*(a*x - 1) + 2*(a^7*c^2*x^7 - 3*a
^5*c^2*x^5 + 3*a^3*c^2*x^3 - a*c^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + c^2)*lo
g(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\operatorname{arcosh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*arccosh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^4\operatorname{acosh}^2(ax)-2a^2x^2\operatorname{acosh}^2(ax)+\operatorname{acosh}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**2/acosh(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**4*acosh(a*x)**2 - 2*a**2*x**2*acosh(a*x)**2 + acosh(a*x
)**2), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2-c\right)^2\operatorname{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^2*arccosh(a*x)^2), x)
```

3.320 $\int \frac{x^3\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=350

$$\frac{\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^4\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^4\sqrt{cx-1}}-\frac{5\sqrt{1-cx}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}\right.}{b}\right.}{16b^2c^4\sqrt{cx-1}}$$

```
[Out] -((x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh[
c*x]))) + (Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(8
*b^2*c^4*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcCosh[
c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*c^4*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*Cos
hIntegral[(5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*c^4*Sqrt[-1 +
c*x]) - (Sqrt[1 - c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b
^2*c^4*Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a
+ b*ArcCosh[c*x]))/b])/(16*b^2*c^4*Sqrt[-1 + c*x]) + (5*Sqrt[1 - c*x]*Cosh[
(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^4*Sqrt[-1 + c*
x])
```

Rubi [A] time = 1.08493, antiderivative size = 429, normalized size of antiderivative = 1.23, number of steps used = 23, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5798, 5778, 5670, 5448, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b^2c^4\sqrt{cx-1}\sqrt{cx+1}}-\frac{3\sqrt{1-c^2x^2}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16b^2c^4\sqrt{cx-1}\sqrt{cx+1}}-\frac{5\sqrt{1-c^2x^2}\sinh\left(\frac{5a}{b}\right)\text{Chi}}{16b^2c^4\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x^3*(1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*
ArcCosh[c*x])) + (Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a
/b])/(8*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*CoshIn
tegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(16*b^2*c^4*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (5*Sqrt[1 - c^2*x^2]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]
]*Sinh[(5*a)/b])/(16*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*
x^2]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b^2*c^4*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b +
```

```
3*ArcCosh[c*x]])/(16*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*Sqrt[1 - c^
2*x^2]*Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b^2*c^4*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\int \frac{x^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(5c\sqrt{1-c^2x^2}\right)\int \frac{}{a+b\,\mathrm{co}}}{b\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)} + \frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}\right.}{bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{16bc^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(5\sqrt{}\right.}{}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(\right.}{8bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3\sqrt{1-c^2x^2}\,\mathrm{C}}{16}$$

Mathematica [A] time = 0.769183, size = 322, normalized size = 0.92

$$\sqrt{1-c^2x^2}\left(2\sinh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3\sinh\left(\frac{3a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathbf{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(16*b*c^3*x^3 - 16*b*c^5*x^5 + 2*(a + b*ArcCosh[c*x])*Co
shIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - 3*(a + b*ArcCosh[c*x])*CoshInteg
ral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - 5*a*CoshIntegral[5*(a/b + ArcCo
sh[c*x])]*Sinh[(5*a)/b] - 5*b*ArcCosh[c*x]*CoshIntegral[5*(a/b + ArcCosh[c*
x])]*Sinh[(5*a)/b] - 2*a*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 2*b*A
rcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 3*a*Cosh[(3*a)/b]*
SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cosh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcCosh[c*x])] + 5*a*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + A
rcCosh[c*x])] + 5*b*ArcCosh[c*x]*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCos
h[c*x])]))/(16*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

Maple [B] time = 0.411, size = 1029, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/32*(-c^2*x^2+1)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6
+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+13*c^2*x^2-1)/(c*x+1)/c^4/(c*x-1)/b/(a+b*arccosh(c*x))-5/32*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arccosh(c
*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b^2+1/32*(-c^2*x
^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c*x+1)/c^4/(c*x-1)/b/(a+b*arccosh(c*x))-
3/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3
*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b^2-1/
32*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(4*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(c*x)*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)
-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)
^(1/2)*b-3*x*b*c)/c^4/b^2/(a+b*arccosh(c*x))-1/32*(-c^2*x^2+1)^(1/2)/(c*x-1
```

```
)^(1/2)/(c*x+1)^(1/2)*(16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*b*c^4+16*x^5*b*c^
5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2-20*x^3*b*c^3+5*arccosh(c*x)*exp(
-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*
a/b)*a+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+5*x*b*c)/c^4/b^2/(a+b*arccosh(c*x))+1/
16*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(arccosh(c*x)*exp(-a/b)*E
i(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arcc
osh(c*x)-a/b)*a+x*b*c)/c^4/b^2/(a+b*arccosh(c*x))-1/16*(-c^2*x^2+1)^(1/2)*(
-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^4/(c*x-1)/b/(a+b*arcc
osh(c*x))+1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2
-1)*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^2x^5-x^3\right)(cx+1)\sqrt{cx-1}+\left(c^3x^6-cx^4\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{\quad}{abc^5x^4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^5 - x^3)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^6 - c*x^4)*sqrt(c*x + 1)
)*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b
*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((5*c^3*x^5 - 2*c*x^3)*(c*x + 1
)^(3/2)*(c*x - 1) + (10*c^4*x^6 - 11*c^2*x^4 + 3*x^2)*(c*x + 1)*sqrt(c*x -
1) + (5*c^5*x^7 - 9*c^3*x^5 + 4*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c
^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c
^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*
(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)
*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}x^3}{b^2\operatorname{arcosh}(cx)^2+2ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^3/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}\,x^3}{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^3/(b*arccosh(c*x) + a)^2, x)
```

3.321 $\int \frac{x^2\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2b^2c^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh[
c*x]))) - (Sqrt[1 - c*x]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b]*Sinh[(4*a
)/b])/(2*b^2*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(4*a)/b]*SinhIntegra
l[(4*(a + b*ArcCosh[c*x]))/b])/(2*b^2*c^3*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.878566, antiderivative size = 185, normalized size of antiderivative = 1.2, number of steps used = 17, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5798, 5778, 5670, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{2b^2c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{2b^2c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{x^2(1-cx)\sqrt{cx+1}\sqrt{1}}{bc\sqrt{cx-1}\left(a+b\cosh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x^2*(1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*
ArcCosh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]]*
Sinh[(4*a)/b])/(2*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2
]*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(2*b^2*c^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{x}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(4c\sqrt{1-c^2x^2}\right)\int \frac{}{a+b\,\mathrm{co}}}{b\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc^3\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(cx)\right)}{bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(4\sqrt{}}{}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{2bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx, x, \cosh^{-}\right.}{2bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x}}{}$$

Mathematica [A] time = 0.4689, size = 130, normalized size = 0.84

$$\frac{\sqrt{1-c^2x^2}\left(-\sinh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathrm{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(\right.\right.\right.}{2b^2c^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-2*b*c^2*x^2*(-1 + c^2*x^2) - (a + b*ArcCosh[c*x])*Cosh
Integral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] + (a + b*ArcCosh[c*x])*Cosh[
(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(2*b^2*c^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

Maple [B] time = 0.235, size = 422, normalized size = 2.7

$$\frac{1}{(16\,cx+16)\,(cx-1)\,c^3\,(a+b\mathrm{arccosh}\,(cx))\,b}\sqrt{-c^2x^2+1}\left(-8\,\sqrt{cx+1}\sqrt{cx-1}x^4c^4+8\,c^5x^5+8\,\sqrt{cx+1}\sqrt{cx-1}x^2c^2-12\,c^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/16*(-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)
+4*c*x)/(c*x+1)/(c*x-1)/c^3/(a+b*arccosh(c*x))/b-1/4*(-c^2*x^2+1)^(1/2)*(-(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b
*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2-1/16/(c*x+1)^(1/2)/(c*x-1)^(1
/2)*(-c^2*x^2+1)^(1/2)*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*b*c^3+8*x^4*b*c^4
-4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*exp(-4*a/b)
*Ei(1,-4*arccosh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+
b)/c^3/b^2/(a+b*arccosh(c*x))+1/8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^
(1/2)/c^3/(a+b*arccosh(c*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^2x^4-x^2\right)(cx+1)\sqrt{cx-1}+\left(c^3x^5-cx^3\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{}{abc^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^4 - x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^5 - c*x^3)*sqrt(c*x + 1)
)*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b
```

```
*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((4*c^3*x^4 - c*x^2)*(c*x + 1)^
(3/2)*(c*x - 1) + 2*(4*c^4*x^5 - 4*c^2*x^3 + x)*(c*x + 1)*sqrt(c*x - 1) + (
4*c^5*x^6 - 7*c^3*x^4 + 3*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4
 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3
 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x -
 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(
c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}x^2}{b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\sqrt{-(cx-1)(cx+1)}}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c^2x^2+1}x^2}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a)^2, x)
```

3.322 $\int \frac{x\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^2\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^2\sqrt{cx-1}}-\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^2\sqrt{cx-1}}$$

```
[Out] -((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh[c*
x]))) + (Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(4*b
^2*c^2*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcCosh[c*
x]))/b]*Sinh[(3*a)/b])/(4*b^2*c^2*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Cosh[a/b
]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(4*b^2*c^2*Sqrt[-1 + c*x]) + (3*Sqr
t[1 - c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b^2*c
^2*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.688778, antiderivative size = 418, normalized size of antiderivative = 1.69, number of steps used = 15, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5798, 5778, 5658, 3303, 3298, 3301, 5670, 5448}

$$-\frac{3\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b^2c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3\sqrt{1-c^2x^2}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+\right.}{4b^2c^2\sqrt{cx-1}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x*(1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*Ar
cCosh[c*x])) - (3*Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a
/b])/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*CoshInte
gral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - (3*Sqrt[1 - c^2*x^2]*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a
)/b])/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*Cosh[
a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (3*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[
c*x]])/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Cosh[a
/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
```

```
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(3c\sqrt{1-c^2x^2}\right)\int \frac{x^2}{a+b\cosh^{-1}(c}}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(}{}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(c\right.}{bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\cosh\left(\right.}{b^2c^2\sqrt{-1}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\cosh\left(\right.}{b^2c^2\sqrt{-1}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{3\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Ch}}{b^2c^2}$$

Mathematica [A] time = 0.421536, size = 217, normalized size = 0.88

$$\frac{\sqrt{1-c^2x^2}\left(\sinh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3\sinh\left(\frac{3a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(4*b*c*x - 4*b*c^3*x^3 + (a + b*ArcCosh[c*x])*CoshIntegr
al[a/b + ArcCosh[c*x]]*Sinh[a/b] - 3*(a + b*ArcCosh[c*x])*CoshIntegral[3*(a
/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - a*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[
c*x]] - b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 3*a*Cos
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cosh[(3*
```

```
a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(a + b*ArcCosh[c*x]))
```

Maple [B] time = 0.23, size = 622, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/8*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c*x+1)/c^2/(c*x-1)/b/(a+b*arc
cosh(c*x))-3/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2
-1)*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x
-1)/b^2-1/8*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(4*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(c*x)*exp(-3*a/b)*Ei(1,-3*arc
cosh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)*a-(c*x+1)^(1/2
)*(c*x-1)^(1/2)*b-3*x*b*c)/c^2/b^2/(a+b*arccosh(c*x))+1/8*(-c^2*x^2+1)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/
b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+x*b*
c)/c^2/b^2/(a+b*arccosh(c*x))-1/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/b/(a+b*arccosh(c*x))+1/8*(-c^2*x
^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+
a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^2x^3-x\right)(cx+1)\sqrt{cx-1}+\left(c^3x^4-cx^2\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{}{abc^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^4 - c*x^2)*sqrt(c*x + 1))*
sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c
 + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x +
```

```
sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*(c*x + 1)^(3/2)*(c*x - 1)*c^3*
x^3 + (6*c^4*x^4 - 5*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (3*c^5*x^5 - 5*
c^3*x^3 + 2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*
x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sq
rt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2
- 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}x}{b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{-(cx-1)(cx+1)}}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c^2x^2+1}x}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a)^2, x)
```

3.323 $\int \frac{\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2} dx$

Optimal. Leaf size=146

$$-\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh[c*x]
))) - (Sqrt[1 - c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/b]
)/(b^2*c*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(2*a)/b]*SinhIntegral[(2*(a
+ b*ArcCosh[c*x]))/b])/(b^2*c*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.335176, antiderivative size = 177, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5713, 5697, 5670, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{cx+1}\sqrt{1-c^2x^2}(1-}{bc\sqrt{cx-1}\left(a+b\cosh^{-}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*ArcC
osh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]]*Sinh
[(2*a)/b])/(b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(
2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b^2*c*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2c\sqrt{1-c^2x^2}\right)\int \frac{x}{a+b\cosh^{-1}(cx)}\,dx}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(cx)\right)}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-1}(\right.}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}}{}$$

Mathematica [A] time = 0.216824, size = 121, normalized size = 0.83

$$-\frac{\sqrt{1-c^2x^2}\left(\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\cosh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx\right.\right.\right.}{b^2c\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*(b*(-1 + c^2*x^2) + (a + b*ArcCosh[c*x])*CoshIntegral[
2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*ArcCosh[c*x])*Cosh[(2*a)/b]*
```

```
SinhIntegral[2*(a/b + ArcCosh[c*x])]))/(b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x])))
```

Maple [B] time = 0.162, size = 361, normalized size = 2.5

$$\frac{1}{(4\,cx+4)\,(cx-1)\,c\,(a+b\mathrm{arccosh}\,(cx))\,b}\sqrt{-c^2x^2+1}\left(-2\,\sqrt{cx+1}\sqrt{cx-1}x^2c^2+2\,c^3x^3+\sqrt{cx-1}\sqrt{cx+1}-2\,cx\right)-\frac{}{(2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/4*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c
*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/(c*x-1)/c/(a+b*arccosh(c*x))/b-1/2
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arc
cosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b^2-1/4/(c*x
+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)*(2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x
*b*c+2*x^2*b*c^2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2
*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a-b)/c/b^2/(a+b*arccosh(c*x))+1/2/
(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)/c/(a+b*arccosh(c*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{}{abc^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c +
 (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))) + integrate(((2*c^2*x^2 + 1)*(c*x + 1)^(3/2)*(c
*x - 1) + 2*(2*c^3*x^3 - c*x)*(c*x + 1)*sqrt(c*x - 1) + (2*c^4*x^4 - 3*c^2*
x^2 + 1)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a
*b*c^2*x^2 - 2*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c
*x - 1) + a*b + (b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*
x^2 + 2*(b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x
```

```
+ sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(cx-1)(cx+1)}}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a)^2, x)
```

3.324 $\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=181

$$\frac{\sqrt{1-cx}\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2\sqrt{cx-1}}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*x*(a + b*ArcCosh[c*
x]))) - (Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b^2
*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x
])/b])/(b^2*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Unintegrable[1/(x^2*(a + b*Arc
Cosh[c*x])), x])/(b*c*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.570332, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*x*Sqrt[-1 + c*x]*(a + b*Ar
cCosh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh
[a/b])/(b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[a/b]*Si
nhIntegral[(a + b*ArcCosh[c*x])/b])/(b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (S
qrt[1 - c^2*x^2]*Defer[Int][1/(x^2*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c\sqrt{1-c^2x^2}\right)\int \frac{}{a+b\,\mathrm{co}}}{b\sqrt{-1+cx}\sqrt{1}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{b^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right.}{}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cos}{b^2\sqrt{-}}$$

Mathematica [A] time = 31.6778, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.434, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{arccosh}(cx)\right)^2}\sqrt{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^3+\sqrt{cx+1}\sqrt{cx-1}abc^2x^2-abcx+\left(b^2c^3x^3+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x^2-b^2cx\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 - a*b*c
*x + (b^2*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 - b^2*c*x)*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((c^3*x^3 + 2*c*x)*(c*x + 1
)^(3/2)*(c*x - 1) + (2*c^4*x^4 + c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^
5*x^5 - c^3*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*
x - 1)*a*b*c^3*x^4 - 2*a*b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x
^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^
3*x^4 - 2*b^2*c^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x
+ 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b^2x\operatorname{arcosh}(cx)^2+2\,abx\operatorname{arcosh}(cx)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x) +
a^2*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(cx-1)(cx+1)}}{x\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*acosh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)^2*x), x)
```

3.325 $\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{1-cx}\text{Unintegrable}\left(\frac{1}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sqrt{1-c^2x^2}}{bcx^2\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*x^2*(a + b*ArcCosh[
c*x]))) + (2*Sqrt[1 - c*x]*Unintegrable[1/(x^3*(a + b*ArcCosh[c*x])), x])/(
b*c*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.493488, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*x^2*Sqrt[-1 + c*x]*(a + b*
ArcCosh[c*x])) + (2*Sqrt[1 - c^2*x^2]*Defer[Int][1/(x^3*(a + b*ArcCosh[c*x]
)), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}\\ &= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx^2\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{1}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

Mathematica [A] time = 9.88206, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.145, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\operatorname{arccosh}(cx)\right)^2}\sqrt{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^4+\sqrt{cx+1}\sqrt{cx-1}abc^2x^3-abcx^2+\left(b^2c^3x^4+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x^3-b^2cx^2\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int -\frac{}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 - a*b*c
*x^2 + (b^2*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 - b^2*c*x^2)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*(c*x + 1)^(3/2)*(c*x
 - 1)*c*x + 2*(2*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqr
```

```
t(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^5 -
 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*sqrt(c*x + 1)*sq
rt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^5 - 2*b^2*c^3*x^
5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b^2x^2\operatorname{arcosh}(cx)^2+2\,abx^2\operatorname{arcosh}(cx)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x
) + a^2*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(cx-1)(cx+1)}}{x^2\left(a+b\operatorname{acosh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*acosh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c^2x^2+1}}{\left(b\operatorname{arcosh}(cx)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")

[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)^2*x^2), x)
```

3.326 $\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.460794, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][(Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(x^3*(a + b*Ar
cCosh[c*x])^2), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 154., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.328, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(a + b \operatorname{arccosh}\left(c x\right)\right)^{2}} \sqrt{-c^{2} x^{2}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^{2} x^{2}-1\right)(c x+1)\sqrt{c x-1}+\left(c^{3} x^{3}-c x\right)\sqrt{c x+1}\right)\sqrt{-c x+1}}{a b c^{3} x^{5}+\sqrt{c x+1}\sqrt{c x-1}\, a b c^{2} x^{4}-a b c x^{3}+\left(b^{2} c^{3} x^{5}+\sqrt{c x+1}\sqrt{c x-1}\, b^{2} c^{2} x^{4}-b^{2} c x^{3}\right)\log\left(c x+\sqrt{c x+1}\sqrt{c x-1}\right)} - \int \frac{}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^4 - a*b*c
*x^3 + (b^2*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^4 - b^2*c*x^3)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((c^3*x^3 - 4*c*x)*(c*x
 + 1)^(3/2)*(c*x - 1) + (2*c^4*x^4 - 7*c^2*x^2 + 3)*(c*x + 1)*sqrt(c*x - 1)
 + (c^5*x^5 - 3*c^3*x^3 + 2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^8
 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^6 - 2*a*b*c^3*x^6 + a*b*c*x^4 + 2*(a*b*c^4
*x^7 - a*b*c^2*x^5)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^8 + (c*x + 1)*
(c*x - 1)*b^2*c^3*x^6 - 2*b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 - b^2*c^
2*x^5)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
```

```
, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b^2x^3\operatorname{arcosh}(cx)^2+2\,abx^3\operatorname{arcosh}(cx)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x
) + a^2*x^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(cx-1)(cx+1)}}{x^3(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**3/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**3*(a + b*acosh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)^2*x^3), x)
```

3.327 $\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.456095, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][(Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(x^4*(a + b*Ar
cCosh[c*x])^2), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [F] time = 180.008, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.431, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(a+b \operatorname{arccosh}\left(c x\right)\right)^{2}} \sqrt{-c^{2} x^{2}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^{2} x^{2}-1\right)\left(c x+1\right) \sqrt{c x-1}+\left(c^{3} x^{3}-c x\right) \sqrt{c x+1}\right) \sqrt{-c x+1}}{a b c^{3} x^{6}+\sqrt{c x+1} \sqrt{c x-1} a b c^{2} x^{5}-a b c x^{4}+\left(b^{2} c^{3} x^{6}+\sqrt{c x+1} \sqrt{c x-1} b^{2} c^{2} x^{5}-b^{2} c x^{4}\right) \log\left(c x+\sqrt{c x+1} \sqrt{c x-1}\right)}-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^5 - a*b*c
*x^4 + (b^2*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^5 - b^2*c*x^4)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((2*c^3*x^3 - 5*c*x)*(c
*x + 1)^(3/2)*(c*x - 1) + 2*(2*c^4*x^4 - 5*c^2*x^2 + 2)*(c*x + 1)*sqrt(c*x
- 1) + (2*c^5*x^5 - 5*c^3*x^3 + 3*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c
^5*x^9 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^7 - 2*a*b*c^3*x^7 + a*b*c*x^5 + 2*(a
*b*c^4*x^8 - a*b*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^9 + (c*x
 + 1)*(c*x - 1)*b^2*c^3*x^7 - 2*b^2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 -
b^2*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b^2x^4\operatorname{arcosh}(cx)^2+2\,abx^4\operatorname{arcosh}(cx)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x
) + a^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**4/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)^2*x^4), x)
```

3.328 $\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=354

$$-\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c^3\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^3\sqrt{cx-1}}$$

```
[Out] -((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*(a + b*ArcCos
h[c*x]))) - (Sqrt[1 - c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2
*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*CoshIntegral[(4*(a + b
*ArcCosh[c*x]))/b]*Sinh[(4*a)/b])/(4*b^2*c^3*Sqrt[-1 + c*x]) + (3*Sqrt[1 -
c*x]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c^3*Sq
rt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh
[c*x]))/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(4*a)/b]*Sinh
Integral[(4*(a + b*ArcCosh[c*x]))/b])/(4*b^2*c^3*Sqrt[-1 + c*x]) - (3*Sqrt[
1 - c*x]*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^
3*Sqrt[-1 + c*x])
```

Rubi [A] time = 1.13541, antiderivative size = 439, normalized size of antiderivative = 1.24, number of steps used = 20, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5798, 5778, 5780, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{16b^2c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{4b^2c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\sqrt{1-c^2x^2}\sinh\left(\frac{6a}{b}\right)}{16b^2c^3\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x^2*(1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a
+ b*ArcCosh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*
x]]*Sinh[(2*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^
2*x^2]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]]*Sinh[(4*a)/b])/(4*b^2*c^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*CoshIntegral[(6*a)/b + 6*
ArcCosh[c*x]]*Sinh[(6*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (S
qrt[1 - c^2*x^2]*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(16*
b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*Si
```

```
nhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(4*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - (3*Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c
*x]])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{x\left(-1+c^2x^2\right)}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(6c\sqrt{1-c^2x^2}\right)\int \frac{x^3(-}{a+b\,\mathrm{co}}}{b\sqrt{-1+cx}\sqrt{1-}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)dx, x, \mathrm{cosh}}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1}}{}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-}\right.}{2bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x}}{}$$

Mathematica [A] time = 1.0655, size = 338, normalized size = 0.95

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-4\sinh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(16*b*c^2*x^2 - 32*b*c^4*x^4 + 16*b*c^6*x^6
- (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] -
 4*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b]
+ 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcCosh[c*x]
*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + a*Cosh[(2*a)/b]*SinhI
ntegral[2*(a/b + ArcCosh[c*x])] + b*ArcCosh[c*x]*Cosh[(2*a)/b]*SinhIntegral
[2*(a/b + ArcCosh[c*x])] + 4*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[
c*x])] + 4*b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])
] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - 3*b*ArcCosh[c*
x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/(16*b^2*c^3*Sqrt[1
- c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Maple [B] time = 0.355, size = 1176, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/64*(-c^2*x^2+1)^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^
7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)/(c*x+1)/(c*x-
1)/c^3/(a+b*arccosh(c*x))/b+3/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+c^2*x^2-1)*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b
)/(c*x+1)/(c*x-1)/c^3/b^2+1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/
2)*(32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*b*c^5+32*x^6*b*c^6-32*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^3*b*c^3-48*x^4*b*c^4+6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c+18
*x^2*b*c^2+6*arccosh(c*x)*exp(-6*a/b)*Ei(1,-6*arccosh(c*x)-6*a/b)*b+6*exp(-
6*a/b)*Ei(1,-6*arccosh(c*x)-6*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))+1/16/(c*
x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(a+b*arccosh(c*x))/b+1/32*(
-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+
```

```
1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x
)/(c*x+1)/(c*x-1)/c^3/(a+b*arccosh(c*x))/b-1/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arcco
sh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2+1/64*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(
c*x+1)/(c*x-1)/c^3/(a+b*arccosh(c*x))/b-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh
(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2-1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^
2*x^2+1)^(1/2)*(2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c+2*x^2*b*c^2+2*arccosh(c
*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)
*exp(-2*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))-1/32/(c*x+1)^(1/2)/(c*x-1)^(1/
2)*(-c^2*x^2+1)^(1/2)*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*b*c^3+8*x^4*b*c^4-
4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*exp(-4*a/b)*
Ei(1,-4*arccosh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+b
)/c^3/b^2/(a+b*arccosh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(c^4x^6-2\,c^2x^4+x^2\right)(cx+1)\sqrt{cx-1}+\left(c^5x^7-2\,c^3x^5+cx^3\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}-\int\frac{}{abc^5x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^6 - 2*c^2*x^4 + x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^7 - 2*c^3*x^5
 + c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c
*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*
c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((6*c^5*
x^6 - 7*c^3*x^4 + c*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(6*c^6*x^7 - 11*c^4*
x^5 + 6*c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + 3*(2*c^7*x^8 - 5*c^5*x^6 + 4
*c^3*x^4 - c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c
*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*s
qrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2
 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c
*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^2x^4-x^2\right)\sqrt{-c^2x^2+1}}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*ar
ccosh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x^2}{\left(b\,\text{arcosh}\,(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a)^2, x)
```

3.329 $\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=348

$$\frac{\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^2\sqrt{cx-1}}-\frac{9\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^2\sqrt{cx-1}}+\frac{5\sqrt{1-cx}\sinh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\cosh^{-1}
}{b}\right)}{16b^2c^2\sqrt{cx-1}}$$

```
[Out] -((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*(a + b*ArcCosh[
c*x]))) + (Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(8
*b^2*c^2*Sqrt[-1 + c*x]) - (9*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcCosh[
c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*c^2*Sqrt[-1 + c*x]) + (5*Sqrt[1 - c*x]*Cos
hIntegral[(5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*c^2*Sqrt[-1 +
c*x]) - (Sqrt[1 - c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b
^2*c^2*Sqrt[-1 + c*x]) + (9*Sqrt[1 - c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a
+ b*ArcCosh[c*x]))/b])/(16*b^2*c^2*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*Cosh[
(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^2*Sqrt[-1 + c*
x])
```

Rubi [A] time = 1.05554, antiderivative size = 429, normalized size of antiderivative = 1.23, number of steps used = 23, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5798, 5778, 5700, 3312, 3303, 3298, 3301, 5780, 5448}

$$\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b^2c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{9\sqrt{1-c^2x^2}\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16b^2c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{5\sqrt{1-c^2x^2}\sinh\left(\frac{5a}{b}\right)\operatorname{Chi}}{16b^2c^2\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x*(1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a +
b*ArcCosh[c*x])) + (Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh
[a/b])/(8*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (9*Sqrt[1 - c^2*x^2]*Cosh
Integral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(16*b^2*c^2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + (5*Sqrt[1 - c^2*x^2]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*
x]]*Sinh[(5*a)/b])/(16*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^
2*x^2]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b^2*c^2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + (9*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b
```

```
+ 3*ArcCosh[c*x]])/(16*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*Sqrt[1 -
c^2*x^2]*Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b^2*c^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5700

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(-d)^p/c, Subst[Int[(a + b*x)^n*Sinh[x]^(2*p + 1), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] &&
IGtQ[p, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{-1+c^2x^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(5c\sqrt{1-c^2x^2}\right)\int \frac{x^2\left(-1+c^2x^2\right.}{a+b\cosh^{-1}(c}}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(5\sqrt{1-c\right.}{}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(i\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)} - \frac{i\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}\right.}{bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(5\sqrt{1-c\right.}{}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{8bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{9\sqrt{1-c^2x^2}\text{Ch}}{16b}$$

Mathematica [A] time = 0.913682, size = 327, normalized size = 0.94

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-2\sinh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+9\sinh\left(\frac{3a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{cosh}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b*c*x + 32*b*c^3*x^3 - 16*b*c^5*x^5 - 2*
(a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] + 9*(a + b*
ArcCosh[c*x])*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - 5*a*Cosh
Integral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 5*b*ArcCosh[c*x]*CoshInteg
ral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 2*a*Cosh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]] + 2*b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x
```

```
]] - 9*a*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - 9*b*ArcCosh[c
*x]*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 5*a*Cosh[(5*a)/b]*
SinhIntegral[5*(a/b + ArcCosh[c*x])] + 5*b*ArcCosh[c*x]*Cosh[(5*a)/b]*SinhI
ntegral[5*(a/b + ArcCosh[c*x])]))/(16*b^2*c^2*Sqrt[1 - c^2*x^2]*(a + b*ArcC
osh[c*x]))
```

Maple [B] time = 0.276, size = 1029, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/32*(-c^2*x^2+1)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^
6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+13*c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/b/(a+b*arccosh(c*x))+5/32*(-c^
2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arccosh(
c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2+1/32*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
4*b*c^4+16*x^5*b*c^5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2-20*x^3*b*c^3+
5*arccosh(c*x)*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1
,-5*arccosh(c*x)-5*a/b)*a+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+5*x*b*c)/c^2/b^2/(a
+b*arccosh(c*x))+3/32*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c*x+1)/c^2/
(c*x-1)/b/(a+b*arccosh(c*x))-9/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/
b)/(c*x+1)/c^2/(c*x-1)/b^2+1/16*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)
^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+x*b*c)/c^2/b^2/(a+b*arccosh(c*
x))-3/32*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(4*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(c*x)*exp(-3*a/b)*Ei(1,-3*arccos
h(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)*a-(c*x+1)^(1/2)*(
c*x-1)^(1/2)*b-3*x*b*c)/c^2/b^2/(a+b*arccosh(c*x))-1/16*(-c^2*x^2+1)^(1/2)*
(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/b/(a+b*arc
cosh(c*x))+1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^
2-1)*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(c^4x^5-2\,c^2x^3+x\right)(cx+1)\sqrt{cx-1}+\left(c^5x^6-2\,c^3x^4+cx^2\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}-\int\frac{}{abc^5x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] ((c^4*x^5 - 2*c^2*x^3 + x)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^6 - 2*c^3*x^4 +
 c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x
 - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^
2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate((5*(c^5*x^
5 - c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + (10*c^6*x^6 - 17*c^4*x^4 + 8*c^2*x
^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (5*c^7*x^7 - 12*c^5*x^5 + 9*c^3*x^3 - 2*c
*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^
3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*s
qrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x
^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(c^2x^3-x\right)\sqrt{-c^2x^2+1}}{b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*arcc
osh(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(-(cx-1)(cx+1)\right)^{\frac{3}{2}}}{\left(a+b\operatorname{acosh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a)^2, x)
```

3.330 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=246

$$-\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c\sqrt{cx-1}}+\frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2b^2c\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}\right.}{b}\right.}{b^2c\sqrt{cx-1}}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*(a + b*ArcCosh[c*
x]))) - (Sqrt[1 - c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/
b])/(b^2*c*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*CoshIntegral[(4*(a + b*ArcCosh[
c*x]))/b]*Sinh[(4*a)/b])/(2*b^2*c*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(2*
a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(b^2*c*Sqrt[-1 + c*x]) - (S
qrt[1 - c*x]*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(2*b^2
*c*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.523821, antiderivative size = 305, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5713, 5697, 5780, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{2b^2c\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Sh}}{b^2c\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*
ArcCosh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]]*
Sinh[(2*a)/b])/(b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Co
shIntegral[(4*a)/b + 4*ArcCosh[c*x]]*Sinh[(4*a)/b])/(2*b^2*c*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2
*ArcCosh[c*x]])/(b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*C
osh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(2*b^2*c*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(4c\sqrt{1-c^2x^2}\right)\int \frac{x\left(-1+c^2x^2\right)}{a+b\cosh^{-1}(cx)}\,dx}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(4\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(4\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)dx, x, \cosh^{-1}\right.}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x}}{}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-1}(\right.}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{C}}{}$$

Mathematica [A] time = 0.591487, size = 232, normalized size = 0.94

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(4\left(\frac{a}{b}+\cos\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-2*b + 4*b*c^2*x^2 - 2*b*c^4*x^4 + 2*(a + b*
ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*A
rcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] - 2*a*Cosh[
(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 2*b*ArcCosh[c*x]*Cosh[(2*a)
/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + a*Cosh[(4*a)/b]*SinhIntegral[4*(
a/b + ArcCosh[c*x])] + b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + A
rcCosh[c*x])]))/(2*b^2*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Maple [B] time = 0.208, size = 737, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/16*(-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+
8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c*x)/(c*x+1)/(c*x-1)/c/(a+b*arccosh(c*x))/b+1/4*(-c^2*x^2+1)^(1/2)*(-(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*
arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b^2+1/16/(c*x+1)^(1/2)/(c*x-1)^(1/2)
*(-c^2*x^2+1)^(1/2)*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*b*c^3+8*x^4*b*c^4-4*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*exp(-4*a/b)*Ei
(1,-4*arccosh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+b)/
c/b^2/(a+b*arccosh(c*x))+3/8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)
/c/(a+b*arccosh(c*x))/b+1/4*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/(c*x-1)/c/
(a+b*arccosh(c*x))/b-1/2*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1
)/(c*x-1)/c/b^2-1/4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)*(2*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*x*b*c+2*x^2*b*c^2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x
)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a-b)/c/b^2
/(a+b*arccosh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(c^4x^4-2\,c^2x^2+1\right)(cx+1)\sqrt{cx-1}+\left(c^5x^5-2\,c^3x^3+cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}-\int\frac{}{abc^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*
x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((4*c^4*x^4
- 3*c^2*x^2 - 1)*(c*x + 1)^(3/2)*(c*x - 1) + 4*(2*c^5*x^5 - 3*c^3*x^3 + c*x
)*(c*x + 1)*sqrt(c*x - 1) + (4*c^6*x^6 - 9*c^4*x^4 + 6*c^2*x^2 - 1)*sqrt(c*
x + 1))*sqrt(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^2*x^2 - 2*a
*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + a*b +
(b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*x^2 + 2*(b^2*c^3
*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b^2\operatorname{arcosh}\left(cx\right)^2+2\,ab\operatorname{arcosh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a)^2, x)
```

3.331 $\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\cosh^{-1}(cx))^2} dx$

Optimal. Leaf size=290

$$-\frac{\sqrt{1-cx}\text{Unintegrable}\left(\frac{c^2x^2-1}{x^2(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{cx-1}}-\frac{9\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2\sqrt{cx-1}}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*x*(a + b*ArcCosh[
c*x]))) - (9*Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/
(4*b^2*Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcCosh[c*
x]))/b]*Sinh[(3*a)/b])/(4*b^2*Sqrt[-1 + c*x]) + (9*Sqrt[1 - c*x]*Cosh[a/b]*
SinhIntegral[(a + b*ArcCosh[c*x])/b])/(4*b^2*Sqrt[-1 + c*x]) - (3*Sqrt[1 -
c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b^2*Sqrt[-1
 + c*x]) - (Sqrt[1 - c*x]*Unintegrable[(-1 + c^2*x^2)/(x^2*(a + b*ArcCosh[c
*x])), x])/(b*c*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.87188, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\cosh^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*x*Sqrt[-1 + c*x]*(a +
b*ArcCosh[c*x])) - (9*Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*Si
nh[a/b])/(4*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*CoshIn
tegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(4*b^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) + (9*Sqrt[1 - c^2*x^2]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
)/(4*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]
*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (Sqrt[1 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)/(x^2*(a + b*ArcCosh[c*x])
), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{-1+c^2x^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(3c\sqrt{1-c^2x^2}\right)\int \ldots}{b\sqrt{-1+cx}\ldots}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \ldots$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3i\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)} - \frac{i\sinh(3x)}{4(a+bx)}\right)dx, x, \ldots\right)}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4b\sqrt{-1+cx}\sqrt{1+cx}} + \ldots$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{-1+c^2x^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(9\sqrt{1-c^2x^2}\cosh\left(\ldots\right.\right.}{\ldots}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{9\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3\sqrt{1-c^2\ldots}}{\ldots}$$

Mathematica [A] time = 32.1354, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.463, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{2}}\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(c^{4}x^{4}-2\,c^{2}x^{2}+1\right)(cx+1)\sqrt{cx-1}+\left(c^{5}x^{5}-2\,c^{3}x^{3}+cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^{3}x^{3}+\sqrt{cx+1}\sqrt{cx-1}abc^{2}x^{2}-abcx+\left(b^{2}c^{3}x^{3}+\sqrt{cx+1}\sqrt{cx-1}b^{2}c^{2}x^{2}-b^{2}cx\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}-\int\frac{}{abc^{5}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^2 - a*b*c*x + (b^2*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*
c^2*x^2 - b^2*c*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((3*
c^5*x^5 - c^3*x^3 - 2*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + (6*c^6*x^6 - 7*c^4*x
^4 + 1)*(c*x + 1)*sqrt(c*x - 1) + 3*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*sqrt(c*
x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^4 - 2*a
*b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x^3)*sqrt(c*x + 1)*sqrt(c
*x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^4 - 2*b^2*c^3*x^4 +
b^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}}{b^{2}x\operatorname{arcosh}\left(cx\right)^{2}+2\,abx\operatorname{arcosh}\left(cx\right)+a^{2}x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
+ a^2*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x*(a + b*acosh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{(b\operatorname{arcosh}(cx)+a)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)^2*x), x)
```

3.332 $\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=156

$$-\frac{2\sqrt{1-cx}\,\text{Unintegrable}\left(\frac{c^2x^2-1}{x^3(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{cx-1}}-\frac{2c\sqrt{1-cx}\,\text{Unintegrable}\left(\frac{c^2x^2-1}{x(a+b\cosh^{-1}(cx))},x\right)}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x\right.}{bcx^2\left(a+b\cosh^{-1}(cx\right.}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*x^2*(a + b*ArcCos
h[c*x]))) - (2*Sqrt[1 - c*x]*Unintegrable[(-1 + c^2*x^2)/(x^3*(a + b*ArcCos
h[c*x])), x])/(b*c*Sqrt[-1 + c*x]) - (2*c*Sqrt[1 - c*x]*Unintegrable[(-1 +
c^2*x^2)/(x*(a + b*ArcCosh[c*x])), x])/(b*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.661832, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*x^2*Sqrt[-1 + c*x]*(a
+ b*ArcCosh[c*x])) - (2*Sqrt[1 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)/(x^3*(a
 + b*ArcCosh[c*x])), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*c*Sqrt[1 -
 c^2*x^2]*Defer[Int][(-1 + c^2*x^2)/(x*(a + b*ArcCosh[c*x])), x])/(b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx^2\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{-1+c^2x^2}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(2c\sqrt{1-c^2x^2}\right)}{b\sqrt{-1}}$$

Mathematica [A] time = 60.6377, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.555, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\operatorname{arccosh}(cx)\right)^2}\left(-c^2x^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(c^4x^4-2\,c^2x^2+1\right)(cx+1)\sqrt{cx-1}+\left(c^5x^5-2\,c^3x^3+cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^4+\sqrt{cx+1}\sqrt{cx-1}abc^2x^3-abcx^2+\left(b^2c^3x^4+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x^3-b^2cx^2\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)} - \int \frac{}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^3 - a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^
2*c^2*x^3 - b^2*c*x^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(
((2*c^5*x^5 + c^3*x^3 - 3*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(2*c^6*x^6 - c
^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (2*c^7*x^7 - 3*c^5*x^5 +
c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1)*a*b*c
^3*x^5 - 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^5 - 2*b^
2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^2\,\text{arcosh}\,(cx)^2+2\,abx^2\,\text{arcosh}\,(cx)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c
*x) + a^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*acosh(c*x))**2,x)
```

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)^2*x^2), x)
```

3.333 $\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{(1-c^2x^2)^{3/2}}{x^3\,(a+b\cosh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.54185, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(x^3*(a
+ b*ArcCosh[c*x])^2), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3\,(a+b\cosh^{-1}(cx))^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^3(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 149.076, size = 0, normalized size = 0.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.648, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(a + b\operatorname{arccosh}\left(cx\right)\right)^{2}} \left(-c^{2}x^{2}+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(c^{4}x^{4}-2\,c^{2}x^{2}+1\right)(cx+1)\sqrt{cx-1}+\left(c^{5}x^{5}-2\,c^{3}x^{3}+cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^{3}x^{5}+\sqrt{cx+1}\sqrt{cx-1}abc^{2}x^{4}-abcx^{3}+\left(b^{2}c^{3}x^{5}+\sqrt{cx+1}\sqrt{cx-1}b^{2}c^{2}x^{4}-b^{2}cx^{3}\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)} - \int \frac{}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^4 - a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^
2*c^2*x^4 - b^2*c*x^3)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(
((c^5*x^5 + 3*c^3*x^3 - 4*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + (2*c^6*x^6 + 3*c
^4*x^4 - 8*c^2*x^2 + 3)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7 - 3*c^3*x^3 + 2*
c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^8 + (c*x + 1)*(c*x - 1)*a*b*c
^3*x^6 - 2*a*b*c^3*x^6 + a*b*c*x^4 + 2*(a*b*c^4*x^7 - a*b*c^2*x^5)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^8 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^6 - 2*b^
2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 - b^2*c^2*x^5)*sqrt(c*x + 1)*sqrt(c*
```

```
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^3\operatorname{arcosh}(cx)^2+2\,abx^3\operatorname{arcosh}(cx)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c
*x) + a^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**3/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arcosh}(cx)+a\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)^2*x^3), x)
```

3.334 $\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=106

$$-\frac{4\sqrt{1-cx}\,\text{Unintegrable}\left(\frac{c^2x^2-1}{x^5(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\,(1-c^2x^2)^{3/2}}{bcx^4\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*x^4*(a + b*ArcCos
h[c*x]))) - (4*Sqrt[1 - c*x]*Unintegrable[(-1 + c^2*x^2)/(x^5*(a + b*ArcCos
h[c*x])), x])/(b*c*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.611748, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*x^4*Sqrt[-1 + c*x]*(a
+ b*ArcCosh[c*x])) - (4*Sqrt[1 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)/(x^5*(a
 + b*ArcCosh[c*x])), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(1-c^2x^2)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^4(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}\\ &= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx^4\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(4\sqrt{1-c^2x^2}\right)\int \frac{-1+c^2x^2}{x^5(a+b\cosh^{-1}(cx))}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}}\end{aligned}$$

Mathematica [F] time = 180.005, size = 0, normalized size = 0.

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.418, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b\operatorname{arccosh}(cx))^2} \left(-c^2x^2 + 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(c^4x^4 - 2\,c^2x^2 + 1\right)(cx + 1)\sqrt{cx - 1} + \left(c^5x^5 - 2\,c^3x^3 + cx\right)\sqrt{cx + 1}\right)\sqrt{-cx + 1}}{abc^3x^6 + \sqrt{cx + 1}\sqrt{cx - 1}abc^2x^5 - abcx^4 + \left(b^2c^3x^6 + \sqrt{cx + 1}\sqrt{cx - 1}b^2c^2x^5 - b^2cx^4\right)\log\left(cx + \sqrt{cx + 1}\sqrt{cx - 1}\right)} - \int \frac{}{abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^5 - a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^
2*c^2*x^5 - b^2*c*x^4)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(
(5*(c^3*x^3 - c*x)*(c*x + 1)^(3/2)*(c*x - 1) + 4*(2*c^4*x^4 - 3*c^2*x^2 + 1
)*(c*x + 1)*sqrt(c*x - 1) + 3*(c^5*x^5 - 2*c^3*x^3 + c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^5*x^9 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^7 - 2*a*b*c^3*x^7
```

```
 + a*b*c*x^5 + 2*(a*b*c^4*x^8 - a*b*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1) +
(b^2*c^5*x^9 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^7 - 2*b^2*c^3*x^7 + b^2*c*x^5
+ 2*(b^2*c^4*x^8 - b^2*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^4\,\text{arcosh}\,(cx)^2+2\,abx^4\,\text{arcosh}\,(cx)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c
*x) + a^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**4/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\,\text{arcosh}\,(cx)+a\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="giac")

[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)^2*x^4), x)
```

3.335 $\int \frac{x^2\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=454

$$-\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8b^2c^3\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^3\sqrt{cx-1}}$$

```
[Out] -((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*(a + b*ArcCos
h[c*x]))) - (Sqrt[1 - c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2
*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*CoshIntegral[(4*(a + b
*ArcCosh[c*x]))/b]*Sinh[(4*a)/b])/(8*b^2*c^3*Sqrt[-1 + c*x]) + (3*Sqrt[1 -
c*x]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c^3*Sq
rt[-1 + c*x]) - (Sqrt[1 - c*x]*CoshIntegral[(8*(a + b*ArcCosh[c*x]))/b]*Sin
h[(8*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(2*a)/b]*Sinh
Integral[(2*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) + (Sqrt[1
 - c*x]*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(8*b^2*c^3*
Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*Arc
Cosh[c*x]))/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(8*a)/b]*
SinhIntegral[(8*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^3*Sqrt[-1 + c*x])
```

Rubi [A] time = 1.51466, antiderivative size = 565, normalized size of antiderivative = 1.24, number of steps used = 29, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5798, 5778, 5780, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{16b^2c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8b^2c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\sqrt{1-c^2x^2}\sinh\left(\frac{6a}{b}\right)}{16b^2c^3\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x^2*(1 - c*x)^3*(1 + c*x)^(5/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a
+ b*ArcCosh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*
x]]*Sinh[(2*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^
2*x^2]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]]*Sinh[(4*a)/b])/(8*b^2*c^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*CoshIntegral[(6*a)/b + 6*
ArcCosh[c*x]]*Sinh[(6*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (S
```

```
qrt[1 - c^2*x^2]*CoshIntegral[(8*a)/b + 8*ArcCosh[c*x]]*Sinh[(8*a)/b])/(16*
b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*Si
nhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*
x]])/(8*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*Cosh[(
6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c*x]])/(16*b^2*c^3*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(8*a)/b]*SinhIntegral[(8*a)/b + 8*
ArcCosh[c*x]])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{x\left(-1+c^2x^2\right)^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(8c\sqrt{1-c^2x^2}\right)\int \frac{x^3\left(-1+\right.}{a+b\cosh}}{b\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{bc^3\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)} - \frac{\sinh(4x)}{8(a+bx)} + \frac{\sinh(6x)}{32(a+bx)}\right) dx\right.}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x}}{}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(5\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\cosh^{-1}\right.}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}}{}$$

Mathematica [A] time = 1.7033, size = 446, normalized size = 0.98

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+2\sinh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(4\left(\frac{a}{b}+\cos\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b*c^2*x^2 + 48*b*c^4*x^4 - 48*b*c^6*x^6
+ 16*b*c^8*x^8 + (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*
Sinh[(2*a)/b] + 2*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]
*Sinh[(4*a)/b] - 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] - 3
*b*ArcCosh[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + a*Cosh
Integral[8*(a/b + ArcCosh[c*x])]*Sinh[(8*a)/b] + b*ArcCosh[c*x]*CoshIntegra
```

```
l[8*(a/b + ArcCosh[c*x])]*Sinh[(8*a)/b] - a*Cosh[(2*a)/b]*SinhIntegral[2*(a
/b + ArcCosh[c*x])] - b*ArcCosh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + Ar
cCosh[c*x])] - 2*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])] - 2*b
*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])] + 3*a*Cosh
[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cosh[(6*a
)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - a*Cosh[(8*a)/b]*SinhIntegral[8*
(a/b + ArcCosh[c*x])] - b*ArcCosh[c*x]*Cosh[(8*a)/b]*SinhIntegral[8*(a/b +
ArcCosh[c*x])]))/(16*b^2*c^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Maple [B] time = 0.43, size = 1676, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/256*(-c^2*x^2+1)^(1/2)*(-128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+128*c^9*
x^9+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6-320*c^7*x^7-160*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^4*c^4+272*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-88*
c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c*x)/(c*x+1)/(c*x-1)/c^3/(a+b*arccosh
(c*x))/b-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-
1)*Ei(1,8*arccosh(c*x)+8*a/b)*exp((b*arccosh(c*x)+8*a)/b)/(c*x+1)/(c*x-1)/c
^3/b^2-1/256/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)*(128*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^7*b*c^7+128*x^8*b*c^8-192*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^5*b*c^5-256*x^6*b*c^6+80*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*b*c^3+160*x^4*b*c
^4-8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c-32*x^2*b*c^2+8*arccosh(c*x)*exp(-8*a
/b)*Ei(1,-8*arccosh(c*x)-8*a/b)*b+8*exp(-8*a/b)*Ei(1,-8*arccosh(c*x)-8*a/b)
*a+b)/c^3/b^2/(a+b*arccosh(c*x))+5/128/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^
2+1)^(1/2)/c^3/(a+b*arccosh(c*x))/b-1/64*(-c^2*x^2+1)^(1/2)*(-32*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4
-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)
*(c*x+1)^(1/2)-6*c*x)/(c*x+1)/(c*x-1)/c^3/(a+b*arccosh(c*x))/b+3/32*(-c^2*x
^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6*arccosh(c*x
)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2+1/64*(-c^2*x^2
+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)/(c*x+1
)/(c*x-1)/c^3/(a+b*arccosh(c*x))/b-1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)
+4*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2+1/64*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/
(c*x-1)/c^3/(a+b*arccosh(c*x))/b-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c
```

```
*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2
*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2-1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1
)^(1/2)*(2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c+2*x^2*b*c^2+2*arccosh(c*x)*Ei(
1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2
*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))-1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^
2*x^2+1)^(1/2)*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*b*c^3+8*x^4*b*c^4-4*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*exp(-4*a/b)*Ei(1,-4
*arccosh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+b)/c^3/b
^2/(a+b*arccosh(c*x))+1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)*(
32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*b*c^5+32*x^6*b*c^6-32*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*b*c^3-48*x^4*b*c^4+6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c+18*x^2
*b*c^2+6*arccosh(c*x)*exp(-6*a/b)*Ei(1,-6*arccosh(c*x)-6*a/b)*b+6*exp(-6*a/
b)*Ei(1,-6*arccosh(c*x)-6*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^6x^8-3\,c^4x^6+3\,c^2x^4-x^2\right)(cx+1)\sqrt{cx-1}+\left(c^7x^9-3\,c^5x^7+3\,c^3x^5-cx^3\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{}{abc^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^8 - 3*c^4*x^6 + 3*c^2*x^4 - x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^
9 - 3*c^5*x^7 + 3*c^3*x^5 - c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x
^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*
x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))) + integrate(((8*c^7*x^8 - 17*c^5*x^6 + 10*c^3*x^4 - c*x^2)*(c*x + 1)^(
3/2)*(c*x - 1) + 2*(8*c^8*x^9 - 22*c^6*x^7 + 21*c^4*x^5 - 8*c^2*x^3 + x)*(c
*x + 1)*sqrt(c*x - 1) + (8*c^9*x^10 - 27*c^7*x^8 + 33*c^5*x^6 - 17*c^3*x^4
+ 3*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)
*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^
2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1)
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^6 - 2\,c^2x^4 + x^2\right)\sqrt{-c^2x^2 + 1}}{b^2\,\text{arcosh}\,(cx)^2 + 2\,ab\,\text{arcosh}\,(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2
 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2 + 1\right)^{\frac{5}{2}}x^2}{\left(b\,\text{arcosh}\,(cx) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a)^2, x)
```

3.336 $\int \frac{x\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=448

$$\frac{5\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64b^2c^2\sqrt{cx-1}}-\frac{27\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64b^2c^2\sqrt{cx-1}}+\frac{25\sqrt{1-cx}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh\right.}{b}\right.}{64b^2c^2\sqrt{cx-1}}$$

```
[Out] -((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*(a + b*ArcCosh[
c*x]))) + (5*Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/
(64*b^2*c^2*Sqrt[-1 + c*x]) - (27*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcC
osh[c*x]))/b]*Sinh[(3*a)/b])/(64*b^2*c^2*Sqrt[-1 + c*x]) + (25*Sqrt[1 - c*x
]*CoshIntegral[(5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/(64*b^2*c^2*Sqrt[
-1 + c*x]) - (7*Sqrt[1 - c*x]*CoshIntegral[(7*(a + b*ArcCosh[c*x]))/b]*Sinh
[(7*a)/b])/(64*b^2*c^2*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*Cosh[a/b]*SinhInt
egral[(a + b*ArcCosh[c*x])/b])/(64*b^2*c^2*Sqrt[-1 + c*x]) + (27*Sqrt[1 - c
*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(64*b^2*c^2*Sqr
t[-1 + c*x]) - (25*Sqrt[1 - c*x]*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCo
sh[c*x]))/b])/(64*b^2*c^2*Sqrt[-1 + c*x]) + (7*Sqrt[1 - c*x]*Cosh[(7*a)/b]*
SinhIntegral[(7*(a + b*ArcCosh[c*x]))/b])/(64*b^2*c^2*Sqrt[-1 + c*x])
```

Rubi [A] time = 1.33071, antiderivative size = 555, normalized size of antiderivative = 1.24, number of steps used = 29, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5798, 5778, 5700, 3312, 3303, 3298, 3301, 5780, 5448}

$$\frac{5\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64b^2c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{27\sqrt{1-c^2x^2}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{64b^2c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{25\sqrt{1-c^2x^2}\sinh\left(\frac{5a}{b}\right)\text{C}}{64b^2c^2\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x*(1 - c*x)^3*(1 + c*x)^(5/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a +
b*ArcCosh[c*x])) + (5*Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*Si
nh[a/b])/(64*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (27*Sqrt[1 - c^2*x^2]*
CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(64*b^2*c^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) + (25*Sqrt[1 - c^2*x^2]*CoshIntegral[(5*a)/b + 5*ArcCo
sh[c*x]]*Sinh[(5*a)/b])/(64*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*Sqrt
```

```
[1 - c^2*x^2]*CoshIntegral[(7*a)/b + 7*ArcCosh[c*x]]*Sinh[(7*a)/b])/(64*b^2
*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*Sqrt[1 - c^2*x^2]*Cosh[a/b]*SinhInt
egral[a/b + ArcCosh[c*x]])/(64*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (27*
Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(64
*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (25*Sqrt[1 - c^2*x^2]*Cosh[(5*a)/b
]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(64*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (7*Sqrt[1 - c^2*x^2]*Cosh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcC
osh[c*x]])/(64*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5700

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(-d)^p/c, Subst[Int[(a + b*x)^n*Sinh[x]^(2*p + 1), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] &&
IGtQ[p, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{5/2}(1+cx)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{\left(-1+c^2x^2\right)^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(7c\sqrt{1-c^2x^2}\right)\int \frac{x^2\left(-1+c^2x\right.}{a+b\cosh^{-}}}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^5(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(7\sqrt{1-\right.}{}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(i\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\left(\frac{5i\sinh(x)}{8(a+bx)} - \frac{5i\sinh(3x)}{16(a+bx)} + \frac{i\sinh(5x)}{16(a+bx)}\right)\right.}{bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{16bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(7\sqrt{1-\right.}{}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(35\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}\right.}{64bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{5\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{64b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{27\sqrt{1-c^2x}}{}$$

Mathematica [A] time = 1.36289, size = 436, normalized size = 0.97

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-5\sinh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+27\sinh\left(\frac{3a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{co}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-64*b*c*x + 192*b*c^3*x^3 - 192*b*c^5*x^5 +
64*b*c^7*x^7 - 5*(a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh
[a/b] + 27*(a + b*ArcCosh[c*x])*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(
3*a)/b] - 25*a*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 25*b*Ar
cCosh[c*x]*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 7*a*CoshInt
egral[7*(a/b + ArcCosh[c*x])]*Sinh[(7*a)/b] + 7*b*ArcCosh[c*x]*CoshIntegral
```

```
[7*(a/b + ArcCosh[c*x])]*Sinh[(7*a)/b] + 5*a*Cosh[a/b]*SinhIntegral[a/b + A
rcCosh[c*x]] + 5*b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
 - 27*a*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - 27*b*ArcCosh[c*
x]*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 25*a*Cosh[(5*a)/b]*
SinhIntegral[5*(a/b + ArcCosh[c*x])] + 25*b*ArcCosh[c*x]*Cosh[(5*a)/b]*Sinh
Integral[5*(a/b + ArcCosh[c*x])] - 7*a*Cosh[(7*a)/b]*SinhIntegral[7*(a/b +
ArcCosh[c*x])] - 7*b*ArcCosh[c*x]*Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCo
sh[c*x])]))/(64*b^2*c^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Maple [B] time = 0.375, size = 1499, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/128*(-c^2*x^2+1)^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^
8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+
1)/(c*x+1)/c^2/(c*x-1)/b/(a+b*arccosh(c*x))-7/128*(-c^2*x^2+1)^(1/2)*(-(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,7*arccosh(c*x)+7*a/b)*exp((b*ar
ccosh(c*x)+7*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2-1/128*(-c^2*x^2+1)^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)*(64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*b*c^6+64*x^7*b*c^7
-80*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*b*c^4-112*x^5*b*c^5+24*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^2*b*c^2+56*x^3*b*c^3+7*arccosh(c*x)*exp(-7*a/b)*Ei(1,-7*arcco
sh(c*x)-7*a/b)*b-(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+7*exp(-7*a/b)*Ei(1,-7*arccos
h(c*x)-7*a/b)*a-7*x*b*c)/c^2/b^2/(a+b*arccosh(c*x))-5/128*(-c^2*x^2+1)^(1/2
)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1
)/(c*x+1)/c^2/(c*x-1)/b/(a+b*arccosh(c*x))+25/128*(-c^2*x^2+1)^(1/2)*(-(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arccosh(c*x)+5*a/b)*exp((b*ar
ccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2+9/128*(-c^2*x^2+1)^(1/2)*(-4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c-5*c^2*x^2+1)/(c*x+1)/c^2/(c*x-1)/b/(a+b*arccosh(c*x))-27/128*(-c^2*x^2+1)
^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arccosh(c*x)+3*a
/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2+5/128*(-c^2*x^2+1)^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x
)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+
x*b*c)/c^2/b^2/(a+b*arccosh(c*x))-9/128*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)*(4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(
c*x)*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arccos
```

```
h(c*x)-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)^(1/2)*b-3*x*b*c)/c^2/b^2/(a+b*arccosh
(c*x))+5/128*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(16*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^4*b*c^4+16*x^5*b*c^5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*
b*c^2-20*x^3*b*c^3+5*arccosh(c*x)*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*b
+5*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*a+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+
5*x*b*c)/c^2/b^2/(a+b*arccosh(c*x))-5/128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/b/(a+b*arccosh(c*x))+5/1
28*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arc
cosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^{6}x^{7}-3\,c^{4}x^{5}+3\,c^{2}x^{3}-x\right)(cx+1)\sqrt{cx-1}+\left(c^{7}x^{8}-3\,c^{5}x^{6}+3\,c^{3}x^{4}-cx^{2}\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^{3}x^{2}+\sqrt{cx+1}\sqrt{cx-1}abc^{2}x-abc+\left(b^{2}c^{3}x^{2}+\sqrt{cx+1}\sqrt{cx-1}b^{2}c^{2}x-b^{2}c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{}{abc^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^7 - 3*c^4*x^5 + 3*c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^8
- 3*c^5*x^6 + 3*c^3*x^4 - c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2
 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x
+ 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)
)) + integrate((7*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1)
 + (14*c^8*x^8 - 37*c^6*x^6 + 33*c^4*x^4 - 11*c^2*x^2 + 1)*(c*x + 1)*sqrt(c
*x - 1) + (7*c^9*x^9 - 23*c^7*x^7 + 27*c^5*x^5 - 13*c^3*x^3 + 2*c*x)*sqrt(c
*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*
a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x -
1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c
 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(
c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^{4}x^{5}-2\,c^{2}x^{3}+x\right)\sqrt{-c^{2}x^{2}+1}}{b^{2}\operatorname{arcosh}\left(cx\right)^{2}+2\,ab\operatorname{arcosh}\left(cx\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 +
 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}x}{(b\,\mathrm{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a)^2, x)
```

3.337 $\int \frac{(1-c^2x^2)^{5/2}}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=351

$$-\frac{15\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+}{}\right.}{16b^2c\sqrt{cx-1}}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*(a + b*ArcCosh[c*
x]))) - (15*Sqrt[1 - c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*
a)/b])/(16*b^2*c*Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*CoshIntegral[(4*(a + b*
ArcCosh[c*x]))/b]*Sinh[(4*a)/b])/(4*b^2*c*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x
]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c*Sqrt[-1
 + c*x]) + (15*Sqrt[1 - c*x]*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c
*x]))/b])/(16*b^2*c*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Cosh[(4*a)/b]*SinhIn
tegral[(4*(a + b*ArcCosh[c*x]))/b])/(4*b^2*c*Sqrt[-1 + c*x]) + (3*Sqrt[1 -
c*x]*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c*Sqrt
[-1 + c*x])
```

Rubi [A] time = 0.651829, antiderivative size = 436, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5713, 5697, 5780, 5448, 3303, 3298, 3301}

$$-\frac{15\sqrt{1-c^2x^2}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{16b^2c\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\sqrt{1-c^2x^2}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{4b^2c\sqrt{cx-1}\sqrt{cx+1}}-\frac{3\sqrt{1-c^2x^2}\sinh\left(\right.}{16b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((1 - c*x)^3*(1 + c*x)^(5/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*
ArcCosh[c*x])) - (15*Sqrt[1 - c^2*x^2]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x
]]*Sinh[(2*a)/b])/(16*b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2
*x^2]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]]*Sinh[(4*a)/b])/(4*b^2*c*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*CoshIntegral[(6*a)/b + 6*Arc
Cosh[c*x]]*Sinh[(6*a)/b])/(16*b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*Sqr
t[1 - c^2*x^2]*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(16*b^
2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*Sinh
```

```
Integral[(4*a)/b + 4*ArcCosh[c*x]])/(4*b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (3*Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c*x]]
)/(16*b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(6c\sqrt{1-c^2x^2}\right)\int \frac{x\left(-1+c^2x^2\right)^2}{a+b\cosh^{-1}(cx)}\,dx}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(6\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(6\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)} - \frac{\sinh(4x)}{8(a+bx)} + \frac{\sinh(6x)}{32(a+bx)}\right) d}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{16bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(3\sqrt{\right.}{}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(15\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh\right.}{16bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{15\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3\sqrt{1-}}{}$$

Mathematica [A] time = 1.03901, size = 343, normalized size = 0.98

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(15\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) - 12\sinh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(4\left(\frac{a}{b}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b + 48*b*c^2*x^2 - 48*b*c^4*x^4 + 16*b*c
^6*x^6 + 15*(a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[
(2*a)/b] - 12*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sin
h[(4*a)/b] + 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + 3*b*A
rcCosh[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] - 15*a*Cosh[
(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 15*b*ArcCosh[c*x]*Cosh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 12*a*Cosh[(4*a)/b]*SinhIntegral
[4*(a/b + ArcCosh[c*x])] + 12*b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(
a/b + ArcCosh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x]
)] - 3*b*ArcCosh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/
(16*b^2*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Maple [B] time = 0.28, size = 1176, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/64*(-c^2*x^2+1)^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7
+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)/(c*x+1)/(c*x-1
)/c/(a+b*arccosh(c*x))/b-3/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+c^2*x^2-1)*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(
c*x+1)/(c*x-1)/c/b^2-1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)*(3
2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*b*c^5+32*x^6*b*c^6-32*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*b*c^3-48*x^4*b*c^4+6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c+18*x^2*
b*c^2+6*arccosh(c*x)*exp(-6*a/b)*Ei(1,-6*arccosh(c*x)-6*a/b)*b+6*exp(-6*a/b
)*Ei(1,-6*arccosh(c*x)-6*a/b)*a-b)/c/b^2/(a+b*arccosh(c*x))+5/16/(c*x+1)^(1
/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)/c/(a+b*arccosh(c*x))/b-3/32*(-c^2*x^2+
1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)/(c*x+1)
/(c*x-1)/c/(a+b*arccosh(c*x))/b+3/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a
)/b)/(c*x+1)/(c*x-1)/c/b^2+15/64*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/(c*x-
1)/c/(a+b*arccosh(c*x))/b-15/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^
```

```
(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)
/(c*x+1)/(c*x-1)/c/b^2-15/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)
*(2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c+2*x^2*b*c^2+2*arccosh(c*x)*Ei(1,-2*ar
ccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a
-b)/c/b^2/(a+b*arccosh(c*x))+3/32/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^
(1/2)*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*b*c^3+8*x^4*b*c^4-4*(c*x-1)^(1/2)*
(c*x+1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*exp(-4*a/b)*Ei(1,-4*arccosh(
c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+b)/c/b^2/(a+b*arc
cosh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^6x^6-3\,c^4x^4+3\,c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^7x^7-3\,c^5x^5+3\,c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{}{abc^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x +
1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
 + integrate(((6*c^6*x^6 - 11*c^4*x^4 + 4*c^2*x^2 + 1)*(c*x + 1)^(3/2)*(c*x
 - 1) + 6*(2*c^7*x^7 - 5*c^5*x^5 + 4*c^3*x^3 - c*x)*(c*x + 1)*sqrt(c*x - 1)
 + (6*c^8*x^8 - 19*c^6*x^6 + 21*c^4*x^4 - 9*c^2*x^2 + 1)*sqrt(c*x + 1))*sqr
t(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^2*x^2 - 2*a*b*c^2*x^2
+ 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + a*b + (b^2*c^4*x^
4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*x^2 + 2*(b^2*c^3*x^3 - b^2*
c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 +
 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a)^2, x)
```

3.338 $\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=385

$$\frac{\sqrt{1-cx}\,\text{Unintegrable}\left(\frac{(c^2x^2-1)^2}{x^2(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{cx-1}}-\frac{25\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2\sqrt{cx-1}}+\frac{25\sqrt{1-cx}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b}{}\right.}{16b^2\sqrt{cx-1}}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*x*(a + b*ArcCosh[
c*x]))) - (25*Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])
/(8*b^2*Sqrt[-1 + c*x]) + (25*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcCosh[
c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*CoshInt
egral[(5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*Sqrt[-1 + c*x]) +
(25*Sqrt[1 - c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b^2*Sq
rt[-1 + c*x]) - (25*Sqrt[1 - c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcC
osh[c*x]))/b])/(16*b^2*Sqrt[-1 + c*x]) + (5*Sqrt[1 - c*x]*Cosh[(5*a)/b]*Sin
hIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b^2*Sqrt[-1 + c*x]) + (Sqrt[1 -
c*x]*Unintegrable[(-1 + c^2*x^2)^2/(x^2*(a + b*ArcCosh[c*x])), x])/(b*c*Sqr
t[-1 + c*x])
```

Rubi [A] time = 0.984997, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)^3*(1 + c*x)^(5/2)*Sqrt[1 - c^2*x^2])/(b*c*x*Sqrt[-1 + c*x]*(a +
b*ArcCosh[c*x])) - (25*Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*S
inh[a/b])/(8*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (25*Sqrt[1 - c^2*x^2]*Cosh
Integral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(16*b^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (5*Sqrt[1 - c^2*x^2]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]]*
Sinh[(5*a)/b])/(16*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (25*Sqrt[1 - c^2*x^2
]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b^2*Sqrt[-1 + c*x]*Sqrt[1
```

```
+ c*x]) - (25*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcC
osh[c*x]])/(16*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*Sqrt[1 - c^2*x^2]*Cos
h[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b^2*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)^2/(x^2*(a + b*
ArcCosh[c*x])), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\sqrt{1-c^2x^2}\int \frac{\left(-1+c^2x^2\right)^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(5c\sqrt{1-c^2x^2}\right)\int \frac{(-1}{a+b\,c}}{b\sqrt{-1+cx}\sqrt{1}}$$

$$=\frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(5\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh^5(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1}}{}$$

$$=\frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(5i\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\left(\frac{5i\sinh(x)}{8(a+bx)}-\frac{5i\sinh(3x)}{16(a+bx)}+\frac{i\sinh(5x)}{16(a+bx)}\right.\right.}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(5\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{16b\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(2}{}$$

$$=\frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\sqrt{1-c^2x^2}\int \frac{\left(-1+c^2x^2\right)^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(25\sqrt{1-c^2x^2}\cosh\left(\right.\right.}{}$$

$$=\frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{25\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{25\sqrt{1-c}}{}$$

Mathematica [A] time = 8.35788, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.553, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{2}}\left(-c^{2}x^{2}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^{6}x^{6}-3\,c^{4}x^{4}+3\,c^{2}x^{2}-1\right)(cx+1)\sqrt{cx-1}+\left(c^{7}x^{7}-3\,c^{5}x^{5}+3\,c^{3}x^{3}-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^{3}x^{3}+\sqrt{cx+1}\sqrt{cx-1}abc^{2}x^{2}-abcx+\left(b^{2}c^{3}x^{3}+\sqrt{cx+1}\sqrt{cx-1}b^{2}c^{2}x^{2}-b^{2}cx\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^3 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 - a*b*c*x + (b^2*c^3*x^3 + sqrt(c*
x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 - b^2*c*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*
x - 1))) + integrate(((5*c^7*x^7 - 8*c^5*x^5 + c^3*x^3 + 2*c*x)*(c*x + 1)^(
3/2)*(c*x - 1) + (10*c^8*x^8 - 23*c^6*x^6 + 15*c^4*x^4 - c^2*x^2 - 1)*(c*x
+ 1)*sqrt(c*x - 1) + 5*(c^9*x^9 - 3*c^7*x^7 + 3*c^5*x^5 - c^3*x^3)*sqrt(c*x
 + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^4 - 2*a*
b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*
x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^4 - 2*b^2*c^3*x^4 + b
^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^4 - 2\,c^2x^2 + 1\right)\sqrt{-c^2x^2 + 1}}{b^2x\,\text{arcosh}\left(cx\right)^2 + 2\,abx\,\text{arcosh}\left(cx\right) + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x*arccosh(c*x)^2
 + 2*a*b*x*arccosh(c*x) + a^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2 + 1\right)^{\frac{5}{2}}}{\left(b\,\text{arcosh}\left(cx\right) + a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)^2*x), x)
```

3.339 $\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=160

$$\frac{2\sqrt{1-cx}\text{Unintegrable}\left(\frac{\left(c^2x^2-1\right)^2}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{cx-1}}+\frac{4c\sqrt{1-cx}\text{Unintegrable}\left(\frac{\left(c^2x^2-1\right)^2}{x\left(a+b\cosh^{-1}(cx)\right)},x\right)}{b\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x\right.}{bcx^2\left(a+b\cosh^{-1}(cx\right.}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*x^2*(a + b*ArcCos
h[c*x]))) + (2*Sqrt[1 - c*x]*Unintegrable[(-1 + c^2*x^2)^2/(x^3*(a + b*ArcC
osh[c*x])), x])/(b*c*Sqrt[-1 + c*x]) + (4*c*Sqrt[1 - c*x]*Unintegrable[(-1
+ c^2*x^2)^2/(x*(a + b*ArcCosh[c*x])), x])/(b*Sqrt[-1 + c*x])
```

Rubi [A] time = 0.711772, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)^3*(1 + c*x)^(5/2)*Sqrt[1 - c^2*x^2])/(b*c*x^2*Sqrt[-1 + c*x]*(a
+ b*ArcCosh[c*x])) + (2*Sqrt[1 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)^2/(x^3*
(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*c*Sqrt[1
 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)^2/(x*(a + b*ArcCosh[c*x])), x])/(b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx^2\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{\left(-1+c^2x^2\right)^2}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(4c\sqrt{1-c^2x^2}\right)\ldots}{b\sqrt{-1+\ldots}}$$

Mathematica [A] time = 16.5788, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.619, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\operatorname{arccosh}(cx)\right)^2}\left(-c^2x^2+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^6x^6-3\,c^4x^4+3\,c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^7x^7-3\,c^5x^5+3\,c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^4+\sqrt{cx+1}\sqrt{cx-1}abc^2x^3-abcx^2+\left(b^2c^3x^4+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x^3-b^2cx^2\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)} + \int \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^4 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 - a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(
c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 - b^2*c*x^2)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))) + integrate(((4*c^7*x^7 - 5*c^5*x^5 - 2*c^3*x^3 + 3*c*x)*(c*x
+ 1)^(3/2)*(c*x - 1) + 2*(4*c^8*x^8 - 8*c^6*x^6 + 3*c^4*x^4 + 2*c^2*x^2 - 1
)*(c*x + 1)*sqrt(c*x - 1) + (4*c^9*x^9 - 11*c^7*x^7 + 9*c^5*x^5 - c^3*x^3 -
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1)*a*b*
c^3*x^5 - 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*sqrt(c*
x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^5 - 2*b
^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x + 1)*sqrt(c
*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^4 - 2\,c^2x^2 + 1\right)\sqrt{-c^2x^2 + 1}}{b^2x^2\,\text{arcosh}\,(cx)^2 + 2\,abx^2\,\text{arcosh}\,(cx) + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^2*arccosh(c*x)
^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)^2*x^2), x)
```

3.340 $\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.529828, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(x^3*(a + b*ArcCosh[c*x])^2), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 20.4402, size = 0, normalized size = 0.

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.685, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(a + b\operatorname{arccosh}\left(cx\right)\right)^{2}} \left(-c^{2}x^{2}+1\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^{6}x^{6}-3\,c^{4}x^{4}+3\,c^{2}x^{2}-1\right)(cx+1)\sqrt{cx-1}+\left(c^{7}x^{7}-3\,c^{5}x^{5}+3\,c^{3}x^{3}-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^{3}x^{5}+\sqrt{cx+1}\sqrt{cx-1}abc^{2}x^{4}-abcx^{3}+\left(b^{2}c^{3}x^{5}+\sqrt{cx+1}\sqrt{cx-1}b^{2}c^{2}x^{4}-b^{2}cx^{3}\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int -\frac{}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^5 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^4 - a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(
c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^4 - b^2*c*x^3)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))) + integrate(((3*c^7*x^7 - 2*c^5*x^5 - 5*c^3*x^3 + 4*c*x)*(c*x
+ 1)^(3/2)*(c*x - 1) + 3*(2*c^8*x^8 - 3*c^6*x^6 - c^4*x^4 + 3*c^2*x^2 - 1)*
(c*x + 1)*sqrt(c*x - 1) + (3*c^9*x^9 - 7*c^7*x^7 + 3*c^5*x^5 + 3*c^3*x^3 -
2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^8 + (c*x + 1)*(c*x - 1)*a*b
*c^3*x^6 - 2*a*b*c^3*x^6 + a*b*c*x^4 + 2*(a*b*c^4*x^7 - a*b*c^2*x^5)*sqrt(c
*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^8 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^6 - 2*
b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 - b^2*c^2*x^5)*sqrt(c*x + 1)*sqrt(
```

```
c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{b^2x^3\,\mathrm{arcosh}\left(cx\right)^2+2\,abx^3\,\mathrm{arcosh}\left(cx\right)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^3*arccosh(c*x)
^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**3/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)^2*x^3), x)
```

3.341 $\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.525628, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(x^4*(a +
b*ArcCosh[c*x])^2), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 158.216, size = 0, normalized size = 0.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.878, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b\mathrm{arccosh}\,(cx))^2} \left(-c^2x^2+1\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^6x^6-3\,c^4x^4+3\,c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^7x^7-3\,c^5x^5+3\,c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^6+\sqrt{cx+1}\sqrt{cx-1}abc^2x^5-abcx^4+\left(b^2c^3x^6+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x^5-b^2cx^4\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^6 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^5 - a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(
c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^5 - b^2*c*x^4)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))) + integrate(((2*c^7*x^7 + c^5*x^5 - 8*c^3*x^3 + 5*c*x)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(2*c^8*x^8 - c^6*x^6 - 6*c^4*x^4 + 7*c^2*x^2 - 2)*(c
*x + 1)*sqrt(c*x - 1) + (2*c^9*x^9 - 3*c^7*x^7 - 3*c^5*x^5 + 7*c^3*x^3 - 3*
c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^9 + (c*x + 1)*(c*x - 1)*a*b*c
^3*x^7 - 2*a*b*c^3*x^7 + a*b*c*x^5 + 2*(a*b*c^4*x^8 - a*b*c^2*x^6)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^9 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^7 - 2*b^
2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 - b^2*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*
```

```
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^4 - 2\,c^2x^2 + 1\right)\sqrt{-c^2x^2 + 1}}{b^2x^4\,\text{arcosh}\,(cx)^2 + 2\,abx^4\,\text{arcosh}\,(cx) + a^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^4*arccosh(c*x)
^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**4/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2 + 1\right)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx) + a)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)^2*x^4), x)
```

3.342 $\int \frac{x^5}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=337

$$-\frac{5\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^6\sqrt{1-cx}}-\frac{15\sqrt{cx-1}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^6\sqrt{1-cx}}-\frac{5\sqrt{cx-1}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^6\sqrt{1-cx}}$$

```
[Out] -((x^5*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (5*Sqrt[
-1 + c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(8*b^2*c^6*Sqrt[1
 - c*x]) - (15*Sqrt[-1 + c*x]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh
[(3*a)/b])/(16*b^2*c^6*Sqrt[1 - c*x]) - (5*Sqrt[-1 + c*x]*CoshIntegral[(5*(
a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*c^6*Sqrt[1 - c*x]) + (5*Sqrt
[-1 + c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b^2*c^6*Sqrt[
1 - c*x]) + (15*Sqrt[-1 + c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh
[c*x]))/b])/(16*b^2*c^6*Sqrt[1 - c*x]) + (5*Sqrt[-1 + c*x]*Cosh[(5*a)/b]*Si
nhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^6*Sqrt[1 - c*x])
```

Rubi [A] time = 0.860358, antiderivative size = 424, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5798, 5775, 5670, 5448, 3303, 3298, 3301}

$$-\frac{5\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b^2c^6\sqrt{1-c^2x^2}}-\frac{15\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16b^2c^6\sqrt{1-c^2x^2}}-\frac{5\sqrt{cx-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x]))) - (5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[a/b + ArcCosh[c*x]]*
Sinh[a/b])/(8*b^2*c^6*Sqrt[1 - c^2*x^2]) - (15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(16*b^2*c^6*Sqrt[1 -
 c^2*x^2]) - (5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[(5*a)/b + 5*ArcCo
sh[c*x]]*Sinh[(5*a)/b])/(16*b^2*c^6*Sqrt[1 - c^2*x^2]) + (5*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b^2*c^6*Sqrt[1
 - c^2*x^2]) + (15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[(3*a)/b]*SinhIntegral[
(3*a)/b + 3*ArcCosh[c*x]])/(16*b^2*c^6*Sqrt[1 - c^2*x^2]) + (5*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*
```

```
b^2*c^6*Sqrt[1 - c^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^5}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^5\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^4}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^5\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{a+bx}$$

$$bc^6\sqrt{1-c^2x^2}$$

$$= -\frac{x^5\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{\sinh(x)}{8(a+bx)} + \frac{3\,\text{s}}{16}$$

$$bc^6\sqrt{1-c^2}$$

$$= -\frac{x^5\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x\right.$$

$$16bc^6\sqrt{1-c^2x^2}$$

$$= -\frac{x^5\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\text{sinh}}{a}$$

$$8bc^6\sqrt{1-c^2x^2}$$

$$= -\frac{x^5\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{5\sqrt{-1+cx}\sqrt{1+cx}\,\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sin}{8b^2c^6\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.66871, size = 190, normalized size = 0.56

$$\sqrt{1-c^2x^2}\left(\frac{16bc^5x^5}{a+b\cosh^{-1}(cx)} + 5\left(2\sinh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + 3\sinh\left(\frac{3a}{b}\right)\textbf{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + \sinh\left(\frac{5a}{b}\right)\textbf{Chi}\left(\right.\right.\right.$$

$$16b$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*((16*b*c^5*x^5)/(a + b*ArcCosh[c*x]) + 5*(2*CoshIntegral
[a/b + ArcCosh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sin
h[(3*a)/b] + CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 2*Cosh[a/
b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcCosh[c*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])])))/(16
*b^2*c^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.39, size = 1046, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/32*(-c^2*x^2+1)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^
6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+13*c^2*x^2-1)/(c^2*x^2-1)/c^6/b/(a+b*arccosh(c*x))-5/32*((c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,5*arccosh(c*x)+
5*a/b)*exp(-(b*arccosh(c*x)-5*a)/b)/c^6/(c^2*x^2-1)/b^2+1/32*(-c^2*x^2+1)^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^6*(16*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^4*b*c^4+16*x^5*b*c^5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2-20*x^
3*b*c^3+5*arccosh(c*x)*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*b+5*exp(-5*a
/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*a+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+5*x*b*c)/b^
2/(a+b*arccosh(c*x))-5/32*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c^2*x^2
-1)/c^6/b/(a+b*arccosh(c*x))-15/32*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2
-1)*(-c^2*x^2+1)^(1/2)*Ei(1,3*arccosh(c*x)+3*a/b)*exp(-(b*arccosh(c*x)-3*a)
/b)/c^6/(c^2*x^2-1)/b^2+5/16*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/(c^2*x^2-1)/c^6*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^
(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+x*b*c)/b^2/(a+b*a
rccosh(c*x))+5/32*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1
)/c^6*(4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(c*x)*e
xp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)
-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)^(1/2)*b-3*x*b*c)/b^2/(a+b*arccosh(c*x))-5/1
6*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c^2*x^2-
1)/c^6/b/(a+b*arccosh(c*x))-5/16*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1
)*(-c^2*x^2+1)^(1/2)*Ei(1,arccosh(c*x)+a/b)*exp(-(b*arccosh(c*x)-a)/b)/c^6/
(c^2*x^2-1)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^8 - cx^6 + \left(c^2x^7 - x^5\right)\sqrt{cx+1}\sqrt{cx-1}}{\left(\left(cx+1\right)\sqrt{cx-1}b^2c^2x + \left(b^2c^3x^2 - b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right) + \left(\left(cx+1\right)\sqrt{cx-1}abc^2x + \left(abc^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^8 - c*x^6 + (c^2*x^7 - x^5)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((5
*c^5*x^9 - 11*c^3*x^7 + 6*c*x^5 + (5*c^3*x^7 - 4*c*x^5)*(c*x + 1)*(c*x - 1)
 + 5*(2*c^4*x^8 - 3*c^2*x^6 + x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)
^(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c
*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x +
1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*
c^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^
4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}x^5}{a^2c^2x^2 + \left(b^2c^2x^2 - b^2\right)\operatorname{arcosh}\left(cx\right)^2 - a^2 + 2\left(abc^2x^2 - ab\right)\operatorname{arcosh}\left(cx\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

3.343 $\int \frac{x^4}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=236

$$-\frac{\sqrt{cx-1}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^5\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2b^2c^5\sqrt{1-cx}}+\frac{\sqrt{cx-1}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^5\sqrt{1-cx}}$$

```
[Out] -((x^4*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (Sqrt[-1
 + c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/b])/(b^2*c^5*Sq
rt[1 - c*x]) - (Sqrt[-1 + c*x]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b]*Sin
h[(4*a)/b])/(2*b^2*c^5*Sqrt[1 - c*x]) + (Sqrt[-1 + c*x]*Cosh[(2*a)/b]*SinhI
ntegral[(2*(a + b*ArcCosh[c*x]))/b])/(b^2*c^5*Sqrt[1 - c*x]) + (Sqrt[-1 + c
*x]*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(2*b^2*c^5*Sqrt
[1 - c*x])
```

Rubi [A] time = 0.786537, antiderivative size = 301, normalized size of antiderivative = 1.28, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5798, 5775, 5670, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^5\sqrt{1-c^2x^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{2b^2c^5\sqrt{1-c^2x^2}}+\frac{\sqrt{cx-1}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x]))) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*
x]]*Sinh[(2*a)/b])/(b^2*c^5*Sqrt[1 - c^2*x^2]) - (Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]]*Sinh[(4*a)/b])/(2*b^2*c^5*Sqrt[1
 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[(2*a)/b]*SinhIntegral[(2*
a)/b + 2*ArcCosh[c*x]])/(b^2*c^5*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(2*b^2*c^5*S
qrt[1 - c^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^4}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x}{a+bx}\right.}{bc^5\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{\sinh(2x)}{4(a+bx)} + \frac{\text{si}}{8}\right.\right.}{bc^5\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, c\right.}{2bc^5\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh}{\quad}\right.}{bc^5\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\text{si}}{b^2c^5\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.485152, size = 149, normalized size = 0.63

$$\frac{\sqrt{1-c^2x^2}\left(\frac{2bc^4x^4}{a+b\cosh^{-1}(cx)} + 2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + \sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) - 2\cosh\left(\frac{2a}{b}\right)\text{Sh}\right.}{2b^2c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*((2*b*c^4*x^4)/(a + b*ArcCosh[c*x]) + 2*CoshIntegral[2*(
a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] + CoshIntegral[4*(a/b + ArcCosh[c*x])]*S
```

```
inh[(4*a)/b] - 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - Cosh[
(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(2*b^2*c^5*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Maple [B] time = 0.372, size = 758, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/16*(-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+
8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c*x)/(c^2*x^2-1)/c^5/(a+b*arccosh(c*x))/b-1/4*((c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,4*arccosh(c*x)+4*a/b)*exp(-(b*ar
ccosh(c*x)-4*a)/b)/c^5/(c^2*x^2-1)/b^2+1/16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-c
^2*x^2+1)^(1/2)/(c^2*x^2-1)/c^5*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*b*c^3+8*
x^4*b*c^4-4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*ex
p(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-
4*a/b)*a+b)/b^2/(a+b*arccosh(c*x))+3/8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-c^2*x^
2+1)^(1/2)/(c^2*x^2-1)/c^5/(a+b*arccosh(c*x))/b-1/4*(-c^2*x^2+1)^(1/2)*(-2*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2
*c*x)/(c^2*x^2-1)/c^5/(a+b*arccosh(c*x))/b-1/2*((c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,2*arccosh(c*x)+2*a/b)*exp(-(b*arcco
sh(c*x)-2*a)/b)/c^5/(c^2*x^2-1)/b^2+1/4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-c^2*x
^2+1)^(1/2)/(c^2*x^2-1)/c^5*(2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*b*c+2*x^2*b*c^
2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccos
h(c*x)-2*a/b)*exp(-2*a/b)*a-b)/b^2/(a+b*arccosh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^7 - cx^5 + \left(c^2x^6 - x^4\right)\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}b^2c^2x + \left(b^2c^3x^2 - b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \left((cx+1)\sqrt{cx-1}abc^2x + \left(abc^3x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^7 - c*x^5 + (c^2*x^6 - x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((4
*c^5*x^8 - 9*c^3*x^6 + 5*c*x^4 + (4*c^3*x^6 - 3*c*x^4)*(c*x + 1)*(c*x - 1)
+ 4*(2*c^4*x^7 - 3*c^2*x^5 + x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^
(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*
x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c
^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4
 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}x^4}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\operatorname{arcosh}\left(cx\right)^2-a^2+2\left(abc^2x^2-ab\right)\operatorname{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

3.344 $\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=237

$$-\frac{3\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^4\sqrt{1-cx}}-\frac{3\sqrt{cx-1}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^4\sqrt{1-cx}}+\frac{3\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\ldots}{b}\right)}{4b^2c^4\sqrt{1-cx}}$$

```
[Out] -((x^3*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (3*Sqrt[
-1 + c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(4*b^2*c^4*Sqrt[1
 - c*x]) - (3*Sqrt[-1 + c*x]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[
(3*a)/b])/(4*b^2*c^4*Sqrt[1 - c*x]) + (3*Sqrt[-1 + c*x]*Cosh[a/b]*SinhInteg
ral[(a + b*ArcCosh[c*x])/b])/(4*b^2*c^4*Sqrt[1 - c*x]) + (3*Sqrt[-1 + c*x]*
Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b^2*c^4*Sqrt[1 -
 c*x])
```

Rubi [A] time = 0.767886, antiderivative size = 298, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5798, 5775, 5670, 5448, 3303, 3298, 3301}

$$-\frac{3\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b^2c^4\sqrt{1-c^2x^2}}-\frac{3\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4b^2c^4\sqrt{1-c^2x^2}}+\frac{3\sqrt{cx-1}\sqrt{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x]))) - (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[a/b + ArcCosh[c*x]]*
Sinh[a/b])/(4*b^2*c^4*Sqrt[1 - c^2*x^2]) - (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(4*b^2*c^4*Sqrt[1 - c
^2*x^2]) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[a/b]*SinhIntegral[a/b + Arc
Cosh[c*x]])/(4*b^2*c^4*Sqrt[1 - c^2*x^2]) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b^2*c^4*Sqrt[1 -
c^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\right.}{bc^4\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sin}{4(}\right.\right.}{bc^4\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x,\right.}{4bc^4\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh}{a}\right.}{4bc^4\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{3\sqrt{-1+cx}\sqrt{1+cx}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sin}{4b^2c^4\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.47508, size = 144, normalized size = 0.61

$$\frac{\sqrt{1-c^2x^2}\left(\frac{4bc^3x^3}{a+b\cosh^{-1}(cx)}+3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}\right.\right.}{4b^2c^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*((4*b*c^3*x^3)/(a + b*ArcCosh[c*x]) + 3*CoshIntegral[a/b
 + ArcCosh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3
```

```
*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Cosh[(3*a)/b]*Sin
hIntegral[3*(a/b + ArcCosh[c*x])]))/(4*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)
```

Maple [B] time = 0.303, size = 634, normalized size = 2.7

$$-\frac{1}{\left(8\,c^{2}x^{2}-8\right)c^{4}b\left(a+b\operatorname{arccosh}\left(cx\right)\right)}\sqrt{-c^{2}x^{2}+1}\left(-4\,\sqrt{cx+1}\sqrt{cx-1}x^{3}c^{3}+4\,c^{4}x^{4}+3\,\sqrt{cx+1}\sqrt{cx-1}xc-5\,c^{2}x^{2}+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/8*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c^2*x^2-1)/c^4/b/(a+b*arccos
h(c*x))-3/8*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*
Ei(1,3*arccosh(c*x)+3*a/b)*exp(-(b*arccosh(c*x)-3*a)/b)/c^4/(c^2*x^2-1)/b^2
+1/8*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^4*(4*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(c*x)*exp(-3*a/b)*Ei
(1,-3*arccosh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)*a-(c*
x+1)^(1/2)*(c*x-1)^(1/2)*b-3*x*b*c)/b^2/(a+b*arccosh(c*x))+3/8*(-c^2*x^2+1)
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^4*(arccosh(c*x)*exp(-a/b)*
Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arc
cosh(c*x)-a/b)*a+x*b*c)/b^2/(a+b*arccosh(c*x))-3/8*(-c^2*x^2+1)^(1/2)*(-(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c^2*x^2-1)/c^4/b/(a+b*arccosh(c*x)
)-3/8*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,a
rccosh(c*x)+a/b)*exp(-(b*arccosh(c*x)-a)/b)/c^4/(c^2*x^2-1)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^{3}x^{6}-cx^{4}+\left(c^{2}x^{5}-x^{3}\right)\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}b^{2}c^{2}x+\left(b^{2}c^{3}x^{2}-b^{2}c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}abc^{2}x+\left(abc^{3}x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^6 - c*x^4 + (c^2*x^5 - x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3
*c^5*x^7 - 7*c^3*x^5 + 4*c*x^3 + (3*c^3*x^5 - 2*c*x^3)*(c*x + 1)*(c*x - 1)
+ 3*(2*c^4*x^6 - 3*c^2*x^4 + x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^
(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*
x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c
^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4
 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}x^3}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\operatorname{arcosh}\left(cx\right)^2-a^2+2\left(abc^2x^2-ab\right)\operatorname{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-c^2x^2+1}(b\,\mathrm{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

3.345 $\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{cx-1}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^3\sqrt{1-cx}}+\frac{\sqrt{cx-1}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^3\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((x^2*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (Sqrt[-1
 + c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/b])/(b^2*c^3*Sq
rt[1 - c*x]) + (Sqrt[-1 + c*x]*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh
[c*x]))/b])/(b^2*c^3*Sqrt[1 - c*x])
```

Rubi [A] time = 0.62644, antiderivative size = 175, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5798, 5775, 5670, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^3\sqrt{1-c^2x^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^3\sqrt{1-c^2x^2}}-\frac{x^2\sqrt{}}{bc\sqrt{1-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x]))) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*
x]]*Sinh[(2*a)/b])/(b^2*c^3*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b^2*c^3*Sqrt[1 -
 c^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\right.}{bc^3\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x,\right.}{bc^3\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, c\right.}{bc^3\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh}{}\right.}{bc^3\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\text{si}}{b^2c^3\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.274179, size = 117, normalized size = 0.86

$$\frac{\sqrt{1-c^2x^2}\left(\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\cosh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx\right.\right.\right.}{b^2c^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(b*c^2*x^2 + (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b +
ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*ArcCosh[c*x])*Cosh[(2*a)/b]*SinhInteg
```

```
ral[2*(a/b + ArcCosh[c*x])]))/(b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*
ArcCosh[c*x]))
```

Maple [B] time = 0.218, size = 377, normalized size = 2.8

$$-\frac{1}{\left(4c^2x^2-4\right)c^3\left(a+b\operatorname{arccosh}(cx)\right)b}\sqrt{-c^2x^2+1}\left(-2\sqrt{cx+1}\sqrt{cx-1}x^2c^2+2c^3x^3+\sqrt{cx-1}\sqrt{cx+1}-2cx\right)-\frac{1}{2c^3\left(c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(
c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c^2*x^2-1)/c^3/(a+b*arccosh(c*x))/b-1/2*
((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,2*arcco
sh(c*x)+2*a/b)*exp(-(b*arccosh(c*x)-2*a)/b)/c^3/(c^2*x^2-1)/b^2+1/4*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c^3*(2*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*x*b*c+2*x^2*b*c^2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*ex
p(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a-b)/b^2/(a+b*arccosh
(c*x))+1/2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c^3/(
a+b*arccosh(c*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^5-cx^3+\left(c^2x^4-x^2\right)\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}b^2c^2x+\left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}abc^2x+\left(abc^3x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^5 - c*x^3 + (c^2*x^4 - x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((2
*c^5*x^6 - 5*c^3*x^4 + (2*c^3*x^4 - c*x^2)*(c*x + 1)*(c*x - 1) + 3*c*x^2 +
2*(2*c^4*x^5 - 3*c^2*x^3 + x)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^(3/2
```

```
)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x -
1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x
^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2
*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}x^2}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\text{arcosh}\left(cx\right)^2-a^2+2\left(abc^2x^2-ab\right)\text{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\left(a+b\,\text{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{-c^2x^2+1}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

3.346 $\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-cx}}+\frac{\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-cx}}-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((x*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (Sqrt[-1 +
 c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b^2*c^2*Sqrt[1 - c*x
]) + (Sqrt[-1 + c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(b^2*c
^2*Sqrt[1 - c*x])
```

Rubi [A] time = 0.43128, antiderivative size = 169, normalized size of antiderivative = 1.3, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5798, 5775, 5658, 3303, 3298, 3301}

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-c^2x^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-c^2x^2}}-\frac{x\sqrt{cx-1}\sqrt{cx+\cdots}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*
x]))) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*
Sinh[a/b])/(b^2*c^2*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh
[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(b^2*c^2*Sqrt[1 - c^2*x^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x$$

$$b^2c^2\sqrt{1-c^2x^2}$$

$$= -\frac{x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh(}{x}$$

$$b^2c^2\sqrt{1-c^2x^2}$$

$$= -\frac{x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}$$

$$b^2c^2\sqrt{1-c^2x^2}$$

Mathematica [A] time = 0.217522, size = 107, normalized size = 0.82

$$\frac{\sqrt{1-c^2x^2}\left(\sinh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\cosh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+bcx}{b^2c^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(b*c*x + (a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh
[c*x]]*Sinh[a/b] - (a + b*ArcCosh[c*x])*Cosh[a/b]*SinhIntegral[a/b + ArcCos
h[c*x]]))/(b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

Maple [B] time = 0.168, size = 283, normalized size = 2.2

$$\frac{1}{2c^2\left(c^2x^2-1\right)b^2\left(a+b\text{arccosh}\,(cx)\right)}\sqrt{-c^2x^2+1}\sqrt{cx-1}\sqrt{cx+1}\left(\text{arccosh}\,(cx)\,e^{-\frac{a}{b}}\text{Ei}\left(1,-\text{arccosh}\,(cx)-\frac{a}{b}\right)b+\sqrt{cx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/(c^2*x^2-1)*(arccosh
(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp
(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+x*b*c)/b^2/(a+b*arccosh(c*x))-1/2*(-c^2*x^
2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/c^2/(c^2*x^2-1)/b/(
a+b*arccosh(c*x))-1/2*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2
+1)^(1/2)*Ei(1,arccosh(c*x)+a/b)*exp(-(b*arccosh(c*x)-a)/b)/b^2/c^2/(c^2*x^
2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^4 - cx^2 + \left(c^2x^3 - x\right)\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}b^2c^2x + \left(b^2c^3x^2 - b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \left((cx+1)\sqrt{cx-1}abc^2x + \left(abc^3x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^4 - c*x^2 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)*
sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x +
1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^
2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((c^5
*x^5 + (c*x + 1)*(c*x - 1)*c^3*x^3 - 3*c^3*x^3 + (2*c^4*x^4 - 3*c^2*x^2 + 1
)*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*c*x)/(((c*x + 1)^(3/2)*(c*x - 1)*b^2*c^3*
x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 -
2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3
- a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c
)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}x}{a^2c^2x^2 + \left(b^2c^2x^2 - b^2\right)\text{arcosh}\left(cx\right)^2 - a^2 + 2\left(abc^2x^2 - ab\right)\text{arcosh}\left(cx\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh(c
*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

3.347 $\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{cx-1}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(Sqrt[-1 + c*x]/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x])))
```

Rubi [A] time = 0.215333, antiderivative size = 50, normalized size of antiderivative = 1.35, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {5713, 5676}

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]
)))
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$
$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}$$

Mathematica [A] time = 0.0313497, size = 50, normalized size = 1.35

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]
)))
```

Maple [A] time = 0.04, size = 57, normalized size = 1.5

$$\frac{1}{c\left(c^2x^2-1\right)(a+b\mathrm{arccosh}\,(cx))\,b}\sqrt{-(cx-1)(cx+1)}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] (-(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(c^2*x^2-1)/(a+b*arc
cosh(c*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^3+\left(c^2x^2-1\right)\sqrt{cx+1}\sqrt{cx-1}-cx}{\left(\left((cx+1)\sqrt{cx-1}b^2c^2x+\left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}abc^2x+\left(abc^3\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(((c*x + 1)*sq
rt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*
x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate(-(c^2*
x^2 - (c*x + 1)*(c*x - 1) - 1)/(((c*x + 1)^(3/2)*(c*x - 1)*b^2*c^2*x^2 + 2*
(b^2*c^3*x^3 - b^2*c*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^4*x^4 - 2*b^2*c^2*
x^2 + b^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*(
c*x + 1)*sqrt(c*x - 1) + (a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*sqrt(c*x + 1))
*sqrt(-c*x + 1)), x)
```

Fricas [B] time = 2.04339, size = 153, normalized size = 4.14

$$\frac{\sqrt{c^2x^2-1}\sqrt{-c^2x^2+1}}{abc^3x^2-abc+\left(b^2c^3x^2-b^2c\right)\log\left(cx+\sqrt{c^2x^2-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 -
b^2*c)*log(c*x + sqrt(c^2*x^2 - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

3.348 $\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{cx-1}\,\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}}{bcx\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(Sqrt[-1 + c*x]/(b*c*x*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (Sqrt[-1 + c
*x]*Unintegrable[1/(x^2*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[1 - c*x])
```

Rubi [A] time = 0.527555, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*
x]))) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^2*(a + b*ArcCosh[c*x]
)), x])/(b*c*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}\\ &= -\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bcx\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{1-c^2x^2}}\end{aligned}$$

Mathematica [A] time = 4.60314, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.231, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\mathrm{arccosh}\left(cx\right)\right)^2}\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^3+\left(c^2x^2-1\right)\sqrt{cx+1}\sqrt{cx-1}-cx}{\left((cx+1)\sqrt{cx-1}b^2c^2x^2+\left(b^2c^3x^3-b^2cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}abc^2x^2+\left(a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((((c*x + 1)*sq
rt(c*x - 1)*b^2*c^2*x^2 + (b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x^2 + (a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrat
e((c^5*x^5 - c^3*x^3 + (c^3*x^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (2*c^4*x^4 -
 3*c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^(3/2)*(c*x - 1)*b^
```

```
2*c^3*x^4 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^
5*x^6 - 2*b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x^4 + 2*(
a*b*c^4*x^5 - a*b*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^6 - 2*a*b*c
^3*x^4 + a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{a^2c^2x^3-a^2x+\left(b^2c^2x^3-b^2x\right)\text{arcosh}\left(cx\right)^2+2\left(abc^2x^3-abx\right)\text{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^3 - a^2*x + (b^2*c^2*x^3 - b^2*x)*a
rccosh(c*x)^2 + 2*(a*b*c^2*x^3 - a*b*x)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{-\left(cx-1\right)\left(cx+1\right)}\left(a+b\,\text{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-c^2x^2+1}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2*x), x)
```

3.349 $\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{2\sqrt{cx-1}\,\text{Unintegrable}\left(\frac{1}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}}{bcx^2\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(Sqrt[-1 + c*x]/(b*c*x^2*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (2*Sqrt[-1
 + c*x]*Unintegrable[1/(x^3*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[1 - c*x])
```

Rubi [A] time = 0.542475, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x]))) - (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^3*(a + b*ArcCosh[
c*x])), x])/(b*c*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}\\ &= -\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bcx^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{1-c^2x^2}}\end{aligned}$$

Mathematica [A] time = 1.50501, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\mathrm{arccosh}\,(cx)\right)^2}\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^3+\left(c^2x^2-1\right)\sqrt{cx+1}\sqrt{cx-1}-cx}{\left((cx+1)\sqrt{cx-1}b^2c^2x^3+\left(b^2c^3x^4-b^2cx^2\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}abc^2x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(((c*x + 1)*sq
rt(c*x - 1)*b^2*c^2*x^3 + (b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*
x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*
b*c^2*x^3 + (a*b*c^3*x^4 - a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - inte
grate((2*c^5*x^5 - 3*c^3*x^3 + (2*c^3*x^3 - 3*c*x)*(c*x + 1)*(c*x - 1) + 2*
(2*c^4*x^4 - 3*c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*x)/(((c*x + 1)^
```

```
(3/2)*(c*x - 1)*b^2*c^3*x^5 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*(c*x + 1)*sqrt(
c*x - 1) + (b^2*c^5*x^7 - 2*b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c*x + 1))*sqrt(-c
*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)
*a*b*c^3*x^5 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (a*b
*c^5*x^7 - 2*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{a^2c^2x^4-a^2x^2+\left(b^2c^2x^4-b^2x^2\right)\operatorname{arcosh}(cx)^2+2\left(abc^2x^4-abx^2\right)\operatorname{arcosh}(cx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^4 - a^2*x^2 + (b^2*c^2*x^4 - b^2*x^
2)*arccosh(c*x)^2 + 2*(a*b*c^2*x^4 - a*b*x^2)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac"
)

[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2*x^2), x)
```

3.350 $\int \frac{x^3}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{x^3}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.568086, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^3/((-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x^3}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 28.8109, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.619, size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+b\operatorname{arccosh}(cx))^2}\left(-c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{cx^4+\sqrt{cx+1}\sqrt{cx-1}x^3}{\left((cx+1)\sqrt{cx-1}b^2c^2x+\left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}abc^2x+\left(abc^3x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] (c*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^3)/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2
*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 -
a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrate((c^5*x^7 - 5*c^3*x^5 + 4*
c*x^3 + (c^3*x^5 - 2*c*x^3)*(c*x + 1)*(c*x - 1) + (2*c^4*x^6 - 7*c^2*x^4 +
3*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/((((b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)
^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^5 - 2*b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sq
rt(c*x - 1) + (b^2*c^7*x^6 - 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*
x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x
^4 - a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^
3 + a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a
*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}x^3}{a^2c^4x^4-2\,a^2c^2x^2+\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\text{arcosh}\,(cx)^2+a^2+2\left(abc^4x^4-2\,abc^2x^2+ab\right)\text{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4
 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x
^2 + a*b)*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\,(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

3.351 $\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{cx-1}\,\text{Unintegrable}\left(\frac{x}{\left(c^2x^2-1\right)^2\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos
h[c*x]))) + (2*Sqrt[-1 + c*x]*Unintegrable[x/((-1 + c^2*x^2)^2*(a + b*ArcCo
sh[c*x])), x])/(b*c*Sqrt[1 - c*x])
```

Rubi [A] time = 0.64553, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^2*Sqrt[-1 + c*x])/(b*c*(1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(a +
b*ArcCosh[c*x]))) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c^2
*x^2)^2*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}\\ &= -\frac{x^2\sqrt{-1+cx}}{bc(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\ }{(-}}{bc\sqrt{1-}}\end{aligned}$$

Mathematica [A] time = 5.78185, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.152, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+b\operatorname{arccosh}(cx)\right)^2}\left(-c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{cx^3+\sqrt{cx+1}\sqrt{cx-1}x^2}{\left(\left((cx+1)\sqrt{cx-1}b^2c^2x+\left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}abc^2x+\left(abc^3x^2\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] (c*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^2)/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2
*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 -
a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3*c^3*x^4 + (c*x + 1)*(c
*x - 1)*c*x^2 - 3*c*x^2 + 2*(2*c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(c*x - 1))/((
```

```
(b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^5 - 2*
b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x^6 - 3*b^2*c^5
*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(
c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^4 - a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x
 - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^3 + a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1)
 + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqr
t(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}x^2}{a^2c^4x^4-2\,a^2c^2x^2+\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\text{arcosh}\,(cx)^2+a^2+2\left(abc^4x^4-2\,abc^2x^2+ab\right)\text{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4
 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x
^2 + a*b)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\,\text{acosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

3.352 $\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.40282, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c*x)^(3/2)*(1 + c*x)^(3
/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 21.5099, size = 0, normalized size = 0.

$$\int \frac{x}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.215, size = 0, normalized size = 0.

$$\int \frac{x}{(a+b\,\mathrm{arccosh}\,(cx))^2}\left(-c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{cx^2+\sqrt{cx+1}\sqrt{cx-1}x}{\big((cx+1)\sqrt{cx-1}b^2c^2x+\big(b^2c^3x^2-b^2c\big)\sqrt{cx+1}\big)\sqrt{-cx+1}\log\big(cx+\sqrt{cx+1}\sqrt{cx-1}\big)+\big((cx+1)\sqrt{cx-1}abc^2x+\big(abc^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] (c*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*x)/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x
 + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*
b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((c^5*x^5 + (c*x + 1)*(c*x -
 1)*c^3*x^3 + c^3*x^3 + (2*c^4*x^4 + c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x -
1) - 2*c*x)/(((b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^
2*c^6*x^5 - 2*b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x
^6 - 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^4 - a*b*c^3*x^2)*(c*x +
 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^3 + a*b*c^2*x)*(c*x + 1)
*sqrt(c*x - 1) + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 - a*b*c)*sqrt
(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}x}{a^2c^4x^4-2\,a^2c^2x^2+\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\text{arcosh}\left(cx\right)^2+a^2+2\left(abc^4x^4-2\,abc^2x^2+ab\right)\text{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 -
 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2
 + a*b)*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

3.353 $\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{2c\sqrt{cx-1}\text{Unintegrable}\left(\frac{x}{\left(c^2x^2-1\right)^2\left(a+b\cosh^{-1}(cx)\right)},x\right)}{b\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*
x]))) + (2*c*Sqrt[-1 + c*x]*Unintegrable[x/((-1 + c^2*x^2)^2*(a + b*ArcCosh
[c*x])), x])/(b*Sqrt[1 - c*x])
```

Rubi [A] time = 0.319595, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -(Sqrt[-1 + c*x]/(b*c*(1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(a + b*ArcC
osh[c*x]))) + (2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c^2*x^2
)^2*(a + b*ArcCosh[c*x])), x])/(b*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$=-\frac{\sqrt{-1+cx}}{bc(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(2c\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{}{(-1}}{b\sqrt{1-c}}$$

Mathematica [A] time = 2.21714, size = 0, normalized size = 0.

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.191, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\operatorname{arccosh}(cx)\right)^2}\left(-c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{cx+\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}b^2c^2x+\left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}abc^2x+\left(abc^3x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] (c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x + (
b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*b*c)
*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((2*c^4*x^4 - c^2*x^2 + (2*c^2*x
^2 - 1)*(c*x + 1)*(c*x - 1) + 2*(2*c^3*x^3 - c*x)*sqrt(c*x + 1)*sqrt(c*x -
1) - 1)/(((b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^
```

```
5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^6*x^6 - 3
*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)^(3/2
)*(c*x - 1) + 2*(a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*(c*x + 1)*sqrt(c*x
- 1) + (a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*sqrt(c*x + 1))*s
qrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{a^2c^4x^4-2\,a^2c^2x^2+\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\operatorname{arcosh}\left(cx\right)^2+a^2+2\left(abc^4x^4-2\,abc^2x^2+ab\right)\operatorname{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 - 2
*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 +
 a*b)*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

3.354 $\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.549333, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x*(-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 21.8446, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.369, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{2}}\left(-c^{2}x^{2}+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{cx+\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}b^{2}c^{2}x^{2}+\left(b^{2}c^{3}x^{3}-b^{2}cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}abc^{2}x^{2}+\left(ab\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] (c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 +
 (b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 + (a*b*c^3*x^3 -
a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3*c^5*x^5 - 3*c^3*x^3
+ (3*c^3*x^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (6*c^4*x^4 - 5*c^2*x^2 + 1)*sqr
t(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^5*x^6 - b^2*c^3*x^4)*(c*x + 1)^(3/2)*(c*
x - 1) + 2*(b^2*c^6*x^7 - 2*b^2*c^4*x^5 + b^2*c^2*x^3)*(c*x + 1)*sqrt(c*x -
 1) + (b^2*c^7*x^8 - 3*b^2*c^5*x^6 + 3*b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x +
1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^6 -
 a*b*c^3*x^4)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^7 - 2*a*b*c^4*x^5 +
a*b*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^8 - 3*a*b*c^5*x^6 + 3*a*b
*c^3*x^4 - a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{a^2c^4x^5-2\,a^2c^2x^3+a^2x+\left(b^2c^4x^5-2\,b^2c^2x^3+b^2x\right)\text{arcosh}\left(cx\right)^2+2\left(abc^4x^5-2\,abc^2x^3+abx\right)\text{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^5 - 2*a^2*c^2*x^3 + a^2*x + (b^2*c^4
*x^5 - 2*b^2*c^2*x^3 + b^2*x)*arccosh(c*x)^2 + 2*(a*b*c^4*x^5 - 2*a*b*c^2*x
^3 + a*b*x)*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2*x), x)
```

3.355 $\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.560465, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^2*(-1 + c*x)^(3/2)*(1 + c*x
)^(3/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 19.8678, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.257, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b\operatorname{arccosh}(cx))^2} \left(-c^2x^2+1\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{cx + \sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}b^2c^2x^3 + \left(b^2c^3x^4 - b^2cx^2\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \left((cx+1)\sqrt{cx-1}abc^2x^3 + \left(abc\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] (c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 +
 (b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 + (a*b*c^3*x^4
- a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((4*c^5*x^5 - 5*c^3*
x^3 + (4*c^3*x^3 - 3*c*x)*(c*x + 1)*(c*x - 1) + 2*(4*c^4*x^4 - 4*c^2*x^2 +
1)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*x)/(((b^2*c^5*x^7 - b^2*c^3*x^5)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^8 - 2*b^2*c^4*x^6 + b^2*c^2*x^4)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^7*x^9 - 3*b^2*c^5*x^7 + 3*b^2*c^3*x^5 - b^2*c*x^3)
*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a
*b*c^5*x^7 - a*b*c^3*x^5)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^8 - 2*a*
b*c^4*x^6 + a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^9 - 3*a*b*c^5
*x^7 + 3*a*b*c^3*x^5 - a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^2x^2+1}}{a^2c^4x^6-2\,a^2c^2x^4+a^2x^2+\left(b^2c^4x^6-2\,b^2c^2x^4+b^2x^2\right)\operatorname{arcosh}\left(cx\right)^2+2\left(abc^4x^6-2\,abc^2x^4+abx^2\right)\operatorname{arcosh}\left(cx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^6 - 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*c
^4*x^6 - 2*b^2*c^2*x^4 + b^2*x^2)*arccosh(c*x)^2 + 2*(a*b*c^4*x^6 - 2*a*b*c
^2*x^4 + a*b*x^2)*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2*x^2), x)
```

3.356 $\int \frac{x^4}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=108

$$-\frac{4\sqrt{cx-1}\,\text{Unintegrable}\left(\frac{x^3}{\left(c^2x^2-1\right)^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{1-cx}}-\frac{x^4\sqrt{cx-1}\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos
h[c*x]))) - (4*Sqrt[-1 + c*x]*Unintegrable[x^3/((-1 + c^2*x^2)^3*(a + b*Arc
Cosh[c*x])), x])/(b*c*Sqrt[1 - c*x])
```

Rubi [A] time = 0.654427, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^4*Sqrt[-1 + c*x])/(b*c*(1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2]*(
a + b*ArcCosh[c*x]))) - (4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^3/((-1
 + c^2*x^2)^3*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x^4}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^4}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$=-\frac{x^4\sqrt{-1+cx}}{bc(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\int}{bc\sqrt{1}}$$

Mathematica [A] time = 5.03117, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^4/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.394, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a+b\operatorname{arccosh}(cx)\right)^2}\left(-c^2x^2+1\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx^5+\sqrt{cx+1}\sqrt{cx-1}x^4}{\left(\left(b^2c^4x^3-b^2c^2x\right)(cx+1)\sqrt{cx-1}+\left(b^2c^5x^4-2\,b^2c^3x^2+b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left(\left(abc^4x^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -(c*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^4)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b
*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)) - integrate((5*c^3*x^6 + 3*(c*x + 1)*(c*x - 1)*
```

```
c*x^4 - 5*c*x^4 + 4*(2*c^2*x^5 - x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c
^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^
8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1)
+ (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 - 4*b^2*c^3*x^2 + b^2*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c
^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^
8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1)
+ (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^5*x^4 - 4*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}x^4}{a^2c^6x^6 - 3\,a^2c^4x^4 + 3\,a^2c^2x^2 + \left(b^2c^6x^6 - 3\,b^2c^4x^4 + 3\,b^2c^2x^2 - b^2\right)\text{arcosh}\left(cx\right)^2 - a^2 + 2\left(abc^6x^6 - 3\,abc^4x^4 + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^4/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x
^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a
^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

3.357 $\int \frac{x^3}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{x^3}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.556879, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^3}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^3/((-1 + c*x)^(5/2)*(1 + c*x)^(5
/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{x^3}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 48.562, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.533, size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+b\,\mathrm{arccosh}\,(cx))^2}\left(-c^2x^2+1\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx^4+\sqrt{cx+1}\sqrt{cx-1}x^3}{\left(\left(b^2c^4x^3-b^2c^2x\right)(cx+1)\sqrt{cx-1}+\left(b^2c^5x^4-2\,b^2c^3x^2+b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left(\left(abc^4x^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -(c*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^3)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b
*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)) - integrate((c^5*x^7 + 3*c^3*x^5 - 4*c*x^3 + (c
^3*x^5 + 2*c*x^3)*(c*x + 1)*(c*x - 1) + (2*c^4*x^6 + 5*c^2*x^4 - 3*x^2)*sqr
t(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*
x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b
^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^
5*x^4 - 4*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*
x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a
*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^
```

```
5*x^4 - 4*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}x^3}{a^2c^6x^6 - 3\,a^2c^4x^4 + 3\,a^2c^2x^2 + \left(b^2c^6x^6 - 3\,b^2c^4x^4 + 3\,b^2c^2x^2 - b^2\right)\operatorname{arcosh}\left(cx\right)^2 - a^2 + 2\left(abc^6x^6 - 3\,abc^4x^4 + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x
^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a
^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

3.358 $\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{x^2}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.556686, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^2/((-1 + c*x)^(5/2)*(1 + c*x)^(5
/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 7.44182, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.517, size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+b\operatorname{arccosh}(cx))^2}\left(-c^2x^2+1\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx^3+\sqrt{cx+1}\sqrt{cx-1}x^2}{\left(\left(b^2c^4x^3-b^2c^2x\right)(cx+1)\sqrt{cx-1}+\left(b^2c^5x^4-2\,b^2c^3x^2+b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left(\left(abc^4x^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -(c*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^2)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b
*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)) - integrate((2*c^5*x^6 + c^3*x^4 + (2*c^3*x^4 +
 c*x^2)*(c*x + 1)*(c*x - 1) - 3*c*x^2 + 2*(2*c^4*x^5 + c^2*x^3 - x)*sqrt(c*
x + 1)*sqrt(c*x - 1))/(((b^2*c^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b^2*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^5*x^
4 - 4*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a*b*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^5*x^
```

```
4 - 4*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}x^2}{a^2c^6x^6-3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6-3\,b^2c^4x^4+3\,b^2c^2x^2-b^2\right)\text{arcosh}\left(cx\right)^2-a^2+2\left(abc^6x^6-3\,abc^4x^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x
^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a
^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

3.359 $\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{x}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.391162, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c*x)^(5/2)*(1 + c*x)^(5/2
)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{x}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 43.8015, size = 0, normalized size = 0.

$$\int \frac{x}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.418, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{2}}\left(-c^{2}x^{2}+1\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx^{2}+\sqrt{cx+1}\sqrt{cx-1}\,x}{\left(\left(b^{2}c^{4}x^{3}-b^{2}c^{2}x\right)(cx+1)\sqrt{cx-1}+\left(b^{2}c^{5}x^{4}-2\,b^{2}c^{3}x^{2}+b^{2}c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left(\left(abc^{4}x^{3}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*x)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x +
 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sq
rt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(
c*x + 1))*sqrt(-c*x + 1)) - integrate((3*c^5*x^5 + 3*(c*x + 1)*(c*x - 1)*c^
3*x^3 - c^3*x^3 + (6*c^4*x^4 - c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2
*c*x)/(((b^2*c^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x -
1) + 2*(b^2*c^8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 - 4*b^2*c^3*x^
2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) + ((a*b*c^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x -
1) + 2*(a*b*c^8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^5*x^4 - 4*a*b*c^3*x^
```

```
2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}x}{a^2c^6x^6-3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6-3\,b^2c^4x^4+3\,b^2c^2x^2-b^2\right)\text{arcosh}\left(cx\right)^2-a^2+2\left(abc^6x^6-3\,abc^4x^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a^2
 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

3.360 $\int \frac{1}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{4c\sqrt{cx-1}\text{Unintegrable}\left(\frac{x}{\left(c^2x^2-1\right)^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{b\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*
x]))) - (4*c*Sqrt[-1 + c*x]*Unintegrable[x/((-1 + c^2*x^2)^3*(a + b*ArcCosh
[c*x])), x])/(b*Sqrt[1 - c*x])
```

Rubi [A] time = 0.304887, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -(Sqrt[-1 + c*x]/(b*c*(1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2]*(a + b*
ArcCosh[c*x]))) - (4*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c^2
*x^2)^3*(a + b*ArcCosh[c*x])), x])/(b*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$=-\frac{\sqrt{-1+cx}}{bc(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(4c\sqrt{-1+cx}\sqrt{1+cx}\right)}{b\sqrt{}}$$

Mathematica [A] time = 3.57803, size = 0, normalized size = 0.

$$\int \frac{1}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.316, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\left(-c^2x^2+1\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx+\sqrt{cx+1}\sqrt{cx-1}}{\left(\left(b^2c^4x^3-b^2c^2x\right)(cx+1)\sqrt{cx-1}+\left(b^2c^5x^4-2\,b^2c^3x^2+b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left(\left(abc^4x^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-
c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b*c^2*x
)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x
+ 1))*sqrt(-c*x + 1)) - integrate((4*c^4*x^4 - 3*c^2*x^2 + (4*c^2*x^2 - 1)*
(c*x + 1)*(c*x - 1) + 4*(2*c^3*x^3 - c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - 1)/
```

```
(((b^2*c^6*x^6 - 2*b^2*c^4*x^4 + b^2*c^2*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2
*(b^2*c^7*x^7 - 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 - b^2*c*x)*(c*x + 1)*sqrt(c*x
 - 1) + (b^2*c^8*x^8 - 4*b^2*c^6*x^6 + 6*b^2*c^4*x^4 - 4*b^2*c^2*x^2 + b^2)
*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a
*b*c^6*x^6 - 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*
b*c^7*x^7 - 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 - a*b*c*x)*(c*x + 1)*sqrt(c*x - 1
) + (a*b*c^8*x^8 - 4*a*b*c^6*x^6 + 6*a*b*c^4*x^4 - 4*a*b*c^2*x^2 + a*b)*sqr
t(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{a^2c^6x^6 - 3\,a^2c^4x^4 + 3\,a^2c^2x^2 + \left(b^2c^6x^6 - 3\,b^2c^4x^4 + 3\,b^2c^2x^2 - b^2\right)\text{arcosh}\left(cx\right)^2 - a^2 + 2\left(abc^6x^6 - 3\,abc^4x^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 +
 (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a^2 +
 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

3.361 $\int \frac{1}{x(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.543159, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x*(-1 + c*x)^(5/2)*(1 + c*x)^(5
/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 39.014, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.662, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{2}}\left(-c^{2}x^{2}+1\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx+\sqrt{cx+1}\sqrt{cx-1}}{\left(\left(b^{2}c^{4}x^{4}-b^{2}c^{2}x^{2}\right)(cx+1)\sqrt{cx-1}+\left(b^{2}c^{5}x^{5}-2\,b^{2}c^{3}x^{3}+b^{2}cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left(\left(abc^{4}x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((((b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^4 - a*b*c
^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*s
qrt(c*x + 1))*sqrt(-c*x + 1)) - integrate((5*c^5*x^5 - 5*c^3*x^3 + (5*c^3*x
^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (10*c^4*x^4 - 7*c^2*x^2 + 1)*sqrt(c*x + 1
)*sqrt(c*x - 1))/(((b^2*c^7*x^8 - 2*b^2*c^5*x^6 + b^2*c^3*x^4)*(c*x + 1)^(3
/2)*(c*x - 1) + 2*(b^2*c^8*x^9 - 3*b^2*c^6*x^7 + 3*b^2*c^4*x^5 - b^2*c^2*x^
3)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^10 - 4*b^2*c^7*x^8 + 6*b^2*c^5*x^6
- 4*b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^8 - 2*a*b*c^5*x^6 + a*b*c^3*x^4)*(c*x
+ 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^9 - 3*a*b*c^6*x^7 + 3*a*b*c^4*x^5 - a*b
*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^10 - 4*a*b*c^7*x^8 + 6*a*b*c
```

```
^5*x^6 - 4*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{a^2c^6x^7 - 3\,a^2c^4x^5 + 3\,a^2c^2x^3 - a^2x + \left(b^2c^6x^7 - 3\,b^2c^4x^5 + 3\,b^2c^2x^3 - b^2x\right)\operatorname{arcosh}\left(cx\right)^2 + 2\left(abc^6x^7 - 3\,abc^4x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^7 - 3*a^2*c^4*x^5 + 3*a^2*c^2*x^3 -
 a^2*x + (b^2*c^6*x^7 - 3*b^2*c^4*x^5 + 3*b^2*c^2*x^3 - b^2*x)*arccosh(c*x)
^2 + 2*(a*b*c^6*x^7 - 3*a*b*c^4*x^5 + 3*a*b*c^2*x^3 - a*b*x)*arccosh(c*x)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2*x), x)
```

3.362 $\int \frac{1}{x^2(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.550756, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^2*(-1 + c*x)^(5/2)*(1 + c*x)^
(5/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 14.4959, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.749, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \operatorname{arccosh}(cx))^2} \left(-c^2x^2 + 1\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx + \sqrt{cx+1}\sqrt{cx-1}}{\left(\left(b^2c^4x^5 - b^2c^2x^3\right)(cx+1)\sqrt{cx-1} + \left(b^2c^5x^6 - 2\,b^2c^3x^4 + b^2cx^2\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \left(\left(abc\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] -(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^4*x^5 - b^2*c^2*x^3)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^6 - 2*b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^5 - a*b
*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^6 - 2*a*b*c^3*x^4 + a*b*c*x^
2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrate((6*c^5*x^5 - 7*c^3*x^3 + 3*(2
*c^3*x^3 - c*x)*(c*x + 1)*(c*x - 1) + 2*(6*c^4*x^4 - 5*c^2*x^2 + 1)*sqrt(c*
x + 1)*sqrt(c*x - 1) + c*x)/(((b^2*c^7*x^9 - 2*b^2*c^5*x^7 + b^2*c^3*x^5)*(
c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^10 - 3*b^2*c^6*x^8 + 3*b^2*c^4*x^6
- b^2*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^11 - 4*b^2*c^7*x^9 + 6*
b^2*c^5*x^7 - 4*b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^9 - 2*a*b*c^5*x^7 + a*b*c^
3*x^5)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^10 - 3*a*b*c^6*x^8 + 3*a*b*
c^4*x^6 - a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^11 - 4*a*b*c^7*
```

```
x^9 + 6*a*b*c^5*x^7 - 4*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x +
 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{a^2c^6x^8 - 3\,a^2c^4x^6 + 3\,a^2c^2x^4 - a^2x^2 + \left(b^2c^6x^8 - 3\,b^2c^4x^6 + 3\,b^2c^2x^4 - b^2x^2\right)\text{arcosh}\,(cx)^2 + 2\left(abc^6x^8 - 3\,abc^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^8 - 3*a^2*c^4*x^6 + 3*a^2*c^2*x^4 -
 a^2*x^2 + (b^2*c^6*x^8 - 3*b^2*c^4*x^6 + 3*b^2*c^2*x^4 - b^2*x^2)*arccosh(
c*x)^2 + 2*(a*b*c^6*x^8 - 3*a*b*c^4*x^6 + 3*a*b*c^2*x^4 - a*b*x^2)*arccosh(
c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arcosh}\,(cx)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2*x^2), x)
```

3.363 $\int \frac{(fx)^m(1-c^2x^2)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=32

$$\text{Unintegrable}\left(\frac{\left(1-c^2x^2\right)^{3/2}(fx)^m}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2, x]
```

Rubi [A] time = 0.53223, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*Defer[Int][((f*x)^m*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/
(a + b*ArcCosh[c*x])^2, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int \frac{(fx)^m\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\sqrt{1-c^2x^2}\int \frac{(fx)^m(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.11353, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2, x]
```

Maple [A] time = 0.875, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{(a + b\operatorname{arccosh}(cx))^2} \left(-c^2x^2 + 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(c^4 f^m x^4 - 2\,c^2 f^m x^2 + f^m\right)(cx+1)\sqrt{cx-1}x^m + \left(c^5 f^m x^5 - 2\,c^3 f^m x^3 + c f^m x\right)\sqrt{cx+1}x^m\right)\sqrt{-cx+1}}{abc^3x^2 + \sqrt{cx+1}\sqrt{cx-1}abc^2x - abc + \left(b^2c^3x^2 + \sqrt{cx+1}\sqrt{cx-1}b^2c^2x - b^2c\right)\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)} - \int \frac{\left(\left(c^5 f^m(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] ((c^4*f^m*x^4 - 2*c^2*f^m*x^2 + f^m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^m
*x^5 - 2*c^3*f^m*x^3 + c*f^m*x)*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^3*
x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c
*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1))) - integrate((((c^5*f^m*(m + 4)*x^5 - c^3*f^m*(2*m + 3)*x^3 + c*f^m*(m
- 1)*x)*(c*x + 1)^(3/2)*(c*x - 1)*x^m + (2*c^6*f^m*(m + 4)*x^6 - c^4*f^m*(5
*m + 12)*x^4 + 4*c^2*f^m*(m + 1)*x^2 - f^m*m)*(c*x + 1)*sqrt(c*x - 1)*x^m +
 (c^7*f^m*(m + 4)*x^7 - 3*c^5*f^m*(m + 3)*x^5 + 3*c^3*f^m*(m + 2)*x^3 - c*f
^m*(m + 1)*x)*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^5*x^5 + (c*x + 1)*(c
*x - 1)*a*b*c^3*x^3 - 2*a*b*c^3*x^3 + a*b*c*x + 2*(a*b*c^4*x^4 - a*b*c^2*x^
2)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^5 + (c*x + 1)*(c*x - 1)*b^2*c^3
```

```
*x^3 - 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*sqrt(c*x + 1
)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^2x^2-1\right)\sqrt{-c^2x^2+1}\left(fx\right)^m}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*(f*x)^m/(b^2*arccosh(c*x)^2 + 2*
a*b*arccosh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(fx\right)^m}{\left(b\,\text{arcosh}\,(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*(f*x)^m/(b*arccosh(c*x) + a)^2, x)
```

3.364 $\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=32

$$\text{Unintegrable}\left(\frac{\sqrt{1-c^2x^2}(fx)^m}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2, x]
```

Rubi [A] time = 0.450238, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][((f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(a + b
*ArcCosh[c*x])^2, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{(fx)^m\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.185017, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2, x]
```

Maple [A] time = 0.865, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{(a+b\operatorname{arccosh}(cx))^2}\sqrt{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(c^2f^mx^2-f^m\right)(cx+1)\sqrt{cx-1}x^m+\left(c^3f^mx^3-cf^mx\right)\sqrt{cx+1}x^m\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{((}{abc^5x^5\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] -((c^2*f^m*x^2 - f^m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^3*f^m*x^3 - c*f^m*x)
*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x -
1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x
 - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((c^3*f^m*(m
+ 2)*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)^(3/2)*(c*x - 1)*x^m + (2*c^4*f^m*(m +
 2)*x^4 - c^2*f^m*(3*m + 2)*x^2 + f^m*m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5
*f^m*(m + 2)*x^5 - c^3*f^m*(2*m + 3)*x^3 + c*f^m*(m + 1)*x)*sqrt(c*x + 1)*x
^m)*sqrt(-c*x + 1)/(a*b*c^5*x^5 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^3 - 2*a*b*c
^3*x^3 + a*b*c*x + 2*(a*b*c^4*x^4 - a*b*c^2*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1
) + (b^2*c^5*x^5 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^3 - 2*b^2*c^3*x^3 + b^2*c*
x + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sq
```

```
rt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,(fx)^m}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*(f*x)^m/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x
) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m\sqrt{-(cx-1)\,(cx+1)}}{(a+b\,\text{acosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((f*x)**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}\,(fx)^m}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*(f*x)^m/(b*arccosh(c*x) + a)^2, x)
```

3.365 $\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{fm\sqrt{cx-1}\text{Unintegrable}\left(\frac{(fx)^{m-1}}{a+b\cosh^{-1}(cx)},x\right)}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}(fx)^m}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((((f*x)^m*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) + (f*m
*Sqrt[-1 + c*x]*Unintegrable[(f*x)^(-1 + m)/(a + b*ArcCosh[c*x]), x])/(b*c*
Sqrt[1 - c*x])
```

Rubi [A] time = 0.52198, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((((f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcC
osh[c*x]))) + (f*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(f*x)^(-1 + m)/(
a + b*ArcCosh[c*x]), x])/(b*c*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$=-\frac{(fx)^m\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(fm\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^{-1+m}}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.635352, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.358, size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^m}{\left(a+b\mathrm{arccosh}\left(cx\right)\right)^2}\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2f^mx^2-f^m\right)\sqrt{cx+1}\sqrt{cx-1}x^m+\left(c^3f^mx^3-cf^mx\right)x^m}{\left((cx+1)\sqrt{cx-1}b^2c^2x+\left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}abc^2x+\left(abc^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] -((c^2*f^m*x^2 - f^m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^3*f^m*x^3 - c*f^
m*x)*x^m)/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(
c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1
)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)) + integrate((((c^3*f^m*m*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^
```

```
m + (2*c^4*f^m*m*x^4 - 3*c^2*f^m*m*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)
*x^m + (c^5*f^m*m*x^5 - c^3*f^m*(2*m + 1)*x^3 + c*f^m*(m + 1)*x)*x^m)/(((c*
x + 1)^(3/2)*(c*x - 1)*b^2*c^3*x^3 + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x
 - 1)*a*b*c^3*x^3 + 2*(a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) +
 (a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,(fx)^m}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\operatorname{arcosh}(cx)^2-a^2+2\left(abc^2x^2-ab\right)\operatorname{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arc
cosh(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(fx)^m}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((f*x)**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(fx)^m}{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] integrate((f*x)^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

3.366 $\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=32

$$\text{Unintegrable}\left(\frac{(fx)^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.55825, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(f*x)^m/((-1 + c*x)^(3/2)*(1 + c
*x)^(3/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.21254, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.5, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{(a+b\operatorname{arccosh}(cx))^2}\left(-c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{cf^m x x^m + \sqrt{cx+1}\sqrt{cx-1}f^m x^m}{\left((cx+1)\sqrt{cx-1}b^2c^2x + \left(b^2c^3x^2 - b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \left((cx+1)\sqrt{cx-1}abc^2x + \left(abc^3x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] (c*f^m*x*x^m + sqrt(c*x + 1)*sqrt(c*x - 1)*f^m*x^m)/(((c*x + 1)*sqrt(c*x -
1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*
c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrate(((c^3*f^m*(m -
2)*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^m + (2*c^4*f^m*(m - 2)*x^4
- c^2*f^m*(3*m - 2)*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^m
*(m - 2)*x^5 - c^3*f^m*(2*m - 1)*x^3 + c*f^m*(m + 1)*x)*x^m)/(((b^2*c^5*x^5
 - b^2*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^6 - 2*b^2*c^4*x^4
+ b^2*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x^7 - 3*b^2*c^5*x^5 + 3*b
^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1)) + ((a*b*c^5*x^5 - a*b*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) +
2*(a*b*c^6*x^6 - 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (a*
b*c^7*x^7 - 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1))*sqrt(-c
```

```
*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,(fx)^m}{a^2c^4x^4-2\,a^2c^2x^2+\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\text{arcosh}\,(cx)^2+a^2+2\left(abc^4x^4-2\,abc^2x^2+ab\right)\text{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4
*x^4 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c
^2*x^2 + a*b)*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] integrate((f*x)^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

3.367 $\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=32

$$\text{Unintegrable}\left(\frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.551211, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(f*x)^m/((-1 + c*x)^(5/2)*(1 + c*x
)^(5/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.68472, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.509, size = 0, normalized size = 0.

$$\int \frac{(fx)^m}{(a + b\,\mathrm{arccosh}\,(cx))^2} \left(-c^2x^2+1\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cf^mxx^m + \sqrt{cx+1}\sqrt{cx-1}f^mx^m}{\left(\left(b^2c^4x^3 - b^2c^2x\right)(cx+1)\sqrt{cx-1} + \left(b^2c^5x^4 - 2\,b^2c^3x^2 + b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \left(\left(abc^4x^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] -(c*f^m*x*x^m + sqrt(c*x + 1)*sqrt(c*x - 1)*f^m*x^m)/(((b^2*c^4*x^3 - b^2*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(
c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4
*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 +
a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate(((c^3*f^m*(m - 4)*x^3 - c
*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^m + (2*c^4*f^m*(m - 4)*x^4 - c^2*f^m*
(3*m - 4)*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^m*(m - 4)*x
^5 - c^3*f^m*(2*m - 3)*x^3 + c*f^m*(m + 1)*x)*x^m)/(((b^2*c^7*x^7 - 2*b^2*c
^5*x^5 + b^2*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^8 - 3*b^2*c^
6*x^6 + 3*b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^9
 - 4*b^2*c^7*x^7 + 6*b^2*c^5*x^5 - 4*b^2*c^3*x^3 + b^2*c*x)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^7 - 2*a
*b*c^5*x^5 + a*b*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^8 - 3*a*
```

```
b*c^6*x^6 + 3*a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9
*x^9 - 4*a*b*c^7*x^7 + 6*a*b*c^5*x^5 - 4*a*b*c^3*x^3 + a*b*c*x)*sqrt(c*x +
1))*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\left(fx\right)^m}{a^2c^6x^6 - 3\,a^2c^4x^4 + 3\,a^2c^2x^2 + \left(b^2c^6x^6 - 3\,b^2c^4x^4 + 3\,b^2c^2x^2 - b^2\right)\operatorname{arcosh}\left(cx\right)^2 - a^2 + 2\left(abc^6x^6 - 3\,abc^4x^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c
^2*x^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2
 - a^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^m}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] integrate((f*x)^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

3.368 $\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=32

$$-\frac{\sqrt{ax-1}}{2a\sqrt{1-ax}\cosh^{-1}(ax)^2}$$

```
[Out] -Sqrt[-1 + a*x]/(2*a*Sqrt[1 - a*x]*ArcCosh[a*x]^2)
```

Rubi [A] time = 0.148091, antiderivative size = 45, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5713, 5676}

$$-\frac{\sqrt{ax-1}\sqrt{ax+1}}{2a\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3),x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}$$

Mathematica [A] time = 0.0251697, size = 45, normalized size = 1.41

$$-\frac{\sqrt{ax-1}\sqrt{ax+1}}{2a\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3),x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)
```

Maple [A] time = 0.046, size = 51, normalized size = 1.6

$$\frac{1}{2\,a\left(a^2x^2-1\right)(\operatorname{arccosh}(ax))^2}\sqrt{-(ax-1)(ax+1)}\sqrt{ax-1}\sqrt{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)/arcc
osh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - a^2*x^2)*(a*x + 1)^(3/2)
*(a*x - 1)^(3/2) + (3*a^5*x^5 - 5*a^3*x^3 + 2*a*x)*(a*x + 1)*(a*x - 1) + (3
*a^6*x^6 - 7*a^4*x^4 + 5*a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x - (
a^5*x^5 - 2*a^3*x^3 - (a^2*x^2 - 1)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) - (a^3*
x^3 - a*x)*(a*x + 1)*(a*x - 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*sqrt(a*x + 1)*sq
rt(a*x - 1) + a*x)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))/(((a*x + 1)^2*(a
*x - 1)^(3/2)*a^4*x^3 + 3*(a^5*x^4 - a^3*x^2)*(a*x + 1)^(3/2)*(a*x - 1) + 3
*(a^6*x^5 - 2*a^4*x^3 + a^2*x)*(a*x + 1)*sqrt(a*x - 1) + (a^7*x^6 - 3*a^5*x
^4 + 3*a^3*x^2 - a)*sqrt(a*x + 1))*sqrt(-a*x + 1)*log(a*x + sqrt(a*x + 1)*s
qrt(a*x - 1))^2) - integrate(-1/2*(2*a^6*x^6 - 3*a^4*x^4 - (2*a^2*x^2 - 3)*
(a*x + 1)^2*(a*x - 1)^2 - 4*(a^3*x^3 - a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2)
 - 4*(a^2*x^2 - 1)*(a*x + 1)*(a*x - 1) + 4*(a^5*x^5 - 2*a^3*x^3 + a*x)*sqrt
(a*x + 1)*sqrt(a*x - 1) + 1)/(((a*x + 1)^(5/2)*(a*x - 1)^2*a^4*x^4 + 4*(a^5
*x^5 - a^3*x^3)*(a*x + 1)^2*(a*x - 1)^(3/2) + 6*(a^6*x^6 - 2*a^4*x^4 + a^2*
x^2)*(a*x + 1)^(3/2)*(a*x - 1) + 4*(a^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 - a*x)*
(a*x + 1)*sqrt(a*x - 1) + (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)
*sqrt(a*x + 1))*sqrt(-a*x + 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [B] time = 1.9688, size = 120, normalized size = 3.75

$$\frac{\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}}{2\left(a^3x^2-a\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)/((a^3*x^2 - a)*log(a*x + sqrt(a^2*
x^2 - 1))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\operatorname{acosh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\mathrm{arcosh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3), x)
```

3.369 $\int \frac{x^3(d-c^2dx^2)}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=259

$$\frac{3\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}de^{\frac{6a}{b}}\text{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}de^{-\frac{6a}{b}}\text{Erf}}{1}$$

```
[Out] (2*d*x^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) +
 (3*d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
])/(16*b^(3/2)*c^4) - (d*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b
*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4) + (3*d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((2*a)/b)) - (d*Sqrt[
(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4
*E^((6*a)/b))
```

Rubi [A] time = 1.74749, antiderivative size = 269, normalized size of antiderivative = 1.04, number of steps used = 27, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5776, 5781, 5448, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}de^{\frac{6a}{b}}\text{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}de^{-\frac{6a}{b}}\text{Erf}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2))/(b*c*Sqrt[a + b*ArcCo
sh[c*x]]) + (3*d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c^4) - (d*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]
*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4) + (3*d*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((2*a)/b))
 - (d*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*
b^(3/2)*c^4*E^((6*a)/b))
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^3\left(d-c^2dx^2\right)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{(6d)\int\frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc}-\frac{(12cd)\int\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,d}{b}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{(6d)\,\mathrm{Subst}\left(\int\frac{\cosh^2(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{bc^4}-$$

$$= -\frac{2dx^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{(6d)\,\mathrm{Subst}\left(\int\left(-\frac{1}{8\sqrt{a+bx}}+\frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx,x,\cosh^{-1}(c}{bc^4}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{(3d)\,\mathrm{Subst}\left(\int\frac{\cosh(2x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{8bc^4}-\frac{(3d)\,\mathrm{Su}}{}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{(3d)\,\mathrm{Subst}\left(\int\frac{e^{-6x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{16bc^4}+\frac{(3d)\,\mathrm{Subs}}{}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{(3d)\,\mathrm{Subst}\left(\int e^{\frac{6a}{b}-\frac{6x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(cx)}\right)}{8b^2c^4}+$$

$$= -\frac{2dx^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{de^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{6}}{}\right.}{16b^{3/2}}$$

Mathematica [A] time = 2.4936, size = 300, normalized size = 1.16

$$\frac{de^{-\frac{6a}{b}}\left(e^{\frac{6a}{b}}\left(-3\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+\sqrt{6}e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{6\left(a+b\cosh^{-1}(cx\right.}{b}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d*(-(Sqrt[6]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCosh
[c*x]))/b]) + 3*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1
/2, (-2*(a + b*ArcCosh[c*x]))/b] + E^((6*a)/b)*(-64*c^3*x^3*Sqrt[(-1 + c*x)
/(1 + c*x)] - 64*c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 3*Sqrt[2]*E^((2*a)/b)
*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (2*(a + b*ArcCosh[c*x]))/b] + Sqrt[6]*
E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x]))/b]
 + 10*Sinh[2*ArcCosh[c*x]] + 8*Sinh[4*ArcCosh[c*x]] + 2*Sinh[6*ArcCosh[c*x]
])))/(32*b*c^4*E^((6*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.237, size = 0, normalized size = 0.

$$\int x^{3} \left(-c^{2} d \,x^{2}+d \right) \left(a +b \operatorname{arccosh}\left(c x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{{\left(c^{2} d x^{2} - d\right)} x^{3}}{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] -integrate((c^2*d*x^2 - d)*x^3/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d\left(\int -\frac{x^3}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx+\int \frac{c^2x^5}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(-x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*aco
sh(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*
acosh(c*x))*acosh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.370 $\int \frac{x^2(d-c^2dx^2)}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=340

$$\frac{\sqrt{\pi}de^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}+\frac{\sqrt{3\pi}de^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}de^{\frac{5a}{b}}\operatorname{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}+\frac{\sqrt{\pi}de^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}$$

```
[Out] (2*d*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) +
 (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3)
 + (d*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
])/(16*b^(3/2)*c^3) - (d*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*Arc
Cosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCos
h[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3*E^(a/b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqr
t[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((3*a)/b)) - (d*Sqrt[5*P
i]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((5*
a)/b))
```

Rubi [A] time = 1.77248, antiderivative size = 350, normalized size of antiderivative = 1.03, number of steps used = 32, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5776, 5781, 5448, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}de^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}+\frac{\sqrt{3\pi}de^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}de^{\frac{5a}{b}}\operatorname{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}+\frac{\sqrt{\pi}de^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2))/(b*c*Sqrt[a + b*ArcCo
sh[c*x]]) + (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*b
^(3/2)*c^3) + (d*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c^3) - (d*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqr
t[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3*E^(a/b)) + (d*Sqrt[3*Pi]*Erfi[(
Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((3*a)/b)) -
```

(d*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)
*c^3*E^((5*a)/b))

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2\left(d-c^2dx^2\right)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\int \frac{x\sqrt{-1+cx}\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} - \frac{(10cd)\int \frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc^3} - (10$$

$$= -\frac{2dx^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\,\mathrm{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}} + \frac{\cosh(3x)}{4\sqrt{a+bx}}\right) dx, x, \cosh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(5d)\,\mathrm{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8bc^3} - (5d)\,\mathrm{Subs}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(5d)\,\mathrm{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{16bc^3} - (5d)\,\mathrm{Subst}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(5d)\,\mathrm{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{8b^2c^3} - (5$$

$$= -\frac{2dx^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} + \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,c}}{\sqrt{}}\right.}{16b^{3/2}c^3}$$

Mathematica [A] time = 1.48846, size = 384, normalized size = 1.13

$$\frac{de^{-\frac{5a}{b}}\left(-2e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)-\sqrt{5}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+\sqrt{3}e^{2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d*(-4*E^((5*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*c*E^((5*a)/b)*x*Sqrt[(-1
+ c*x)/(1 + c*x)] - 2*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b +
 ArcCosh[c*x]] - Sqrt[5]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-5*(a
+ b*ArcCosh[c*x]))/b] + Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]
*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] + 2*E^((4*a)/b)*Sqrt[-((a + b*ArcC
osh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - Sqrt[3]*E^((8*a)/b)*S
qrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] + Sqrt[5]*E^
((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a + b*ArcCosh[c*x]))/b]
- 2*E^((5*a)/b)*Sinh[3*ArcCosh[c*x]] + 2*E^((5*a)/b)*Sinh[5*ArcCosh[c*x]]))
/(16*b*c^3*E^((5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.275, size = 0, normalized size = 0.

$$\int x^{2}\left(-c^{2}dx^{2}+d\right)\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(c^{2}dx^{2}-d\right)x^{2}}{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] -integrate((c^2*d*x^2 - d)*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d\left(\int -\frac{x^2}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx+\int \frac{c^2x^4}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(-x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*aco
sh(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*
acosh(c*x))*acosh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.371 $\int \frac{x(d-c^2dx^2)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{\sqrt{\pi}de^{\frac{4a}{b}}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}-\frac{\sqrt{\pi}de^{-\frac{4a}{b}}\,\mathrm{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{2b^{3/2}c}$$

```
[Out] (2*d*x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) - (
d*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2
)*c^2) + (d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/S
qrt[b]])/(2*b^(3/2)*c^2) - (d*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) + (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
b*ArcCosh[c*x]])/Sqrt[b]])/(2*b^(3/2)*c^2*E^((2*a)/b))
```

Rubi [A] time = 1.15412, antiderivative size = 251, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5776, 5701, 3312, 3307, 2180, 2204, 2205, 5781, 5448}

$$-\frac{\sqrt{\pi}de^{\frac{4a}{b}}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}-\frac{\sqrt{\pi}de^{-\frac{4a}{b}}\,\mathrm{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{2b^{3/2}c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2))/(b*c*Sqrt[a + b*ArcCosh
[c*x]]) - (d*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]
)/(4*b^(3/2)*c^2) + (d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCo
sh[c*x]])/Sqrt[b]])/(2*b^(3/2)*c^2) - (d*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCos
h[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) + (d*Sqrt[Pi/2]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(2*b^(3/2)*c^2*E^((2*a)/b))
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
```

```
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(d-c^2dx^2\right)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2d)\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} - \frac{(8cd)\int \frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2dx\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc^2} - \frac{(8d)\,\mathrm{Subst}}{}$$

$$= -\frac{2dx\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\,\mathrm{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx, x, \cosh^{-1}(cx)\right)}{bc^2} -$$

$$= -\frac{2dx\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc^2} - \frac{d\,\mathrm{Subst}\left(\int \right.}{}$$

$$= -\frac{2dx\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{2bc^2} + \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-}}{\sqrt{a-}}\right.}{}$$

$$= -\frac{2dx\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d\,\mathrm{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2c^2} + \frac{d\,\mathrm{Sub}}{}$$

$$= -\frac{2dx\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{de^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cos}}{\sqrt{b}}\right.}{2b^{3/2}c^2}$$

Mathematica [A] time = 4.05243, size = 331, normalized size = 1.37

$$de^{-\frac{4a}{b}}\left(\frac{\sqrt{b}\left(-\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)-\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+e^{\frac{4a}{b}}\left(\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\mathrm{Gamma}\right.\right.}{\sqrt{a+b\cosh^{-1}(cx)}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d*(2*E^((6*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
] + 2*E^((2*a)/b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
```

```
]] + (Sqrt[b]*(-(Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcC
osh[c*x]))/b]) - Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[
1/2, (-2*(a + b*ArcCosh[c*x]))/b] + E^((4*a)/b)*(8*c*x*((-1 + c*x)/(1 + c*x
))^(3/2)*(1 + c*x)^3 + Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1
/2, (2*(a + b*ArcCosh[c*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, (4*(a + b*ArcCosh[c*x]))/b])))/Sqrt[a + b*ArcCosh[c*x]]))/(4*b^(3/2)
*c^2*E^((4*a)/b))
```

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int x\left(-c^{2}d x^{2}+d\right)\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(c^{2}d x^{2}-d\right)x}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*d*x^2 - d)*x/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d\left(\int -\frac{x}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx+\int \frac{c^2x^3}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(-x/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*acosh(
c*x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*aco
sh(c*x))*acosh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.372 $\int \frac{d-c^2dx^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=233

$$\frac{3\sqrt{\pi}de^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}de^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{3\sqrt{\pi}de^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}de^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\cdots}{\cdots}\right.}{4b^{3/2}c}$$

```
[Out] (2*d*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (3*
d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c) - (
d*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(
4*b^(3/2)*c) + (3*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(
3/2)*c*E^(a/b)) - (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]])/(4*b^(3/2)*c*E^((3*a)/b))
```

Rubi [A] time = 0.722464, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5695, 5781, 5448, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}de^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}de^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{3\sqrt{\pi}de^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}de^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\cdots}{\cdots}\right.}{4b^{3/2}c}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (2*d*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (3*
d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c) - (
d*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(
4*b^(3/2)*c) + (3*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(
3/2)*c*E^(a/b)) - (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]])/(4*b^(3/2)*c*E^((3*a)/b))
```

Rule 5695

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((-d)^p*(-1 + c*x)^(p + 1/2)*(1 + c*x)^(p + 1/2)*(a + b*Ar
```

```
cCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(-d)^p*(2*p + 1))/(b*(n +
1)), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n
, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{d-c^2dx^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6cd)\int \frac{x\sqrt{-1+cx}\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6d)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6d)\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}}+\frac{\cosh(3x)}{4\sqrt{a+bx}}\right)dx, x, \cosh^{-1}(cx)\right)}{bc}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(3d)\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{2bc} - \frac{(3d)\,\text{Subst}\left(\int \frac{\cosh}{\sqrt{a}}\right.}{}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(3d)\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4bc} + \frac{(3d)\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+b}}\right.}{4}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(3d)\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{2b^2c} + \frac{(3d)\,\text{Subs}}{}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{3de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{4b^{3/2}c}$$

Mathematica [A] time = 1.61916, size = 246, normalized size = 1.06

$$\frac{e^{-\frac{3a}{b}}\left(-3de^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)-\sqrt{3}d\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+de^{\frac{2a}{b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-3*d*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] -
 Sqrt[3]*d*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*
x]))/b] + d*E^((2*a)/b)*(3*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a
+ b*ArcCosh[c*x])/b)] + E^(a/b)*(-6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) +
Sqrt[3]*E^((3*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c
*x]))/b] + 2*Sinh[3*ArcCosh[c*x]])))/(4*b*c*E^((3*a)/b)*Sqrt[a + b*ArcCosh[
c*x]])
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int \left(-c^{2} d \,x^{2}+d \right) \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c^{2} d x^{2} - d}{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*d*x^2 - d)/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d\left(\int \frac{c^2x^2}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\,dx+\int -\frac{1}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(c**2*x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))
*acosh(c*x)), x) + Integral(-1/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*aco
sh(c*x))*acosh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.373 $\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2d\text{Unintegrable}\left(\frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}},x\right)}{bc}-\frac{\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{2}{b}$$

```
[Out] (2*d*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*x*Sqrt[a + b*ArcCosh[c*x]]) - (
d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/b
^(3/2) - (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(b
^(3/2)*E^((2*a)/b)) + (2*d*Unintegrable[1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*Sqrt[a + b*ArcCosh[c*x]]), x])/(b*c)
```

Rubi [A] time = 1.6619, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] (-2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2))/(b*c*x*Sqrt[a + b*ArcCosh
[c*x]]) - (d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/
Sqrt[b]])/b^(3/2) - (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/S
qrt[b]])/(b^(3/2)*E^((2*a)/b)) + (2*d*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]]), x])/(b*c)
```

Rubi steps

$$\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^2\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} - \frac{(4cd)\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(4d)\,\mathrm{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{b} - \frac{(2d)\int \Big(}{}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\,\mathrm{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx, x, \cosh^{-1}(cx)\right)}{b}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{b} + \frac{(2d)\int}{}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{b} - \frac{d\,\mathrm{Subst}\Big(\int}{}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\,\mathrm{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2} - $$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} - \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\Big(\frac{\sqrt{2}\sqrt{}}{}}{b^{3/2}}$$

Mathematica [A] time = 4.12511, size = 0, normalized size = 0.

$$\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Maple [A] time = 0.222, size = 0, normalized size = 0.

$$\int \frac{-c^2 d x^2 + d}{x} (a + b \operatorname{arccosh}(cx))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c^2 d x^2 - d}{(b \operatorname{arcosh}(cx) + a)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*d*x^2 - d)/((b*arccosh(c*x) + a)^(3/2)*x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-d\left(\int \frac{c^2 x^2}{ax\sqrt{a + b \operatorname{acosh}(cx)} + bx\sqrt{a + b \operatorname{acosh}(cx)} \operatorname{acosh}(cx)}\, dx + \int -\frac{1}{ax\sqrt{a + b \operatorname{acosh}(cx)} + bx\sqrt{a + b \operatorname{acosh}(cx)} \operatorname{aco}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)/x/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(c**2*x**2/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a + b*acosh(c
*x))*acosh(c*x)), x) + Integral(-1/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a
 + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.374 $\int \frac{x^3(d-c^2dx^2)^2}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=479

$$-\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}d^2e^{\frac{8a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\mathrm{Erf}\left(\right.}{32b}$$

```
[Out] (-2*d^2*x^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]
) - (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(3
2*b^(3/2)*c^4) + (3*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcC
osh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((8*a)/b)*Sqrt[Pi/2]*Erf[(2*
Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) - (d^2*E^((6*a
)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^
(3/2)*c^4) - (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*
b^(3/2)*c^4*E^((4*a)/b)) + (3*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCo
sh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((2*a)/b)) + (d^2*Sqrt[Pi/2]*Erfi[(2*
Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((8*a)/b)) -
(d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b
^(3/2)*c^4*E^((6*a)/b))
```

Rubi [A] time = 2.13268, antiderivative size = 491, normalized size of antiderivative = 1.03, number of steps used = 32, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5776, 5781, 5448, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}d^2e^{\frac{8a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\mathrm{Erf}\left(\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^2)/(b*c*Sqrt[a + b*A
rcCosh[c*x]]) - (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/
Sqrt[b]])/(32*b^(3/2)*c^4) + (3*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqr
t[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((8*a)/b)*Sqrt[P
```

```
i/2]*Erf[(2*Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) -
(d^2*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt
[b]])/(32*b^(3/2)*c^4) - (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(32*b^(3/2)*c^4*E^((4*a)/b)) + (3*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((2*a)/b)) + (d^2*Sqrt[Pi
/2]*Erfi[(2*Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^(
(8*a)/b)) - (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]])/(32*b^(3/2)*c^4*E^((6*a)/b))
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^3 \left(d-c^2 d x^2\right)^2}{\left(a+b \cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(6d^2\right)\int \frac{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} + \frac{\left(16cd^2\right)\int \frac{x^4(-1+cx}{\sqrt{a+b\,c}}}{b}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(6d^2\right)\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc^4}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(6d^2\right)\text{Subst}\left(\int \left(\frac{1}{16\sqrt{a+bx}} - \frac{\cosh(2x)}{32\sqrt{a+bx}} - \frac{\cosh(4x)}{16\sqrt{a+bx}} + \frac{c}{3}\right.\right.}{bc^4}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(8x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8bc^4} + \frac{\left(3d^2\right)\text{S}}{}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{-8x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{16bc^4} + \frac{d^2\,\text{Subst}}{}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int e^{\frac{8a}{b}-\frac{8x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{8b^2c^4} +$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{}}{}\right.}{32b^{3/}}$$

<u>Mathematica</u> [A] time = 3.58386, size = 527, normalized size = 1.1

$$-\frac{d^2e^{-\frac{8a}{b}}\left(-\sqrt{2}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{8\left(a+b\cosh^{-1}(cx)\right)}{b}\right) + \sqrt{6}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right) + 2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] -(d^2*(128*c^3*E^((8*a)/b)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 128*c^4*E^((8*a
)/b)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - Sqrt[2]*Sqrt[-((a + b*ArcCosh[c*x])/b
```

```
)]*Gamma[1/2, (-8*(a + b*ArcCosh[c*x]))/b] + Sqrt[6]*E^((2*a)/b)*Sqrt[-((a
+ b*ArcCosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCosh[c*x]))/b] + 2*E^((4*a)/
b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c*x]))/b]
- 3*Sqrt[2]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-2*(a +
 b*ArcCosh[c*x]))/b] + 3*Sqrt[2]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, (2*(a + b*ArcCosh[c*x]))/b] - 2*E^((12*a)/b)*Sqrt[a/b + ArcCosh[c*x]
]*Gamma[1/2, (4*(a + b*ArcCosh[c*x]))/b] - Sqrt[6]*E^((14*a)/b)*Sqrt[a/b +
ArcCosh[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x]))/b] + Sqrt[2]*E^((16*a)/b)
*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (8*(a + b*ArcCosh[c*x]))/b] - 26*E^((8
*a)/b)*Sinh[2*ArcCosh[c*x]] - 18*E^((8*a)/b)*Sinh[4*ArcCosh[c*x]] - 2*E^((8
*a)/b)*Sinh[6*ArcCosh[c*x]] + E^((8*a)/b)*Sinh[8*ArcCosh[c*x]]))/(64*b*c^4*
E^((8*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.396, size = 0, normalized size = 0.

$$\int x^{3}\left(-c^{2}d x^{2}+d\right)^{2}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^{2}dx^{2}-d\right)^{2}x^{3}}{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x^3/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{x^3}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\,dx+\int -\frac{2c^2x^5}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*ac
osh(c*x)), x) + Integral(-2*c**2*x**5/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a
+ b*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*acosh(c*
x)) + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.375 $\int \frac{x^2(d-c^2dx^2)^2}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=462

$$\frac{5\sqrt{\pi}d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{7\pi}d^2e^{\frac{7a}{b}}\text{Erf}\left(\right.}{64b}$$

```
[Out] (-2*d^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]
) + (5*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(64*b^(3
/2)*c^3) + (d^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]
])/Sqrt[b]])/(64*b^(3/2)*c^3) - (3*d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*
Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) + (d^2*E^((7*a)/b)*Sqr
t[7*Pi]*Erf[(Sqrt[7]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) +
 (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3*E^
(a/b)) + (d^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(64*b^(3/2)*c^3*E^((3*a)/b)) - (3*d^2*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*A
rcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((5*a)/b)) + (d^2*Sqrt[7*Pi]*Erfi
[(Sqrt[7]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((7*a)/b))
```

Rubi [A] time = 2.26657, antiderivative size = 474, normalized size of antiderivative = 1.03, number of steps used = 42, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5776, 5781, 5448, 3307, 2180, 2204, 2205}

$$\frac{5\sqrt{\pi}d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{7\pi}d^2e^{\frac{7a}{b}}\text{Erf}\left(\right.}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^2)/(b*c*Sqrt[a + b*A
rcCosh[c*x]]) + (5*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b
]])/(64*b^(3/2)*c^3) + (d^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*
ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (3*d^2*E^((5*a)/b)*Sqrt[5*Pi]*E
rf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) + (d^2*E^(
```

```
(7*a)/b)*Sqrt[7*Pi]*Erf[(Sqrt[7]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^
(3/2)*c^3) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(64*b^
(3/2)*c^3*E^(a/b)) + (d^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]]
)/Sqrt[b]])/(64*b^(3/2)*c^3*E^((3*a)/b)) - (3*d^2*Sqrt[5*Pi]*Erfi[(Sqrt[5]*
Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((5*a)/b)) + (d^2*Sqr
t[7*Pi]*Erfi[(Sqrt[7]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E
^((7*a)/b))
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
```

```
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2\left(d-c^2dx^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(4d^2\right)\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} + \frac{\left(14cd^2\right)\int \frac{x^3(-1+cx)}{\sqrt{a+b\cosh}}}{b}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(4d^2\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc^3} +$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(4d^2\right)\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{a+bx}} - \frac{3\cosh(3x)}{16\sqrt{a+bx}} + \frac{\cosh(5x)}{16\sqrt{a+bx}}\right) dx}{bc^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{32bc^3} + \frac{\left(7d^2\right.}{}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{e^{-7x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{64bc^3} - \frac{\left(7d^2\right)}{}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(7d^2\right)\text{Subst}\left(\int e^{\frac{7a}{b}-\frac{7x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{32b^2c^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\right.}{64b^{3}}$$

Mathematica [A] time = 2.9055, size = 498, normalized size = 1.08

$$-\frac{d^2e^{-\frac{7a}{b}}\left(5e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)-\sqrt{7}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{7\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+3\sqrt{5}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] -(d^2*(10*E^((7*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] + 10*c*E^((7*a)/b)*x*Sqrt[
(-1 + c*x)/(1 + c*x)] + 5*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a
```

```
/b + ArcCosh[c*x]] - Sqrt[7]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-7
*(a + b*ArcCosh[c*x]))/b] + 3*Sqrt[5]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x
])/b)]*Gamma[1/2, (-5*(a + b*ArcCosh[c*x]))/b] - Sqrt[3]*E^((4*a)/b)*Sqrt[-
((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] - 5*E^((6
*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)
] + Sqrt[3]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcC
osh[c*x]))/b] - 3*Sqrt[5]*E^((12*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2,
(5*(a + b*ArcCosh[c*x]))/b] + Sqrt[7]*E^((14*a)/b)*Sqrt[a/b + ArcCosh[c*x]]
*Gamma[1/2, (7*(a + b*ArcCosh[c*x]))/b] + 2*E^((7*a)/b)*Sinh[3*ArcCosh[c*x]
] - 6*E^((7*a)/b)*Sinh[5*ArcCosh[c*x]] + 2*E^((7*a)/b)*Sinh[7*ArcCosh[c*x]]
))/(64*b*c^3*E^((7*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.487, size = 0, normalized size = 0.

$$\int x^2 \left(-c^2 d x^2 + d\right)^2 (a + b\operatorname{arccosh}(cx))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2 d x^2 - d\right)^2 x^2}{(b \operatorname{arcosh}(cx) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{x^2}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx+\int -\frac{2c^2x^4}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*ac
osh(c*x)), x) + Integral(-2*c**2*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a
+ b*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*acosh(c*
x)) + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.376 $\int \frac{x(d-c^2dx^2)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=363

$$-\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\operatorname{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}-\frac{\sqrt{\pi}d^2e^{-\frac{4a}{b}}\operatorname{Erfi}(}{4b^{3}}$$

```
[Out] (-2*d^2*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]])
- (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b
^(3/2)*c^2) + (5*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) + (d^2*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(S
qrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) - (d^2*Sqrt[Pi]
*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) +
(5*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(
3/2)*c^2*E^((2*a)/b)) + (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCos
h[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2*E^((6*a)/b))
```

Rubi [A] time = 1.77547, antiderivative size = 375, normalized size of antiderivative = 1.03, number of steps used = 32, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5776, 5701, 3312, 3307, 2180, 2204, 2205, 5781, 5448}

$$-\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\operatorname{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}-\frac{\sqrt{\pi}d^2e^{-\frac{4a}{b}}\operatorname{Erfi}(}{4b^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^2)/(b*c*Sqrt[a + b*Arc
Cosh[c*x]]) - (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(4*b^(3/2)*c^2) + (5*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a
 + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) + (d^2*E^((6*a)/b)*Sqrt[(3*P
i)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) - (
d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^(
(4*a)/b)) + (5*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
```

```
b]])/(16*b^(3/2)*c^2*E^((2*a)/b)) + (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[
a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2*E^((6*a)/b))
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(d-c^2dx^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(2d^2\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} + \frac{\left(12cd^2\right)\int \frac{x^2(-1+cx)^{3/2}}{\sqrt{a+b\cosh}}}{b}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(2d^2\right)\operatorname{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc^2} + \frac{\left(12d^2\right.}{}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(2d^2\right)\operatorname{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}} + \frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx, x\right.}{bc^2}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2\operatorname{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4bc^2} - \frac{\left(3d^2\right)\operatorname{Su}}{}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2\operatorname{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8bc^2} - \frac{d^2\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2\operatorname{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{4b^2c^2} - \frac{d}{}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}}{}\right.}{16b^{3/2}}$$

Mathematica [A] time = 6.96287, size = 508, normalized size = 1.4

$$\frac{d^2e^{-\frac{6a}{b}}\left(\frac{\sqrt{b}\left(\sqrt{6}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\operatorname{Gamma}\left(\frac{1}{2},-\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)-8e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\operatorname{Gamma}\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)-11\sqrt{2}e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\operatorname{Gamm}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d^2*(16*E^((8*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt
[b]] + 16*E^((4*a)/b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sq
```

```
rt[b]] + (Sqrt[b]*(128*c^3*E^((6*a)/b)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 128
*c^4*E^((6*a)/b)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] + Sqrt[6]*Sqrt[-((a + b*Arc
Cosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCosh[c*x]))/b] - 8*E^((2*a)/b)*Sqrt
[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c*x]))/b] - 11*Sq
rt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-2*(a + b*Arc
Cosh[c*x]))/b] + 11*Sqrt[2]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2,
 (2*(a + b*ArcCosh[c*x]))/b] + 8*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, (4*(a + b*ArcCosh[c*x]))/b] - Sqrt[6]*E^((12*a)/b)*Sqrt[a/b + ArcCos
h[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x]))/b] - 42*E^((6*a)/b)*Sinh[2*ArcC
osh[c*x]] - 8*E^((6*a)/b)*Sinh[4*ArcCosh[c*x]] - 2*E^((6*a)/b)*Sinh[6*ArcCo
sh[c*x]]))/Sqrt[a + b*ArcCosh[c*x]]))/(32*b^(3/2)*c^2*E^((6*a)/b))
```

Maple [F] time = 0.318, size = 0, normalized size = 0.

$$\int x\left(-c^2dx^2+d\right)^2 (a+b\operatorname{arccosh}(cx))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2-d\right)^2 x}{(b\operatorname{arcosh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2 \left(\int \frac{x}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(cx)}\,dx + \int -\frac{2c^2x^3}{a\sqrt{a+b\,\text{acosh}\,(cx)}+b\sqrt{a+b\,\text{acosh}\,(cx)}\,\text{acosh}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*acosh
(c*x)), x) + Integral(-2*c**2*x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b
*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*acosh(c*x))
 + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.377 $\int \frac{(d-c^2dx^2)^2}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=351

$$\frac{5\sqrt{\pi}d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{5\sqrt{\pi}d^2e^{-\frac{a}{b}}\text{Erfi}}{8b}$$

```
[Out] (-2*d^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) +
(5*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c
) - (5*d^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(16*b^(3/2)*c) + (d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a +
b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c*E^(a/b)) - (5*d^2*Sqrt[3*Pi]*Erfi[(Sq
rt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((3*a)/b)) + (d^2
*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c
*E^((5*a)/b))
```

Rubi [A] time = 0.924336, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5695, 5781, 5448, 3307, 2180, 2204, 2205}

$$\frac{5\sqrt{\pi}d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{5\sqrt{\pi}d^2e^{-\frac{a}{b}}\text{Erfi}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^2/(a + b*ArcCosh[c*x])^(3/2),x]
```


```
[Out] (-2*d^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) +
(5*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c
) - (5*d^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(16*b^(3/2)*c) + (d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a +
b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c*E^(a/b)) - (5*d^2*Sqrt[3*Pi]*Erfi[(Sq
rt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((3*a)/b)) + (d^2
```

```
*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c
*E^((5*a)/b))
```

Rule 5695

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((-d)^p*(-1 + c*x)^(p + 1/2)*(1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(-d)^p*(2*p + 1))/(b*(n +
1)), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n
, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(10cd^2\right)\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(10d^2\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(10d^2\right)\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{a+bx}} - \frac{3\cosh(3x)}{16\sqrt{a+bx}} + \frac{\cosh(5x)}{16\sqrt{a+bx}}\right) dx, x, \cosh^{-1}\right.}{bc}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8bc} + \frac{\left(5d^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{16bc} + \frac{\left(5d^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(5d^2\right)\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{8b^2c} + \frac{\left(5d^2\right)\text{S}}{}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh}}{\sqrt{b}}\right.}{16b^{3/2}c}$$

Mathematica [A] time = 1.93068, size = 387, normalized size = 1.1

$$-\frac{d^2e^{-\frac{5a}{b}}\left(10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)-\sqrt{5}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+5\sqrt{\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^2/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] -(d^2*(20*E^((5*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] + 20*c*E^((5*a)/b)*x*Sqrt[
(-1 + c*x)/(1 + c*x)] + 10*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2,
a/b + ArcCosh[c*x]] - Sqrt[5]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-
5*(a + b*ArcCosh[c*x]))/b] + 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*
x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] - 10*E^((4*a)/b)*Sqrt[-((a
+ b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - 5*Sqrt[3]*E^(
(8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] +
Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a + b*ArcCosh[
c*x]))/b] - 10*E^((5*a)/b)*Sinh[3*ArcCosh[c*x]] + 2*E^((5*a)/b)*Sinh[5*ArcC
osh[c*x]]))/(16*b*c*E^((5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.28, size = 0, normalized size = 0.

$$\int \left(-c^2dx^2+d\right)^2 (a+b\operatorname{arccosh}(cx))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2-d\right)^2}{(b\operatorname{arcosh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 - d)^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int -\frac{2c^2x^2}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\,dx+\int \frac{c^4x^4}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(-2*c**2*x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(
c*x))*acosh(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sq
rt(a + b*acosh(c*x))*acosh(c*x)), x) + Integral(1/(a*sqrt(a + b*acosh(c*x))
 + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.378 $\int \frac{(d-c^2dx^2)^2}{x(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=288

$$\frac{2d^2\text{Unintegrable}\left(\frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}},x\right)}{bc}+\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}}+$$

```
[Out] (-2*d^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*x*Sqrt[a + b*ArcCosh[c*x]])
+ (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b
^(3/2)) - (3*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(2*b^(3/2)) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])
/Sqrt[b]])/(4*b^(3/2)*E^((4*a)/b)) - (3*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a
 + b*ArcCosh[c*x]])/Sqrt[b]])/(2*b^(3/2)*E^((2*a)/b)) + (2*d^2*Unintegrable
[1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]]), x])/(b*c)
```

Rubi [A] time = 2.49589, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\cosh^{-1}(cx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] (-2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^2)/(b*c*x*Sqrt[a + b*Arc
Cosh[c*x]]) + (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(4*b^(3/2)) + (d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*A
rcCosh[c*x]])/Sqrt[b]])/(2*b^(3/2)) - (d^2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt
[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/b^(3/2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqr
t[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2)*E^((4*a)/b)) + (d^2*Sqrt[Pi/2]*
Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(2*b^(3/2)*E^((2*a)/b)) -
 (d^2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(b^(3/2)
*E^((2*a)/b)) + (2*d^2*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[
a + b*ArcCosh[c*x]]), x])/(b*c)
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(2d^2\right)\int\frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^2\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} + \frac{\left(8cd^2\right)\int\frac{(-1+cx)^{3/2}(1+c}{\sqrt{a+b\cosh^{-1}(}}}{b}$$

$$= -\frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(8d^2\right)\text{Subst}\left(\int\frac{\sinh^4(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{b} + \frac{\left(2d^2\right)}{}$$

$$= -\frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(8d^2\right)\text{Subst}\left(\int\left(\frac{3}{8\sqrt{a+bx}}-\frac{\cosh(2x)}{2\sqrt{a+bx}}+\frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx,x,}{b}$$

$$= -\frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\cosh^{-1}(cx)}}{b^2} + \frac{d^2\,\text{Subst}\left(\int\frac{\cosh(4x)}{\sqrt{a+bx}}\,dx,x,\right.}{b}$$

$$= -\frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\cosh^{-1}(cx)}}{b^2} + \frac{d^2\,\text{Subst}\left(\int\frac{e^{-4x}}{\sqrt{a+bx}}\,dx,x,\text{c}\right.}{2b}$$

$$= -\frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2} + \frac{d^2}{}$$

$$= -\frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} - \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{b^{3/2}}$$

$$= -\frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} - \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{b^{3/2}}$$

$$= -\frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} + \frac{d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a}}{}\right.}{2b^{3/2}}$$

Mathematica [A] time = 3.08532, size = 0, normalized size = 0.

$$\int \frac{\left(d-c^2dx^2\right)^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Maple [A] time = 0.326, size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^2}{x}\left(a+b\operatorname{arccosh}(cx)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2-d\right)^2}{\left(b\operatorname{arcosh}(cx)+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c^2*d*x^2 - d)^2/((b*arccosh(c*x) + a)^(3/2)*x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int -\frac{2c^2x^2}{ax\sqrt{a+b\operatorname{acosh}(cx)}+bx\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx+\int \frac{c^4x^4}{ax\sqrt{a+b\operatorname{acosh}(cx)}+bx\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2/x/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(-2*c**2*x**2/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a + b*ac
osh(c*x))*acosh(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*acosh(c*x))
+ b*x*sqrt(a + b*acosh(c*x))*acosh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*a
cosh(c*x)) + b*x*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.379 $\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=351

$$-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\operatorname{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] (3*c*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/8 + (x*(c - a^2*c*x^2)^(3/2)
*Sqrt[ArcCosh[a*x]])/4 - (c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(4*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[Ar
cCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (c*Sqrt[Pi/2]*Sqrt[c -
 a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]) + (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(256*a*
Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt
[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.723019, antiderivative size = 363, normalized size of antiderivative = 1.03, number of steps used = 25, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5713, 5685, 5683, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205, 5780}

$$-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\operatorname{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (3*c*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/8 + (c*x*(1 - a*x)*(1 + a*x)
*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/4 - (c*Sqrt[c - a^2*c*x^2]*ArcCosh
[a*x]^(3/2))/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi]*Sqrt[c - a^2*
c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (c
*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]) + (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[Arc
Cosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi/2]*Sqrt[c -
a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 +
a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
```

```
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\sqrt{\cosh^{-1}(ax)}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int\sqrt{-1+ax}\sqrt{1+ax}\sqrt{}}{4\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\left(3c\sqrt{c}\right.}{}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{c\sqrt{c-}}{4a\sqrt{}}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{c\sqrt{c-}}{4a\sqrt{}}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{c\sqrt{c-}}{4a\sqrt{}}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{c\sqrt{c-}}{4a\sqrt{}}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{c\sqrt{c-}}{4a\sqrt{}}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{c\sqrt{c-}}{4a\sqrt{}}$$

Mathematica [A] time = 0.239761, size = 154, normalized size = 0.44

$$-\frac{c\sqrt{c-a^2cx^2}\left(-\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2},-4\cosh^{-1}(ax)\right)+8\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2},-2\cosh^{-1}(ax)\right)+\sqrt{\cosh}\right.}{128a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{\cosh^{-1}(a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -(c*Sqrt[c - a^2*c*x^2]*(-(Sqrt[-ArcCosh[a*x]]*Gamma[3/2, -4*ArcCosh[a*x]])
 + 8*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[3/2, -2*ArcCosh[a*x]] + Sqrt[ArcCosh
[a*x]]*(32*ArcCosh[a*x]^(3/2) + 8*Sqrt[2]*Gamma[3/2, 2*ArcCosh[a*x]] - Gamm
a[3/2, 4*ArcCosh[a*x]])))/(128*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[
ArcCosh[a*x]])
```

Maple [F] time = 0.354, size = 0, normalized size = 0.

$$\int \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\operatorname{arccosh}\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arcosh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.380 $\int \sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c-a}$$

```
[Out] (x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh
[a*x]^(3/2))/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*
c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
 - (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*
Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.374543, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5713, 5683, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c-a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh
[a*x]^(3/2))/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*
c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
 - (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*
Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}\,dx = \frac{\sqrt{c-a^2cx^2}\int \sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(a\sqrt{c-a^2cx^2}\right)\int \frac{x}{\sqrt{\cosh^{-1}(a}}}{4\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int \frac{\cosh(x)\mathrm{si}}{\sqrt{x}}\right.}{4a\sqrt{-1+ax}\sqrt{}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\right.}{4a\sqrt{-1+ax}\sqrt{1}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\right.}{8a\sqrt{-1+ax}\sqrt{1}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,\right.}{16a\sqrt{-1+ax}\sqrt{1+}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int e^{-2x^2}\,dx\right.}{8a\sqrt{-1+ax}\sqrt{1}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\mathrm{erf}\left(\sqrt{2}\sqrt{\cosh}\right.}{16a\sqrt{-1+ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.132218, size = 117, normalized size = 0.57

$$-\frac{\sqrt{-c(ax-1)(ax+1)}\left(3\sqrt{2}\sqrt{\cosh^{-1}(ax)}\mathrm{Gamma}\left(\frac{3}{2},2\cosh^{-1}(ax)\right)+3\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\mathrm{Gamma}\left(\frac{3}{2},-2\cosh^{-1}(ax)\right)+\right.}{48a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(16*ArcCosh[a*x]^2 + 3*Sqrt[2]*Sqrt[-ArcC
osh[a*x]]*Gamma[3/2, -2*ArcCosh[a*x]] + 3*Sqrt[2]*Sqrt[ArcCosh[a*x]]*Gamma[
3/2, 2*ArcCosh[a*x]]))/(48*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcC
osh[a*x]])
```

Maple [F] time = 0.526, size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\sqrt{\operatorname{arccosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\sqrt{\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c(ax-1)(ax+1)}\sqrt{\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*acosh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.381 $\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(3*a*Sqrt[c - a^2*c*x^2
])
```

Rubi [A] time = 0.163926, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcCosh[a*x]]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(3*a*Sqrt[c - a^2*c*x^2
])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0316465, size = 48, normalized size = 1.

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcCosh[a*x]]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(3*a*Sqrt[c - a^2*c*x^2
])
```

Maple [A] time = 0.05, size = 41, normalized size = 0.9

$$\frac{2}{3\,a}\left(\mathrm{arccosh}\left(ax\right)\right)^{\frac{3}{2}}\sqrt{ax-1}\sqrt{ax+1}\frac{1}{\sqrt{-\left(ax-1\right)\left(ax+1\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/3*arccosh(a*x)^(3/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\mathrm{arcosh}\left(ax\right)}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x))/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{acosh}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(acosh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.382 $\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{a\sqrt{ax-1}\sqrt{ax+1}\,\text{Unintegrable}\left(\frac{x}{(1-a^2x^2)\sqrt{\cosh^{-1}(ax)}},x\right)}{2c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{\cosh^{-1}(ax)}}{c\sqrt{c-a^2cx^2}}$$

```
[Out] (x*Sqrt[ArcCosh[a*x]])/(c*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*Unintegrable[x/((1 - a^2*x^2)*Sqrt[ArcCosh[a*x]]), x])/(2*c*Sqrt[c -
a^2*c*x^2])
```

Rubi [A] time = 0.243686, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcCosh[a*x]])/(c*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*Defer[Int][x/((1 - a^2*x^2)*Sqrt[ArcCosh[a*x]]), x])/(2*c*Sqrt[c - a^
2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{x\sqrt{\cosh^{-1}(ax)}}{c\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x}{(1-a^2x^2)\sqrt{\cosh^{-1}(ax)}}\,dx}{2c\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 1.58432, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 0.336, size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{arccosh}(ax)}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arcosh}(ax)}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x))/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{acosh}(ax)}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(acosh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.383 $\int \frac{\sqrt{\cosh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=192

$$\frac{a\sqrt{ax-1}\sqrt{ax+1}\text{Unintegrable}\left(\frac{x}{(1-a^2x^2)\sqrt{\cosh^{-1}(ax)}},x\right)}{3c^2\sqrt{c-a^2cx^2}}+\frac{a\sqrt{ax-1}\sqrt{ax+1}\text{Unintegrable}\left(\frac{x}{(a^2x^2-1)^2\sqrt{\cosh^{-1}(ax)}},x\right)}{6c^2\sqrt{c-a^2cx^2}}+\frac{2x}{3c}$$

```
[Out] (x*Sqrt[ArcCosh[a*x]])/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x*Sqrt[ArcCosh[a*x]
])/(3*c^2*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Unintegrab
le[x/((1 - a^2*x^2)*Sqrt[ArcCosh[a*x]]), x])/(3*c^2*Sqrt[c - a^2*c*x^2]) +
(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Unintegrable[x/((-1 + a^2*x^2)^2*Sqrt[ArcCo
sh[a*x]]), x])/(6*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.432165, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (2*x*Sqrt[ArcCosh[a*x]])/(3*c^2*Sqrt[c - a^2*c*x^2]) + (x*Sqrt[ArcCosh[a*x]
])/(3*c^2*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*Defer[Int][x/((1 - a^2*x^2)*Sqrt[ArcCosh[a*x]]), x])/(3*c^2*Sqrt[
c - a^2*c*x^2]) + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2*x
^2)^2*Sqrt[ArcCosh[a*x]]), x])/(6*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{x\sqrt{\cosh^{-1}(ax)}}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}} - \frac{\left(2\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{3c^2\sqrt{c-a^2cx^2}} + \frac{\left(a\sqrt{-1+ax}\sqrt{1+a\right.}{6c^2}$$

$$= \frac{2x\sqrt{\cosh^{-1}(ax)}}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\sqrt{\cosh^{-1}(ax)}}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\left(a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x}{\left(-1+a^2x^2\right)^2\sqrt{\cosh^{-1}(ax)}}\,d}{6c^2\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 2.12787, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(5/2), x]
```

Maple [A] time = 0.376, size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{arccosh}(ax)}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arcosh}(ax)}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x))/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.384 $\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=511

$$-\frac{3\sqrt{\pi}c\sqrt{c-a^2cx^2}\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a\sqrt{ax-1}\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}-\frac{3\sqrt{\pi}c\sqrt{c-a^2cx^2}\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] (27*c*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]) - (9*a*c*x^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(32*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]) + (3*c*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh
[a*x]])/(32*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt[c - a^2*c*x^2]*Ar
cCosh[a*x]^(3/2))/8 + (x*(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2))/4 - (3*c
*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/(20*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
) - (3*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(2048*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[
2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (3*c*Sqrt[Pi]
*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(2048*a*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]) + (3*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh
[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 1.09759, antiderivative size = 523, normalized size of antiderivative = 1.02, number of steps used = 27, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {5713, 5685, 5683, 5676, 5664, 5781, 3312, 3307, 2180, 2204, 2205, 5716, 5701}

$$-\frac{3\sqrt{\pi}c\sqrt{c-a^2cx^2}\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a\sqrt{ax-1}\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}-\frac{3\sqrt{\pi}c\sqrt{c-a^2cx^2}\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (27*c*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]) - (9*a*c*x^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(32*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]) + (3*c*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh
[a*x]])/(32*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt[c - a^2*c*x^2]*Ar
cCosh[a*x]^(3/2))/8 + (c*x*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]*ArcCosh[
a*x]^(3/2))/4 - (3*c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/(20*a*Sqrt[-1
+ a*x]*Sqrt[1 + a*x]) - (3*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCos
```

```
h[a*x]]])/(2048*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*Sqrt[Pi/2]*Sqrt[c -
a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a
*x]) - (3*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(2048*
a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[
Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{3/2}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int\sqrt{-1+ax}\sqrt{1+ax}}{4\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} + \frac{1}{4}cx(1-ax$$

$$= -\frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c}$$

$$= -\frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c}$$

$$= -\frac{9c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-}}{32a\sqrt{-1+}}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-}}{32a\sqrt{-1+}}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-}}{32a\sqrt{-1+}}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-}}{32a\sqrt{-1+}}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-}}{32a\sqrt{-1+}}$$

Mathematica [A] time = 0.461414, size = 198, normalized size = 0.39

$$c\sqrt{c-a^2cx^2}\left(5\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\tfrac{5}{2},4\cosh^{-1}(ax)\right) - 5\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\tfrac{5}{2},-4\cosh^{-1}(ax)\right) + 60\sqrt{2\pi}\sqrt{\cosh^{-}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-384*ArcCosh[a*x]^3 - 480*ArcCosh[a*x]*Cosh[2*ArcCo
sh[a*x]] + 60*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]]
 + 60*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] - 5*Sq
rt[-ArcCosh[a*x]]*Gamma[5/2, -4*ArcCosh[a*x]] + 5*Sqrt[ArcCosh[a*x]]*Gamma[
5/2, 4*ArcCosh[a*x]] + 640*ArcCosh[a*x]^2*Sinh[2*ArcCosh[a*x]]))/(2560*a*Sq
rt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int \left(-a^2 c x^2 + c\right)^{\frac{3}{2}} (\operatorname{arccosh}(ax))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2 c x^2 + c\right)^{\frac{3}{2}} \operatorname{arcosh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.385 $\int \sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=302

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}{5a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c}$$

```
[Out] (3*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]) - (3*a*x^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(8*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]) + (x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/2 - (Sqrt[c - a^2
*c*x^2]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*Sqrt[Pi
/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]) + (3*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[Arc
Cosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.61828, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5713, 5683, 5676, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}{5a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2),x]
```


```
[Out] (3*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]) - (3*a*x^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(8*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]) + (x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/2 - (Sqrt[c - a^2
*c*x^2]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*Sqrt[Pi
/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]) + (3*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[Arc
Cosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
```

```
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} - \frac{\sqrt{c-a^2cx^2}\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(3a\sqrt{c-a^2cx^2}\right)\int x\sqrt{\cos}}{4\sqrt{-1+ax}\sqrt{1+}}$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)}{5a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)}{5a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)}{5a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3}$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3}$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3}$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3}$$

Mathematica [A] time = 0.393647, size = 136, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\left(15\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+15\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)-8\sqrt{\cosh^{-1}(ax)}\left(16\cosh^{-1}(ax)^2+15\cosh\left(\right.\right.\right.}{640a\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 15*Sq
rt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] - 8*Sqrt[ArcCosh[a*x]]*(16*ArcCos
```

```
h[a*x]^2 + 15*Cosh[2*ArcCosh[a*x]] - 20*ArcCosh[a*x]*Sinh[2*ArcCosh[a*x]]))
)/(640*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [F] time = 0.499, size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\,(\mathrm{arccosh}\,(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\,\mathrm{arcosh}\,(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.386 $\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[c - a^2*c*x^2
])
```

Rubi [A] time = 0.161481, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(3/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[c - a^2*c*x^2
])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0435926, size = 48, normalized size = 1.

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^(3/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[c - a^2*c*x^2
])
```

Maple [A] time = 0.047, size = 41, normalized size = 0.9

$$\frac{2}{5\,a}\left(\operatorname{arccosh}(ax)\right)^{\frac{5}{2}}\sqrt{ax-1}\sqrt{ax+1}\frac{1}{\sqrt{-(ax-1)(ax+1)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/5*arccosh(a*x)^(5/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}(ax)}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**(3/2)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.387 $\int \frac{\cosh^{-1}(ax)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{3a\sqrt{ax-1}\sqrt{ax+1}\,\text{Unintegrable}\left(\frac{x\sqrt{\cosh^{-1}(ax)}}{1-a^2x^2},x\right)}{2c\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)^{3/2}}{c\sqrt{c-a^2cx^2}}$$

```
[Out] (x*ArcCosh[a*x]^(3/2))/(c*Sqrt[c - a^2*c*x^2]) + (3*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*Unintegrable[(x*Sqrt[ArcCosh[a*x]])/(1 - a^2*x^2), x])/(2*c*Sqrt[c
- a^2*c*x^2])
```

Rubi [A] time = 0.231715, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^(3/2))/(c*Sqrt[c - a^2*c*x^2]) + (3*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*Defer[Int][(x*Sqrt[ArcCosh[a*x]])/(1 - a^2*x^2), x])/(2*c*Sqrt[c -
a^2*c*x^2])
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{3/2}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$=\frac{x\cosh^{-1}(ax)^{3/2}}{c\sqrt{c-a^2cx^2}}+\frac{\left(3a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\sqrt{\cosh^{-1}(ax)}}{1-a^2x^2}\,dx}{2c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 1.67427, size = 0, normalized size = 0.

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcCosh[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 0.302, size = 0, normalized size = 0.

$$\int (\operatorname{arccosh}(ax))^{\frac{3}{2}}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.388 $\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=580

$$-\frac{15\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{16384a\sqrt{ax-1}\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}+\frac{15\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erfi}\left(2\sqrt{\cosh}\right.}{16384a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] (225*c*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/512 + (15*c*x*(1 - a*x)*(1
 + a*x)*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/256 + (45*c*Sqrt[c - a^2*c*
x^2]*ArcCosh[a*x]^(3/2))/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*a*c*x^2
*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(32*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
+ (5*c*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(32*a*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/8
+ (x*(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2))/4 - (3*c*Sqrt[c - a^2*c*x^2]
*ArcCosh[a*x]^(7/2))/(28*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*c*Sqrt[Pi]*S
qrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(16384*a*Sqrt[-1 + a*x]*Sqrt[
1 + a*x]) + (15*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a
*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c*Sqrt[Pi]*Sqrt[c - a^2*c
*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(16384*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) -
(15*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(256
*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 1.52602, antiderivative size = 592, normalized size of antiderivative = 1.02, number of steps used = 40, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5713, 5685, 5683, 5676, 5664, 5759, 5670, 5448, 12, 3308, 2180, 2204, 2205, 5716, 5780}

$$-\frac{15\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{16384a\sqrt{ax-1}\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}+\frac{15\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erfi}\left(2\sqrt{\cosh}\right.}{16384a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (225*c*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/512 + (15*c*x*(1 - a*x)*(1
 + a*x)*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/256 + (45*c*Sqrt[c - a^2*c*
x^2]*ArcCosh[a*x]^(3/2))/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*a*c*x^2
*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(32*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
+ (5*c*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(32*a*Sqrt[-
```

```
1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/8
+ (c*x*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/4 - (3*c
*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(7/2))/(28*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
) - (15*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(16384*a*
Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sq
rt[2]*Sqrt[ArcCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c*Sqr
t[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(16384*a*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]) - (15*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[
ArcCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
```

```
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{5/2}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2} + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int\sqrt{-1+ax}\sqrt{1+a}}{4\sqrt{-1+ax}\sqrt{1+a}}$$

$$= \frac{5c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{32a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2} + \frac{1}{4}cx(1-$$

$$= \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{15acx^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{32\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} -$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} +$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} +$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} +$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} +$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} +$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} +$$

Mathematica [A] time = 0.484845, size = 213, normalized size = 0.37

$$c\sqrt{c-a^2cx^2}\left(7\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},4\cosh^{-1}(ax)\right) + 7\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-4\cosh^{-1}(ax)\right) + 420\sqrt{2\pi}\sqrt{\cos}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-1536*ArcCosh[a*x]^4 - 4480*ArcCosh[a*x]^2*Cosh[2*A
rcCosh[a*x]] + 420*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a
*x]]] - 420*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]
+ 7*Sqrt[-ArcCosh[a*x]]*Gamma[7/2, -4*ArcCosh[a*x]] + 7*Sqrt[ArcCosh[a*x]]*
Gamma[7/2, 4*ArcCosh[a*x]] + 3360*ArcCosh[a*x]*Sinh[2*ArcCosh[a*x]] + 3584*
ArcCosh[a*x]^3*Sinh[2*ArcCosh[a*x]]))/(14336*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(
1 + a*x)*Sqrt[ArcCosh[a*x]])
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int \left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \left(\operatorname{arccosh}\left(a x\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} \operatorname{arcosh}\left(a x\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.389 $\int \sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=330

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{7/2}}{7a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c$$

```
[Out] (15*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/32 + (5*Sqrt[c - a^2*c*x^2]*A
rcCosh[a*x]^(3/2))/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (5*a*x^2*Sqrt[c -
a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Sqrt[c
 - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(7/
2))/(7*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]
*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (1
5*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(256*a*S
qrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.707116, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5713, 5683, 5676, 5664, 5759, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{7/2}}{7a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2),x]
```


```
[Out] (15*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/32 + (5*Sqrt[c - a^2*c*x^2]*A
rcCosh[a*x]^(3/2))/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (5*a*x^2*Sqrt[c -
a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Sqrt[c
 - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(7/
2))/(7*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]
*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (1
5*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(256*a*S
qrt[-1 + a*x]*Sqrt[1 + a*x])
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
```

```
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
```

```
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2} - \frac{\sqrt{c-a^2cx^2}\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(5a\sqrt{c-a^2cx^2}\right)\int x\cos}{4\sqrt{-1+ax}\sqrt{1}}$$

$$= -\frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(}{7a\sqrt{-1+ax}\sqrt{1+}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(}{8\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(}{8\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(}{8\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(}{8\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(}{8\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(}{8\sqrt{-1+ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.476998, size = 148, normalized size = 0.45

$$-\frac{\sqrt{-c(ax-1)(ax+1)}\left(-105\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+105\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+8\sqrt{\cosh^{-1}(ax)}\left(64\cosh^{-1}\right.\right.}{3584a\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2),x]
```

```
[Out] -(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(-105*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh
[a*x]]] + 105*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 8*Sqrt[ArcCosh[
a*x]]*(64*ArcCosh[a*x]^3 + 140*ArcCosh[a*x]*Cosh[2*ArcCosh[a*x]] - 7*(15 +
16*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]])))/(3584*a*Sqrt[(-1 + a*x)/(1 + a*x
)]*(1 + a*x))
```

Maple [F] time = 0.51, size = 0, normalized size = 0.

$$\int \sqrt{-a^{2} c x^{2}+c}\, (\operatorname{arccosh}(a x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^{2} c x^{2}+c}\, \operatorname{arcosh}(a x)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.390 $\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(7/2))/(7*a*Sqrt[c - a^2*c*x^2
])
```

Rubi [A] time = 0.153482, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(5/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(7/2))/(7*a*Sqrt[c - a^2*c*x^2
])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0318298, size = 48, normalized size = 1.

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^(5/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(7/2))/(7*a*Sqrt[c - a^2*c*x^2
])
```

Maple [A] time = 0.047, size = 41, normalized size = 0.9

$$\frac{2}{7a}(\operatorname{arccosh}(ax))^{\frac{7}{2}}\sqrt{ax-1}\sqrt{ax+1}\frac{1}{\sqrt{-(ax-1)(ax+1)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/7*arccosh(a*x)^(7/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{5}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(5/2)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(5/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.391 $\int \frac{\cosh^{-1}(ax)^{5/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{5a\sqrt{ax-1}\sqrt{ax+1}\text{Unintegrable}\left(\frac{x\cosh^{-1}(ax)^{3/2}}{1-a^2x^2},x\right)}{2c\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)^{5/2}}{c\sqrt{c-a^2cx^2}}$$

```
[Out] (x*ArcCosh[a*x]^(5/2))/(c*Sqrt[c - a^2*c*x^2]) + (5*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*Unintegrable[(x*ArcCosh[a*x]^(3/2))/(1 - a^2*x^2), x])/(2*c*Sqrt[c
- a^2*c*x^2])
```

Rubi [A] time = 0.227844, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^(5/2))/(c*Sqrt[c - a^2*c*x^2]) + (5*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*Defer[Int][(x*ArcCosh[a*x]^(3/2))/(1 - a^2*x^2), x])/(2*c*Sqrt[c -
a^2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^{5/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{5/2}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{x\cosh^{-1}(ax)^{5/2}}{c\sqrt{c-a^2cx^2}}+\frac{\left(5a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^{3/2}}{1-a^2x^2}\,dx}{2c\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 1.52244, size = 0, normalized size = 0.

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcCosh[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 0.299, size = 0, normalized size = 0.

$$\int \left(\operatorname{arccosh}(ax)\right)^{\frac{5}{2}}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{5}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(5/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.392 $\int \left(a^2-x^2\right)^{3/2} \sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\, dx$

Optimal. Leaf size=368

$$-\frac{\sqrt{\pi}a^3\sqrt{a^2-x^2}\operatorname{Erf}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2-x^2}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{\sqrt{\pi}a^3\sqrt{a^2-x^2}\operatorname{Erfi}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-$$

```
[Out] (3*a^2*x*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/8 + (x*(a^2 - x^2)^(3/2)*Sqrt[
ArcCosh[x/a]])/4 - (a^3*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/(4*Sqrt[-1 + x/
a]*Sqrt[1 + x/a]) - (a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erf[2*Sqrt[ArcCosh[x/a]]]
)/(256*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (a^3*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erf[
Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (a^3*Sqrt[
Pi]*Sqrt[a^2 - x^2]*Erfi[2*Sqrt[ArcCosh[x/a]]])/(256*Sqrt[-1 + x/a]*Sqrt[1
+ x/a]) - (a^3*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])
/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a])
```

Rubi [A] time = 0.778797, antiderivative size = 376, normalized size of antiderivative = 1.02, number of steps used = 25, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5713, 5685, 5683, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205, 5780}

$$-\frac{\sqrt{\pi}a^3\sqrt{a^2-x^2}\operatorname{Erf}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2-x^2}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{\sqrt{\pi}a^3\sqrt{a^2-x^2}\operatorname{Erfi}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - x^2)^(3/2)*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] (3*a^2*x*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/8 + ((a - x)*x*(a + x)*Sqrt[a^
2 - x^2]*Sqrt[ArcCosh[x/a]])/4 - (a^3*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/(
4*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) - (a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erf[2*Sqrt[
ArcCosh[x/a]]])/(256*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (a^3*Sqrt[Pi/2]*Sqrt[a
^2 - x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a]
) + (a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erfi[2*Sqrt[ArcCosh[x/a]]])/(256*Sqrt[-1
+ x/a]*Sqrt[1 + x/a]) - (a^3*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[A
rcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
```

```
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(a^2-x^2\right)^{3/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx = -\frac{\left(a^2\sqrt{a^2-x^2}\right)\int \left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{\left(a\sqrt{a^2-x^2}\right)\int \frac{x\left(-1+\frac{x^2}{a^2}\right)}{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}\,dx}{8\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{\left(3a^2\sqrt{a^2-\right.}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{\left(3a\sqrt{a^2-x\right.}{16\sqrt{-}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3\sqrt{a^2-x^2}}{4\sqrt{-1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3\sqrt{a^2-x^2}}{4\sqrt{-1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3\sqrt{a^2-x^2}}{4\sqrt{-1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3\sqrt{a^2-x^2}}{4\sqrt{-1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3\sqrt{a^2-x^2}}{4\sqrt{-1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3\sqrt{a^2-x^2}}{4\sqrt{-1+}}$$

Mathematica [A] time = 0.270036, size = 165, normalized size = 0.45

$$-\frac{a^4\sqrt{a^2-x^2}\left(-\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{3}{2},-4\cosh^{-1}\left(\frac{x}{a}\right)\right)+8\sqrt{2}\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{3}{2},-2\cosh^{-1}\left(\frac{x}{a}\right)\right)+\sqrt{\cosh}\right.}{128\sqrt{\frac{x-a}{a+x}}(a+x)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 - x^2)^(3/2)*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] -(a^4*Sqrt[a^2 - x^2]*(-(Sqrt[-ArcCosh[x/a]]*Gamma[3/2, -4*ArcCosh[x/a]]) +
 8*Sqrt[2]*Sqrt[-ArcCosh[x/a]]*Gamma[3/2, -2*ArcCosh[x/a]] + Sqrt[ArcCosh[x
/a]]*(32*ArcCosh[x/a]^(3/2) + 8*Sqrt[2]*Gamma[3/2, 2*ArcCosh[x/a]] - Gamma[
3/2, 4*ArcCosh[x/a]])))/(128*Sqrt[(-a + x)/(a + x)]*(a + x)*Sqrt[ArcCosh[x/
a]])
```

Maple [F] time = 0.336, size = 0, normalized size = 0.

$$\int \left(a^2-x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x)
```

```
[Out] int((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2-x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 - x^2)^(3/2)*sqrt(arccosh(x/a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(3/2)*acosh(x/a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.393 $\int \sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx$

Optimal. Leaf size=211

$$\frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2-x^2}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2-x^2}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{}$$

```
[Out] (x*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/2 - (a*Sqrt[a^2 - x^2]*ArcCosh[x/a]^
(3/2))/(3*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (a*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erf
[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) - (a*Sqrt[P
i/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*S
qrt[1 + x/a])
```

Rubi [A] time = 0.392167, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5713, 5683, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2-x^2}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2-x^2}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]],x]
```


```
[Out] (x*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/2 - (a*Sqrt[a^2 - x^2]*ArcCosh[x/a]^
(3/2))/(3*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (a*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erf
[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) - (a*Sqrt[P
i/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*S
qrt[1 + x/a])
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx = \frac{\sqrt{a^2-x^2}\int \sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx}{2\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{\sqrt{a^2-x^2}\int \frac{x}{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}\,dx}{4a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{\left(a\sqrt{a^2-x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sin}{\sqrt{x}}\right.}{4\sqrt{-1+\frac{x}{a}}\sqrt{1}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{\left(a\sqrt{a^2-x^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,d\right.}{4\sqrt{-1+\frac{x}{a}}\sqrt{1+}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{\left(a\sqrt{a^2-x^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,d\right.}{8\sqrt{-1+\frac{x}{a}}\sqrt{1+}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{\left(a\sqrt{a^2-x^2}\right)\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x\right.}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{\left(a\sqrt{a^2-x^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx,\right.}{8\sqrt{-1+\frac{x}{a}}\sqrt{1+}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2-x^2}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}}\right.}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

Mathematica [A] time = 0.137909, size = 121, normalized size = 0.57

$$-\frac{a^2\sqrt{a^2-x^2}\left(3\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{3}{2},2\cosh^{-1}\left(\frac{x}{a}\right)\right)+3\sqrt{2}\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{3}{2},-2\cosh^{-1}\left(\frac{x}{a}\right)\right)+16\cosh\right.}{48\sqrt{\frac{x-a}{a+x}}(a+x)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] -(a^2*Sqrt[a^2 - x^2]*(16*ArcCosh[x/a]^2 + 3*Sqrt[2]*Sqrt[-ArcCosh[x/a]]*Ga
mma[3/2, -2*ArcCosh[x/a]] + 3*Sqrt[2]*Sqrt[ArcCosh[x/a]]*Gamma[3/2, 2*ArcCo
sh[x/a]]))/(48*Sqrt[(-a + x)/(a + x)]*(a + x)*Sqrt[ArcCosh[x/a]])
```

Maple [F] time = 0.47, size = 0, normalized size = 0.

$$\int \sqrt{a^2 - x^2}\sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x)
```

```
[Out] int((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 - x^2}\sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2 - x^2)*sqrt(arccosh(x/a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(-a+x\right)\left(a+x\right)}\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(1/2)*acosh(x/a)**(1/2),x)
```

```
[Out] Integral(sqrt(-(-a + x)*(a + x))*sqrt(acosh(x/a)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.394 $\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=50

$$\frac{2a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(3/2))/(3*Sqrt[a^2 - x^2])
```

Rubi [A] time = 0.174926, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcCosh[x/a]]/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(3/2))/(3*Sqrt[a^2 - x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx = \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx}{\sqrt{a^2-x^2}}$$
$$= \frac{2a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

Mathematica [A] time = 0.0466195, size = 50, normalized size = 1.

$$\frac{2a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcCosh[x/a]]/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(3/2))/(3*Sqrt[a^2 - x^2])
```

Maple [A] time = 0.055, size = 44, normalized size = 0.9

$$\frac{2\,a}{3}\left(\operatorname{arccosh}\left(\frac{x}{a}\right)\right)^{\frac{3}{2}}\sqrt{\frac{-a+x}{a}}\sqrt{\frac{a+x}{a}}\frac{1}{\sqrt{-(-a+x)(a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x)
```

```
[Out] 2/3*arccosh(x/a)^(3/2)*a/(-(-a+x)*(a+x))^(1/2)*((-a+x)/a)^(1/2)*((a+x)/a)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(x/a))/sqrt(a^2 - x^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\sqrt{-\left(-a+x\right)\left(a+x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(1/2)/(a**2-x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(acosh(x/a))/sqrt(-(-a + x)*(a + x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.395 $\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\text{Unintegrable}\left(\frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}},x\right)}{2a^3\sqrt{a^2-x^2}}+\frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}$$

```
[Out] (x*Sqrt[ArcCosh[x/a]])/(a^2*Sqrt[a^2 - x^2]) + (Sqrt[-1 + x/a]*Sqrt[1 + x/a
]*Unintegrable[x/((1 - x^2/a^2)*Sqrt[ArcCosh[x/a]]), x])/(2*a^3*Sqrt[a^2 -
x^2])
```

Rubi [A] time = 0.257602, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcCosh[x/a]])/(a^2*Sqrt[a^2 - x^2]) + (Sqrt[-1 + x/a]*Sqrt[1 + x/a
]*Defer[Int][x/((1 - x^2/a^2)*Sqrt[ArcCosh[x/a]]), x])/(2*a^3*Sqrt[a^2 - x^
2])
```

Rubi steps

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx}{a^2\sqrt{a^2-x^2}}$$

$$= \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}} + \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}\,dx}{2a^3\sqrt{a^2-x^2}}$$

Mathematica [A] time = 0.903274, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(3/2), x]
```

Maple [A] time = 0.283, size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}\left(a^2-x^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x)
```

```
[Out] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(x/a))/(a^2 - x^2)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(1/2)/(a**2-x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(acosh(x/a))/(-(-a + x)*(a + x))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.396 $\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=198

$$\frac{\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\text{Unintegrable}\left(\frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}},x\right)}{3a^5\sqrt{a^2-x^2}}+\frac{\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\text{Unintegrable}\left(\frac{x}{\left(\frac{x^2}{a^2}-1\right)^2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}},x\right)}{6a^5\sqrt{a^2-x^2}}+\frac{2x\sqrt{\cosh^{-1}}}{3a^4\sqrt{a^2-x}}$$

```
[Out] (x*Sqrt[ArcCosh[x/a]])/(3*a^2*(a^2 - x^2)^(3/2)) + (2*x*Sqrt[ArcCosh[x/a]])
/(3*a^4*Sqrt[a^2 - x^2]) + (Sqrt[-1 + x/a]*Sqrt[1 + x/a]*Unintegrable[x/((1
 - x^2/a^2)*Sqrt[ArcCosh[x/a]]), x])/(3*a^5*Sqrt[a^2 - x^2]) + (Sqrt[-1 + x
/a]*Sqrt[1 + x/a]*Unintegrable[x/((-1 + x^2/a^2)^2*Sqrt[ArcCosh[x/a]]), x])
/(6*a^5*Sqrt[a^2 - x^2])
```

Rubi [A] time = 0.47886, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(5/2),x]
```

```
[Out] (2*x*Sqrt[ArcCosh[x/a]])/(3*a^4*Sqrt[a^2 - x^2]) + (x*Sqrt[ArcCosh[x/a]])/(
3*a^2*(a - x)*(a + x)*Sqrt[a^2 - x^2]) + (Sqrt[-1 + x/a]*Sqrt[1 + x/a]*Defe
r[Int][x/((1 - x^2/a^2)*Sqrt[ArcCosh[x/a]]), x])/(3*a^5*Sqrt[a^2 - x^2]) +
(Sqrt[-1 + x/a]*Sqrt[1 + x/a]*Defer[Int][x/((-1 + x^2/a^2)^2*Sqrt[ArcCosh[x
/a]]), x])/(6*a^5*Sqrt[a^2 - x^2])
```

Rubi steps

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(-1+\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx}{a^4\sqrt{a^2-x^2}}$$

$$= \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^2(a-x)(a+x)\sqrt{a^2-x^2}} + \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x}{\left(-1+\frac{x^2}{a^2}\right)^2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2-x^2}} - \frac{\left(2\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int}{3a^4\sqrt{a^2}}$$

$$= \frac{2x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2-x^2}} + \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^2(a-x)(a+x)\sqrt{a^2-x^2}} + \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x}{\left(-1+\frac{x^2}{a^2}\right)^2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2-x^2}} + \frac{(\sqrt{}}{}$$

Mathematica [A] time = 2.0983, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(5/2),x]
```

```
[Out] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(5/2), x]
```

Maple [A] time = 0.321, size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}\left(a^2-x^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x)
```

```
[Out] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(x/a))/(a^2 - x^2)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(1/2)/(a**2-x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.397 $\int \left(a^2-x^2\right)^{3/2} \cosh^{-1}\left(\frac{x}{a}\right)^{3/2} \, dx$

Optimal. Leaf size=525

$$-\frac{3\sqrt{\pi}a^3\sqrt{a^2-x^2}\text{Erf}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2-x^2}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{3\sqrt{\pi}a^3\sqrt{a^2-x^2}\text{Erfi}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}$$

```
[Out] (27*a^3*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(256*Sqrt[-1 + x/a]*Sqrt[1 + x/
a]) - (9*a*x^2*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(32*Sqrt[-1 + x/a]*Sqrt[
1 + x/a]) + (3*(a^2 - x^2)^(5/2)*Sqrt[ArcCosh[x/a]])/(32*a*Sqrt[-1 + x/a]*S
qrt[1 + x/a]) + (3*a^2*x*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/8 + (x*(a^2 -
x^2)^(3/2)*ArcCosh[x/a]^(3/2))/4 - (3*a^3*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(5/2
))/(20*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) - (3*a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erf[
2*Sqrt[ArcCosh[x/a]]])/(2048*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (3*a^3*Sqrt[Pi
/2]*Sqrt[a^2 - x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[-1 + x/a]*Sqr
t[1 + x/a]) - (3*a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erfi[2*Sqrt[ArcCosh[x/a]]])/(
2048*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erfi
[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[-1 + x/a]*Sqrt[1 + x/a])
```

Rubi [A] time = 1.28204, antiderivative size = 533, normalized size of antiderivative = 1.02, number of steps used = 27, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {5713, 5685, 5683, 5676, 5664, 5781, 3312, 3307, 2180, 2204, 2205, 5716, 5701}

$$-\frac{3\sqrt{\pi}a^3\sqrt{a^2-x^2}\text{Erf}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2-x^2}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{3\sqrt{\pi}a^3\sqrt{a^2-x^2}\text{Erfi}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - x^2)^(3/2)*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (27*a^3*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(256*Sqrt[-1 + x/a]*Sqrt[1 + x/
a]) - (9*a*x^2*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(32*Sqrt[-1 + x/a]*Sqrt[
1 + x/a]) + (3*(a^2 - x^2)^(5/2)*Sqrt[ArcCosh[x/a]])/(32*a*Sqrt[-1 + x/a]*S
qrt[1 + x/a]) + (3*a^2*x*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/8 + ((a - x)*x
*(a + x)*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/4 - (3*a^3*Sqrt[a^2 - x^2]*Arc
Cosh[x/a]^(5/2))/(20*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) - (3*a^3*Sqrt[Pi]*Sqrt[a
^2 - x^2]*Erf[2*Sqrt[ArcCosh[x/a]]])/(2048*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) +
(3*a^3*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt
```

```
[-1 + x/a]*Sqrt[1 + x/a]) - (3*a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erfi[2*Sqrt[Arc
Cosh[x/a]]])/(2048*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a
^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[-1 + x/a]*Sqrt[1 + x/a
])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \left(a^2-x^2\right)^{3/2} \cosh^{-1}\left(\frac{x}{a}\right)^{3/2} dx = -\frac{\left(a^2\sqrt{a^2-x^2}\right)\int \left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} dx}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{\left(3a\sqrt{a^2-x^2}\right)\int x\left(-1+\frac{x^2}{a^2}\right)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} dx}{8\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3}{8}a^2x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x$$

$$= -\frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3}{8}a^2x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right.$$

$$= -\frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3}{8}a^2x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right.$$

$$= -\frac{9a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right.}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right.}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right.}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right.}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right.}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

Mathematica [A] time = 0.442251, size = 219, normalized size = 0.42

$$a^4\sqrt{a^2-x^2}\left(5\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{5}{2},4\cosh^{-1}\left(\frac{x}{a}\right)\right)-5\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{5}{2},-4\cosh^{-1}\left(\frac{x}{a}\right)\right)+60\sqrt{2\pi}\sqrt{\cosh^{-}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 - x^2)^(3/2)*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (a^4*Sqrt[a^2 - x^2]*(-384*ArcCosh[x/a]^3 - 480*ArcCosh[x/a]*Cosh[2*ArcCosh
[x/a]] + 60*Sqrt[2*Pi]*Sqrt[ArcCosh[x/a]]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]] +
 60*Sqrt[2*Pi]*Sqrt[ArcCosh[x/a]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]] - 5*Sqrt
[-ArcCosh[x/a]]*Gamma[5/2, -4*ArcCosh[x/a]] + 5*Sqrt[ArcCosh[x/a]]*Gamma[5/
2, 4*ArcCosh[x/a]] + 640*ArcCosh[x/a]^2*Sinh[2*ArcCosh[x/a]]))/(2560*Sqrt[(
-a + x)/(a + x)]*(a + x)*Sqrt[ArcCosh[x/a]])
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int \left(a^2-x^2\right)^{\frac{3}{2}} \left(\text{arccosh}\left(\frac{x}{a}\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x)
```

```
[Out] int((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2-x^2\right)^{\frac{3}{2}} \operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 - x^2)^(3/2)*arccosh(x/a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(3/2)*acosh(x/a)**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x, algorithm="giac")

[Out] sage0*x
```

3.398 $\int \sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}\,dx$

Optimal. Leaf size=316

$$\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2-x^2}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2-x^2}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{2}x\sqrt{a^2-}$$

```
[Out] (3*a*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a])
- (3*x^2*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(8*a*Sqrt[-1 + x/a]*Sqrt[1 + x
/a]) + (x*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/2 - (a*Sqrt[a^2 - x^2]*ArcCos
h[x/a]^(5/2))/(5*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 -
 x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) +
(3*a*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[
-1 + x/a]*Sqrt[1 + x/a])
```

Rubi [A] time = 0.727819, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5713, 5683, 5676, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2-x^2}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2-x^2}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{2}x\sqrt{a^2-}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2),x]
```



```
[Out] (3*a*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a])
- (3*x^2*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(8*a*Sqrt[-1 + x/a]*Sqrt[1 + x
/a]) + (x*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/2 - (a*Sqrt[a^2 - x^2]*ArcCos
h[x/a]^(5/2))/(5*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 -
 x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) +
(3*a*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[
-1 + x/a]*Sqrt[1 + x/a])
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
```

```
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} dx = \frac{\sqrt{a^2-x^2}\int \sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} dx}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{\sqrt{a^2-x^2}\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} dx}{2\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{\left(3\sqrt{a^2-x^2}\right)\int x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{4a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= -\frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + -$$

$$= -\frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} +$$

$$= -\frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} +$$

$$= \frac{3a\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} -$$

$$= \frac{3a\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} -$$

$$= \frac{3a\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} -$$

$$= \frac{3a\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} -$$

Mathematica [A] time = 0.363241, size = 144, normalized size = 0.46

$$\frac{a^2\sqrt{a^2-x^2}\left(15\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)+15\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)-8\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\left(16\cosh^{-1}\left(\frac{x}{a}\right)^2+15\cosh\left(2\right.\right.\right.}{640\sqrt{\frac{x-a}{a+x}}(a+x)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (a^2*Sqrt[a^2 - x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]] + 15*Sq
rt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]] - 8*Sqrt[ArcCosh[x/a]]*(16*ArcCos
h[x/a]^2 + 15*Cosh[2*ArcCosh[x/a]] - 20*ArcCosh[x/a]*Sinh[2*ArcCosh[x/a]]))
)/(640*Sqrt[(-a + x)/(a + x)]*(a + x))
```

Maple [F] time = 0.448, size = 0, normalized size = 0.

$$\int \left(\operatorname{arccosh}\left(\frac{x}{a}\right)\right)^{\frac{3}{2}} \sqrt{a^2 - x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x)
```

```
[Out] int(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 - x^2}\, \operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2 - x^2)*arccosh(x/a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(3/2)*(a**2-x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.399 $\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=50

$$\frac{2a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Rubi [A] time = 0.167808, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```


```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx}{\sqrt{a^2-x^2}}$$
$$= \frac{2a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Mathematica [A] time = 0.0510657, size = 50, normalized size = 1.

$$\frac{2a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Maple [A] time = 0.054, size = 44, normalized size = 0.9

$$\frac{2a}{5}\left(\operatorname{arccosh}\left(\frac{x}{a}\right)\right)^{\frac{5}{2}}\sqrt{\frac{-a+x}{a}}\sqrt{\frac{a+x}{a}}\frac{1}{\sqrt{-(-a+x)(a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x)
```

```
[Out] 2/5*arccosh(x/a)^(5/2)*a/(-(-a+x)*(a+x))^(1/2)*((-a+x)/a)^(1/2)*((a+x)/a)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(x/a)^(3/2)/sqrt(a^2 - x^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{-\left(-a+x\right)\left(a+x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(3/2)/(a**2-x**2)**(1/2),x)
```

```
[Out] Integral(acosh(x/a)**(3/2)/sqrt(-(-a + x)*(a + x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.400 $\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{3\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\text{Unintegrable}\left(\frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{1-\frac{x^2}{a^2}},x\right)}{2a^3\sqrt{a^2-x^2}}+\frac{x\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2-x^2}}$$

```
[Out] (x*ArcCosh[x/a]^(3/2))/(a^2*Sqrt[a^2 - x^2]) + (3*Sqrt[-1 + x/a]*Sqrt[1 + x
/a]*Unintegrable[(x*Sqrt[ArcCosh[x/a]])/(1 - x^2/a^2), x])/(2*a^3*Sqrt[a^2
- x^2])
```

Rubi [A] time = 0.24817, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[x/a]^(3/2)/(a^2 - x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[x/a]^(3/2))/(a^2*Sqrt[a^2 - x^2]) + (3*Sqrt[-1 + x/a]*Sqrt[1 + x
/a]*Defer[Int][(x*Sqrt[ArcCosh[x/a]])/(1 - x^2/a^2), x])/(2*a^3*Sqrt[a^2 -
x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx}{a^2\sqrt{a^2-x^2}}\\ &= \frac{x\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2-x^2}}+\frac{\left(3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{1-\frac{x^2}{a^2}}\,dx}{2a^3\sqrt{a^2-x^2}}\end{aligned}$$

Mathematica [A] time = 0.96622, size = 0, normalized size = 0.

$$\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[x/a]^(3/2)/(a^2 - x^2)^(3/2),x]
```

```
[Out] Integrate[ArcCosh[x/a]^(3/2)/(a^2 - x^2)^(3/2), x]
```

Maple [A] time = 0.236, size = 0, normalized size = 0.

$$\int \left(\operatorname{arccosh}\left(\frac{x}{a}\right)\right)^{\frac{3}{2}}\left(a^2-x^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x)
```

```
[Out] int(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2-x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(x/a)^(3/2)/(a^2 - x^2)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(3/2)/(a**2-x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.401 $\int \frac{x}{\sqrt{1-x^2}\sqrt{\cosh^{-1}(x)}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}\sqrt{x-1}\,\text{Erf}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x}}+\frac{\sqrt{\pi}\sqrt{x-1}\,\text{Erfi}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x}}$$

```
[Out] (Sqrt[Pi]*Sqrt[-1 + x]*Erf[Sqrt[ArcCosh[x]]])/(2*Sqrt[1 - x]) + (Sqrt[Pi]*S
qrt[-1 + x]*Erfi[Sqrt[ArcCosh[x]]])/(2*Sqrt[1 - x])
```

Rubi [A] time = 0.191704, antiderivative size = 83, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5798, 5781, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{x-1}\sqrt{x+1}\,\text{Erf}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x^2}}+\frac{\sqrt{\pi}\sqrt{x-1}\sqrt{x+1}\,\text{Erfi}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^2]*Sqrt[ArcCosh[x]]),x]
```

```
[Out] (Sqrt[Pi]*Sqrt[-1 + x]*Sqrt[1 + x]*Erf[Sqrt[ArcCosh[x]]])/(2*Sqrt[1 - x^2])
 + (Sqrt[Pi]*Sqrt[-1 + x]*Sqrt[1 + x]*Erfi[Sqrt[ArcCosh[x]]])/(2*Sqrt[1 - x
^2])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
```

```
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\cosh^{-1}(x)}}\,dx = \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\sqrt{\cosh^{-1}(x)}}\,dx}{\sqrt{1-x^2}}$$

$$= \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(x)\right)}{\sqrt{1-x^2}}$$

$$= \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(x)\right)}{2\sqrt{1-x^2}} + \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, c\right.}{2\sqrt{1-x^2}}$$

$$= \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\cosh^{-1}(x)}\right)}{\sqrt{1-x^2}} + \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int e^{x^2}\,dx,\right.}{\sqrt{1-x^2}}$$

$$= \frac{\sqrt{\pi}\sqrt{-1+x}\sqrt{1+x}\,\text{erf}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x^2}} + \frac{\sqrt{\pi}\sqrt{-1+x}\sqrt{1+x}\,\text{erfi}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x^2}}$$

Mathematica [A] time = 0.0987592, size = 72, normalized size = 1.11

$$-\frac{\sqrt{-(x-1)(x+1)}\left(\sqrt{-\cosh^{-1}(x)}\,\text{Gamma}\left(\frac{1}{2}, -\cosh^{-1}(x)\right) - \sqrt{\cosh^{-1}(x)}\,\text{Gamma}\left(\frac{1}{2}, \cosh^{-1}(x)\right)\right)}{2\sqrt{\frac{x-1}{x+1}}(x+1)\sqrt{\cosh^{-1}(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 - x^2]*Sqrt[ArcCosh[x]]),x]
```

```
[Out] -(Sqrt[-((-1 + x)*(1 + x))]*(Sqrt[-ArcCosh[x]]*Gamma[1/2, -ArcCosh[x]] - Sq
rt[ArcCosh[x]]*Gamma[1/2, ArcCosh[x]]))/(2*Sqrt[(-1 + x)/(1 + x)]*(1 + x)*S
qrt[ArcCosh[x]])
```

Maple [F] time = 0.312, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{-x^2+1}}\frac{1}{\sqrt{\text{arccosh}\,(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x)
```

```
[Out] int(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^2+1}\sqrt{\operatorname{arccosh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^2 + 1)*sqrt(arccosh(x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)(x+1)}\sqrt{\operatorname{acosh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2)/acosh(x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(acosh(x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^2+1}\,\sqrt{\operatorname{arccosh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^2 + 1)*sqrt(arccosh(x))), x)
```

3.402 $\int \frac{\left(c-a^2cx^2\right)^{5/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=438

$$-\frac{3\sqrt{\pi}c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{6}}c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{6}\sqrt{\cosh\right.}}{64a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] (-5*c^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(8*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(6
4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*
Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (c^2
*Sqrt[Pi/6]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[6]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt
[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2*Sqrt[Pi/2]*S
qrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (c^2*Sqrt[Pi/6]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[6]*Sqrt[ArcCo
sh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.444848, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5713, 5701, 3312, 3307, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{6}}c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{6}\sqrt{\cosh\right.}}{64a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/Sqrt[ArcCosh[a*x]],x]
```


```
[Out] (-5*c^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(8*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(6
4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*
Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (c^2
*Sqrt[Pi/6]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[6]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt
[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2*Sqrt[Pi/2]*S
qrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*S
```

```
qrt[1 + a*x]) + (c^2*Sqrt[Pi/6]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[6]*Sqrt[ArcCo
sh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{5/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{5/2}(1+ax)^{5/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh^6(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{5}{16\sqrt{x}}-\frac{15\cosh(2x)}{32\sqrt{x}}+\frac{3\cosh(4x)}{16\sqrt{x}}-\frac{\cosh(6x)}{32\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(6x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{32a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(3c^2\sqrt{c-a\right.}}{}$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{64a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2cx^2}\right.}{6}$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2\right.}}{}$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3c^2\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{erf}\left(\right.}{64a\sqrt{-1+ax}}$$

Mathematica [A] time = 0.422279, size = 209, normalized size = 0.48

$$-\frac{c^2\sqrt{c-a^2cx^2}\left(45\sqrt{2}\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, 2\cosh^{-1}(ax)\right) - 18\sqrt{\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, 4\cosh^{-1}(ax)\right) + \sqrt{6}\sqrt{\cosh\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -(c^2*Sqrt[c - a^2*c*x^2]*(240*ArcCosh[a*x] - Sqrt[6]*Sqrt[-ArcCosh[a*x]]*G
amma[1/2, -6*ArcCosh[a*x]] + 18*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a
*x]] - 45*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + 45*Sqrt
[2]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 2*ArcCosh[a*x]] - 18*Sqrt[ArcCosh[a*x]]*G
amma[1/2, 4*ArcCosh[a*x]] + Sqrt[6]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 6*ArcCosh
[a*x]]))/(384*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

Maple [F] time = 0.316, size = 0, normalized size = 0.

$$\int \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \frac{1}{\sqrt{\operatorname{arccosh}\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2} c x^{2} + c\right)^{\frac{5}{2}}}{\sqrt{\operatorname{arcosh}\left(a x\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.403 $\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=294

$$-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] (-3*c*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]) - (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(32*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]) + (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]
*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi]*Sqrt
[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(32*a*Sqrt[-1 + a*x]*Sqrt[1 + a
*x]) + (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/
(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.339293, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5713, 5701, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (-3*c*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]) - (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(32*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]) + (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]
*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi]*Sqrt
[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(32*a*Sqrt[-1 + a*x]*Sqrt[1 + a
*x]) + (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/
(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
```

```
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{3/2}(1+ax)^{3/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{8\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{3c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{8a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\left(c\sqrt{c-a^2c\right.}{}}{}$$

$$= -\frac{3c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(c\sqrt{c-a^2cx^2}\right)}{16}$$

$$= -\frac{3c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(c\sqrt{c-a^2c\right.}{}}{}$$

$$= -\frac{3c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{c\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\right.}{}}{4a\sqrt{-1+ax}\sqrt{1+}}$$

Mathematica [A] time = 0.248487, size = 153, normalized size = 0.52

$$-\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -4\cosh^{-1}(ax)\right) - 4\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -2\cosh^{-1}(ax)\right) + \sqrt{\cosh\right.}{}}{32a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{\cosh^{-1}(a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -(c*Sqrt[c - a^2*c*x^2]*(Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] -
4*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + Sqrt[ArcCosh[a*
x]]*(24*Sqrt[ArcCosh[a*x]] + 4*Sqrt[2]*Gamma[1/2, 2*ArcCosh[a*x]] - Gamma[1
/2, 4*ArcCosh[a*x]])))/(32*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcC
osh[a*x]])
```

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$\int \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \frac{1}{\sqrt{\operatorname{arccosh}\left(a x \right)}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}}}{\sqrt{\operatorname{arcosh}\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\sqrt{\operatorname{acosh}\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.404 $\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=175

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] -((Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(4*a*S
qrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]
*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.253809, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5713, 5701, 3312, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/Sqrt[ArcCosh[a*x]],x]
```


```
[Out] -((Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(4*a*S
qrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]
*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{-1+ax}\sqrt{1+ax}}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}-\frac{\cosh(2x)}{2\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int\right.}{4a\sqrt{-1+a}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}}{2a\sqrt{-}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\cos\right.}{4a\sqrt{-1+ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.142362, size = 114, normalized size = 0.65

$$-\frac{\sqrt{-c(ax-1)(ax+1)}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, 2\cosh^{-1}(ax)\right)-\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -2\cosh^{-1}(ax)\right)+8\right.}{8a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(8*ArcCosh[a*x] - Sqrt[2]*Sqrt[-ArcCosh[a
*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + Sqrt[2]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 2*
ArcCosh[a*x]]))/(8*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]
])
```

Maple [F] time = 0.504, size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\,\frac{1}{\sqrt{\operatorname{arccosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)(ax+1)}}{\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt(acosh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.405 $\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}$$

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.159642, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]),x]
```


```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[c - a^2*c*x^2])
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0370626, size = 46, normalized size = 1.

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.046, size = 41, normalized size = 0.9

$$2\,\frac{\sqrt{\operatorname{arccosh}(ax)}\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{-(ax-1)(ax+1)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] 2*arccosh(a*x)^(1/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\sqrt{\operatorname{arccosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt(arccosh(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c\,(ax-1)\,(ax+1)}\sqrt{\operatorname{acosh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*sqrt(acosh(a*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.406 $\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]), x]
```

Rubi [A] time = 0.198037, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] -((Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][1/((-1 + a*x)^(3/2)*(1 + a*x)^(3
/2)*Sqrt[ArcCosh[a*x]]), x])/(c*Sqrt[c - a^2*c*x^2]))
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx=-\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{3/2}(1+ax)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx}{c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 1.7483, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]), x]
```

Maple [A] time = 0.3, size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{-\frac{3}{2}} \frac{1}{\sqrt{\operatorname{arccosh}\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\operatorname{arcosh}\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt(arccosh(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}\sqrt{\operatorname{acosh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/((-c*(a*x - 1)*(a*x + 1))**(3/2)*sqrt(acosh(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.407 $\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{\left(c-a^2cx^2\right)^{5/2}\sqrt{\cosh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]), x]
```

Rubi [A] time = 0.203432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][1/((-1 + a*x)^(5/2)*(1 + a*x)^(5/2
)*Sqrt[ArcCosh[a*x]]), x])/(c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx=\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{5/2}(1+ax)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 2.38179, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]), x]
```

Maple [A] time = 0.378, size = 0, normalized size = 0.

$$\int \left(-a^{2} c\,x^{2}+c\right)^{-\frac{5}{2}} \frac{1}{\sqrt{\operatorname{arccosh}\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} \sqrt{\operatorname{arcosh}\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt(arccosh(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.408 $\int \frac{\left(c-a^2cx^2\right)^{5/2}}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=433

$$\frac{3\sqrt{\pi}c^2\sqrt{c-a^2cx^2}\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{c-a^2cx^2}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\frac{3\pi}{2}}c^2\sqrt{c-a^2cx^2}\text{Erf}\left(\sqrt{6}\sqrt{\cosh^{-1}}\right.}{16a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c - a^2*c*x^2)^(5/2))/(a*Sqrt[ArcCosh[a*x
]]) + (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(8*a*S
qrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*c^2*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[S
qrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c^2*Sqrt
[(3*Pi)/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[6]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[
ArcCosh[a*x]]])/(8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2*Sqrt[Pi/2]*Sqr
t[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]) + (c^2*Sqrt[(3*Pi)/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[6]*Sqrt[Arc
Cosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.460933, antiderivative size = 444, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5713, 5697, 5780, 5448, 3308, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}c^2\sqrt{c-a^2cx^2}\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{c-a^2cx^2}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\frac{3\pi}{2}}c^2\sqrt{c-a^2cx^2}\text{Erf}\left(\sqrt{6}\sqrt{\cosh^{-1}}\right.}{16a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (2*c^2*(1 - a*x)^3*(1 + a*x)^(5/2)*Sqrt[c - a^2*c*x^2])/(a*Sqrt[-1 + a*x]*S
qrt[ArcCosh[a*x]]) + (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh
[a*x]]])/(8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*c^2*Sqrt[Pi/2]*Sqrt[c - a
^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]) - (c^2*Sqrt[(3*Pi)/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[6]*Sqrt[ArcCosh[a*x]
]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^
2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2
*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqr
```

```
t[-1 + a*x]*Sqrt[1 + a*x]) + (c^2*Sqrt[(3*Pi)/2]*Sqrt[c - a^2*c*x^2]*Erfi[S
qrt[6]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{5/2}}{\cosh^{-1}(ax)^{3/2}}\,dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{5/2}(1+ax)^{5/2}}{\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(12ac^2\sqrt{c-a^2cx^2}\right)\int \frac{x\left(-1+a^2x^2\right)^2}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c-a^2cx^2}\right)\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c-a^2cx^2}\right)\mathrm{Subst}\left(\int \left(\frac{5\sinh(2x)}{32\sqrt{x}} - \frac{\sinh(4x)}{8\sqrt{x}} + \frac{\sinh(6x)}{32\sqrt{x}}\right) \ldots\right.}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(3c^2\sqrt{c-a^2cx^2}\right)\mathrm{Subst}\left(\int \frac{\sinh(6x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{8a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(3c \ldots\right.}{\ldots}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c-a^2cx^2}\right)\mathrm{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{16a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\left(3c^2\sqrt{c \ldots\right.}{\ldots}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c-a^2cx^2}\right)\mathrm{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\left(3c \ldots\right.}{\ldots}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{3c^2\sqrt{\pi}\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx \ldots}}{16a\sqrt{-1 \ldots}}$$

Mathematica [A] time = 1.19608, size = 411, normalized size = 0.95

$$\frac{c^2\sqrt{c-a^2cx^2}e^{-6\cosh^{-1}(ax)}\left(\sqrt{6}e^{6\cosh^{-1}(ax)}\sqrt{-\cosh^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},-6\cosh^{-1}(ax)\right) - 12e^{6\cosh^{-1}(ax)}\sqrt{-\cosh^{-1}(ax)}\mathrm{Gar} \ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(-1 + 6*E^(2*ArcCosh[a*x]) + E^(4*ArcCosh[a*x]) +
52*E^(6*ArcCosh[a*x]) + E^(8*ArcCosh[a*x]) + 6*E^(10*ArcCosh[a*x]) - E^(12*
ArcCosh[a*x]) - 64*a^2*E^(6*ArcCosh[a*x])*x^2 - 16*E^(6*ArcCosh[a*x])*Sqrt[
2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 16*E^(6*ArcCosh[
a*x])*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Sqrt
[6]*E^(6*ArcCosh[a*x])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -6*ArcCosh[a*x]] - 12
*E^(6*ArcCosh[a*x])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] - Sqrt[
2]*E^(6*ArcCosh[a*x])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] - Sqr
t[2]*E^(6*ArcCosh[a*x])*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 2*ArcCosh[a*x]] - 12*
E^(6*ArcCosh[a*x])*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 4*ArcCosh[a*x]] + Sqrt[6]*
E^(6*ArcCosh[a*x])*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 6*ArcCosh[a*x]]))/(32*a*E^
(6*ArcCosh[a*x])*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int \left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} \left(\operatorname{arccosh}\left(a x\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}}}{\operatorname{arcosh}\left(a x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.409 $\int \frac{(c-a^2cx^2)^{3/2}}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}+$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c - a^2*c*x^2)^(3/2))/(a*Sqrt[ArcCosh[a*x
]]) + (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sq
rt[ArcCosh[a*x]]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi]*Sqrt[c -
a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) +
 (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(a*Sqr
t[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.348836, antiderivative size = 295, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5713, 5697, 5780, 5448, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (2*c*(1 - a*x)^2*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])/(a*Sqrt[-1 + a*x]*Sqr
t[ArcCosh[a*x]]) + (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]
])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*E
rf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[
Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]) + (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a
*x]]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
```

art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]

Rule 5697

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]

Rule 5780

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]

Rule 5448

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3308

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 2180

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\cosh^{-1}(ax)^{3/2}}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{3/2}(1+ax)^{3/2}}{\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8ac\sqrt{c-a^2cx^2}\right)\int \frac{x\left(-1+a^2x^2\right)}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{x}}+\frac{\sinh(4x)}{8\sqrt{x}}\right) dx, x, \cosh^{-1}(a\right.}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\left(2c\sqrt{c-\right.}{}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(c\sqrt{c-a^2cx\right.}{}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(c\sqrt{c-a\right.}{}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{c\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{erf}\left(\sqrt{2\right.}{a\sqrt{-1+ax}\sqrt{1}}$$

Mathematica [A] time = 0.467264, size = 239, normalized size = 0.84

$$-\frac{c\sqrt{c-a^2cx^2}e^{-4\cosh^{-1}(ax)}\left(2e^{4\cosh^{-1}(ax)}\sqrt{-\cosh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4\cosh^{-1}(ax)\right)+2e^{4\cosh^{-1}(ax)}\sqrt{\cosh^{-1}(ax)}\text{Gamma}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] -(c*Sqrt[c - a^2*c*x^2]*(-1 - 14*E^(4*ArcCosh[a*x]) - E^(8*ArcCosh[a*x]) +
16*a^2*E^(4*ArcCosh[a*x])*x^2 + 4*E^(4*ArcCosh[a*x])*Sqrt[2*Pi]*Sqrt[ArcCos
h[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] - 4*E^(4*ArcCosh[a*x])*Sqrt[2*Pi]*S
qrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 2*E^(4*ArcCosh[a*x])*S
qrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] + 2*E^(4*ArcCosh[a*x])*Sqrt[
ArcCosh[a*x]]*Gamma[1/2, 4*ArcCosh[a*x]]))/(8*a*E^(4*ArcCosh[a*x])*Sqrt[(-1
 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\operatorname{arccosh}(ax)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.410 $\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\sqrt{ax+1}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}\sqrt{c-a^2cx^2}}{a\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2])/(a*Sqrt[ArcCosh[a*x]]
) - (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(a*Sqr
t[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*S
qrt[ArcCosh[a*x]]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.238779, antiderivative size = 176, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5713, 5697, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2\sqrt{ax+1}(1-ax)\sqrt{c-a^2cx^2}}{a\sqrt{ax-1}\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (2*(1 - a*x)*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2])/(a*Sqrt[-1 + a*x]*Sqrt[ArcC
osh[a*x]]) - (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]
])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[
Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{3/2}}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{-1+ax}\sqrt{1+ax}}{\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4a\sqrt{c-a^2cx^2}\right)\int \frac{x}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(2\sqrt{c-a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{c-a^2cx^2}\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\operatorname{Subst}}{a\sqrt{-1}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\left(2\sqrt{c-a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\left(2\sqrt{c-a^2c\right.}{}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\right.}{a\sqrt{-1+ax}\sqrt{1+}}$$

Mathematica [A] time = 0.255774, size = 127, normalized size = 0.75

$$\frac{\sqrt{c-a^2cx^2}\left(-4a^2x^2-\sqrt{2\pi}\sqrt{\cosh^{-1}(ax)}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+\sqrt{2\pi}\sqrt{\cosh^{-1}(ax)}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+4\right)}{2a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(4 - 4*a^2*x^2 - Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqr
t[2]*Sqrt[ArcCosh[a*x]]] + Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[
ArcCosh[a*x]]]))/(2*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x
]])
```

Maple [F] time = 0.487, size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\,(\operatorname{arccosh}(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\operatorname{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\operatorname{acosh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(3/2),x)

[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/acosh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="giac")

[Out] sage0*x
```

3.411 $\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]
)
```

Rubi [A] time = 0.15355, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]
)
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}$$

Mathematica [A] time = 0.0343056, size = 46, normalized size = 1.

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]
)
```

Maple [A] time = 0.046, size = 41, normalized size = 0.9

$$-2\,\frac{\sqrt{ax-1}\sqrt{ax+1}}{\sqrt{\operatorname{arccosh}(ax)}\,a\sqrt{-(ax-1)(ax+1)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x)
```

```
[Out] -2/arccosh(a*x)^(1/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(3/2)), x)
```

Fricas [A] time = 2.23985, size = 131, normalized size = 2.85

$$\frac{2\sqrt{-a^2cx^2+c}\sqrt{a^2x^2-1}}{\left(a^3cx^2-ac\right)\sqrt{\log\left(ax+\sqrt{a^2x^2-1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-a^2*c*x^2 + c)*sqrt(a^2*x^2 - 1)/((a^3*c*x^2 - a*c)*sqrt(log(a*x +
sqrt(a^2*x^2 - 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c(ax-1)(ax+1)}\operatorname{acosh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*acosh(a*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.412 $\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=109

$$\frac{4a\sqrt{ax-1}\sqrt{ax+1}\text{Unintegrable}\left(\frac{x}{\left(a^2x^2-1\right)^2\sqrt{\cosh^{-1}(ax)}},x\right)}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x
]]) + (4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Unintegrable[x/((-1 + a^2*x^2)^2*Sq
rt[ArcCosh[a*x]]), x])/(c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.233607, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x])/(a*c*(1 - a*x)*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2]*Sqrt[A
rcCosh[a*x]]) + (4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2*x
^2)^2*Sqrt[ArcCosh[a*x]]), x])/(c*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$=-\frac{2\sqrt{-1+ax}}{ac(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}+\frac{\left(4a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x}{\left(-1+a^2x^2\right)^2\sqrt{c}}}{c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 1.72746, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2)), x]
```

Maple [A] time = 0.289, size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{-\frac{3}{2}}\left(\operatorname{arccosh}(ax)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.413 $\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{8a\sqrt{ax-1}\sqrt{ax+1}\,\text{Unintegrable}\left(\frac{x}{\left(a^2x^2-1\right)^3\sqrt{\cosh^{-1}(ax)}},x\right)}{c^2\sqrt{c-a^2cx^2}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\left(c-a^2cx^2\right)^{5/2}\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*(c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x
]]) - (8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Unintegrable[x/((-1 + a^2*x^2)^3*Sq
rt[ArcCosh[a*x]]), x])/(c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.247513, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x])/(a*c^2*(1 - a*x)^2*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2]*
Sqrt[ArcCosh[a*x]]) - (8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 +
 a^2*x^2)^3*Sqrt[ArcCosh[a*x]]), x])/(c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx=\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$=-\frac{2\sqrt{-1+ax}}{ac^2(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}-\frac{\left(8a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{}{(-1+a^2}}{c^2\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 2.25184, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(3/2)), x]
```

Maple [A] time = 0.363, size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{-\frac{5}{2}}(\mathrm{arccosh}\,(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\mathrm{arcosh}\,(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.414 $\int \frac{(c-a^2cx^2)^{3/2}}{\cosh^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=329

$$-\frac{2\sqrt{\pi}c\sqrt{c-a^2cx^2}\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2\sqrt{2\pi}c\sqrt{c-a^2cx^2}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}-\frac{2\sqrt{\pi}c\sqrt{c-a^2cx^2}\text{Erfi}\left(2\sqrt{\cosh^{-1}(a\right.}}{3a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c - a^2*c*x^2)^(3/2))/(3*a*ArcCosh[a*x]^(
3/2)) - (16*c*x*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2])/(3*Sqrt[ArcCosh[a*
x]]) - (2*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(3*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]) + (2*c*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[
2]*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (2*c*Sqrt[Pi]*
Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]) + (2*c*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x
]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.745105, antiderivative size = 337, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5713, 5697, 5776, 5701, 3312, 3307, 2180, 2204, 2205, 5781, 5448}

$$-\frac{2\sqrt{\pi}c\sqrt{c-a^2cx^2}\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2\sqrt{2\pi}c\sqrt{c-a^2cx^2}\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}-\frac{2\sqrt{\pi}c\sqrt{c-a^2cx^2}\text{Erfi}\left(2\sqrt{\cosh^{-1}(a\right.}}{3a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (2*c*(1 - a*x)^2*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[-1 + a*x]*A
rcCosh[a*x]^(3/2)) - (16*c*x*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*Sqrt[Arc
Cosh[a*x]]) - (2*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/
(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (2*c*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Er
f[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (2*c*Sq
rt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]) + (2*c*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcC
osh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\cosh^{-1}(ax)^{5/2}}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{3/2}(1+ax)^{3/2}}{\cosh^{-1}(ax)^{5/2}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{\left(8ac\sqrt{c-a^2cx^2}\right)\int \frac{x\left(-1+a^2x^2\right)}{\cosh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(16c\sqrt{c-a^2cx^2}\right)\int \frac{\sqrt{-1+ax}\sqrt{1+a}}{\sqrt{\cosh^{-1}(ax)}}}{3\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(16c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh}{\sqrt{}}\right.}{3a\sqrt{-1+ax}\sqrt{}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\left(16c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int\left(\frac{1}{2\sqrt{}}\right.\right.}{3a\sqrt{-1+}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(8c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(}{\sqrt{x}}\right.}{3a\sqrt{-1+ax}\sqrt{1}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\left(4c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\right.}{3a\sqrt{-1+ax}\sqrt{1+}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\right.}{3a\sqrt{-1+ax}\sqrt{1}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2c\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-}}\right.}{3a\sqrt{-1+ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.580683, size = 317, normalized size = 0.96

$$-\frac{c\sqrt{c-a^2cx^2}e^{-4\cosh^{-1}(ax)}\left(-16e^{4\cosh^{-1}(ax)}\left(-\cosh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-4\cosh^{-1}(ax)\right)+16\sqrt{2}e^{4\cosh^{-1}(ax)}\left(-\cosh^{-1}(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(5/2),x]
```

```
[Out] -(c*Sqrt[c - a^2*c*x^2]*(-1 - 14*E^(4*ArcCosh[a*x]) - E^(8*ArcCosh[a*x]) +
16*a^2*E^(4*ArcCosh[a*x])*x^2 + 8*ArcCosh[a*x] - 8*E^(8*ArcCosh[a*x])*ArcCo
sh[a*x] + 64*a*E^(4*ArcCosh[a*x])*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]
 + 64*a^2*E^(4*ArcCosh[a*x])*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] -
16*E^(4*ArcCosh[a*x])*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -4*ArcCosh[a*x]] + 1
6*Sqrt[2]*E^(4*ArcCosh[a*x])*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -2*ArcCosh[a*
x]] + 16*Sqrt[2]*E^(4*ArcCosh[a*x])*ArcCosh[a*x]^(3/2)*Gamma[1/2, 2*ArcCosh
[a*x]] - 16*E^(4*ArcCosh[a*x])*ArcCosh[a*x]^(3/2)*Gamma[1/2, 4*ArcCosh[a*x]
]))/(24*a*E^(4*ArcCosh[a*x])*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a
*x]^(3/2))
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}} \left(\operatorname{arccosh}(ax)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arcosh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.415 $\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=201

$$\frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}-\frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2])/(3*a*ArcCosh[a*x]^(3/
2)) - (8*x*Sqrt[c - a^2*c*x^2])/(3*Sqrt[ArcCosh[a*x]]) + (2*Sqrt[2*Pi]*Sqrt
[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]) + (2*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]
]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.223367, antiderivative size = 207, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5713, 5697, 5666, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}-\frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}}+\frac{2(1-ax)\sqrt{ax+}}{3a\sqrt{ax-1}\cos}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (2*(1 - a*x)*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[-1 + a*x]*ArcCosh
[a*x]^(3/2)) - (8*x*Sqrt[c - a^2*c*x^2])/(3*Sqrt[ArcCosh[a*x]]) + (2*Sqrt[2
*Pi]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]) + (2*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[Arc
Cosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{5/2}}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{-1+ax}\sqrt{1+ax}}{\cosh^{-1}(ax)^{5/2}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} + \frac{\left(4a\sqrt{c-a^2cx^2}\right)\int \frac{x}{\cosh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(8\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(a$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{3a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(8\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(ax}\right.}{3a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\mathbf{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{2\sqrt{2\pi}}{}$$

Mathematica [A] time = 0.300379, size = 141, normalized size = 0.7

$$-\frac{2\sqrt{c-a^2cx^2}\left(\sqrt{2}\left(-\cosh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-2\cosh^{-1}(ax)\right)+\sqrt{2}\cosh^{-1}(ax)^{3/2}\text{Gamma}\left(\frac{1}{2},2\cosh^{-1}(ax)\right)+(ax+\right.}{3a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\cosh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[c - a^2*c*x^2]*((1 + a*x)*(-1 + a*x + 4*a*x*Sqrt[(-1 + a*x)/(1 + a
*x)]*ArcCosh[a*x]) + Sqrt[2]*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -2*ArcCosh[a*
x]] + Sqrt[2]*ArcCosh[a*x]^(3/2)*Gamma[1/2, 2*ArcCosh[a*x]]))/(3*a*Sqrt[(-1
 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]^(3/2))
```

Maple [F] time = 0.499, size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\,(\operatorname{arccosh}(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\operatorname{arcosh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.416 $\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/
2))
```

Rubi [A] time = 0.154337, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/
2))
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}$$

Mathematica [A] time = 0.0328619, size = 48, normalized size = 1.

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/
2))
```

Maple [A] time = 0.048, size = 41, normalized size = 0.9

$$-\frac{2}{3a}\sqrt{ax-1}\sqrt{ax+1}\left(\operatorname{arccosh}(ax)\right)^{-\frac{3}{2}}\frac{1}{\sqrt{-(ax-1)(ax+1)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] -2/3/arccosh(a*x)^(3/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(5/2)), x)
```

Fricas [A] time = 1.96189, size = 134, normalized size = 2.79

$$\frac{2\sqrt{-a^2cx^2+c}\sqrt{a^2x^2-1}}{3\left(a^3cx^2-ac\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-a^2*c*x^2 + c)*sqrt(a^2*x^2 - 1)/((a^3*c*x^2 - a*c)*log(a*x + sqr
t(a^2*x^2 - 1))^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.417 $\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{4a\sqrt{ax-1}\sqrt{ax+1}\,\text{Unintegrable}\left(\frac{x}{\left(a^2x^2-1\right)^2\cosh^{-1}(ax)^{3/2}},x\right)}{3c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(
3/2)) + (4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Unintegrable[x/((-1 + a^2*x^2)^2*
ArcCosh[a*x]^(3/2)), x])/(3*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.228131, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x])/(3*a*c*(1 - a*x)*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2]*ArcC
osh[a*x]^(3/2)) + (4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2
*x^2)^2*ArcCosh[a*x]^(3/2)), x])/(3*c*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx=-\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$=-\frac{2\sqrt{-1+ax}}{3ac(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}+\frac{\left(4a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{}{\left(-1+a^2x^2\right.}}{3c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 1.73978, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2)), x]
```

Maple [A] time = 0.3, size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{-\frac{3}{2}}(\mathrm{arccosh}\,(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\mathrm{arcosh}\,(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.418 $\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{8a\sqrt{ax-1}\sqrt{ax+1}\,\text{Unintegrable}\left(\frac{x}{\left(a^2x^2-1\right)^3\cosh^{-1}(ax)^{3/2}},x\right)}{3c^2\sqrt{c-a^2cx^2}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{3/2}}$$

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*(c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(
3/2)) - (8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Unintegrable[x/((-1 + a^2*x^2)^3*
ArcCosh[a*x]^(3/2)), x])/(3*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.225575, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x])/(3*a*c^2*(1 - a*x)^2*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2
]*ArcCosh[a*x]^(3/2)) - (8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1
 + a^2*x^2)^3*ArcCosh[a*x]^(3/2)), x])/(3*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx=\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$=-\frac{2\sqrt{-1+ax}}{3ac^2(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}-\frac{\left(8a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{}{(-1+a}}{3c^2\sqrt{c-a^2cx}}$$

Mathematica [A] time = 2.26372, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(5/2)), x]
```

Maple [A] time = 0.377, size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{-\frac{5}{2}}(\mathrm{arccosh}\,(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\mathrm{arcosh}\,(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.419 $\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=253

$$\frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{d-c^2d}}{}$$

```
[Out] -(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*c^3*(1 + n)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma
[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c^3*E^((4*a)/b)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^((4*a)/b)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/
b])/(2^(2*(3 + n))*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b
)^n)
```

Rubi [A] time = 0.620448, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5798, 5781, 5448, 3307, 2181}

$$\frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{d-c^2d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```


```
[Out] -(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*c^3*(1 + n)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma
[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c^3*E^((4*a)/b)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^((4*a)/b)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/
b])/(2^(2*(3 + n))*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b
)^n)
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
```

```
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\sqrt{d-c^2dx^2}\int x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh^2(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int\left(-\frac{1}{8}(a+bx)^n+\frac{1}{8}(a+bx)^n\cosh(4x)\right)dx,x,\cosh^{-1}(cx)\right.}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh(4\right.}{8c^3\sqrt{-1+cx}\sqrt{1+c}}$$

$$=-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int e^{-4x}(a+bx)^n\,dx\right.}{16c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4^{-3-n}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{c^3\sqrt{-}}$$

Mathematica [A] time = 0.995498, size = 181, normalized size = 0.72

$$-\frac{d\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(4^{-n}e^{-\frac{4a}{b}}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\text{Gamma}\left(n+1,-\frac{4\left(a+b\cosh^{-1}(cx)\right.}{b}\right.\right.\right.}{64c^3\sqrt{-d(cx-1)(cx+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -(d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*((-8*(a + b
*ArcCosh[c*x]))/(b + b*n) + ((a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b
*ArcCosh[c*x]))/b] - E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n,
 (4*(a + b*ArcCosh[c*x]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^
2))^n)))/(64*c^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

Maple [F] time = 0.454, size = 0, normalized size = 0.

$$\int x^2\left(a+b\text{arccosh}\left(cx\right)\right)^n\sqrt{-c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.420 $\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=379

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{}$$

```
[Out] (3^(-1 - n)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a
+ b*ArcCosh[c*x]))/b])/(8*c^2*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((
a + b*ArcCosh[c*x])/b))^n) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Ga
mma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*c^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (E^(a/b)*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*c^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x])
)/b])/(8*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 0.679973, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5781, 5448, 3307, 2181}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```


```
[Out] (3^(-1 - n)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a
+ b*ArcCosh[c*x]))/b])/(8*c^2*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((
a + b*ArcCosh[c*x])/b))^n) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Ga
mma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*c^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (E^(a/b)*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*c^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x])
)/b])/(8*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```


Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx = \frac{\sqrt{d-c^2dx^2}\int x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^2(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int\left(-\frac{1}{4}(a+bx)^n\cosh(x)+\frac{1}{4}(a+bx)^n\cosh(3x)\right)dx,x,\cosh}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh(x)\,dx,x,\cosh^{-1}(cx)\right)}{4c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{d-c^2dx^2}\,\text{Subs}}{}$$

$$= \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{8c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int e\right.}{8c^2\sqrt{-}}$$

$$= \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3\left(a+b\cosh\right.}{b}\right.}{8c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.24698, size = 241, normalized size = 0.64

$$-\frac{de^{-\frac{3a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\left(-3^{-n}e^{\frac{6a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{2n}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\text{Gamma}\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -(d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*((3*E^((4*a
)/b)*Gamma[1 + n, a/b + ArcCosh[c*x]])/(a/b + ArcCosh[c*x])^n + (Gamma[1 +
n, (-3*(a + b*ArcCosh[c*x]))/b]/3^n - 3*E^((2*a)/b)*Gamma[1 + n, -((a + b*A
rcCosh[c*x])/b)] - (E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 +
 n, (3*(a + b*ArcCosh[c*x]))/b])/(3^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n))/(
-((a + b*ArcCosh[c*x])/b))^n))/(24*c^2*E^((3*a)/b)*Sqrt[-(d*(-1 + c*x)*(1 +
 c*x))])
```

Maple [F] time = 0.391, size = 0, normalized size = 0.

$$\int x\left(a+b\text{arccosh}\left(cx\right)\right)^n\sqrt{-c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-d(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.421 $\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=253

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a\right.}{}$$

```
[Out] -(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c*(1 + n)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (2^(-3 - n)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x
]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 0.375673, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5713, 5701, 3312, 3307, 2181}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```


```
[Out] -(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c*(1 + n)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (2^(-3 - n)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x
]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```


Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx = \frac{\sqrt{d-c^2dx^2}\int\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int(a+bx)^n\sinh^2(x)\,dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int\left(\frac{1}{2}(a+bx)^n-\frac{1}{2}(a+bx)^n\cosh(2x)\right)dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int(a+bx)^n\cosh(2x)\right.}{2c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int e^{-2x}(a+bx)^n\,dx,x\right.}{4c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c\sqrt{-1}}$$

Mathematica [A] time = 0.687661, size = 214, normalized size = 0.85

$$\frac{d2^{-n-3}e^{-\frac{2a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(-b(n+1)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\mathrm{Gamma}\left(n+1,-\frac{2(a+b}{}\right.\right.}{bc(n+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-3 - n)*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(
2^(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^
n - b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))
/b] + b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(
a + b*ArcCosh[c*x]))/b]))/(b*c*E^((2*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int(a+b\mathrm{arccosh}\,(cx))^n\sqrt{-c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2 + d}(b\operatorname{arcosh}(cx) + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2 + d}(b\operatorname{arcosh}(cx) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-d(cx-1)(cx+1)}\,(a + b\operatorname{acosh}(cx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.422 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$

Optimal. Leaf size=211

$$d\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}},x\right)-\frac{de^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\right.}{2\sqrt{d-c^2dx^2}}$$

```
[Out] -(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a +
 b*ArcCosh[c*x])/b)])/(2*E^(a/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x]
)/b))^n) + (d*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*G
amma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh
[c*x])/b)^n) + d*Unintegrable[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]
), x]
```

Rubi [A] time = 1.0825, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c
*x])/b)])/(2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b
))^n) - (E^(a/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a
 + b*ArcCosh[c*x])/b])/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x]
)/b)^n) - (Sqrt[d - c^2*d*x^2]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d-c^2dx^2}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int (a+bx)^n\cosh(x)\,dx,\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int e^{-x}(a+bx)^n\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int e\right.}{2\sqrt{-1}}$$

$$= \frac{e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.226385, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x, x]
```

Maple [A] time = 0.342, size = 0, normalized size = 0.

$$\int \frac{(a+b\mathrm{arccosh}\,(cx))^n}{x}\sqrt{-c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.423 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=91

$$d\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}},x\right)-\frac{cd\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{b(n+1)\sqrt{d-c^2dx^2}}$$

```
[Out] -((c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*(1 + n
)*Sqrt[d - c^2*d*x^2])) + d*Unintegrable[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d
 - c^2*d*x^2]), x]
```

Rubi [A] time = 0.901684, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] (c*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(b*(1 + n)*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*x^2]*Defer[Int][(a + b*ArcCosh[c*x])^n
/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int\left(\frac{c^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\int\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d-c^2dx^2}\right)\int\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\int\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.225363, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

Maple [A] time = 0.344, size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^n}{x^2}\sqrt{-c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.424 $\int x^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=658

$$-\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{d2^{-2n-7}e^{-\frac{4a}{b}}\sqrt{d}}{}$$

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(16*b*c^3*(1 + n)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (2^(-7 - n)*3^(-1 - n)*d*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((6
*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-7
 - 2*n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a +
b*ArcCosh[c*x]))/b])/(c^3*E^((4*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a +
b*ArcCosh[c*x])/b))^n) + (2^(-7 - n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7 - n)*d*E^((2*a
)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcC
osh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n
) - (2^(-7 - 2*n)*d*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*
Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*((a + b*ArcCosh[c*x])/b)^n) + (2^(-7 - n)*3^(-1 - n)*d*E^((6*a)/b)*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b
])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 1.05162, antiderivative size = 658, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5798, 5781, 5448, 3307, 2181}

$$-\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{d2^{-2n-7}e^{-\frac{4a}{b}}\sqrt{d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(16*b*c^3*(1 + n)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (2^(-7 - n)*3^(-1 - n)*d*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((6
*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-7
```

```
 - 2*n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a +
b*ArcCosh[c*x]))/b])/(c^3*E^((4*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a +
b*ArcCosh[c*x])/b))^n) + (2^(-7 - n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7 - n)*d*E^((2*a
)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcC
osh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n
) - (2^(-7 - 2*n)*d*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*
Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*((a + b*ArcCosh[c*x])/b)^n) + (2^(-7 - n)*3^(-1 - n)*d*E^((6*a)/b)*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b
])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh^4(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{16}(a+bx)^n-\frac{1}{32}(a+bx)^n\cosh(2x)-\frac{1}{16}(a+bx\right.\right.}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\,c\right.}{32c^3\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int e^{-6x}(a+b\right.}{64c^3\sqrt{-1+cx}\sqrt{1}}$$

$$= -\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\right.}{}$$

Mathematica [A] time = 3.18452, size = 438, normalized size = 0.67

$$\frac{d^2 2^{-2n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2^ne^{\frac{6a}{b}}\left(2^{n+3}3^{n+1}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{\left(a+b\cosh\right.}{b^2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-7 - 2*n)*3^(-1 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*Ar
cCosh[c*x])^n*(2^n*b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b
```

```
*ArcCosh[c*x]))/b] - 3^(1 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n
*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 2^n*3^(1 + n)*b*E^((4*a)/b)*(1
 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b] + 2^
n*3^(1 + n)*b*E^((8*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n
, (2*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((10*a)/b)*(1 + n)*(-((a + b*
ArcCosh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b] + 2^n*E^((6*a)
/b)*(2^(3 + n)*3^(1 + n)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2
))^n - b*E^((6*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (6*
(a + b*ArcCosh[c*x]))/b])))/(b*c^3*E^((6*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*
(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int x^{2} \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^{2} d x^{2} + d\right)^{\frac{3}{2}} \left(b \operatorname{arcosh}\left(c x\right) + a\right)^{n} x^{2}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(-{\left(c^{2} d x^{4} - d x^{2}\right)} \sqrt{-c^{2} d x^{2} + d} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="frica
s")
```

```
[Out] integral(-(c^2*d*x^4 - d*x^2)*sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.425 $\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=578

$$-\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{d-c^2dx}}{}$$

```
[Out] -(5^(-1 - n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-5*
(a + b*ArcCosh[c*x]))/b])/(32*c^2*E^((5*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(-((a + b*ArcCosh[c*x])/b))^n) + (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(32*3^n*c^2*E^((3*a)/b)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (d*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(16*c
^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (d
*E^(a/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*Arc
Cosh[c*x])/b])/(16*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b
)^n) - (d*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 +
n, (3*(a + b*ArcCosh[c*x]))/b])/(32*3^n*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((
a + b*ArcCosh[c*x])/b)^n) + (5^(-1 - n)*d*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(32*c^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 0.825303, antiderivative size = 578, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5781, 5448, 3307, 2181}

$$-\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{d-c^2dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```


```
[Out] -(5^(-1 - n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-5*
(a + b*ArcCosh[c*x]))/b])/(32*c^2*E^((5*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(-((a + b*ArcCosh[c*x])/b))^n) + (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(32*3^n*c^2*E^((3*a)/b)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (d*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(16*c
```

```
^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (d
*E^(a/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*Arc
Cosh[c*x])/b])/(16*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b
)^n) - (d*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 +
n, (3*(a + b*ArcCosh[c*x]))/b])/(32*3^n*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((
a + b*ArcCosh[c*x])/b)^n) + (5^(-1 - n)*d*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(32*c^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
```

```
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^4(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{8}(a+bx)^n\cosh(x)-\frac{3}{16}(a+bx)^n\cosh(3x)+\right.\right.}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh(5x)\,dx,x,\cosh^{-1}(cx)\right)}{16c^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(d\sqrt{d-\right.}{}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int e^{-5x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{32c^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(d\sqrt{d-c^2dx^2}\right.}{}$$

$$=-\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\right.}{32c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 2.09443, size = 500, normalized size = 0.87

$$-\frac{d^2 15^{-n-1}e^{-\frac{5a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-3n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\left(-3^{n+1}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{2n}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -(15^(-1 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])
^n*(2*15^(1 + n)*E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCos
h[c*x])^2/b^2))^(2*n)*Gamma[1 + n, a/b + ArcCosh[c*x]] + (a/b + ArcCosh[c*x
])^n*(-(3^(1 + n)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-5*(a
 + b*ArcCosh[c*x]))/b]) + 3*5^(1 + n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*x])^2
/b^2))^(2*n)*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] - 2*15^(1 + n)*E^((4
```

```
*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, -((a + b*ArcCosh[
c*x])/b)] + 5^(1 + n)*E^((8*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[
c*x])/b))^(3*n)*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b] - 4*5^(1 + n)*E^((
8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*(-((a + b*ArcCosh[c*x])^2/b^2))^n
*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*E^((10*a)/b)*(a/b + A
rcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (5*(a + b*Arc
Cosh[c*x]))/b])))/(32*c^2*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh
[c*x])^2/b^2))^(3*n))
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int x\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(a+b\operatorname{arccosh}\left(cx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(c^2dx^3-dx\right)\sqrt{-c^2dx^2+d}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas"
)
```

```
[Out] integral(-(c^2*d*x^3 - d*x)*sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.426 $\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=450

$$-\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{d2^{-n-3}e^{-\frac{2a}{b}}\sqrt{d-c^2}}{}$$

```
[Out] (-3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*c*(1 + n)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*G
amma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c*E^((4*a)/b)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-3 - n)*d*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x
]))/b])/(c*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])
/b))^n) - (2^(-3 - n)*d*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*((a + b*ArcCosh[c*x])/b)^n) + (d*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 0.563549, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5713, 5701, 3312, 3307, 2181}

$$-\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{d2^{-n-3}e^{-\frac{2a}{b}}\sqrt{d-c^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```


```
[Out] (-3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*c*(1 + n)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*G
amma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c*E^((4*a)/b)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-3 - n)*d*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x
]))/b])/(c*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])
/b))^n) - (2^(-3 - n)*d*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*((a + b*ArcCosh[c*x])/b)^n) + (d*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*
```

```
ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\sinh^4(x)\,dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{3}{8}(a+bx)^n-\frac{1}{2}(a+bx)^n\cosh(2x)+\frac{1}{8}(a+bx)^n\cos\right.\right.}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cos\right.}{8c\sqrt{-1+cx}\sqrt{1-}}$$

$$= -\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int e^{-4x}(a+bx)\right.}{16c\sqrt{-1+cx}\sqrt{1+c}}$$

$$= -\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{4^{-3-n}de^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{c\sqrt{}}$$

Mathematica [A] time = 2.0866, size = 384, normalized size = 0.85

$$\frac{d^2 4^{-n-3}e^{-\frac{4a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(b\left(-2^{n+3}\right)(n+1)e^{\frac{2a}{b}}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\left(\frac{a}{b}+\cosh^{-}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (4^(-3 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n
*(3*2^(3 + 2*n)*E^((4*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/
b^2))^(2*n) + b*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh[c*x])/
b))^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 2^(3 + n)*b*E^((2*a)/b)*(
1 + n)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n
, (-2*(a + b*ArcCosh[c*x]))/b] + 2^(3 + n)*b*E^((6*a)/b)*(1 + n)*(-((a + b*
ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a +
b*ArcCosh[c*x]))/b] - b*E^((8*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n*(-((a +
b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b]))/(b*c*E
^((4*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n
))
```

Maple [F] time = 0.216, size = 0, normalized size = 0.

$$\int \left(-c^{2} d x^{2}+d\right)^{\frac{3}{2}} \left(a +b \operatorname{arccosh}\left(c x \right)\right)^{n}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^{2} d x^{2}+d\right)^{\frac{3}{2}} \left(b \operatorname{arcosh}\left(c x\right)+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(-c^{2} d x^{2}+d\right)^{\frac{3}{2}} \left(b \operatorname{arcosh}\left(c x\right)+a\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.427 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^n}{x}\,dx$

Optimal. Leaf size=414

$$d^2\text{Unintegrable}\left(\frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{d-c^2dx^2}},x\right)+\frac{d^2 3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamm}}{8\sqrt{d-c^2dx^2}}$$

```
[Out] (3^(-1 - n)*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1
 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(8*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])/b))^n) - (5*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*Arc
Cosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*E^(a/b)*Sqrt[d - c
^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) + (5*d^2*E^(a/b)*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8
*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*d^2*E^((3*a)
/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a
+ b*ArcCosh[c*x]))/b])/(8*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) +
 d^2*Unintegrable[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]), x]
```

Rubi [A] time = 1.83564, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] -(3^(-1 - n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*
(a + b*ArcCosh[c*x]))/b])/(8*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a
 + b*ArcCosh[c*x])/b))^n) + (5*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n
*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (5*d*E^(a/b)*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) + (3^(-1 - n)*d*E^((3*a)/b)*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]
))/b])/(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (d*Sqr
t[d - c^2*d*x^2]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[-1 + c*x]*Sqrt[1
```

```
+ c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^4x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(2c^2d\sqrt{d-c^2dx^2}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,d}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int (a+bx)^n\cos\right.}{\sqrt{-1+cx}\sqrt{1+c}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int e^{-x}(a+bx)^n\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{de^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{de^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{3^{-1-n}de^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+}{}\right.}{8\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.279811, size = 0, normalized size = 0.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x, x]
```

Maple [A] time = 0.244, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{n}}{x}\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{n}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.428 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=291

$$d^2\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}},x\right)+\frac{cd^2 2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamm}}{\sqrt{d-c^2dx^2}}$$

```
[Out] (-3*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*(
1 + n)*Sqrt[d - c^2*d*x^2]) + (2^(-3 - n)*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(E^((2*
a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*c*d^
2*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 +
 n, (2*(a + b*ArcCosh[c*x]))/b])/(Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])
/b)^n) + d^2*Unintegrable[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]),
 x]
```

Rubi [A] time = 1.51746, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] (3*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*(1 + n)*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (2^(-3 - n)*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(E^((2*a)/b)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-3 - n)*c*d*E^((2
*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*Ar
cCosh[c*x]))/b])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
- (d*Sqrt[d - c^2*d*x^2]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(-\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^4x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(2c^2d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{2cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} -$$

$$= \frac{2cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} -$$

$$= \frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} -$$

$$= \frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} -$$

$$= \frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2^{-3-n}cde^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{\sqrt{}}$$

Mathematica [A] time = 0.561489, size = 0, normalized size = 0.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

Maple [A] time = 0.272, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{n}}{x^{2}}\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}\left(b\,\operatorname{arcosh}\left(cx\right)+a\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}\left(b\,\operatorname{arcosh}\left(cx\right)+a\right)^{n}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="frica
s")
```

```
[Out] integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.429 $\int x^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=870

result too large to display

```
[Out] (-5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(128*b*c^3*(1 + n
)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2^(-11 - 3*n)*d^2*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((8*
a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7
- n)*3^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n,
 (-6*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((6*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(-((a + b*ArcCosh[c*x])/b))^n) + (d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(4 + n))*c^3*E^((4
*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-7
 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a +
b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a +
b*ArcCosh[c*x])/b))^n) - (2^(-7 - n)*d^2*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (d^2*E^((4*a)/b)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/
b])/(2^(2*(4 + n))*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b
)^n) + (2^(-7 - n)*3^(-1 - n)*d^2*E^((6*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (2^(-11 - 3*n)*d^2*E^((8*a)/b)
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (8*(a + b*ArcCosh[
c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 1.24155, antiderivative size = 870, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5798, 5781, 5448, 3307, 2181}

$$\frac{2^{-3n-11}d^2e^{-\frac{8a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n \text{Gamma}\left(n+1,-\frac{8\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}}{c^3\sqrt{cx-1}\sqrt{cx+1}} - \frac{2^{-n-7}3^{-n-1}d^2e^{-\frac{6a}{b}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```


```
[Out] (-5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(128*b*c^3*(1 + n
)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2^(-11 - 3*n)*d^2*Sqrt[d - c^2*d*x^2]*(a
```

```
+ b*ArcCosh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((8*
a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7
- n)*3^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n,
(-6*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((6*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(-((a + b*ArcCosh[c*x])/b))^n) + (d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(4 + n))*c^3*E^((4
*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-7
- n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a +
b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a +
b*ArcCosh[c*x])/b))^n) - (2^(-7 - n)*d^2*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (d^2*E^((4*a)/b)*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/
b])/(2^(2*(4 + n))*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b
)^n) + (2^(-7 - n)*3^(-1 - n)*d^2*E^((6*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (2^(-11 - 3*n)*d^2*E^((8*a)/b)
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (8*(a + b*ArcCosh[
c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh^6(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{5}{128}(a+bx)^n+\frac{1}{32}(a+bx)^n\cosh(2x)+\frac{1}{32}(a\right.\right.}{c^3\sqrt{-}}$$

$$=-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+b\right.}{128c^3\sqrt{-1+}}$$

$$=-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int e^{-8x}(a\right.}{256c^3\sqrt{-1+c}}$$

$$=-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d-c^2dx^2}\left(a+b\right.}{}$$

Mathematica [A] time = 6.88134, size = 677, normalized size = 0.78

$$d^32^{-3n-11}3^{-n-1}e^{-\frac{8a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(e^{\frac{8a}{b}}\left(b3^{n+1}4^{n+2}(n+1)e^{\frac{2a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*A
rcCosh[c*x])^n*(-(3^(1 + n)*b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (
-8*(a + b*ArcCosh[c*x]))/b]) + 4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcCo
sh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b] - 2^(3 + n)*3^(1 + n)*
b*E^((4*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCos
h[c*x]))/b] - 3^(1 + n)*4^(2 + n)*b*E^((6*a)/b)*(1 + n)*(a/b + ArcCosh[c*x]
)^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b] + E^((8*a)/b)*(5*2^(4 + 3*n)*
3^(1 + n)*a*(-((a + b*ArcCosh[c*x])^2/b^2))^n + 5*2^(4 + 3*n)*3^(1 + n)*b*A
rcCosh[c*x]*(-((a + b*ArcCosh[c*x])^2/b^2))^n + 3^(1 + n)*4^(2 + n)*b*E^((2
*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*ArcCosh
[c*x]))/b] + 2^(3 + n)*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*
x])/b))^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b] - 4^(2 + n)*b*E^((6*a)/b
)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b] -
4^(2 + n)*b*E^((6*a)/b)*n*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (6*(a
+ b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))
^n*Gamma[1 + n, (8*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*n*(-(
(a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (8*(a + b*ArcCosh[c*x]))/b])))/(b*c
^3*E^((8*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^
n)
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int x^2 \left(-c^2 d x^2 + d\right)^{\frac{5}{2}} \left(a + b \operatorname{arccosh}(cx)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2 d x^2 + d\right)^{\frac{5}{2}} \left(b \operatorname{arcosh}(cx) + a\right)^n x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^4d^2x^6 - 2\,c^2d^2x^4 + d^2x^2\right)\sqrt{-c^2dx^2 + d}(b\,\text{arcosh}\,(cx) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="frica
s")
```

```
[Out] integral((c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(-c^2*d*x^2 + d)*(b*ar
ccosh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.430 $\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=793

$$\frac{d^2 7^{-n-1} e^{-\frac{7a}{b}} \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{7\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{128c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2 5^{-n} e^{-\frac{5a}{b}}\sqrt{d-c^2dx}}{}$$

```
[Out] (7^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-7
*(a + b*ArcCosh[c*x]))/b])/(128*c^2*E^((7*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(-((a + b*ArcCosh[c*x])/b))^n) - (d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^n*Gamma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b])/(128*5^n*c^2*E^((5*a)/b)
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (3^(1 - n)*d
^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCo
sh[c*x]))/b])/(128*c^2*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*A
rcCosh[c*x])/b))^n) - (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gam
ma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(128*c^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (5*d^2*E^(a/b)*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(128*c^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (3^(1 - n)*d^2*E^((
3*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*A
rcCosh[c*x]))/b])/(128*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x
])/b)^n) + (d^2*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamm
a[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(128*5^n*c^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*((a + b*ArcCosh[c*x])/b)^n) - (7^(-1 - n)*d^2*E^((7*a)/b)*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcCosh[c*x]))/b])/(1
28*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 1.05402, antiderivative size = 793, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5781, 5448, 3307, 2181}

$$\frac{d^2 7^{-n-1} e^{-\frac{7a}{b}} \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{7\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{128c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2 5^{-n} e^{-\frac{5a}{b}}\sqrt{d-c^2dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```



```
[Out] (7^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-7
*(a + b*ArcCosh[c*x]))/b])/(128*c^2*E^((7*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
```

```
]*(-((a + b*ArcCosh[c*x])/b))^n) - (d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^n*Gamma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b])/(128*5^n*c^2*E^((5*a)/b)
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (3^(1 - n)*d
^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCo
sh[c*x]))/b])/(128*c^2*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*A
rcCosh[c*x])/b))^n) - (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gam
ma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(128*c^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (5*d^2*E^(a/b)*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(128*c^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (3^(1 - n)*d^2*E^((
3*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*A
rcCosh[c*x]))/b])/(128*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x
])/b)^n) + (d^2*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamm
a[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(128*5^n*c^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*((a + b*ArcCosh[c*x])/b)^n) - (7^(-1 - n)*d^2*E^((7*a)/b)*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcCosh[c*x]))/b])/(1
28*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
```

```
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^6(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{5}{64}(a+bx)^n\cosh(x)+\frac{9}{64}(a+bx)^n\cosh(3x)-\right.\right.}{c^2\sqrt{-1+cx}\sqrt{1}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh(7x)\,dx,x,\cosh^{-1}(cx)\right)}{64c^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(5d^2\sqrt{d}\right.}{}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int e^{-7x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx\right.}{}$$

$$=\frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{7}{}\right.}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 3.80858, size = 633, normalized size = 0.8

$$\frac{d^35^{-n}21^{-n-1}e^{-\frac{7a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-3n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\left(e^{\frac{2a}{b}}\left(21^{n+1}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)}{b^2}\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (21^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^
n*(-(105^(1 + n)*E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCos
h[c*x])^2/b^2))^(2*n)*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + ArcCosh[c*
x])^n*(-(3^(1 + n)*5^n*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (
-7*(a + b*ArcCosh[c*x]))/b]) + E^((2*a)/b)*(21^(1 + n)*(-((a + b*ArcCosh[c*
x])^2/b^2))^(2*n)*Gamma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b] - 9*5^n*7^(1 +
n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-3*(a +
b*ArcCosh[c*x]))/b] + 105^(1 + n)*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2
))^(2*n)*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)] - 5^n*7^(2 + n)*E^((8*a)/b
)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (3*
(a + b*ArcCosh[c*x]))/b] + 16*5^n*7^(1 + n)*E^((8*a)/b)*(-((a + b*ArcCosh[c
*x])/b))^(2*n)*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (3*(a + b*Arc
Cosh[c*x]))/b] - 21^(1 + n)*E^((10*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*A
rcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)
*5^n*E^((12*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*
Gamma[1 + n, (7*(a + b*ArcCosh[c*x]))/b]))))/(128*5^n*c^2*E^((7*a)/b)*Sqrt[
d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^(3*n))
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int x\left(-c^2dx^2+d\right)^{\frac{5}{2}}(a+b\operatorname{arccosh}(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^4d^2x^5 - 2\,c^2d^2x^3 + d^2x\right)\sqrt{-c^2dx^2 + d}(b\,\text{arcosh}\,(cx) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)*sqrt(-c^2*d*x^2 + d)*(b*arcc
osh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.431 $\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=674

$$\frac{d^2 2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{3d^2 2^{-2n-7}e^{-\frac{4a}{b}}\sqrt{}}{}$$

[Out] (-5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(16*b*c*(1 + n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2^(-7 - n)*3^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b])/(c*E^((6*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (3*2^(-7 - 2*n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(c*E^((4*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (15*2^(-7 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (15*2^(-7 - n)*d^2*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) + (3*2^(-7 - 2*n)*d^2*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d^2*E^((6*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)

Rubi [A] time = 0.703305, antiderivative size = 674, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5713, 5701, 3312, 3307, 2181}

$$\frac{d^2 2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{3d^2 2^{-2n-7}e^{-\frac{4a}{b}}\sqrt{}}{}$$

Antiderivative was successfully verified.

[In] Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]

[Out] (-5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(16*b*c*(1 + n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2^(-7 - n)*3^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b])/(c*E^(

```
(6*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (3*2
^(-7 - 2*n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4
*(a + b*ArcCosh[c*x]))/b])/(c*E^((4*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((
a + b*ArcCosh[c*x])/b))^n) + (15*2^(-7 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c*E^((2*a)/b)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (15*2^(-7 - n)
*d^2*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2
*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[
c*x])/b)^n) + (3*2^(-7 - 2*n)*d^2*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d^2*E^((6
*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (6*(a + b*Ar
cCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n
)
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\sinh^6(x)\,dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{5}{16}(a+bx)^n-\frac{15}{32}(a+bx)^n\cosh(2x)+\frac{3}{16}(a+bx)^n\right.\right.}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\right.}{32c\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int e^{-6x}(a+b\right.}{64c\sqrt{-1+cx}\sqrt{1+}}$$

$$= -\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\right.}{}$$

Mathematica [A] time = 5.3658, size = 538, normalized size = 0.8

$$\frac{d^3 2^{-2n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(-5b2^n3^{n+2}(n+1)e^{\frac{4a}{b}}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\left(\frac{a}{b}+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-7 - 2*n)*3^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*Ar
cCosh[c*x])^n*(-(2^n*b*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh
```

```
[c*x])/b))^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b]) + 3^(2 + n)*b*E^((2
*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh[c*x])/b))^n*Gam
ma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 5*2^n*3^(2 + n)*b*E^((4*a)/b)*(1 +
 n)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (
-2*(a + b*ArcCosh[c*x]))/b] + 5*2^n*3^(2 + n)*b*E^((8*a)/b)*(1 + n)*(-((a +
 b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a
 + b*ArcCosh[c*x]))/b] - 3^(2 + n)*b*E^((10*a)/b)*(1 + n)*(a/b + ArcCosh[c*
x])^n*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]
))/b] + 2^n*E^((6*a)/b)*(5*2^(3 + n)*3^(1 + n)*(a + b*ArcCosh[c*x])*(-((a +
 b*ArcCosh[c*x])^2/b^2))^(2*n) + b*E^((6*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])
^n*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/
b])))/(b*c*E^((6*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^
2/b^2))^(2*n))
```

Maple [F] time = 0.228, size = 0, normalized size = 0.

$$\int \left(-c^{2} d x^{2}+d\right)^{\frac{5}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^{2} d x^{2} + d\right)^{\frac{5}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(c^{4} d^{2} x^{4} - 2 \, c^{2} d^{2} x^{2} + d^{2}\right)} \sqrt{-c^{2} d x^{2} + d} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arccos
h(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.432 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$

Optimal. Leaf size=804

result too large to display

```
[Out] -(5^(-1 - n)*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[
1 + n, (-5*(a + b*ArcCosh[c*x]))/b])/(32*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(-
((a + b*ArcCosh[c*x])/b))^n) - (5*3^(-1 - n)*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(32*E^
((3*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) + (d^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCos
h[c*x]))/b])/(8*3^n*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])
/b))^n) - (11*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma
[1 + n, -((a + b*ArcCosh[c*x])/b)])/(16*E^(a/b)*Sqrt[d - c^2*d*x^2]*(-((a +
 b*ArcCosh[c*x])/b))^n) + (11*d^3*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(16*Sqrt[d - c^2*d
*x^2]*((a + b*ArcCosh[c*x])/b)^n) + (5*3^(-1 - n)*d^3*E^((3*a)/b)*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c
*x]))/b])/(32*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) - (d^3*E^((3*
a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(
a + b*ArcCosh[c*x]))/b])/(8*3^n*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b
)^n) + (5^(-1 - n)*d^3*E^((5*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(32*Sqrt[d - c^2*d*x^
2]*((a + b*ArcCosh[c*x])/b)^n) + d^3*Unintegrable[(a + b*ArcCosh[c*x])^n/(x
*Sqrt[d - c^2*d*x^2]), x]
```

Rubi [A] time = 2.5547, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] (5^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-5
*(a + b*ArcCosh[c*x]))/b])/(32*E^((5*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-(
```

```
(a + b*ArcCosh[c*x])/b))^n) + (5*3^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(32*E^((3*a)/b)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (d^2*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b
])/(8*3^n*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/
b))^n) + (11*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -(
(a + b*ArcCosh[c*x])/b)])/(16*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a +
b*ArcCosh[c*x])/b))^n) - (11*d^2*E^(a/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(16*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*((a + b*ArcCosh[c*x])/b)^n) - (5*3^(-1 - n)*d^2*E^((3*a)/b)*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b])/
(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) + (d^2*E^((3*a
)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcC
osh[c*x]))/b])/(8*3^n*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)
^n) - (5^(-1 - n)*d^2*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^
n*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*((a + b*ArcCosh[c*x])/b)^n) - (d^2*Sqrt[d - c^2*d*x^2]*Defer[Int][(a + b*
ArcCosh[c*x])^n/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3c^4x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(3c^2d^2\sqrt{d-c^2dx^2}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\right.}{\sqrt{-1+cx}\sqrt{1}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{5}{8}(a+bx\right.\right.}{}$$

$$= \frac{3d^2e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}}{b}\right.}{2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{3d^2e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}}{b}\right.}{2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{5^{-1-n}d^2e^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a}{}\right.}{32\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.301884, size = 0, normalized size = 0.

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x, x]
```

Maple [A] time = 0.273, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{arccosh}(cx))^n}{x}\left(-c^2dx^2 + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx) + a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^4d^2x^4 - 2\,c^2d^2x^2 + d^2\right)\sqrt{-c^2dx^2 + d}(b\operatorname{arcosh}(cx) + a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arccos
h(c*x) + a)^n/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.433 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))^n}{x^2}\,dx$

Optimal. Leaf size=485

$$d^3\text{Unintegrable}\left(\frac{(a+b\cosh^{-1}(cx))^n}{x^2\sqrt{d-c^2dx^2}},x\right)-\frac{cd^3 2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamm}}{\sqrt{d-c^2dx^2}}$$

```
[Out] (-15*c*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*
(1 + n)*Sqrt[d - c^2*d*x^2]) - (c*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*A
rcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*E^
((4*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-2 - n)*
c*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*
(a + b*ArcCosh[c*x]))/b])/(E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCos
h[c*x])/b))^n) - (2^(-2 - n)*c*d^3*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(Sqrt[d -
 c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) + (c*d^3*E^((4*a)/b)*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))
/b])/(2^(2*(3 + n))*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) + d^3*U
nintegrable[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]), x]
```

Rubi [A] time = 2.18818, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] (15*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*(1 + n)*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x
])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*E^((4*a)/b)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-2 - n)*c
*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*Arc
Cosh[c*x]))/b])/(E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh
[c*x])/b))^n) + (2^(-2 - n)*c*d^2*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(Sqrt[-1 + c*x]*Sqr
```

```
t[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (c*d^2*E^((4*a)/b)*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(2^(2
*(3 + n))*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (d^2*S
qrt[d - c^2*d*x^2]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3c^4x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(3c^2d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{3cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,d}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{3cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,d}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8b(1+n)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4^{-3-n}cd^2e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{}$$

Mathematica [A] time = 0.584664, size = 0, normalized size = 0.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

Maple [A] time = 0.27, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{arccosh}(cx))^n}{x^2} \left(-c^2dx^2 + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2 + d\right)^{\frac{5}{2}} (b\operatorname{arcosh}(cx) + a)^n}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4d^2x^4 - 2\,c^2d^2x^2 + d^2\right)\sqrt{-c^2dx^2 + d}(b\operatorname{arcosh}(cx) + a)^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="frica
s")
```

```
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arccos
h(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.434 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=323

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{1-cx}}+\frac{3e^{-\frac{a}{b}}\sqrt{cx-1}\left(a+b\cosh^{-1}\right.}{}$$

```
[Out] (3^(-1 - n)*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*A
rcCosh[c*x]))/b])/(8*c^4*E^((3*a)/b)*Sqrt[1 - c*x]*(-((a + b*ArcCosh[c*x])/
b))^n) + (3*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*Arc
Cosh[c*x])/b)])/(8*c^4*E^(a/b)*Sqrt[1 - c*x]*(-((a + b*ArcCosh[c*x])/b))^n)
 - (3*E^(a/b)*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*Arc
Cosh[c*x])/b])/(8*c^4*Sqrt[1 - c*x]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 -
n)*E^((3*a)/b)*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b
*ArcCosh[c*x]))/b])/(8*c^4*Sqrt[1 - c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 0.732098, antiderivative size = 375, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5798, 5781, 3312, 3307, 2181}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{1-c^2x^2}}+\frac{3e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (3^(-1 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n
, (-3*(a + b*ArcCosh[c*x]))/b])/(8*c^4*E^((3*a)/b)*Sqrt[1 - c^2*x^2]*(-((a
+ b*ArcCosh[c*x])/b))^n) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c
*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*c^4*E^(a/b)*Sqrt[1 - c^2
*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (3*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*c^4*Sqr
t[1 - c^2*x^2]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCos
h[c*x]))/b])/(8*c^4*Sqrt[1 - c^2*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n \cosh^3(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{3}{4}(a+bx)^n \cosh(x) + \frac{1}{4}(a+bx)^n \cosh(3x)\right) dx, x, \cosh^{-1}(\right.}{c^4\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n \cosh(3x)\,dx, x, \cosh^{-1}(cx)\right)}{4c^4\sqrt{1-c^2x^2}} + \frac{\left(3\sqrt{-1+cx}\sqrt{1+c\right.}{}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx, x, \cosh^{-1}(cx)\right)}{8c^4\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Sub}}{8}$$

$$= \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n \left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n, -\frac{3\left(a+b\cosh^{-1}(c\right.}{b}\right.}{8c^4\sqrt{1-c^2x^2}}$$

<u>Mathematica</u> [A] time = 1.273, size = 292, normalized size = 0.9

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(3^{n+2}e^{\frac{4a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n \text{Gamma}\left(n\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (3^(-1 - n)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^n*(3^(2 + n)*E^((4*a)/b)
*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 +
n, a/b + ArcCosh[c*x]] - (a/b + ArcCosh[c*x])^n*((-((a + b*ArcCosh[c*x])^2/
b^2))^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] + 3^(2 + n)*E^((2*a)/b)*(
-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)] -
E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcCos
h[c*x]))/b])))/(8*c^4*E^((3*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((
a + b*ArcCosh[c*x])^2/b^2))^(2*n))
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int x^{3} \left(a+b \,\mathrm{arccosh}\left(c x\right)\right)^{n} \frac{1}{\sqrt{-c^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x^{3}}{\sqrt{-c^{2} x^{2} + 1}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^3/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(-\frac{\sqrt{-c^{2} x^{2} + 1} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x^{3}}{c^{2} x^{2} - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x^3/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.435 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=211

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{1-cx}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{cx-1}\left(a+b\cosh\right.}{}$$

```
[Out] (Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c^3*(1 + n)*Sqrt[1 - c*x
]) + (2^(-3 - n)*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a
+ b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1 - c*x]*(-((a + b*ArcCosh[c*x
])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*G
amma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[1 - c*x]*((a + b*ArcCosh
[c*x])/b)^n)
```

Rubi [A] time = 0.615718, antiderivative size = 250, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5798, 5781, 3312, 3307, 2181}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{1-c^2x^2}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{cx-1}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c^3*(1 + n
)*Sqrt[1 - c^2*x^2]) + (2^(-3 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqr
t[1 - c^2*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*Arc
Cosh[c*x]))/b])/(c^3*Sqrt[1 - c^2*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{x^2 \left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh^2(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{1}{2}(a+bx)^n + \frac{1}{2}(a+bx)^n\cosh(2x)\right)dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n \cosh\right.}{2c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-2x}(a+bx)^n\right.}{4c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{1-c^2x^2}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx\right.}{c^3\sqrt{}}$$

Mathematica [A] time = 0.787952, size = 212, normalized size = 1.

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(b(n+1)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\text{Gamma}\left(n+1,-\frac{2(a+b\cos}{b}\right.\right.}{bc^3(n+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (2^(-3 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2^
(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n
+ b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b
] - b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(a
+ b*ArcCosh[c*x]))/b]))/(b*c^3*E^((2*a)/b)*(1 + n)*Sqrt[1 - c^2*x^2]*(-((a
+ b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int x^2\left(a+b\text{arccosh}\left(cx\right)\right)^n\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^n x^2}{\sqrt{-c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2 + 1}(b \operatorname{arcosh}(cx) + a)^n x^2}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x^2/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{acosh}(cx))^n}{\sqrt{-(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.436 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=154

$$\frac{e^{-\frac{a}{b}}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}-\frac{e^{a/b}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b}{\ldots}\right.}{2c\ldots}$$

```
[Out] (Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/
b)])/(2*c^2*E^(a/b)*Sqrt[1 - c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^(a/b)
*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b]
)/(2*c^2*Sqrt[1 - c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 0.421691, antiderivative size = 180, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5798, 5781, 3307, 2181}

$$\frac{e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-c^2x^2}}-\frac{e^{a/b}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\,\text{c}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*
ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[1 - c^2*x^2]*(-((a + b*ArcCosh[c*x])
/b))^n) - (E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamm
a[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[1 - c^2*x^2]*((a + b*ArcCosh[
c*x])/b)^n)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int (a+bx)^n\cosh(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int e^{-x}(a+bx)^n\,dx, x, \cosh^{-1}(cx)\right)}{2c^2\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Subs}}{2c}$$
$$= \frac{e^{-\frac{a}{b}}\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-c^2x^2}} - \frac{e}{}$$

Mathematica [A] time = 0.242509, size = 154, normalized size = 1.

$$-\frac{e^{-\frac{a}{b}}\sqrt{-(cx-1)(cx+1)}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\mathrm{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)-\right.}{2c^2\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] -(Sqrt[-((-1 + c*x)*(1 + c*x))]*(a + b*ArcCosh[c*x])^n*(-(E^((2*a)/b)*(-((a
 + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + ArcCosh
[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(2*c^2*E^(a/b)*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F] time = 0.278, size = 0, normalized size = 0.

$$\int x\left(a+b\mathrm{arccosh}\left(cx\right)\right)^{n}\frac{1}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}x}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^{2}x^{2}+1}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}x}{c^{2}x^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a + b \operatorname{acosh}\left(cx\right)\right)^{n}}{\sqrt{-\left(cx - 1\right)\left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.437 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{1-cx}}$$

```
[Out] (Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[1 - c*x])
```

Rubi [A] time = 0.207832, antiderivative size = 56, normalized size of antiderivative = 1.3, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {5713, 5676}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^n/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[1 - c^2*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$
$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.0389282, size = 56, normalized size = 1.3

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[1 - c^2*x^2])
```

Maple [A] time = 0.034, size = 53, normalized size = 1.2

$$\frac{(a+b\operatorname{arccosh}(cx))^{1+n}}{cb(1+n)}\sqrt{cx-1}\sqrt{cx+1}\frac{1}{\sqrt{-(cx-1)(cx+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] (a+b*arccosh(c*x))^(1+n)/b/(1+n)/c/(-(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Fricas [B] time = 2.60406, size = 487, normalized size = 11.33

$$\frac{\left(\sqrt{c^2x^2-1}\sqrt{-c^2x^2+1}b\log\left(cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}\sqrt{-c^2x^2+1}a\right)\cosh\left(n\log\left(b\log\left(cx+\sqrt{c^2x^2-1}\right)+a\right)\right)+\left(\sqrt{c^2\cdots\right.}{bcn-\left(bc^3n+bc^3\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] ((sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*b*log(c*x + sqrt(c^2*x^2 - 1)) + sqr
t(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*a)*cosh(n*log(b*log(c*x + sqrt(c^2*x^2 -
1)) + a)) + (sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*b*log(c*x + sqrt(c^2*x^2
- 1)) + sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*a)*sinh(n*log(b*log(c*x + sqrt
(c^2*x^2 - 1)) + a)))/(b*c*n - (b*c^3*n + b*c^3)*x^2 + b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{acosh}(cx))^n}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.438 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]), x]
```

Rubi [A] time = 0.435239, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{1-c^2x^2}}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 2.48225, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]), x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int \frac{(a + b\mathrm{arccosh}\,(cx))^n}{x} \frac{1}{\sqrt{-c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)^n}{\sqrt{-c^2x^2 + 1}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2 + 1}(b\,\mathrm{arcosh}\,(cx) + a)^n}{c^2x^3 - x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n/(c^2*x^3 - x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))^n}{x\sqrt{-(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x*sqrt(-(c*x - 1)*(c*x + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.439 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]), x]
```

Rubi [A] time = 0.445807, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.29318, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]), x]
```

Maple [A] time = 0.232, size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{arccosh}(cx))^n}{x^2}\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{-c^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2x^2+1}(b\operatorname{arcosh}(cx)+a)^n}{c^2x^4-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n/(c^2*x^4 - x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acosh}(cx))^n}{x^2 \sqrt{-(cx-1)(cx+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x**2*sqrt(-(c*x - 1)*(c*x + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.440 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=379

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{d-c^2dx^2}}+\frac{3e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+}}{}$$

```
[Out] (3^(-1 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n
, (-3*(a + b*ArcCosh[c*x]))/b])/(8*c^4*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])/b))^n) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*c^4*E^(a/b)*Sqrt[d - c
^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (3*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*c^4
*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*
ArcCosh[c*x]))/b])/(8*c^4*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 0.80871, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5798, 5781, 3312, 3307, 2181}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{d-c^2dx^2}}+\frac{3e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (3^(-1 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n
, (-3*(a + b*ArcCosh[c*x]))/b])/(8*c^4*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])/b))^n) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*c^4*E^(a/b)*Sqrt[d - c
^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (3*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*c^4
*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*
ArcCosh[c*x]))/b])/(8*c^4*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh^3(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^4\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{3}{4}(a+bx)^n\cosh(x)+\frac{1}{4}(a+bx)^n\cosh(3x)\right)dx,x,\cosh^{-1}(\cdots}{c^4\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh(3x)\,dx,x,\cosh^{-1}(cx)\right)}{4c^4\sqrt{d-c^2dx^2}} + \frac{\left(3\sqrt{-1+cx}\sqrt{1+c\cdots\right.}{\cdots}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{8c^4\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Sub}\cdots}{8\cdots}$$

$$= \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3\left(a+b\cosh^{-1}(c\cdots\right.}{b}\cdots}{8c^4\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.07402, size = 291, normalized size = 0.77

$$-\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(3^{n+2}e^{\frac{4a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n \text{Gamm}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -(3^(-1 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(3
^(2 + n)*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^
2/b^2))^n*Gamma[1 + n, a/b + ArcCosh[c*x]] - (a/b + ArcCosh[c*x])^n*((-((a
+ b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] + 3^(
2 + n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, -((a + b*
ArcCosh[c*x])/b)] - E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 +
 n, (3*(a + b*ArcCosh[c*x]))/b])))/(8*c^4*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(
-((a + b*ArcCosh[c*x])^2/b^2))^(2*n))
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int x^{3} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{n} \frac{1}{\sqrt{-c^{2} d \,x^{2}+d}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x^{3}}{\sqrt{-c^{2} d x^{2} + d}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^3/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(-\frac{\sqrt{-c^{2} d x^{2} + d} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x^{3}}{c^{2} d x^{2} - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^3/(c^2*d*x^2 - d),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.441 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=253

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{d-c^2dx^2}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{cx-1}\ldots}{\ldots}$$

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c^3*(1 + n
)*Sqrt[d - c^2*d*x^2]) + (2^(-3 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*S
qrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b
*ArcCosh[c*x]))/b])/(c^3*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 0.63997, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5798, 5781, 3312, 3307, 2181}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{d-c^2dx^2}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{cx-1}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c^3*(1 + n
)*Sqrt[d - c^2*d*x^2]) + (2^(-3 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*S
qrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b
*ArcCosh[c*x]))/b])/(c^3*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh^2(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{1}{2}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)\right)dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{d-c^2dx^2}}$$
$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n \cosh\right.}{2c^3\sqrt{d-c^2dx^2}}$$
$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-2x}(a+bx)^n\right.}{4c^3\sqrt{d-c^2dx^2}}$$
$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx\right.}{c^3\sqrt{d}}$$

Mathematica [A] time = 0.754513, size = 213, normalized size = 0.84

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(b(n+1)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\text{Gamma}\left(n+1,-\frac{2\left(a+b\cos\right.}{b}\right.\right.}{bc^3(n+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2^(-3 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2^
(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n
+ b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b
] - b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(a
+ b*ArcCosh[c*x]))/b]))/(b*c^3*E^((2*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int x^2\left(a+b\operatorname{arccosh}(cx)\right)^n\frac{1}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^n x^2}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n x^2}{c^2dx^2-d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2/(c^2*d*x^2 - d),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2(a+b\operatorname{acosh}(cx))^n}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.442 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{d-c^2dx^2}}-\frac{e^{a/b}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right.}{}$$

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*
ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x
])/b))^n) - (E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Ga
mma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[d - c^2*d*x^2]*((a + b*ArcC
osh[c*x])/b)^n)
```

Rubi [A] time = 0.41948, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5781, 3307, 2181}

$$\frac{e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{d-c^2dx^2}}-\frac{e^{a/b}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*
ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x
])/b))^n) - (E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Ga
mma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[d - c^2*d*x^2]*((a + b*ArcC
osh[c*x])/b)^n)
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int (a+bx)^n\cosh(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int e^{-x}(a+bx)^n\,dx, x, \cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Subs}}{2c}$$

$$= \frac{e^{-\frac{a}{b}}\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{d-c^2dx^2}} - \frac{e^{}}{}$$

Mathematica [A] time = 0.223898, size = 153, normalized size = 0.84

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)-e^{\frac{2a}{b}}\left(-\frac{a+\cdots}{\cdots}\right.\right.}{2c^2\sqrt{-d(cx-1)(cx+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(-(E^((2*a)/b)
*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + A
rcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(2*c^2*E^(a/b)*Sqr
t[-(d*(-1 + c*x)*(1 + c*x))]*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F] time = 0.281, size = 0, normalized size = 0.

$$\int x\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{n}\frac{1}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}x}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)^n x}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,(a+b\,\text{acosh}\,(cx))^n}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.443 $\int \frac{(a+b\cosh^{-1}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{d-c^2dx^2}}$$

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[d - c^2*d*x^2])
```

Rubi [A] time = 0.193784, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5713, 5676}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^n/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[d - c^2*d*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
```

]

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.0448806, size = 57, normalized size = 1.

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[d - c^2*d*x^2])
```

Maple [A] time = 0.035, size = 54, normalized size = 1.

$$\frac{(a+b\text{arccosh}\,(cx))^{1+n}}{cb\,(1+n)}\sqrt{cx-1}\sqrt{cx+1}\frac{1}{\sqrt{-(cx-1)\,(cx+1)\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] (a+b*arccosh(c*x))^(1+n)/b/(1+n)/c/(-(c*x-1)*(c*x+1)*d)^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^n}{\sqrt{-c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [B] time = 2.68469, size = 509, normalized size = 8.93

$$\frac{\left(\sqrt{-c^2dx^2 + d}\sqrt{c^2x^2 - 1}b\log\left(cx + \sqrt{c^2x^2 - 1}\right) + \sqrt{-c^2dx^2 + d}\sqrt{c^2x^2 - 1}a\right)\cosh\left(n\log\left(b\log\left(cx + \sqrt{c^2x^2 - 1}\right) + a\right)\right) + \left(\right.}{bcdn + bcd - \left(bc^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] ((sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*log(c*x + sqrt(c^2*x^2 - 1)) + s
qrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a)*cosh(n*log(b*log(c*x + sqrt(c^2*x^
2 - 1)) + a)) + (sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*log(c*x + sqrt(c^
2*x^2 - 1)) + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a)*sinh(n*log(b*log(c*
x + sqrt(c^2*x^2 - 1)) + a)))/(b*c*d*n + b*c*d - (b*c^3*d*n + b*c^3*d)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acosh}(cx))^n}{\sqrt{-d(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.444 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]), x]
```

Rubi [A] time = 0.442886, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x])/Sqrt[d - c^2*d*x^2]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.313692, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]), x]
```

Maple [A] time = 0.304, size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^n}{x}\frac{1}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)^n}{c^2dx^3-dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^2*d*x^3 - d*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acosh}(cx))^n}{x\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.445 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]), x]
```

Rubi [A] time = 0.464772, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[d - c^2*d*x^2]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.336044, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]), x]
```

Maple [A] time = 0.282, size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{arccosh}(cx))^n}{x^2}\frac{1}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{-c^2dx^2+d}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)^n}{c^2dx^4-dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^2*d*x^4 - d*x^2),
x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(cx))^n}{x^2\sqrt{-d(cx-1)(cx+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.446 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [A] time = 0.52532, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(x^2*(a + b*ArcCosh[c*x])^n)/((-
1 + c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.623505, size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Maple [A] time = 0.313, size = 0, normalized size = 0.

$$\int x^{2} \left(a + b \operatorname{arccosh}\left(c x\right)\right)^{n} \left(-c^{2} d x^{2} + d\right)^{-\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{arcosh}\left(c x\right) + a\right)^{n} x^{2}}{\left(-c^{2} d x^{2} + d\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^{2} d x^{2} + d} \left(b \operatorname{arcosh}\left(c x\right) + a\right)^{n} x^{2}}{c^{4} d^{2} x^{4} - 2 \, c^{2} d^{2} x^{2} + d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2/(c^4*d^2*x^4 - 2*c
^2*d^2*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.447 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [A] time = 0.368062, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(x*(a + b*ArcCosh[c*x])^n)/((-1
+ c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.361164, size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Maple [A] time = 0.28, size = 0, normalized size = 0.

$$\int x \left(a+b\operatorname{arccosh}\left(cx\right)\right)^{n} \left(-c^{2}d x^{2}+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{n} x}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^{2}dx^{2}+d}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{n} x}{c^{4}d^{2}x^{4}-2\,c^{2}d^{2}x^{2}+d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x/(c^4*d^2*x^4 - 2*c^2
*d^2*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.448 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [A] time = 0.222932, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/((-1 + c*
x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.0782835, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2), x]
```

Maple [A] time = 0.22, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(cx))^{n} \left(-c^{2} d x^{2}+d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^{n}}{\left(-c^{2} d x^{2}+d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^{2} d x^{2}+d}(b\operatorname{arcosh}(cx)+a)^{n}}{c^{4} d^{2} x^{4}-2\, c^{2} d^{2} x^{2}+d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 - 2*c^2*d
^2*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.449 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)), x]
```

Rubi [A] time = 0.520825, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*(-1 +
c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.38015, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)), x]
```

Maple [A] time = 0.276, size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{arccosh}(cx))^{n}}{x}\left(-c^{2}dx^{2}+d\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^{n}}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-c^{2}dx^{2}+d}(b\operatorname{arcosh}(cx)+a)^{n}}{c^{4}d^{2}x^{5}-2\,c^{2}d^{2}x^{3}+d^{2}x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^5 - 2*c^2*d
^2*x^3 + d^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.450 $\int \frac{(a+b\cosh^{-1}(cx))^n}{x^2(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)), x]
```

Rubi [A] time = 0.524975, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*(-1
+ c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.413668, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)), x]
```

Maple [A] time = 0.267, size = 0, normalized size = 0.

$$\int \frac{(a+b\mathrm{arccosh}\,(cx))^n}{x^2}\left(-c^2dx^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^n}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)^n}{c^4d^2x^6-2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^6 - 2*c^2*d
^2*x^4 + d^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.451 $\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=32

$$\text{Unintegrable}\left(\frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2], x]
```

Rubi [A] time = 0.416793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.442976, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2], x]
```

Maple [A] time = 0.311, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(a + b\operatorname{arccosh}\left(cx\right)\right)^{n} \frac{1}{\sqrt{-c^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^{m} \left(b \operatorname{arcosh}\left(cx\right) + a\right)^{n}}{\sqrt{-c^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^{2}x^{2}+1} \left(fx\right)^{m} \left(b \operatorname{arcosh}\left(cx\right) + a\right)^{n}}{c^{2}x^{2}-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*x^2 - 1),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] sage0*x
```

3.452 $\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\left(d-c^2dx^2\right)^2 (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n,x\right)$$

```
[Out] Unintegrable[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n, x]
```

Rubi [A] time = 0.0900236, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Defer[Int][(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^n \, dx = \int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Mathematica [A] time = 1.11284, size = 0, normalized size = 0.

$$\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n, x]
```

Maple [A] time = 0.305, size = 0, normalized size = 0.

$$\int (fx)^m \left(-c^2dx^2+d\right)^2 (a+b\operatorname{arccosh}(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2-d\right)^2 (fx)^m (b\operatorname{arcosh}(cx)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c^4d^2x^4-2\,c^2d^2x^2+d^2\right)(fx)^m (b\operatorname{arcosh}(cx)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="frica
s")
```

```
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*(f*x)^m*(b*arccosh(c*x) + a)^n
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.453 $\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\left(d-c^2dx^2\right)(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n,x\right)$$

```
[Out] Unintegrable[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n, x]
```

Rubi [A] time = 0.0572402, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Defer[Int][(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx = \int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Mathematica [A] time = 0.58422, size = 0, normalized size = 0.

$$\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n, x]
```

Maple [A] time = 0.209, size = 0, normalized size = 0.

$$\int (fx)^m \left(-c^2dx^2+d\right)(a+b\mathrm{arccosh}\,(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \left(c^2dx^2-d\right)(fx)^m (b\,\mathrm{arcosh}\,(cx)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="maxima"
)
```

```
[Out] -integrate((c^2*d*x^2 - d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(c^2dx^2-d\right)(fx)^m (b\,\mathrm{arcosh}\,(cx)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="fricas"
)
```

```
[Out] integral(-(c^2*d*x^2 - d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.454 $\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left((fx)^m \left(a+b\cosh^{-1}(cx)\right)^n, x\right)$$

```
[Out] Unintegrable[(f*x)^m*(a + b*ArcCosh[c*x])^n, x]
```

Rubi [A] time = 0.0225902, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Defer[Int][(f*x)^m*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps

$$\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n dx = \int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n dx$$

Mathematica [A] time = 0.0191948, size = 0, normalized size = 0.

$$\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Integrate[(f*x)^m*(a + b*ArcCosh[c*x])^n, x]
```

Maple [A] time = 0.167, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(a+b\mathrm{arccosh}\left(cx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(fx\right)^{m} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n,x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**n, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.455 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2), x]
```

Rubi [A] time = 0.102393, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2),x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$$

Mathematica [A] time = 0.587033, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2), x]
```

Maple [A] time = 0.258, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b\operatorname{arccosh}(cx))^n}{-c^2dx^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(fx)^m (b \operatorname{arcosh}(cx) + a)^n}{c^2dx^2 - d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="maxima"
)
```

```
[Out] -integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(fx)^m (b \operatorname{arcosh}(cx) + a)^n}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="fricas"
)
```

```
[Out] integral(-(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.456 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2, x]
```

Rubi [A] time = 0.0992411, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2,x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$$

Mathematica [A] time = 0.907385, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2, x]
```

Maple [A] time = 0.271, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b\text{arccosh}(cx))^n}{(-c^2dx^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (b\,\text{arcosh}(cx) + a)^n}{(c^2dx^2 - d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx)^m (b\,\text{arcosh}(cx) + a)^n}{c^4d^2x^4 - 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="frica
s")
```

```
[Out] integral((f*x)^m*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.457 $\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\left(d-c^2dx^2\right)^{3/2}(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n,x\right)$$

```
[Out] Unintegrable[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n, x]
```

Rubi [A] time = 0.49809, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -((d*Sqrt[d - c^2*d*x^2]*Defer[Int][(f*x)^m*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2
)*(a + b*ArcCosh[c*x])^n, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int (fx)^m(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n \,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.600705, size = 0, normalized size = 0.

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n, x]
```

Maple [A] time = 0.326, size = 0, normalized size = 0.

$$\int (fx)^m \left(-c^2dx^2+d\right)^{\frac{3}{2}} (a+b\,\mathrm{arccosh}\,(cx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(-c^2dx^2+d\right)^{\frac{3}{2}} (fx)^m (b\,\mathrm{arcosh}\,(cx)+a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="m
axima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(c^2dx^2-d\right)\sqrt{-c^2dx^2+d}\,(fx)^m (b\,\mathrm{arcosh}\,(cx)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="f
ricas")
```

```
[Out] integral(-(c^2*d*x^2 - d)*sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)
^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="g
iac")
```

```
[Out] sage0*x
```

3.458 $\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\sqrt{d-c^2dx^2}(fx)^m \left(a+b\cosh^{-1}(cx)\right)^n, x\right)$$

```
[Out] Unintegrable[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n, x]
```

Rubi [A] time = 0.401181, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*Defer[Int][(f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])^n, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx = \frac{\sqrt{d-c^2dx^2} \int (fx)^m \sqrt{-1+cx}\sqrt{1+cx} \left(a+b\cosh^{-1}(cx)\right)^n \, dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.104005, size = 0, normalized size = 0.

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n, x]
```

Maple [A] time = 0.401, size = 0, normalized size = 0.

$$\int (fx)^m \sqrt{-c^2dx^2+d}\,(a+b\,\mathrm{arccosh}\,(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c^2dx^2+d}\,(fx)^m\,(b\,\mathrm{arcosh}\,(cx)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-c^2dx^2+d}\,(fx)^m\,(b\,\mathrm{arcosh}\,(cx)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="g
iac")
```

```
[Out] sage0*x
```

3.459 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2], x]
```

Rubi [A] time = 0.436233, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[d - c^2*d*x^2]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.412742, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2], x]
```

Maple [A] time = 0.331, size = 0, normalized size = 0.

$$\int (fx)^m (a + b\operatorname{arccosh}(cx))^n \frac{1}{\sqrt{-c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (b\operatorname{arcosh}(cx) + a)^n}{\sqrt{-c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2dx^2 + d}\,(fx)^m (b\operatorname{arcosh}(cx) + a)^n}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 -
d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

3.460 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [A] time = 0.502943, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)
/((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.648258, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Maple [A] time = 0.298, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{n} \left(-c^{2}d x^{2}+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^{m} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}}{\left(-c^{2}d x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^{2}d x^{2}+d}\left(fx\right)^{m} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}}{c^{4}d^{2}x^{4}-2\,c^{2}d^{2}x^{2}+d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 -
 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

3.461 $\int x^4 (d+ex^2) (a+b\cosh^{-1}(cx)) dx$

Optimal. Leaf size=177

$$\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\cosh^{-1}(cx)\right)-\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}\left(49c^2d+30e\right)}{1225c^3}-\frac{4bx^2\sqrt{cx-1}\sqrt{cx+1}\left(49c^2d\right.}{3675c^5}$$

```
[Out] (-8*b*(49*c^2*d + 30*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^7) - (4*b*(49
*c^2*d + 30*e)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^5) - (b*(49*c^2*d
+ 30*e)*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1225*c^3) - (b*e*x^6*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/(49*c) + (d*x^5*(a + b*ArcCosh[c*x]))/5 + (e*x^7*(a + b*
ArcCosh[c*x]))/7
```

Rubi [A] time = 0.141917, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5786, 460, 100, 12, 74}

$$\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\cosh^{-1}(cx)\right)-\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}\left(49c^2d+30e\right)}{1225c^3}-\frac{4bx^2\sqrt{cx-1}\sqrt{cx+1}\left(49c^2d\right.}{3675c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (-8*b*(49*c^2*d + 30*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^7) - (4*b*(49
*c^2*d + 30*e)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^5) - (b*(49*c^2*d
+ 30*e)*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1225*c^3) - (b*e*x^6*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/(49*c) + (d*x^5*(a + b*ArcCosh[c*x]))/5 + (e*x^7*(a + b*
ArcCosh[c*x]))/7
```


Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^4 \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{35}(bc)\int\frac{x^5\left(7d+5ex^2\right.}{\sqrt{-1+cx}\sqrt{1+}}$$

$$= -\frac{bex^6\sqrt{-1+cx}\sqrt{1+cx}}{49c}+\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\cosh^{-1}(cx)\right)+$$

$$= -\frac{b\left(49c^2d+30e\right)x^4\sqrt{-1+cx}\sqrt{1+cx}}{1225c^3}-\frac{bex^6\sqrt{-1+cx}\sqrt{1+cx}}{49c}+\frac{1}{5}dx^5\left(a+b\,\mathrm{c}\right.$$

$$= -\frac{b\left(49c^2d+30e\right)x^4\sqrt{-1+cx}\sqrt{1+cx}}{1225c^3}-\frac{bex^6\sqrt{-1+cx}\sqrt{1+cx}}{49c}+\frac{1}{5}dx^5\left(a+b\,\mathrm{c}\right.$$

$$= -\frac{4b\left(49c^2d+30e\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{3675c^5}-\frac{b\left(49c^2d+30e\right)x^4\sqrt{-1+cx}\sqrt{1+cx}}{1225c^3}-$$

$$= -\frac{4b\left(49c^2d+30e\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{3675c^5}-\frac{b\left(49c^2d+30e\right)x^4\sqrt{-1+cx}\sqrt{1+cx}}{1225c^3}-$$

$$= -\frac{8b\left(49c^2d+30e\right)\sqrt{-1+cx}\sqrt{1+cx}}{3675c^7}-\frac{4b\left(49c^2d+30e\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{3675c^5}-$$

Mathematica [A] time = 0.106008, size = 122, normalized size = 0.69

$$\frac{1}{35}ax^5\left(7d+5ex^2\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(3c^6\left(49dx^4+25ex^6\right)+2c^4\left(98dx^2+45ex^4\right)+8c^2\left(49d+15ex^2\right)+240e\right)}{3675c^7}+\frac{1}{35}b$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (a*x^5*(7*d + 5*e*x^2))/35 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(240*e + 8*c^2
*(49*d + 15*e*x^2) + 2*c^4*(98*d*x^2 + 45*e*x^4) + 3*c^6*(49*d*x^4 + 25*e*x
^6)))/(3675*c^7) + (b*x^5*(7*d + 5*e*x^2)*ArcCosh[c*x])/35
```

Maple [A] time = 0.029, size = 133, normalized size = 0.8

$$\frac{1}{c^5}\left(\frac{a}{c^2}\left(\frac{ec^7x^7}{7}+\frac{c^7x^5d}{5}\right)+\frac{b}{c^2}\left(\frac{\mathrm{arccosh}\,(cx)\,ec^7x^7}{7}+\frac{\mathrm{arccosh}\,(cx)\,c^7x^5d}{5}-\frac{75\,c^6ex^6+147\,c^6dx^4+90\,c^4ex^4+196\,c^4dx^2}{3675}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(a/c^2*(1/7*e*c^7*x^7+1/5*c^7*x^5*d)+b/c^2*(1/7*arccosh(c*x)*e*c^7*x^
7+1/5*arccosh(c*x)*c^7*x^5*d-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*e*x
^6+147*c^6*d*x^4+90*c^4*e*x^4+196*c^4*d*x^2+120*c^2*e*x^2+392*c^2*d+240*e))
)
```

Maxima [A] time = 1.16551, size = 240, normalized size = 1.36

$$\frac{1}{7}\,aex^7+\frac{1}{5}\,adx^5+\frac{1}{75}\left(15\,x^5\,\operatorname{arcosh}\left(cx\right)-\left(\frac{3\,\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd+\frac{1}{245}\left(35\,x^7\,\operatorname{arcosh}\left(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1
)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d +
 1/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2
 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*
b*e
```

Fricas [A] time = 2.49662, size = 329, normalized size = 1.86

$$\frac{525\,ac^7ex^7+735\,ac^7dx^5+105\left(5\,bc^7ex^7+7\,bc^7dx^5\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(75\,bc^6ex^6+3\left(49\,bc^6d+30\,bc^4e\right)x^4+392\,b\right.}{3675\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*e*x^7 + 735*a*c^7*d*x^5 + 105*(5*b*c^7*e*x^7 + 7*b*c^7*d*
x^5)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*e*x^6 + 3*(49*b*c^6*d + 30*b*
c^4*e)*x^4 + 392*b*c^2*d + 4*(49*b*c^4*d + 30*b*c^2*e)*x^2 + 240*b*e)*sqrt(
c^2*x^2 - 1))/c^7
```

Sympy [A] time = 9.52757, size = 230, normalized size = 1.3

$$\begin{cases} \frac{adx^5}{5} + \frac{aex^7}{7} + \frac{bdx^5 \operatorname{acosh}(cx)}{5} + \frac{bex^7 \operatorname{acosh}(cx)}{7} - \frac{bdx^4\sqrt{c^2x^2-1}}{25c} - \frac{bex^6\sqrt{c^2x^2-1}}{49c} - \frac{4bdx^2\sqrt{c^2x^2-1}}{75c^3} - \frac{6bex^4\sqrt{c^2x^2-1}}{245c^3} - \frac{8bd\sqrt{c^2x^2-1}}{75c^5} - \frac{8bex^2\sqrt{c^2 \dots}}{245c^5} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{dx^5}{5} + \frac{ex^7}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*acosh(c*x)/5 + b*e*x**7*acosh
(c*x)/7 - b*d*x**4*sqrt(c**2*x**2 - 1)/(25*c) - b*e*x**6*sqrt(c**2*x**2 - 1
)/(49*c) - 4*b*d*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 6*b*e*x**4*sqrt(c**2*
x**2 - 1)/(245*c**3) - 8*b*d*sqrt(c**2*x**2 - 1)/(75*c**5) - 8*b*e*x**2*sqr
t(c**2*x**2 - 1)/(245*c**5) - 16*b*e*sqrt(c**2*x**2 - 1)/(245*c**7), Ne(c,
0)), ((a + I*pi*b/2)*(d*x**5/5 + e*x**7/7), True))
```

Giac [A] time = 1.27582, size = 232, normalized size = 1.31

$$\frac{1}{5}\,adx^5 + \frac{1}{75}\left(15\,x^5\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{3\left(c^2x^2-1\right)^{\frac{5}{2}} + 10\left(c^2x^2-1\right)^{\frac{3}{2}} + 15\,\sqrt{c^2x^2-1}}{c^5}\right)bd + \frac{1}{245}\left(35\,ax^7 + \left(35\,x^7\log \dots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*a*d*x^5 + 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1)^
(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))/c^5)*b*d + 1/245*(35
*a*x^7 + (35*x^7*log(c*x + sqrt(c^2*x^2 - 1)) - (5*(c^2*x^2 - 1)^(7/2) + 21
*(c^2*x^2 - 1)^(5/2) + 35*(c^2*x^2 - 1)^(3/2) + 35*sqrt(c^2*x^2 - 1))/c^7)*
b)*e
```

3.462 $\int x^3 \left(d+ex^2\right) \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=161

$$\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\cosh^{-1}(cx)\right)-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}\left(9c^2d+5e\right)}{144c^3}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}\left(9c^2d+5e\right)}{96c^5}-$$

```
[Out] -(b*(9*c^2*d + 5*e)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96*c^5) - (b*(9*c^2*d
+ 5*e)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(144*c^3) - (b*e*x^5*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])/(36*c) - (b*(9*c^2*d + 5*e)*ArcCosh[c*x])/(96*c^6) + (d*x^
4*(a + b*ArcCosh[c*x]))/4 + (e*x^6*(a + b*ArcCosh[c*x]))/6
```

Rubi [A] time = 0.136614, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5786, 460, 100, 12, 90, 52}

$$\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\cosh^{-1}(cx)\right)-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}\left(9c^2d+5e\right)}{144c^3}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}\left(9c^2d+5e\right)}{96c^5}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```


```
[Out] -(b*(9*c^2*d + 5*e)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96*c^5) - (b*(9*c^2*d
+ 5*e)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(144*c^3) - (b*e*x^5*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])/(36*c) - (b*(9*c^2*d + 5*e)*ArcCosh[c*x])/(96*c^6) + (d*x^
4*(a + b*ArcCosh[c*x]))/4 + (e*x^6*(a + b*ArcCosh[c*x]))/6
```


Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
```

```
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x^3(d+ex^2)(a+b\cosh^{-1}(cx))\,dx = \frac{1}{4}dx^4(a+b\cosh^{-1}(cx))+\frac{1}{6}ex^6(a+b\cosh^{-1}(cx))-\frac{1}{24}(bc)\int\frac{x^4(6d+4ex^2)}{\sqrt{-1+cx}\sqrt{1+c}}$$

$$=-\frac{bex^5\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{4}dx^4(a+b\cosh^{-1}(cx))+\frac{1}{6}ex^6(a+b\cosh^{-1}(cx))-$$

$$=-\frac{b(9c^2d+5e)x^3\sqrt{-1+cx}\sqrt{1+cx}}{144c^3}-\frac{bex^5\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{4}dx^4(a+b\cosh$$

$$=-\frac{b(9c^2d+5e)x^3\sqrt{-1+cx}\sqrt{1+cx}}{144c^3}-\frac{bex^5\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{4}dx^4(a+b\cosh$$

$$=-\frac{b(9c^2d+5e)x\sqrt{-1+cx}\sqrt{1+cx}}{96c^5}-\frac{b(9c^2d+5e)x^3\sqrt{-1+cx}\sqrt{1+cx}}{144c^3}-\frac{bex^5\sqrt{-}}{}$$

$$=-\frac{b(9c^2d+5e)x\sqrt{-1+cx}\sqrt{1+cx}}{96c^5}-\frac{b(9c^2d+5e)x^3\sqrt{-1+cx}\sqrt{1+cx}}{144c^3}-\frac{bex^5\sqrt{-}}{}$$

Mathematica [A] time = 0.166379, size = 140, normalized size = 0.87

$$\frac{24ac^6x^4(3d+2ex^2)-bcx\sqrt{cx-1}\sqrt{cx+1}(2c^4(9dx^2+4ex^4)+c^2(27d+10ex^2)+15e)+24bc^6x^4\cosh^{-1}(cx)(3d+2ex^2}{288c^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (24*a*c^6*x^4*(3*d + 2*e*x^2) - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(15*e +
c^2*(27*d + 10*e*x^2) + 2*c^4*(9*d*x^2 + 4*e*x^4)) + 24*b*c^6*x^4*(3*d + 2*
e*x^2)*ArcCosh[c*x] - 6*b*(9*c^2*d + 5*e)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)
]])/(288*c^6)
```

Maple [A] time = 0.019, size = 250, normalized size = 1.6

$$\frac{aex^6}{6}+\frac{ax^4d}{4}+\frac{b\mathrm{arccosh}(cx)ex^6}{6}+\frac{b\mathrm{arccosh}(cx)x^4d}{4}-\frac{bex^5}{36c}\sqrt{cx-1}\sqrt{cx+1}-\frac{bdx^3}{16c}\sqrt{cx-1}\sqrt{cx+1}-\frac{5bex^3}{144c^3}\sqrt{cx-1}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/6*a*e*x^6+1/4*a*x^4*d+1/6*b*arccosh(c*x)*e*x^6+1/4*b*arccosh(c*x)*x^4*d-1
/36*b*e*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/16*b*d*x^3*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c-5/144/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e*x^3-3/32*b*d*x*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/c^3-3/32/c^4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^
(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d-5/96/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e*
x-5/96/c^6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^
2-1)^(1/2))
```

Maxima [A] time = 1.1482, size = 289, normalized size = 1.8

$$\frac{1}{6}\,aex^6+\frac{1}{4}\,adx^4+\frac{1}{32}\left(8\,x^4\,\text{arcosh}\,(cx)-\left(\frac{2\,\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\,\sqrt{c^2x^2-1}x}{c^4}+\frac{3\,\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^4}\right)c\right)bd+\frac{1}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)
*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*
sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*d + 1/288*(48*x^6*arccosh(c*x) - (8*sqrt(c
^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x
/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^6))*c)*
b*e
```

Fricas [A] time = 2.31987, size = 308, normalized size = 1.91

$$\frac{48\,ac^6ex^6+72\,ac^6dx^4+3\left(16\,bc^6ex^6+24\,bc^6dx^4-9\,bc^2d-5\,be\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(8\,bc^5ex^5+2\left(9\,bc^5d+5\,bc^3e\right)\right.}{288\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*e*x^6 + 72*a*c^6*d*x^4 + 3*(16*b*c^6*e*x^6 + 24*b*c^6*d*x^4
 - 9*b*c^2*d - 5*b*e)*log(c*x + sqrt(c^2*x^2 - 1)) - (8*b*c^5*e*x^5 + 2*(9*
b*c^5*d + 5*b*c^3*e)*x^3 + 3*(9*b*c^3*d + 5*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^
6
```

Sympy [A] time = 6.39488, size = 212, normalized size = 1.32

$$\begin{cases} \frac{adx^4}{4} + \frac{aex^6}{6} + \frac{bdx^4 \operatorname{acosh}(cx)}{4} + \frac{bex^6 \operatorname{acosh}(cx)}{6} - \frac{bdx^3\sqrt{c^2x^2-1}}{16c} - \frac{bex^5\sqrt{c^2x^2-1}}{36c} - \frac{3bdx\sqrt{c^2x^2-1}}{32c^3} - \frac{5bex^3\sqrt{c^2x^2-1}}{144c^3} - \frac{3bd \operatorname{acosh}(cx)}{32c^4} - \frac{5bex\sqrt{c^2x^2}}{96c^5} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{dx^4}{4} + \frac{ex^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*acosh(c*x)/4 + b*e*x**6*acosh
(c*x)/6 - b*d*x**3*sqrt(c**2*x**2 - 1)/(16*c) - b*e*x**5*sqrt(c**2*x**2 - 1
)/(36*c) - 3*b*d*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 5*b*e*x**3*sqrt(c**2*x**
2 - 1)/(144*c**3) - 3*b*d*acosh(c*x)/(32*c**4) - 5*b*e*x*sqrt(c**2*x**2 - 1
)/(96*c**5) - 5*b*e*acosh(c*x)/(96*c**6), Ne(c, 0)), ((a + I*pi*b/2)*(d*x**
4/4 + e*x**6/6), True))
```

Giac [A] time = 1.34462, size = 267, normalized size = 1.66

$$\frac{1}{4}\,adx^4 + \frac{1}{32}\left(8\,x^4\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(\sqrt{c^2x^2-1}x\left(\frac{2\,x^2}{c^2} + \frac{3}{c^4}\right) - \frac{3\,\log\left(\left|-x|c| + \sqrt{c^2x^2-1}\right|\right)}{c^4|c|}\right)c\right)bd + \frac{1}{288}\left(48\,ax^6 + \left(4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*a*d*x^4 + 1/32*(8*x^4*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)
*x*(2*x^2/c^2 + 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*abs
(c)))*c)*b*d + 1/288*(48*a*x^6 + (48*x^6*log(c*x + sqrt(c^2*x^2 - 1)) - (sq
rt(c^2*x^2 - 1)*(2*x^2*(4*x^2/c^2 + 5/c^4) + 15/c^6)*x - 15*log(abs(-x*abs(
c) + sqrt(c^2*x^2 - 1)))/(c^6*abs(c)))*c)*b)*e
```

3.463 $\int x^2 \left(d+ex^2\right) \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=138

$$\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\cosh^{-1}(cx)\right)-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(25c^2d+12e\right)}{225c^3}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(25c^2d+1\right.}{225c^5}$$

```
[Out] (-2*b*(25*c^2*d + 12*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^5) - (b*(25*c^
2*d + 12*e)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^3) - (b*e*x^4*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/(25*c) + (d*x^3*(a + b*ArcCosh[c*x]))/3 + (e*x^5*(a +
 b*ArcCosh[c*x]))/5
```

Rubi [A] time = 0.121313, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5786, 460, 100, 12, 74}

$$\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\cosh^{-1}(cx)\right)-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(25c^2d+12e\right)}{225c^3}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(25c^2d+1\right.}{225c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (-2*b*(25*c^2*d + 12*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^5) - (b*(25*c^
2*d + 12*e)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^3) - (b*e*x^4*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/(25*c) + (d*x^3*(a + b*ArcCosh[c*x]))/3 + (e*x^5*(a +
 b*ArcCosh[c*x]))/5
```


Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
```

```
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^2\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)dx = \frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{15}(bc)\int\frac{x^3\left(5d+3ex^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}$$

$$= -\frac{bex^4\sqrt{-1+cx}\sqrt{1+cx}}{25c}+\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\cosh^{-1}(cx)\right)-$$

$$= -\frac{b\left(25c^2d+12e\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{225c^3}-\frac{bex^4\sqrt{-1+cx}\sqrt{1+cx}}{25c}+\frac{1}{3}dx^3\left(a+b\cos\right.$$

$$= -\frac{b\left(25c^2d+12e\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{225c^3}-\frac{bex^4\sqrt{-1+cx}\sqrt{1+cx}}{25c}+\frac{1}{3}dx^3\left(a+b\cos\right.$$

$$= -\frac{2b\left(25c^2d+12e\right)\sqrt{-1+cx}\sqrt{1+cx}}{225c^5}-\frac{b\left(25c^2d+12e\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{225c^3}-\frac{be}{}$$

Mathematica [A] time = 0.101433, size = 101, normalized size = 0.73

$$\frac{1}{225}\left(15ax^3\left(5d+3ex^2\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^4\left(25dx^2+9ex^4\right)+2c^2\left(25d+6ex^2\right)+24e\right)}{c^5}+15bx^3\cosh^{-1}(cx)\left(5d+3ex\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (15*a*x^3*(5*d + 3*e*x^2) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(24*e + 2*c^2*(
25*d + 6*e*x^2) + c^4*(25*d*x^2 + 9*e*x^4)))/c^5 + 15*b*x^3*(5*d + 3*e*x^2)
*ArcCosh[c*x])/225
```

Maple [A] time = 0.01, size = 115, normalized size = 0.8

$$\frac{1}{c^3}\left(\frac{a}{c^2}\left(\frac{c^5x^5e}{5}+\frac{c^5x^3d}{3}\right)+\frac{b}{c^2}\left(\frac{\operatorname{arccosh}(cx)\,c^5x^5e}{5}+\frac{\operatorname{arccosh}(cx)\,c^5x^3d}{3}-\frac{9\,c^4ex^4+25\,c^4dx^2+12\,x^2c^2e+50\,c^2d+24\,e}{225}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(a/c^2*(1/5*c^5*x^5*e+1/3*c^5*x^3*d)+b/c^2*(1/5*arccosh(c*x)*c^5*x^5*
e+1/3*arccosh(c*x)*c^5*x^3*d-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*e*x^4
+25*c^4*d*x^2+12*c^2*e*x^2+50*c^2*d+24*e)))
```

Maxima [A] time = 1.14329, size = 188, normalized size = 1.36

$$\frac{1}{5}aex^5+\frac{1}{3}adx^3+\frac{1}{9}\left(3\,x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*
x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d + 1/75*(15*x^5*arccosh(c*x) - (3*sq
rt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)
```

```
/c^6)*c)*b*e
```

Fricas [A] time = 2.40362, size = 271, normalized size = 1.96

$$\frac{45\,ac^5ex^5+75\,ac^5dx^3+15\left(3\,bc^5ex^5+5\,bc^5dx^3\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,bc^4ex^4+50\,bc^2d+\left(25\,bc^4d+12\,bc^2e\right)x^2+24\right.}{225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*e*x^5 + 75*a*c^5*d*x^3 + 15*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3)
*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^4*e*x^4 + 50*b*c^2*d + (25*b*c^4*d +
 12*b*c^2*e)*x^2 + 24*b*e)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] time = 3.01693, size = 178, normalized size = 1.29

$$\begin{cases}\frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\operatorname{acosh}(cx)}{3}+\frac{bex^5\operatorname{acosh}(cx)}{5}-\frac{bdx^2\sqrt{c^2x^2-1}}{9c}-\frac{bex^4\sqrt{c^2x^2-1}}{25c}-\frac{2bd\sqrt{c^2x^2-1}}{9c^3}-\frac{4bex^2\sqrt{c^2x^2-1}}{75c^3}-\frac{8be\sqrt{c^2x^2-1}}{75c^5} & \text{for } c\neq 0\\ \left(a+\frac{i\pi b}{2}\right)\left(\frac{dx^3}{3}+\frac{ex^5}{5}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*acosh(c*x)/3 + b*e*x**5*acosh
(c*x)/5 - b*d*x**2*sqrt(c**2*x**2 - 1)/(9*c) - b*e*x**4*sqrt(c**2*x**2 - 1)
/(25*c) - 2*b*d*sqrt(c**2*x**2 - 1)/(9*c**3) - 4*b*e*x**2*sqrt(c**2*x**2 -
1)/(75*c**3) - 8*b*e*sqrt(c**2*x**2 - 1)/(75*c**5), Ne(c, 0)), ((a + I*pi*b
/2)*(d*x**3/3 + e*x**5/5), True))
```

Giac [A] time = 1.2535, size = 194, normalized size = 1.41

$$\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{\left(c^2x^2-1\right)^{\frac{3}{2}}+3\,\sqrt{c^2x^2-1}}{c^3}\right)bd+\frac{1}{75}\left(15\,ax^5+\left(15\,x^5\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{3\left(c\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*a*d*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3/2
) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*d + 1/75*(15*a*x^5 + (15*x^5*log(c*x + sqrt
(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(
c^2*x^2 - 1))/c^5)*b)*e
```

3.464 $\int x\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=122

$$\frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4\left(a+b\cosh^{-1}(cx)\right)-\frac{bx\sqrt{cx-1}\sqrt{cx+1}\left(8c^2d+3e\right)}{32c^3}-\frac{b\left(8c^2d+3e\right)\cosh^{-1}(cx)}{32c^4}-\frac{bex^3\sqrt{}}{}$$

```
[Out] -(b*(8*c^2*d + 3*e)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c^3) - (b*e*x^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/(16*c) - (b*(8*c^2*d + 3*e)*ArcCosh[c*x])/(32*c^
4) + (d*x^2*(a + b*ArcCosh[c*x]))/2 + (e*x^4*(a + b*ArcCosh[c*x]))/4
```

Rubi [A] time = 0.109358, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5786, 460, 90, 52}

$$\frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4\left(a+b\cosh^{-1}(cx)\right)-\frac{bx\sqrt{cx-1}\sqrt{cx+1}\left(8c^2d+3e\right)}{32c^3}-\frac{b\left(8c^2d+3e\right)\cosh^{-1}(cx)}{32c^4}-\frac{bex^3\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```



```
[Out] -(b*(8*c^2*d + 3*e)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c^3) - (b*e*x^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/(16*c) - (b*(8*c^2*d + 3*e)*ArcCosh[c*x])/(32*c^
4) + (d*x^2*(a + b*ArcCosh[c*x]))/2 + (e*x^4*(a + b*ArcCosh[c*x]))/4
```


Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
```

```
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)dx=\frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{8}(bc)\int\frac{x^2\left(4d+2ex^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{bex^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}+\frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4\left(a+b\cosh^{-1}(cx)\right)-$$

$$=-\frac{b\left(8c^2d+3e\right)x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{bex^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}+\frac{1}{2}dx^2\left(a+b\cosh^{-1}\right.$$

$$=-\frac{b\left(8c^2d+3e\right)x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{bex^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{b\left(8c^2d+3e\right)\cosh}{32c^4}$$

Mathematica [A] time = 0.131741, size = 120, normalized size = 0.98

$$\frac{cx\left(8ac^3x\left(2d+ex^2\right)-b\sqrt{cx-1}\sqrt{cx+1}\left(2c^2\left(4d+ex^2\right)+3e\right)\right)+8bc^4x^2\cosh^{-1}(cx)\left(2d+ex^2\right)-2b\left(8c^2d+3e\right)\tanh^{-1}}{32c^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (c*x*(8*a*c^3*x*(2*d + e*x^2) - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3*e + 2*c^2
*(4*d + e*x^2))) + 8*b*c^4*x^2*(2*d + e*x^2)*ArcCosh[c*x] - 2*b*(8*c^2*d +
3*e)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(32*c^4)
```

Maple [A] time = 0.013, size = 202, normalized size = 1.7

$$\frac{ax^4e}{4}+\frac{dax^2}{2}+\frac{b\mathrm{arccosh}\,(cx)\,x^4e}{4}+\frac{d b\mathrm{arccosh}\,(cx)\,x^2}{2}-\frac{bex^3}{16\,c}\sqrt{cx-1}\sqrt{cx+1}-\frac{bdx}{4\,c}\sqrt{cx-1}\sqrt{cx+1}-\frac{bd}{4\,c^2}\sqrt{cx-1}\sqrt{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/4*a*x^4*e+1/2*d*a*x^2+1/4*b*arccosh(c*x)*x^4*e+1/2*d*b*arccosh(c*x)*x^2-1
/16*b*e*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4*b*d*x*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/c-1/4/c^2*d*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^
2*x^2-1)^(1/2))-3/32/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e*x-3/32/c^4*b*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.10462, size = 235, normalized size = 1.93

$$\frac{1}{4}\,aex^4+\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right)\right)bd+\frac{1}{32}\left(8\,x^4\,\mathrm{arcosh}\,(cx)-\left(\frac{2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*
x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^2)))*b*d
+ 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2
- 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4)
)*c)*b*e
```

Fricas [A] time = 2.42342, size = 255, normalized size = 2.09

$$\frac{8\,ac^4ex^4+16\,ac^4dx^2+\left(8\,bc^4ex^4+16\,bc^4dx^2-8\,bc^2d-3\,be\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(2\,bc^3ex^3+\left(8\,bc^3d+3\,bce\right)x\right)\sqrt{c^2x^2}}{32\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*c^4*e*x^4 + 16*a*c^4*d*x^2 + (8*b*c^4*e*x^4 + 16*b*c^4*d*x^2 - 8*
b*c^2*d - 3*b*e)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^3*e*x^3 + (8*b*c^3*d
 + 3*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] time = 1.89702, size = 160, normalized size = 1.31

$$\begin{cases} \frac{adx^2}{2} + \frac{aex^4}{4} + \frac{bdx^2 \operatorname{acosh}(cx)}{2} + \frac{bex^4 \operatorname{acosh}(cx)}{4} - \frac{bdx\sqrt{c^2x^2-1}}{4c} - \frac{bex^3\sqrt{c^2x^2-1}}{16c} - \frac{bd \operatorname{acosh}(cx)}{4c^2} - \frac{3bex\sqrt{c^2x^2-1}}{32c^3} - \frac{3be \operatorname{acosh}(cx)}{32c^4} & \text{for } c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{dx^2}{2} + \frac{ex^4}{4}\right) & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*acosh(c*x)/2 + b*e*x**4*acosh
(c*x)/4 - b*d*x*sqrt(c**2*x**2 - 1)/(4*c) - b*e*x**3*sqrt(c**2*x**2 - 1)/(1
6*c) - b*d*acosh(c*x)/(4*c**2) - 3*b*e*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 3*
b*e*acosh(c*x)/(32*c**4), Ne(c, 0)), ((a + I*pi*b/2)*(d*x**2/2 + e*x**4/4),
 True))
```

Giac [A] time = 1.343, size = 238, normalized size = 1.95

$$\frac{1}{2}adx^2 + \frac{1}{4}\left(2x^2\log\left(cx + \sqrt{c^2x^2-1}\right) - c\left(\frac{\sqrt{c^2x^2-1}x}{c^2} - \frac{\log\left(\left|-x|c| + \sqrt{c^2x^2-1}\right|\right)}{c^2|c|}\right)\right)bd + \frac{1}{32}\left(8ax^4 + \left(8x^4\log\left(cx + \sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 - 1)) - c*(sqrt(c^2*x^2 - 1
)*x/c^2 - log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^2*abs(c))))*b*d + 1/32
*(8*a*x^4 + (8*x^4*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*x*(2*x
^2/c^2 + 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*abs(c)))*c
)*b)*e
```

3.465 $\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=94

$$dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(9c^2d+2e\right)}{9c^3}-\frac{bex^2\sqrt{cx-1}\sqrt{cx+1}}{9c}$$

```
[Out] -(b*(9*c^2*d + 2*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3) - (b*e*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])/(9*c) + d*x*(a + b*ArcCosh[c*x]) + (e*x^3*(a + b*Ar
cCosh[c*x]))/3
```

Rubi [A] time = 0.0799358, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5705, 460, 74}

$$dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(9c^2d+2e\right)}{9c^3}-\frac{bex^2\sqrt{cx-1}\sqrt{cx+1}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```


```
[Out] -(b*(9*c^2*d + 2*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3) - (b*e*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])/(9*c) + d*x*(a + b*ArcCosh[c*x]) + (e*x^3*(a + b*Ar
cCosh[c*x]))/3
```


Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
```

```
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)dx &= dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)-(bc)\int \frac{x\left(d+\frac{ex^2}{3}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}dx\\ &= -\frac{bex^2\sqrt{-1+cx}\sqrt{1+cx}}{9c}+dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{9}\Big(b\\ &= -\frac{b\left(9c^2d+2e\right)\sqrt{-1+cx}\sqrt{1+cx}}{9c^3}-\frac{bex^2\sqrt{-1+cx}\sqrt{1+cx}}{9c}+dx\left(a+b\cosh^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.0855492, size = 76, normalized size = 0.81

$$\frac{1}{9}\left(3ax\left(3d+ex^2\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(9d+ex^2\right)+2e\right)}{c^3}+3bx\cosh^{-1}(cx)\left(3d+ex^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*a*x*(3*d + e*x^2) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*e + c^2*(9*d + e*
x^2)))/c^3 + 3*b*x*(3*d + e*x^2)*ArcCosh[c*x])/9
```

Maple [A] time = 0.008, size = 90, normalized size = 1.

$$\frac{1}{c}\left(\frac{a}{c^2}\left(\frac{c^3x^3e}{3}+c^3dx\right)+\frac{b}{c^2}\left(\frac{\operatorname{arccosh}(cx)\,c^3x^3e}{3}+\operatorname{arccosh}(cx)\,c^3dx-\frac{x^2c^2e+9\,c^2d+2\,e}{9}\sqrt{cx-1}\sqrt{cx+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(a/c^2*(1/3*c^3*x^3*e+c^3*d*x)+b/c^2*(1/3*arccosh(c*x)*c^3*x^3*e+arccos
h(c*x)*c^3*d*x-1/9*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*e*x^2+9*c^2*d+2*e)))
```

Maxima [A] time = 1.08782, size = 123, normalized size = 1.31

$$\frac{1}{3}\,aex^3 + \frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\left(cx\right) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)be + adx + \frac{\left(cx\,\mathrm{arcosh}\left(cx\right) - \sqrt{c^2x^2-1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sq
rt(c^2*x^2 - 1)/c^4))*b*e + a*d*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*
b*d/c
```

Fricas [A] time = 2.3047, size = 208, normalized size = 2.21

$$\frac{3\,ac^3ex^3 + 9\,ac^3dx + 3\left(bc^3ex^3 + 3\,bc^3dx\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(bc^2ex^2 + 9\,bc^2d + 2\,be\right)\sqrt{c^2x^2-1}}{9\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*log(c*x +
sqrt(c^2*x^2 - 1)) - (b*c^2*e*x^2 + 9*b*c^2*d + 2*b*e)*sqrt(c^2*x^2 - 1))/c
^3
```

Sympy [A] time = 0.911373, size = 116, normalized size = 1.23

$$\begin{cases} adx + \frac{aex^3}{3} + bdx\,\mathrm{acosh}\left(cx\right) + \frac{bex^3\,\mathrm{acosh}\left(cx\right)}{3} - \frac{bd\sqrt{c^2x^2-1}}{c} - \frac{bex^2\sqrt{c^2x^2-1}}{9c} - \frac{2be\sqrt{c^2x^2-1}}{9c^3} & \text{for}\ c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(dx + \frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**3/3 + b*d*x*acosh(c*x) + b*e*x**3*acosh(c*x)/3 -
b*d*sqrt(c**2*x**2 - 1)/c - b*e*x**2*sqrt(c**2*x**2 - 1)/(9*c) - 2*b*e*sqrt
(c**2*x**2 - 1)/(9*c**3), Ne(c, 0)), ((a + I*pi*b/2)*(d*x + e*x**3/3), True
))
```

Giac [A] time = 1.23011, size = 146, normalized size = 1.55

$$\left(x \log\left(cx + \sqrt{c^2x^2 - 1}\right) - \frac{\sqrt{c^2x^2 - 1}}{c}\right)bd + adx + \frac{1}{9}\left(3\,ax^3 + \left(3\,x^3 \log\left(cx + \sqrt{c^2x^2 - 1}\right) - \frac{\left(c^2x^2 - 1\right)^{\frac{3}{2}} + 3\,\sqrt{c^2x^2 - 1}}{c^3}\right)b\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d + a*d*x + 1/9*(3
*a*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3/2) + 3*sqr
t(c^2*x^2 - 1))/c^3)*b)*e
```

3.466 $\int \frac{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=264

$$-\frac{ibd\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+d\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd\sqrt{1-c^2x^2}\sin^{-1}(cx)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -(b*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (b*e*ArcCosh[c*x])/(4*c^2) +
(e*x^2*(a + b*ArcCosh[c*x]))/2 - ((I/2)*b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2
)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1
 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*(a + b*ArcCos
h[c*x])*Log[x] - (b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - ((I/2)*b*d*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c
*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.67577, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {14, 5790, 12, 6742, 90, 52, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{ibd\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+d\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd\sqrt{1-c^2x^2}\sin^{-1}(cx)}{2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```


```
[Out] -(b*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (b*e*ArcCosh[c*x])/(4*c^2) +
(e*x^2*(a + b*ArcCosh[c*x]))/2 - ((I/2)*b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2
)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1
 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*(a + b*ArcCos
h[c*x])*Log[x] - (b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - ((I/2)*b*d*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c
*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x}\,dx = \frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-(bc)\int \frac{ex^2+2d\log(x)}{2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-\frac{1}{2}(bc)\int \frac{ex^2+2d\log(x)}{\sqrt{-1+cx}\sqrt{1+c}}$$

$$= \frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-\frac{1}{2}(bc)\int \left(\frac{ex^2}{\sqrt{-1+cx}\sqrt{1+}}\right.$$

$$= \frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-(bcd)\int \frac{\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-$$

$$= -\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh\right.$$

$$= -\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh\right.$$

$$= -\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd\sqrt{1-c^2x^2}}{2\sqrt{-1+c}}$$

$$= -\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd\sqrt{1-c^2x^2}}{2\sqrt{-1+c}}$$

$$= -\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd\sqrt{1-c^2x^2}}{2\sqrt{-1+c}}$$

$$= -\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd\sqrt{1-c^2x^2}}{2\sqrt{-1+c}}$$

Mathematica [A] time = 0.255012, size = 119, normalized size = 0.45

$$\frac{1}{2}\left(-bd\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+2ad\log(x)+aex^2-\frac{be\left(cx\sqrt{cx-1}\sqrt{cx+1}+2\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)\right)}{2c^2}+bd\cosh^{-1}(cx)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (a*e*x^2 + b*e*x^2*ArcCosh[c*x] - (b*e*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] +
2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(2*c^2) + b*d*ArcCosh[c*x]*(ArcCosh
[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) + 2*a*d*Log[x] - b*d*PolyLog[2, -E^
(-2*ArcCosh[c*x])])/2
```

Maple [A] time = 0.112, size = 130, normalized size = 0.5

$$\frac{ax^2e}{2} + da\ln(cx) - \frac{db(\operatorname{arccosh}(cx))^2}{2} + \frac{b\operatorname{arccosh}(cx)\,x^2e}{2} - \frac{bex}{4c}\sqrt{cx-1}\sqrt{cx+1} - \frac{be\operatorname{arccosh}(cx)}{4c^2} + db\operatorname{arccosh}(cx)\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/2*a*x^2*e+d*a*ln(c*x)-1/2*d*b*arccosh(c*x)^2+1/2*b*arccosh(c*x)*x^2*e-1/4
*b*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4*b*e*arccosh(c*x)/c^2+d*b*arccosh(c
*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)+1/2*d*b*polylog(2,-(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}aex^2 + ad\log(x) + \int bex\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \frac{bd\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + a*d*log(x) + integrate(b*e*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)) + b*d*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{aex^2 + ad + \left(bex^2 + bd\right)\operatorname{arcosh}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acosh}(cx)) \left(d + ex^2\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b \operatorname{arcosh}(cx) + a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x, x)
```

3.467 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(cx)\right)+bcd\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{be\sqrt{cx-1}\sqrt{cx+1}}{c}$$

```
[Out] -((b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c) - (d*(a + b*ArcCosh[c*x]))/x + e*x*
(a + b*ArcCosh[c*x]) + b*c*d*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rubi [A] time = 0.0991049, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5786, 460, 92, 205}

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(cx)\right)+bcd\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{be\sqrt{cx-1}\sqrt{cx+1}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```



```
[Out] -((b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c) - (d*(a + b*ArcCosh[c*x]))/x + e*x*
(a + b*ArcCosh[c*x]) + b*c*d*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```


Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
```

```
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(cx)\right)+(bc)\int \frac{d-ex^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= -\frac{be\sqrt{-1+cx}\sqrt{1+cx}}{c}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(cx)\right)+(bcd)\int -$$

$$= -\frac{be\sqrt{-1+cx}\sqrt{1+cx}}{c}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(cx)\right)+\left(bc^2d\right)\text{Su}$$

$$= -\frac{be\sqrt{-1+cx}\sqrt{1+cx}}{c}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(cx)\right)+bcd\tan^{-1}$$

Mathematica [A] time = 0.131937, size = 105, normalized size = 1.4

$$-\frac{ad}{x}+aex+\frac{bcd\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bd\cosh^{-1}(cx)}{x}-\frac{be\sqrt{cx-1}\sqrt{cx+1}}{c}+bex\cosh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x - (b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (b*d*ArcCosh[c*
x])/x + b*e*x*ArcCosh[c*x] + (b*c*d*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2
*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [A] time = 0.017, size = 95, normalized size = 1.3

$$axe-\frac{ad}{x}+b\operatorname{arccosh}(cx)\,xe-\frac{bd\operatorname{arccosh}(cx)}{x}-cbd\sqrt{cx-1}\sqrt{cx+1}\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\frac{1}{\sqrt{c^2x^2-1}}-\frac{be}{c}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] a*x*e-a*d/x+b*arccosh(c*x)*x*e-b*arccosh(c*x)*d/x-c*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/(c^2*x^2-1)^(1/2)*d*arctan(1/(c^2*x^2-1)^(1/2))-b*e*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/c
```

Maxima [A] time = 1.71594, size = 88, normalized size = 1.17

$$-\left(c\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\operatorname{arcosh}(cx)}{x}\right)bd+aex+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)be}{c}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -(c*arcsin(1/(sqrt(c^2)*abs(x))) + arccosh(c*x)/x)*b*d + a*e*x + (c*x*arcco
sh(c*x) - sqrt(c^2*x^2 - 1))*b*e/c - a*d/x
```

Fricas [A] time = 2.61705, size = 298, normalized size = 3.97

$$\frac{2\,bc^2dx\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+acex^2-\sqrt{c^2x^2-1}bex-acd+(bcd-bce)x\log\left(-cx+\sqrt{c^2x^2-1}\right)+\left(bcex^2-bcd+(\right.}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (2*b*c^2*d*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) + a*c*e*x^2 - sqrt(c^2*x^2 -
1)*b*e*x - a*c*d + (b*c*d - b*c*e)*x*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*e
```

```
*x^2 - b*c*d + (b*c*d - b*c*e)*x)*log(c*x + sqrt(c^2*x^2 - 1)))/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(cx))\left(d + ex^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b\operatorname{arcosh}(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^2, x)
```

3.468 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^3} dx$

Optimal. Leaf size=251

$$-\frac{ibe\sqrt{1-c^2x^2}\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\log(x)\left(a+b\cosh^{-1}(cx)\right)-\frac{ibe\sqrt{1-c^2x^2}\sin^{-1}(cx)^2}{2\sqrt{cx-1}\sqrt{cx+1}}+$$

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (d*(a + b*ArcCosh[c*x]))/(2*x^
2) - ((I/2)*b*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (b*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + e*(a + b*ArcCosh[c*x])*Log[x] - (b*e*Sqrt[1
 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/2)*b*e
*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rubi [A] time = 0.611472, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {14, 5790, 6742, 95, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{ibe\sqrt{1-c^2x^2}\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\log(x)\left(a+b\cosh^{-1}(cx)\right)-\frac{ibe\sqrt{1-c^2x^2}\sin^{-1}(cx)^2}{2\sqrt{cx-1}\sqrt{cx+1}}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```


```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (d*(a + b*ArcCosh[c*x]))/(2*x^
2) - ((I/2)*b*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (b*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + e*(a + b*ArcCosh[c*x])*Log[x] - (b*e*Sqrt[1
 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/2)*b*e
*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + e\left(a+b\cosh^{-1}(cx)\right)\log(x) - (bc)\int \frac{-\frac{d}{2x^2}+e\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + e\left(a+b\cosh^{-1}(cx)\right)\log(x) - (bc)\int \left(-\frac{d}{2x^2\sqrt{-1+cx}\sqrt{1+}}\right.$$

$$= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + e\left(a+b\cosh^{-1}(cx)\right)\log(x) + \frac{1}{2}(bcd)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + e\left(a+b\cosh^{-1}(cx)\right)\log(x) - \frac{(bce}{}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + e\left(a+b\cosh^{-1}(cx)\right)\log(x) - \frac{be\sqrt{1}}{}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + e\left(a+b\cosh^{-1}(cx)\right)\log(x) - \frac{be\sqrt{1}}{}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{ibe\sqrt{1-c^2x^2}\sin^{-1}(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}} + e\left(a+b\right.$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{ibe\sqrt{1-c^2x^2}\sin^{-1}(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be\sqrt{1-}}{}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{ibe\sqrt{1-c^2x^2}\sin^{-1}(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be\sqrt{1-}}{}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{ibe\sqrt{1-c^2x^2}\sin^{-1}(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be\sqrt{1-}}{}$$

Mathematica [A] time = 0.137312, size = 101, normalized size = 0.4

$$\frac{-bex^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right) - ad + 2aex^2\log(x) - b\cosh^{-1}(cx)\left(d-2ex^2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\right) + bcdx\sqrt{cx-1}\sqrt{}}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-(a*d) + b*c*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*e*x^2*ArcCosh[c*x]^2 - b
*ArcCosh[c*x]*(d - 2*e*x^2*Log[1 + E^(-2*ArcCosh[c*x])]) + 2*a*e*x^2*Log[x]
```

```
- b*e*x^2*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(2*x^2)
```

Maple [A] time = 0.147, size = 126, normalized size = 0.5

$$ae\ln(cx) - \frac{da}{2x^2} - \frac{b(\operatorname{arccosh}(cx))^2 e}{2} + \frac{bcd}{2x}\sqrt{cx-1}\sqrt{cx+1} - \frac{c^2db}{2} - \frac{bd\operatorname{arccosh}(cx)}{2x^2} + be\operatorname{arccosh}(cx)\ln\left(\left(cx+\sqrt{cx-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] a*e*ln(c*x)-1/2*d*a/x^2-1/2*b*arccosh(c*x)^2*e+1/2*b*c*d*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/x-1/2*c^2*d*b-1/2*d*b*arccosh(c*x)/x^2+b*e*arccosh(c*x)*ln((c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)+1/2*b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}bd\left(\frac{\sqrt{c^2x^2-1}c}{x} - \frac{\operatorname{arcosh}(cx)}{x^2}\right) + be\int \frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{x}\,dx + ae\log(x) - \frac{ad}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*d*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) + b*e*integrate(log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{aex^2 + ad + \left(bex^2 + bd\right)\operatorname{arcosh}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(cx))\left(d + ex^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b\operatorname{arcosh}(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^3, x)
```

3.469 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=94

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{1}{6}bc\left(c^2d+6e\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (d*(a + b*ArcCosh[c*x]))/(3*
x^3) - (e*(a + b*ArcCosh[c*x]))/x + (b*c*(c^2*d + 6*e)*ArcTan[Sqrt[-1 + c*x
]*Sqrt[1 + c*x]])/6
```

Rubi [A] time = 0.104277, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5786, 454, 92, 205}

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{1}{6}bc\left(c^2d+6e\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (d*(a + b*ArcCosh[c*x]))/(3*
x^3) - (e*(a + b*ArcCosh[c*x]))/x + (b*c*(c^2*d + 6*e)*ArcTan[Sqrt[-1 + c*x
]*Sqrt[1 + c*x]])/6
```

Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
```

```
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{x} - \frac{1}{3}(bc)\int \frac{-d-3ex^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{1}{6}\left(bc\left(c^2d\right.\right.$$
$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{1}{6}\left(bc^2\left(c^2d\right.\right.$$
$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{1}{6}bc\left(c^2d+\right.$$

Mathematica [A] time = 0.253088, size = 128, normalized size = 1.36

$$\frac{\frac{-2a\sqrt{cx-1}\sqrt{cx+1}\left(d+3ex^2\right)+bcx^3\sqrt{c^2x^2-1}\left(c^2d+6e\right)\tan^{-1}\left(\sqrt{c^2x^2-1}\right)+bcdx\left(c^2x^2-1\right)}{\sqrt{cx-1}\sqrt{cx+1}}-2b\cosh^{-1}(cx)\left(d+3ex^2\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (-2*b*(d + 3*e*x^2)*ArcCosh[c*x] + (b*c*d*x*(-1 + c^2*x^2) - 2*a*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(d + 3*e*x^2) + b*c*(c^2*d + 6*e)*x^3*Sqrt[-1 + c^2*x^2]
*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(6*x^3)
```

Maple [A] time = 0.019, size = 146, normalized size = 1.6

$$-\frac{ae}{x}-\frac{da}{3\,x^3}-\frac{b\mathrm{arccosh}\,(cx)\,e}{x}-\frac{bd\mathrm{arccosh}\,(cx)}{3\,x^3}-\frac{c^3db}{6}\sqrt{cx-1}\sqrt{cx+1}\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\frac{1}{\sqrt{c^2x^2-1}}+\frac{bcd}{6\,x^2}\sqrt{cx-1}\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] -a*e/x-1/3*d*a/x^3-b*arccosh(c*x)*e/x-1/3*d*b*arccosh(c*x)/x^3-1/6*c^3*d*b*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))+1
/6*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2-c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c
^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))*e
```

Maxima [A] time = 1.70314, size = 120, normalized size = 1.28

$$-\frac{1}{6}\left(\left(\left(c^2\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\,\mathrm{arcosh}\,(cx)}{x^3}\right)bd-\left(c\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)be-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*arcsin(1/(sqrt(c^2)*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arcc
osh(c*x)/x^3)*b*d - (c*arcsin(1/(sqrt(c^2)*abs(x))) + arccosh(c*x)/x)*b*e -
 a*e/x - 1/3*a*d/x^3
```

Fricas [A] time = 2.72737, size = 325, normalized size = 3.46

$$\frac{2\left(bc^3d+6\,bce\right)x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\,(bd+3\,be)x^3\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}bcdx-6\,aex^2-2\,ad-2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(b*c^3*d + 6*b*c*e)*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*d +
3*b*e)*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^2 - 1)*b*c*d*x - 6*a*
e*x^2 - 2*a*d - 2*(3*b*e*x^2 - (b*d + 3*b*e)*x^3 + b*d)*log(c*x + sqrt(c^2*
x^2 - 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(cx))\left(d + ex^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b\operatorname{arcosh}(cx) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^4, x)
```

3.470 $\int x^4 \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=319

$$\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{b\left(1-c^2x^2\right)^3\left(21c^4d^2+90c^2de+70e^2\right)}{525c^9\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*(1 - c^2*x^2))/(315*c^9*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (2*b*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^2
)/(945*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(21*c^4*d^2 + 90*c^2*d*e + 70
*e^2)*(1 - c^2*x^2)^3)/(525*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*e*(9*c
^2*d + 14*e)*(1 - c^2*x^2)^4)/(441*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e
^2*(1 - c^2*x^2)^5)/(81*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^5*(a + b
*ArcCosh[c*x]))/5 + (2*d*e*x^7*(a + b*ArcCosh[c*x]))/7 + (e^2*x^9*(a + b*Ar
cCosh[c*x]))/9
```

Rubi [A] time = 0.411012, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5790, 12, 520, 1251, 897, 1153}

$$\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{b\left(1-c^2x^2\right)^3\left(21c^4d^2+90c^2de+70e^2\right)}{525c^9\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (b*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*(1 - c^2*x^2))/(315*c^9*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (2*b*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^2
)/(945*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(21*c^4*d^2 + 90*c^2*d*e + 70
*e^2)*(1 - c^2*x^2)^3)/(525*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*e*(9*c
^2*d + 14*e)*(1 - c^2*x^2)^4)/(441*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e
^2*(1 - c^2*x^2)^5)/(81*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^5*(a + b
*ArcCosh[c*x]))/5 + (2*d*e*x^7*(a + b*ArcCosh[c*x]))/7 + (e^2*x^9*(a + b*Ar
cCosh[c*x]))/9
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^4 \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx\right.$$

$$= \frac{b\left(63c^4d^2+90c^2de+35e^2\right)\left(1-c^2x^2\right)}{315c^9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b\left(63c^4d^2+135c^2de+70e^2\right)\left(1-c^2x^2\right.}{945c^9\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.263065, size = 192, normalized size = 0.6

$$\frac{315ax^5\left(63d^2+90dex^2+35e^2x^4\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^8\left(3969d^2x^4+4050dex^6+1225e^2x^8\right)+4c^6\left(1323d^2x^2+1215dex^4+350e^2x^6\right)+24c^4\left(441d^2+270dex^2+\right.\right.}{c^9}}{99225}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (315*a*x^5*(63*d^2 + 90*d*e*x^2 + 35*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(4480*e^2 + 160*c^2*e*(81*d + 14*e*x^2) + 24*c^4*(441*d^2 + 270*d*e*x^
2 + 70*e^2*x^4) + 4*c^6*(1323*d^2*x^2 + 1215*d*e*x^4 + 350*e^2*x^6) + c^8*(
```

```
3969*d^2*x^4 + 4050*d*e*x^6 + 1225*e^2*x^8)))/c^9 + 315*b*x^5*(63*d^2 + 90*
d*e*x^2 + 35*e^2*x^4)*ArcCosh[c*x])/99225
```

Maple [A] time = 0.014, size = 227, normalized size = 0.7

$$\frac{1}{c^5}\left(\frac{a}{c^4}\left(\frac{e^2c^9x^9}{9}+\frac{2\,c^9dex^7}{7}+\frac{c^9x^5d^2}{5}\right)+\frac{b}{c^4}\left(\frac{\operatorname{arccosh}(cx)\,e^2c^9x^9}{9}+\frac{2\operatorname{arccosh}(cx)\,c^9dex^7}{7}+\frac{\operatorname{arccosh}(cx)\,c^9x^5d^2}{5}-\frac{1225\ldots}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(a/c^4*(1/9*e^2*c^9*x^9+2/7*c^9*d*e*x^7+1/5*c^9*x^5*d^2)+b/c^4*(1/9*a
rccosh(c*x)*e^2*c^9*x^9+2/7*arccosh(c*x)*c^9*d*e*x^7+1/5*arccosh(c*x)*c^9*x
^5*d^2-1/99225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*e^2*x^8+4050*c^8*d*e*x
^6+3969*c^8*d^2*x^4+1400*c^6*e^2*x^6+4860*c^6*d*e*x^4+5292*c^6*d^2*x^2+1680
*c^4*e^2*x^4+6480*c^4*d*e*x^2+10584*c^4*d^2+2240*c^2*e^2*x^2+12960*c^2*d*e+
4480*e^2)))
```

Maxima [A] time = 1.02647, size = 412, normalized size = 1.29

$$\frac{1}{9}\,ae^2x^9+\frac{2}{7}\,adex^7+\frac{1}{5}\,ad^2x^5+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd^2+\frac{2}{245}\left(3\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*e^2*x^9 + 2/7*a*d*e*x^7 + 1/5*a*d^2*x^5 + 1/75*(15*x^5*arccosh(c*x) -
 (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 - 1)/c^6)*c)*b*d^2 + 2/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^
6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt
(c^2*x^2 - 1)/c^8)*c)*b*d*e + 1/2835*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x
^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c
^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*e^2
```

Fricas [A] time = 2.38452, size = 562, normalized size = 1.76

$$\frac{11025\,ac^9e^2x^9 + 28350\,ac^9dex^7 + 19845\,ac^9d^2x^5 + 315\left(35\,bc^9e^2x^9 + 90\,bc^9dex^7 + 63\,bc^9d^2x^5\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(12\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*e^2*x^9 + 28350*a*c^9*d*e*x^7 + 19845*a*c^9*d^2*x^5 +
315*(35*b*c^9*e^2*x^9 + 90*b*c^9*d*e*x^7 + 63*b*c^9*d^2*x^5)*log(c*x + sqrt
(c^2*x^2 - 1)) - (1225*b*c^8*e^2*x^8 + 10584*b*c^4*d^2 + 50*(81*b*c^8*d*e +
 28*b*c^6*e^2)*x^6 + 12960*b*c^2*d*e + 3*(1323*b*c^8*d^2 + 1620*b*c^6*d*e +
 560*b*c^4*e^2)*x^4 + 4480*b*e^2 + 4*(1323*b*c^6*d^2 + 1620*b*c^4*d*e + 560
*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^9
```

Sympy [A] time = 31.7405, size = 422, normalized size = 1.32

$$\begin{cases} \frac{ad^2x^5}{5} + \frac{2adex^7}{7} + \frac{ae^2x^9}{9} + \frac{bd^2x^5\operatorname{acosh}(cx)}{5} + \frac{2bdex^7\operatorname{acosh}(cx)}{7} + \frac{be^2x^9\operatorname{acosh}(cx)}{9} - \frac{bd^2x^4\sqrt{c^2x^2-1}}{25c} - \frac{2bdex^6\sqrt{c^2x^2-1}}{49c} - \frac{be^2x^8\sqrt{c^2x^2-1}}{81c} - \frac{4bd^2x^{\cdots}}{\cdots} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^2x^5}{5} + \frac{2dex^7}{7} + \frac{e^2x^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**5/5 + 2*a*d*e*x**7/7 + a*e**2*x**9/9 + b*d**2*x**5*aco
sh(c*x)/5 + 2*b*d*e*x**7*acosh(c*x)/7 + b*e**2*x**9*acosh(c*x)/9 - b*d**2*x
**4*sqrt(c**2*x**2 - 1)/(25*c) - 2*b*d*e*x**6*sqrt(c**2*x**2 - 1)/(49*c) -
b*e**2*x**8*sqrt(c**2*x**2 - 1)/(81*c) - 4*b*d**2*x**2*sqrt(c**2*x**2 - 1)/
(75*c**3) - 12*b*d*e*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 8*b*e**2*x**6*sq
rt(c**2*x**2 - 1)/(567*c**3) - 8*b*d**2*sqrt(c**2*x**2 - 1)/(75*c**5) - 16*
b*d*e*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**2*x**4*sqrt(c**2*x**2 -
 1)/(945*c**5) - 32*b*d*e*sqrt(c**2*x**2 - 1)/(245*c**7) - 64*b*e**2*x**2*s
qrt(c**2*x**2 - 1)/(2835*c**7) - 128*b*e**2*sqrt(c**2*x**2 - 1)/(2835*c**9)
, Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**5/5 + 2*d*e*x**7/7 + e**2*x**9/9), Tr
ue))
```

Giac [A] time = 1.36795, size = 383, normalized size = 1.2

$$\frac{1}{5}\,ad^2x^5+\frac{1}{75}\left(15\,x^5\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{3\left(c^2x^2-1\right)^{\frac{5}{2}}+10\left(c^2x^2-1\right)^{\frac{3}{2}}+15\,\sqrt{c^2x^2-1}}{c^5}\right)bd^2+\frac{1}{2835}\left(315\,ax^9+\left(315\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*a*d^2*x^5 + 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1
)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))/c^5)*b*d^2 + 1/283
5*(315*a*x^9 + (315*x^9*log(c*x + sqrt(c^2*x^2 - 1)) - (35*(c^2*x^2 - 1)^(9
/2) + 180*(c^2*x^2 - 1)^(7/2) + 378*(c^2*x^2 - 1)^(5/2) + 420*(c^2*x^2 - 1)
^(3/2) + 315*sqrt(c^2*x^2 - 1))/c^9)*b)*e^2 + 2/245*(35*a*d*x^7 + (35*x^7*l
og(c*x + sqrt(c^2*x^2 - 1)) - (5*(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5/
2) + 35*(c^2*x^2 - 1)^(3/2) + 35*sqrt(c^2*x^2 - 1))/c^7)*b*d)*e
```

3.471 $\int x^3 \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=341

$$\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\cosh^{-1}(cx)\right)+\frac{bx^3\left(1-c^2x^2\right)\left(288c^4d^2+320c^2de+\cdots\right)}{4608c^5\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*x*(1 - c^2*x^2))/(3072*c^7*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) + (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*x^3*(1 -
c^2*x^2))/(4608*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e*(64*c^2*d + 21*e)*
x^5*(1 - c^2*x^2))/(1152*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^2*x^7*(1
- c^2*x^2))/(64*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^4*(a + b*ArcCosh[c
*x]))/4 + (d*e*x^6*(a + b*ArcCosh[c*x]))/3 + (e^2*x^8*(a + b*ArcCosh[c*x]))
/8 - (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*Sqrt[-1 + c^2*x^2]*ArcTanh[(c
*x)/Sqrt[-1 + c^2*x^2]])/(3072*c^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.360308, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {266, 43, 5790, 12, 520, 1267, 459, 321, 217, 206}

$$\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\cosh^{-1}(cx)\right)+\frac{bx^3\left(1-c^2x^2\right)\left(288c^4d^2+320c^2de+\cdots\right)}{4608c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*x*(1 - c^2*x^2))/(3072*c^7*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) + (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*x^3*(1 -
c^2*x^2))/(4608*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e*(64*c^2*d + 21*e)*
x^5*(1 - c^2*x^2))/(1152*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^2*x^7*(1
- c^2*x^2))/(64*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^4*(a + b*ArcCosh[c
*x]))/4 + (d*e*x^6*(a + b*ArcCosh[c*x]))/3 + (e^2*x^8*(a + b*ArcCosh[c*x]))
/8 - (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*Sqrt[-1 + c^2*x^2]*ArcTanh[(c
*x)/Sqrt[-1 + c^2*x^2]])/(3072*c^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\cosh^{-1}(c\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\cosh^{-1}(c\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\cosh^{-1}(c\right.$$

$$=\frac{be^2x^7\left(1-c^2x^2\right)}{64c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)$$

$$=\frac{be\left(64c^2d+21e\right)x^5\left(1-c^2x^2\right)}{1152c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^2x^7\left(1-c^2x^2\right)}{64c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(\right.$$

$$=\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)x^3\left(1-c^2x^2\right)}{4608c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be\left(64c^2d+21e\right)x^5\left(1-c^2x^2\right)}{1152c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)}{4608c^5\sqrt{-1+cx}\sqrt{1+}}$$

$$=\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)}{4608c^5\sqrt{-1+cx}\sqrt{1+}}$$

$$=\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)}{4608c^5\sqrt{-1+cx}\sqrt{1+}}$$

Mathematica [A] time = 0.335491, size = 214, normalized size = 0.63

$$384ac^8x^4\left(6d^2+8dex^2+3e^2x^4\right)-bcx\sqrt{cx-1}\sqrt{cx+1}\left(16c^6\left(36d^2x^2+32dex^4+9e^2x^6\right)+8c^4\left(108d^2+80dex^2+21e^2x^4\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (384*a*c^8*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) - b*c*x*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(315*e^2 + 30*c^2*e*(32*d + 7*e*x^2) + 8*c^4*(108*d^2 + 80*d*e*x^2
 + 21*e^2*x^4) + 16*c^6*(36*d^2*x^2 + 32*d*e*x^4 + 9*e^2*x^6)) + 384*b*c^8*
x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x] - 6*b*(288*c^4*d^2 + 320*c
```

```
^2*d*e + 105*e^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(9216*c^8)
```

Maple [A] time = 0.019, size = 440, normalized size = 1.3

$$\frac{ae^2x^8}{8}+\frac{adex^6}{3}+\frac{d^2ax^4}{4}+\frac{b\operatorname{arccosh}(cx)\,e^2x^8}{8}+\frac{b\operatorname{arccosh}(cx)\,dex^6}{3}+\frac{d^2b\operatorname{arccosh}(cx)\,x^4}{4}-\frac{be^2x^7}{64\,c}\sqrt{cx-1}\sqrt{cx+1}-\frac{bx^5}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/8*a*e^2*x^8+1/3*a*d*e*x^6+1/4*d^2*a*x^4+1/8*b*arccosh(c*x)*e^2*x^8+1/3*b*
arccosh(c*x)*d*e*x^6+1/4*d^2*b*arccosh(c*x)*x^4-1/64/c*b*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*e^2*x^7-1/18/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5*d*e-1/16*b*d^2*x
^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-7/384/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^
2*x^5-5/72/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e*x^3-3/32*b*d^2*x*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/c^3-3/32/c^4*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-
1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))-35/1536/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2
)*e^2*x^3-5/48/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e*x-5/48/c^6*b*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-35/1024/
c^7*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2*x-35/1024/c^8*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/(c^2*x^2-1)^(1/2)*e^2*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.14553, size = 485, normalized size = 1.42

$$\frac{1}{8}\,ae^2x^8+\frac{1}{3}\,adex^6+\frac{1}{4}\,ad^2x^4+\frac{1}{32}\left(8\,x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}x}{c^4}+\frac{3\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/32*(8*x^4*arccosh(c*x) -
(2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x +
2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*d^2 + 1/144*(48*x^6*ar
ccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 +
15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2)
)/(sqrt(c^2)*c^6))*c)*b*d*e + 1/3072*(384*x^8*arccosh(c*x) - (48*sqrt(c^2*x
```

```
^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c
^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sq
rt(c^2))/(sqrt(c^2)*c^8))*c)*b*e^2
```

Fricas [A] time = 2.54593, size = 539, normalized size = 1.58

$$\frac{1152\,ac^8e^2x^8 + 3072\,ac^8dex^6 + 2304\,ac^8d^2x^4 + 3\left(384\,bc^8e^2x^8 + 1024\,bc^8dex^6 + 768\,bc^8d^2x^4 - 288\,bc^4d^2 - 320\,bc^2de - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9216*(1152*a*c^8*e^2*x^8 + 3072*a*c^8*d*e*x^6 + 2304*a*c^8*d^2*x^4 + 3*(3
84*b*c^8*e^2*x^8 + 1024*b*c^8*d*e*x^6 + 768*b*c^8*d^2*x^4 - 288*b*c^4*d^2 -
 320*b*c^2*d*e - 105*b*e^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (144*b*c^7*e^2*x
^7 + 8*(64*b*c^7*d*e + 21*b*c^5*e^2)*x^5 + 2*(288*b*c^7*d^2 + 320*b*c^5*d*e
 + 105*b*c^3*e^2)*x^3 + 3*(288*b*c^5*d^2 + 320*b*c^3*d*e + 105*b*c*e^2)*x)*
sqrt(c^2*x^2 - 1))/c^8
```

Sympy [A] time = 19.7484, size = 389, normalized size = 1.14

$$\begin{cases} \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\operatorname{acosh}(cx)}{4} + \frac{bdex^6\operatorname{acosh}(cx)}{3} + \frac{be^2x^8\operatorname{acosh}(cx)}{8} - \frac{bd^2x^3\sqrt{c^2x^2-1}}{16c} - \frac{bdex^5\sqrt{c^2x^2-1}}{18c} - \frac{be^2x^7\sqrt{c^2x^2-1}}{64c} - \frac{3bd^2x\sqrt{\cdots}}{32\cdots} \cdots \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^2x^4}{4} + \frac{dex^6}{3} + \frac{e^2x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*acosh
(c*x)/4 + b*d*e*x**6*acosh(c*x)/3 + b*e**2*x**8*acosh(c*x)/8 - b*d**2*x**3*
sqrt(c**2*x**2 - 1)/(16*c) - b*d*e*x**5*sqrt(c**2*x**2 - 1)/(18*c) - b*e**2
*x**7*sqrt(c**2*x**2 - 1)/(64*c) - 3*b*d**2*x*sqrt(c**2*x**2 - 1)/(32*c**3)
 - 5*b*d*e*x**3*sqrt(c**2*x**2 - 1)/(72*c**3) - 7*b*e**2*x**5*sqrt(c**2*x**
2 - 1)/(384*c**3) - 3*b*d**2*acosh(c*x)/(32*c**4) - 5*b*d*e*x*sqrt(c**2*x**
2 - 1)/(48*c**5) - 35*b*e**2*x**3*sqrt(c**2*x**2 - 1)/(1536*c**5) - 5*b*d*e
*acosh(c*x)/(48*c**6) - 35*b*e**2*x*sqrt(c**2*x**2 - 1)/(1024*c**7) - 35*b*
e**2*acosh(c*x)/(1024*c**8), Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**4/4 + d*e*
```

```
x**6/3 + e**2*x**8/8), True))
```

Giac [A] time = 1.44553, size = 432, normalized size = 1.27

$$\frac{1}{4}ad^2x^4 + \frac{1}{32}\left(8x^4\log\left(cx + \sqrt{c^2x^2 - 1}\right) - \left(\sqrt{c^2x^2 - 1}x\left(\frac{2x^2}{c^2} + \frac{3}{c^4}\right) - \frac{3\log\left(\left|-x|c| + \sqrt{c^2x^2 - 1}\right|\right)}{c^4|c|}\right)c\right)bd^2 + \frac{1}{3072}\left(384ax^8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^2*x^4 + 1/32*(8*x^4*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 -
1)*x*(2*x^2/c^2 + 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*a
bs(c)))*c)*b*d^2 + 1/3072*(384*a*x^8 + (384*x^8*log(c*x + sqrt(c^2*x^2 - 1)
) - (sqrt(c^2*x^2 - 1)*(2*(4*x^2*(6*x^2/c^2 + 7/c^4) + 35/c^6)*x^2 + 105/c^
8)*x - 105*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^8*abs(c)))*c)*b)*e^2
+ 1/144*(48*a*d*x^6 + (48*x^6*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2
- 1)*(2*x^2*(4*x^2/c^2 + 5/c^4) + 15/c^6)*x - 15*log(abs(-x*abs(c) + sqrt(c
^2*x^2 - 1)))/(c^6*abs(c)))*c)*b*d)*e
```

3.472 $\int x^2 \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=260

$$\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{b\left(1-c^2x^2\right)^2\left(35c^4d^2+84c^2de+45\right.}{315c^7\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(1 - c^2*x^2))/(105*c^7*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (b*(35*c^4*d^2 + 84*c^2*d*e + 45*e^2)*(1 - c^2*x^2)^2)/(
315*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e*(14*c^2*d + 15*e)*(1 - c^2*x^2
)^3)/(175*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^2*(1 - c^2*x^2)^4)/(49*c
^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^3*(a + b*ArcCosh[c*x]))/3 + (2*d*
e*x^5*(a + b*ArcCosh[c*x]))/5 + (e^2*x^7*(a + b*ArcCosh[c*x]))/7
```

Rubi [A] time = 0.318493, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 5790, 12, 520, 1251, 771}

$$\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{b\left(1-c^2x^2\right)^2\left(35c^4d^2+84c^2de+45\right.}{315c^7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (b*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(1 - c^2*x^2))/(105*c^7*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (b*(35*c^4*d^2 + 84*c^2*d*e + 45*e^2)*(1 - c^2*x^2)^2)/(
315*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e*(14*c^2*d + 15*e)*(1 - c^2*x^2
)^3)/(175*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^2*(1 - c^2*x^2)^4)/(49*c
^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^3*(a + b*ArcCosh[c*x]))/3 + (2*d*
e*x^5*(a + b*ArcCosh[c*x]))/5 + (e^2*x^7*(a + b*ArcCosh[c*x]))/7
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\cosh^{-1}(c\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\cosh^{-1}(c\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\cosh^{-1}(c\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\cosh^{-1}(c\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\cosh^{-1}(c\right.$$

$$=\frac{b\left(35c^4d^2+42c^2de+15e^2\right)\left(1-c^2x^2\right)}{105c^7\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b\left(35c^4d^2+84c^2de+45e^2\right)\left(1-c^2x^2\right)}{315c^7\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.211636, size = 163, normalized size = 0.63

$$\frac{105ax^3\left(35d^2+42dex^2+15e^2x^4\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^6\left(1225d^2x^2+882dex^4+225e^2x^6\right)+2c^4\left(1225d^2+588dex^2+135e^2x^4\right)+24c^2e\left(98d+15ex^2\right)+720e^2\right)}{c^7}}{11025}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (105*a*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(720*e^2 + 24*c^2*e*(98*d + 15*e*x^2) + 2*c^4*(1225*d^2 + 588*d*e*x^2
+ 135*e^2*x^4) + c^6*(1225*d^2*x^2 + 882*d*e*x^4 + 225*e^2*x^6)))/c^7 + 105
*b*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCosh[c*x])/11025
```

Maple [A] time = 0.013, size = 195, normalized size = 0.8

$$\frac{1}{c^3}\left(\frac{a}{c^4}\left(\frac{e^2c^7x^7}{7}+\frac{2\,c^7dex^5}{5}+\frac{x^3c^7d^2}{3}\right)+\frac{b}{c^4}\left(\frac{\operatorname{arccosh}(cx)\,e^2c^7x^7}{7}+\frac{2\operatorname{arccosh}(cx)\,c^7dex^5}{5}+\frac{\operatorname{arccosh}(cx)\,c^7x^3d^2}{3}-\frac{225}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(a/c^4*(1/7*e^2*c^7*x^7+2/5*c^7*d*e*x^5+1/3*x^3*c^7*d^2)+b/c^4*(1/7*a
rccosh(c*x)*e^2*c^7*x^7+2/5*arccosh(c*x)*c^7*d*e*x^5+1/3*arccosh(c*x)*c^7*x
^3*d^2-1/11025*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(225*c^6*e^2*x^6+882*c^6*d*e*x^4
+1225*c^6*d^2*x^2+270*c^4*e^2*x^4+1176*c^4*d*e*x^2+2450*c^4*d^2+360*c^2*e^2
*x^2+2352*c^2*d*e+720*e^2)))
```

Maxima [A] time = 1.14611, size = 333, normalized size = 1.28

$$\frac{1}{7}ae^2x^7 + \frac{2}{5}adex^5 + \frac{1}{3}ad^2x^3 + \frac{1}{9}\left(3x^3\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^2 + \frac{2}{75}\left(15x^5\operatorname{arcosh}(cx) - \left(\frac{3\sqrt{c}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) - c
*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2 + 2/75*(15*x^
5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4
 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d*e + 1/245*(35*x^7*arccosh(c*x) - (5*sqrt
(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x
^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*e^2
```

Fricas [A] time = 2.38416, size = 471, normalized size = 1.81

$$\frac{1575\,ac^7e^2x^7 + 4410\,ac^7dex^5 + 3675\,ac^7d^2x^3 + 105\left(15\,bc^7e^2x^7 + 42\,bc^7dex^5 + 35\,bc^7d^2x^3\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(225\,bc\right.}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/11025*(1575*a*c^7*e^2*x^7 + 4410*a*c^7*d*e*x^5 + 3675*a*c^7*d^2*x^3 + 105
*(15*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3)*log(c*x + sqrt(c^
2*x^2 - 1)) - (225*b*c^6*e^2*x^6 + 2450*b*c^4*d^2 + 2352*b*c^2*d*e + 18*(49
*b*c^6*d*e + 15*b*c^4*e^2)*x^4 + 720*b*e^2 + (1225*b*c^6*d^2 + 1176*b*c^4*d
*e + 360*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] time = 23.2328, size = 340, normalized size = 1.31

$$\begin{cases} \frac{ad^2x^3}{3} + \frac{2adex^5}{5} + \frac{ae^2x^7}{7} + \frac{bd^2x^3\operatorname{acosh}(cx)}{3} + \frac{2bdex^5\operatorname{acosh}(cx)}{5} + \frac{be^2x^7\operatorname{acosh}(cx)}{7} - \frac{bd^2x^2\sqrt{c^2x^2-1}}{9c} - \frac{2bdex^4\sqrt{c^2x^2-1}}{25c} - \frac{be^2x^6\sqrt{c^2x^2-1}}{49c} - \frac{2bd^2 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^2x^3}{3} + \frac{2dex^5}{5} + \frac{e^2x^7}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*aco
sh(c*x)/3 + 2*b*d*e*x**5*acosh(c*x)/5 + b*e**2*x**7*acosh(c*x)/7 - b*d**2*x
**2*sqrt(c**2*x**2 - 1)/(9*c) - 2*b*d*e*x**4*sqrt(c**2*x**2 - 1)/(25*c) - b
*e**2*x**6*sqrt(c**2*x**2 - 1)/(49*c) - 2*b*d**2*sqrt(c**2*x**2 - 1)/(9*c**
3) - 8*b*d*e*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 6*b*e**2*x**4*sqrt(c**2*x
**2 - 1)/(245*c**3) - 16*b*d*e*sqrt(c**2*x**2 - 1)/(75*c**5) - 8*b*e**2*x**
2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**2*sqrt(c**2*x**2 - 1)/(245*c**7)
, Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**3/3 + 2*d*e*x**5/5 + e**2*x**7/7), Tr
ue))
```

Giac [A] time = 1.35315, size = 328, normalized size = 1.26

$$\frac{1}{3}ad^2x^3 + \frac{1}{9}\left(3x^3\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{\left(c^2x^2-1\right)^{\frac{3}{2}} + 3\sqrt{c^2x^2-1}}{c^3}\right)bd^2 + \frac{1}{245}\left(35ax^7 + \left(35x^7\log\left(cx + \sqrt{c^2x^2-1}\right) -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3
/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*d^2 + 1/245*(35*a*x^7 + (35*x^7*log(c*x +
 sqrt(c^2*x^2 - 1)) - (5*(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5/2) + 35*
(c^2*x^2 - 1)^(3/2) + 35*sqrt(c^2*x^2 - 1))/c^7)*b)*e^2 + 2/75*(15*a*d*x^5
+ (15*x^5*log(c*x + sqrt(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x
^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))/c^5)*b*d)*e
```

3.473 $\int x\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=269

$$\frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}+\frac{bx\left(1-c^2x^2\right)\left(44c^4d^2+44c^2de+15e^2\right)}{288c^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\sqrt{c^2x^2-1}\left(2c^2d+e\right)\left(8c^4d^2+8c^2de+5e^2\right)\text{ta}}{96c^6e\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*(44*c^4*d^2 + 44*c^2*d*e + 15*e^2)*x*(1 - c^2*x^2))/(288*c^5*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (5*b*(2*c^2*d + e)*x*(1 - c^2*x^2)*(d + e*x^2))/(144*c
^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(36*c*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/(6*e)
- (b*(2*c^2*d + e)*(8*c^4*d^2 + 8*c^2*d*e + 5*e^2)*Sqrt[-1 + c^2*x^2]*ArcTa
nh[(c*x)/Sqrt[-1 + c^2*x^2]])/(96*c^6*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.248621, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5788, 902, 416, 528, 388, 217, 206}

$$\frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}+\frac{bx\left(1-c^2x^2\right)\left(44c^4d^2+44c^2de+15e^2\right)}{288c^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\sqrt{c^2x^2-1}\left(2c^2d+e\right)\left(8c^4d^2+8c^2de+5e^2\right)\text{ta}}{96c^6e\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (b*(44*c^4*d^2 + 44*c^2*d*e + 15*e^2)*x*(1 - c^2*x^2))/(288*c^5*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (5*b*(2*c^2*d + e)*x*(1 - c^2*x^2)*(d + e*x^2))/(144*c
^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(36*c*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/(6*e)
- (b*(2*c^2*d + e)*(8*c^4*d^2 + 8*c^2*d*e + 5*e^2)*Sqrt[-1 + c^2*x^2]*ArcTa
nh[(c*x)/Sqrt[-1 + c^2*x^2]])/(96*c^6*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 902

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^FracPart[m])/(d*
f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e*f + d*g, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^3}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{6e}$$

$$=\frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{\left(d+ex^2\right)^3}{\sqrt{-1+c^2x^2}}dx}{6e\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^2}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}-\frac{\left(b\sqrt{-1+c^2x^2}\right)\int\frac{(d+e}{}}{36ce\sqrt{-1}}$$

$$=\frac{5b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)}{144c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^2}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d+ex^2\right)^3(a+}{6e}$$

$$=\frac{b\left(44c^4d^2+44c^2de+15e^2\right)x\left(1-c^2x^2\right)}{288c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)}{144c^3\sqrt{-1+cx}\sqrt{1+cx}}+$$

$$=\frac{b\left(44c^4d^2+44c^2de+15e^2\right)x\left(1-c^2x^2\right)}{288c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)}{144c^3\sqrt{-1+cx}\sqrt{1+cx}}+$$

$$=\frac{b\left(44c^4d^2+44c^2de+15e^2\right)x\left(1-c^2x^2\right)}{288c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)}{144c^3\sqrt{-1+cx}\sqrt{1+cx}}+$$

Mathematica [A] time = 0.306075, size = 183, normalized size = 0.68

$$\frac{cx\left(48ac^5x\left(3d^2+3dex^2+e^2x^4\right)-b\sqrt{cx-1}\sqrt{cx+1}\left(4c^4\left(18d^2+9dex^2+2e^2x^4\right)+2c^2e\left(27d+5ex^2\right)+15e^2\right)\right)+48bc^6x^2}{288c^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (c*x*(48*a*c^5*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) - b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(15*e^2 + 2*c^2*e*(27*d + 5*e*x^2) + 4*c^4*(18*d^2 + 9*d*e*x^2 + 2*e^2
*x^4))) + 48*b*c^6*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCosh[c*x] - 6*b*(24
*c^4*d^2 + 18*c^2*d*e + 5*e^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(288*c^
6)
```

Maple [A] time = 0.016, size = 363, normalized size = 1.4

$$\frac{ae^2x^6}{6}+\frac{adex^4}{2}+\frac{ax^2d^2}{2}+\frac{b\operatorname{arccosh}(cx)\,e^2x^6}{6}+\frac{b\operatorname{arccosh}(cx)\,dex^4}{2}+\frac{b\operatorname{arccosh}(cx)\,x^2d^2}{2}-\frac{be^2x^5}{36\,c}\sqrt{cx-1}\sqrt{cx+1}-\frac{bd}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/6*a*e^2*x^6+1/2*a*d*e*x^4+1/2*a*x^2*d^2+1/6*b*arccosh(c*x)*e^2*x^6+1/2*b*
arccosh(c*x)*d*e*x^4+1/2*b*arccosh(c*x)*x^2*d^2-1/36/c*b*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*e^2*x^5-1/8/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e*x^3-1/4*b*d^2*x*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4/c^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2
-1)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-5/144/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*e^2*x^3-3/16/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e*x-3/16/c^4*b*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-5/96/
c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2*x-5/96/c^6*b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/(c^2*x^2-1)^(1/2)*e^2*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.13256, size = 405, normalized size = 1.51

$$\frac{1}{6}\,ae^2x^6+\frac{1}{2}\,adex^4+\frac{1}{2}\,ad^2x^2+\frac{1}{4}\left(2\,x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right)\right)bd^2+\frac{1}{16}\left(8\,x^4\operatorname{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arccosh(c*x) - c
*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(s
qrt(c^2)*c^2)))*b*d^2 + 1/16*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3
/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt
(c^2))/(sqrt(c^2)*c^4))*c)*b*d*e + 1/288*(48*x^6*arccosh(c*x) - (8*sqrt(c^2
*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c
^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^6))*c)*b*
e^2
```

Fricas [A] time = 2.574, size = 436, normalized size = 1.62

$$\frac{48\,ac^6e^2x^6+144\,ac^6dex^4+144\,ac^6d^2x^2+3\left(16\,bc^6e^2x^6+48\,bc^6dex^4+48\,bc^6d^2x^2-24\,bc^4d^2-18\,bc^2de-5\,be^2\right)\log\left(cx\right.}{288\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*e^2*x^6 + 144*a*c^6*d*e*x^4 + 144*a*c^6*d^2*x^2 + 3*(16*b*c
^6*e^2*x^6 + 48*b*c^6*d*e*x^4 + 48*b*c^6*d^2*x^2 - 24*b*c^4*d^2 - 18*b*c^2*
d*e - 5*b*e^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (8*b*c^5*e^2*x^5 + 2*(18*b*c^
5*d*e + 5*b*c^3*e^2)*x^3 + 3*(24*b*c^5*d^2 + 18*b*c^3*d*e + 5*b*c*e^2)*x)*s
qrt(c^2*x^2 - 1))/c^6
```

Sympy [A] time = 11.7074, size = 306, normalized size = 1.14

$$\begin{cases}\frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\operatorname{acosh}(cx)}{2}+\frac{bdex^4\operatorname{acosh}(cx)}{2}+\frac{be^2x^6\operatorname{acosh}(cx)}{6}-\frac{bd^2x\sqrt{c^2x^2-1}}{4c}-\frac{bdex^3\sqrt{c^2x^2-1}}{8c}-\frac{be^2x^5\sqrt{c^2x^2-1}}{36c}-\frac{bd^2\operatorname{acosh}}{4c^2} \\ \left(a+\frac{i\pi b}{2}\right)\left(\frac{d^2x^2}{2}+\frac{dex^4}{2}+\frac{e^2x^6}{6}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*acosh
(c*x)/2 + b*d*e*x**4*acosh(c*x)/2 + b*e**2*x**6*acosh(c*x)/6 - b*d**2*x*sqr
t(c**2*x**2 - 1)/(4*c) - b*d*e*x**3*sqrt(c**2*x**2 - 1)/(8*c) - b*e**2*x**5
*sqrt(c**2*x**2 - 1)/(36*c) - b*d**2*acosh(c*x)/(4*c**2) - 3*b*d*e*x*sqrt(c
**2*x**2 - 1)/(16*c**3) - 5*b*e**2*x**3*sqrt(c**2*x**2 - 1)/(144*c**3) - 3*
b*d*e*acosh(c*x)/(16*c**4) - 5*b*e**2*x*sqrt(c**2*x**2 - 1)/(96*c**5) - 5*b
*e**2*acosh(c*x)/(96*c**6), Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**2/2 + d*e*x
**4/2 + e**2*x**6/6), True))
```

Giac [A] time = 1.45162, size = 387, normalized size = 1.44

$$\frac{1}{2}\,ad^2x^2+\frac{1}{4}\left(2\,x^2\log\left(cx+\sqrt{c^2x^2-1}\right)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}-\frac{\log\left(\left|-x|c|+\sqrt{c^2x^2-1}\right|\right)}{c^2|c|}\right)\right)bd^2+\frac{1}{288}\left(48\,ax^6+\left(48\,x^6\log\left(cx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*a*d^2*x^2 + 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 - 1)) - c*(sqrt(c^2*x^2 -
 1)*x/c^2 - log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^2*abs(c))))*b*d^2 +
1/288*(48*a*x^6 + (48*x^6*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)
*(2*x^2*(4*x^2/c^2 + 5/c^4) + 15/c^6)*x - 15*log(abs(-x*abs(c) + sqrt(c^2*x
^2 - 1)))/(c^6*abs(c)))*c)*b)*e^2 + 1/16*(8*a*d*x^4 + (8*x^4*log(c*x + sqrt
(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*x*(2*x^2/c^2 + 3/c^4) - 3*log(abs(-x*ab
s(c) + sqrt(c^2*x^2 - 1)))/(c^4*abs(c)))*c)*b*d)*e
```

3.474 $\int \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=196

$$d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{b\left(1-c^2x^2\right)\left(15c^4d^2+10c^2de+3e^2\right)}{15c^5\sqrt{cx-1}\sqrt{cx+1}}-$$

```
[Out] (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*(1 - c^2*x^2))/(15*c^5*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (2*b*e*(5*c^2*d + 3*e)*(1 - c^2*x^2)^2)/(45*c^5*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + (b*e^2*(1 - c^2*x^2)^3)/(25*c^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + d^2*x*(a + b*ArcCosh[c*x]) + (2*d*e*x^3*(a + b*ArcCosh[c*x]))/3
+ (e^2*x^5*(a + b*ArcCosh[c*x]))/5
```

Rubi [A] time = 0.203285, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {194, 5705, 12, 520, 1247, 698}

$$d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{b\left(1-c^2x^2\right)\left(15c^4d^2+10c^2de+3e^2\right)}{15c^5\sqrt{cx-1}\sqrt{cx+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*(1 - c^2*x^2))/(15*c^5*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (2*b*e*(5*c^2*d + 3*e)*(1 - c^2*x^2)^2)/(45*c^5*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + (b*e^2*(1 - c^2*x^2)^3)/(25*c^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + d^2*x*(a + b*ArcCosh[c*x]) + (2*d*e*x^3*(a + b*ArcCosh[c*x]))/3
+ (e^2*x^5*(a + b*ArcCosh[c*x]))/5
```


Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
```

```
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx = d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh^{-1}(cx)\right)-(b$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{15}$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh^{-1}(cx)\right)-$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh^{-1}(cx)\right)-$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh^{-1}(cx)\right)-$$

$$= \frac{b\left(15c^4d^2+10c^2de+3e^2\right)\left(1-c^2x^2\right)}{15c^5\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2be\left(5c^2d+3e\right)\left(1-c^2x^2\right)^2}{45c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^2\left(1-c\right.}{25c^5\sqrt{-1+c}}$$

Mathematica [A] time = 0.191845, size = 130, normalized size = 0.66

$$\frac{1}{225}\left(15ax\left(15d^2+10dex^2+3e^2x^4\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^4\left(225d^2+50dex^2+9e^2x^4\right)+4c^2e\left(25d+3ex^2\right)+24e^2\right)}{c^5}+15bx\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4
)))/c^5 + 15*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x])/225
```

Maple [A] time = 0.012, size = 157, normalized size = 0.8

$$\frac{1}{c}\left(\frac{a}{c^4}\left(\frac{e^2c^5x^5}{5}+\frac{2\,c^5dex^3}{3}+xc^5d^2\right)+\frac{b}{c^4}\left(\frac{\operatorname{arccosh}(cx)\,e^2c^5x^5}{5}+\frac{2\operatorname{arccosh}(cx)\,c^5dex^3}{3}+\operatorname{arccosh}(cx)\,c^5xd^2-\frac{9\,c^4e^2x^4+}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(a/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b/c^4*(1/5*arccosh(c
*x)*e^2*c^5*x^5+2/3*arccosh(c*x)*c^5*d*e*x^3+arccosh(c*x)*c^5*x*d^2-1/225*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*e^2*x^4+50*c^4*d*e*x^2+225*c^4*d^2+12*c^2
*e^2*x^2+100*c^2*d*e+24*e^2)))
```

Maxima [A] time = 1.18332, size = 243, normalized size = 1.24

$$\frac{1}{5}ae^2x^5+\frac{2}{3}adex^3+\frac{2}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bde+\frac{1}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 -
 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d*e + 1/75*(15*x^5*arccosh(c*x) -
 (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 - 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d
^2/c
```

Fricas [A] time = 2.42728, size = 370, normalized size = 1.89

$$\frac{45ac^5e^2x^5+150ac^5dex^3+225ac^5d^2x+15\left(3bc^5e^2x^5+10bc^5dex^3+15bc^5d^2x\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9bc^4e^2x^4+22\right.}{225c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^5
*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 - 1))
- (9*b*c^4*e^2*x^4 + 225*b*c^4*d^2 + 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*c^4
*d*e + 6*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] time = 3.7601, size = 246, normalized size = 1.26

$$\begin{cases} ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\operatorname{acosh}(cx) + \frac{2bdex^3\operatorname{acosh}(cx)}{3} + \frac{be^2x^5\operatorname{acosh}(cx)}{5} - \frac{bd^2\sqrt{c^2x^2-1}}{c} - \frac{2bdex^2\sqrt{c^2x^2-1}}{9c} - \frac{be^2x^4\sqrt{c^2x^2-1}}{25c} - \frac{4bde}{} \\ \left(a + \frac{i\pi b}{2}\right)\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*acosh(c*x)
+ 2*b*d*e*x**3*acosh(c*x)/3 + b*e**2*x**5*acosh(c*x)/5 - b*d**2*sqrt(c**2*x
**2 - 1)/c - 2*b*d*e*x**2*sqrt(c**2*x**2 - 1)/(9*c) - b*e**2*x**4*sqrt(c**2
*x**2 - 1)/(25*c) - 4*b*d*e*sqrt(c**2*x**2 - 1)/(9*c**3) - 4*b*e**2*x**2*sq
rt(c**2*x**2 - 1)/(75*c**3) - 8*b*e**2*sqrt(c**2*x**2 - 1)/(75*c**5), Ne(c,
 0)), ((a + I*pi*b/2)*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Giac [A] time = 1.29177, size = 262, normalized size = 1.34

$$\left(x\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{\sqrt{c^2x^2-1}}{c}\right)bd^2 + ad^2x + \frac{1}{75}\left(15\,ax^5 + \left(15\,x^5\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{3\left(c^2x^2-1\right)^{\frac{5}{2}} + 10\left(c^2x^2 - \right.}{c^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d^2 + a*d^2*x + 1/
75*(15*a*x^5 + (15*x^5*log(c*x + sqrt(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1)^(5/2
) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))/c^5)*b)*e^2 + 2/9*(3*a*d
*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3/2) + 3*sqrt(
c^2*x^2 - 1))/c^3)*b*d)*e
```

3.475 $\int \frac{(d+ex^2)^2 (a+b\cosh^{-1}(cx))}{x} \, dx$

Optimal. Leaf size=342

$$-\frac{ibd^2\sqrt{1-c^2x^2}\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+d^2\log(x)\left(a+b\cosh^{-1}(cx)\right)+dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}\right.$$

```
[Out] -(b*e*(16*c^2*d + 3*e)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c^3) - (b*e^2*x^
3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c) - (b*e*(16*c^2*d + 3*e)*ArcCosh[c*x]
)/(32*c^4) + d*e*x^2*(a + b*ArcCosh[c*x]) + (e^2*x^4*(a + b*ArcCosh[c*x]))/
4 - ((I/2)*b*d^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (b*d^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])]
)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*(a + b*ArcCosh[c*x])*Log[x] - (b*d^2
*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I
/2)*b*d^2*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.786414, antiderivative size = 369, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {266, 43, 5790, 6742, 90, 52, 100, 12, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{ibd^2\sqrt{1-c^2x^2}\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+d^2\log(x)\left(a+b\cosh^{-1}(cx)\right)+dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -(b*d*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c) - (3*b*e^2*x*Sqrt[-1 + c*x]*S
qrt[1 + c*x])/(32*c^3) - (b*e^2*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c) -
(b*d*e*ArcCosh[c*x])/(2*c^2) - (3*b*e^2*ArcCosh[c*x])/(32*c^4) + d*e*x^2*(a
 + b*ArcCosh[c*x]) + (e^2*x^4*(a + b*ArcCosh[c*x]))/4 - ((I/2)*b*d^2*Sqrt[1
 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*Sqrt[1 -
 c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) + d^2*(a + b*ArcCosh[c*x])*Log[x] - (b*d^2*Sqrt[1 - c^2*x^2]*ArcS
in[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/2)*b*d^2*Sqrt[1 - c^2*
x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
```

```
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)+d^2\left(a+b\cosh^{-1}(cx)\right)\log(x$$

$$= dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)+d^2\left(a+b\cosh^{-1}(cx)\right)\log(x$$

$$= dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)+d^2\left(a+b\cosh^{-1}(cx)\right)\log(x$$

$$= -\frac{bdex\sqrt{-1+cx}\sqrt{1+cx}}{2c}-\frac{be^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}+dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}$$

$$= -\frac{bdex\sqrt{-1+cx}\sqrt{1+cx}}{2c}-\frac{be^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{bde\cosh^{-1}(cx)}{2c^2}+dex^2\left(a+\right.$$

$$= -\frac{bdex\sqrt{-1+cx}\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{b}{}$$

$$= -\frac{bdex\sqrt{-1+cx}\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{b}{}$$

$$= -\frac{bdex\sqrt{-1+cx}\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{b}{}$$

$$= -\frac{bdex\sqrt{-1+cx}\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{b}{}$$

$$= -\frac{bdex\sqrt{-1+cx}\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{b}{}$$

Mathematica [A] time = 0.415412, size = 217, normalized size = 0.63

$$-\frac{1}{2}bd^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+ad^2\log(x)+adex^2+\frac{1}{4}ae^2x^4-\frac{bde\left(cx\sqrt{cx-1}\sqrt{cx+1}+2\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)\right)}{2c^2}-\frac{be^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + b*d*e*x^2*ArcCosh[c*x] + (b*e^2*x^4*ArcCosh[c*x
])/4 - (b*d*e*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sqrt[(-1 + c*x)
```

```
/(1 + c*x)]]]))/(2*c^2) - (b*e^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 + 2*c^
2*x^2) + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(32*c^4) + (b*d^2*ArcCosh[
c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]))/2 + a*d^2*Log[x] - (b
*d^2*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

Maple [A] time = 0.239, size = 225, normalized size = 0.7

$$\frac{ae^2x^4}{4} + ax^2de + ad^2\ln(cx) - \frac{bdex}{2c}\sqrt{cx-1}\sqrt{cx+1} - \frac{b(\operatorname{arccosh}(cx))^2 d^2}{2} + \frac{b\operatorname{arccosh}(cx)e^2x^4}{4} + b\operatorname{arccosh}(cx)x^2de +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/4*a*e^2*x^4+a*x^2*d*e+a*d^2*ln(c*x)-1/2*b*d*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c-1/2*b*arccosh(c*x)^2*d^2+1/4*b*arccosh(c*x)*e^2*x^4+b*arccosh(c*x)*x^2
*d*e+1/2*b*d^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/16*b*e^2*x
^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/32*b*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
^3+b*d^2*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)-1/2*b*d*e*a
rccosh(c*x)/c^2-3/32*b*e^2*arccosh(c*x)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}ae^2x^4 + adex^2 + ad^2\log(x) + \int be^2x^3\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + 2\,bdex\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \frac{bd^2\log\left(cx + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + integrate(b*e^2*x^3*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1)) + 2*b*d*e*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
)) + b*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\operatorname{arcosh}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccosh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(cx))\left(d + ex^2\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2(b\operatorname{arcosh}(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x, x)
```

3.476 $\int \frac{(d+ex^2)^2(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=160

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+2dex\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{be\left(1-c^2x^2\right)\left(6c^2d+e\right)}{3c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{be^2\left(1-c^2x^2\right.}{9c^3\sqrt{cx-1}\sqrt{c}}$$

```
[Out] (b*e*(6*c^2*d + e)*(1 - c^2*x^2))/(3*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*e^2*(1 - c^2*x^2)^2)/(9*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*Ar
cCosh[c*x]))/x + 2*d*e*x*(a + b*ArcCosh[c*x]) + (e^2*x^3*(a + b*ArcCosh[c*x
]))/3 + b*c*d^2*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rubi [A] time = 0.302226, antiderivative size = 185, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5790, 520, 1251, 897, 1153, 205}

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+2dex\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{bcd^2\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{be\left(1\right.}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (b*e*(6*c^2*d + e)*(1 - c^2*x^2))/(3*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*e^2*(1 - c^2*x^2)^2)/(9*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*Ar
cCosh[c*x]))/x + 2*d*e*x*(a + b*ArcCosh[c*x]) + (e^2*x^3*(a + b*ArcCosh[c*x
]))/3 + (b*c*d^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 +
c*x]*Sqrt[1 + c*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\cosh^{-1}(cx))}{x^2}\,dx = -\frac{d^2 (a+b\cosh^{-1}(cx))}{x} + 2dex (a+b\cosh^{-1}(cx)) + \frac{1}{3}e^2x^3 (a+b\cosh^{-1}(cx)) - (b$$

$$= -\frac{d^2 (a+b\cosh^{-1}(cx))}{x} + 2dex (a+b\cosh^{-1}(cx)) + \frac{1}{3}e^2x^3 (a+b\cosh^{-1}(cx)) - \frac{(b}{}$$

$$= -\frac{d^2 (a+b\cosh^{-1}(cx))}{x} + 2dex (a+b\cosh^{-1}(cx)) + \frac{1}{3}e^2x^3 (a+b\cosh^{-1}(cx)) - \frac{(b}{}$$

$$= -\frac{d^2 (a+b\cosh^{-1}(cx))}{x} + 2dex (a+b\cosh^{-1}(cx)) + \frac{1}{3}e^2x^3 (a+b\cosh^{-1}(cx)) - \frac{(b}{}$$

$$= -\frac{d^2 (a+b\cosh^{-1}(cx))}{x} + 2dex (a+b\cosh^{-1}(cx)) + \frac{1}{3}e^2x^3 (a+b\cosh^{-1}(cx)) - \frac{(b}{}$$

$$= \frac{be (6c^2d+e) (1-c^2x^2)}{3c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^2 (1-c^2x^2)^2}{9c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2 (a+b\cosh^{-1}(cx))}{x} + 2dex (a$$

$$= \frac{be (6c^2d+e) (1-c^2x^2)}{3c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^2 (1-c^2x^2)^2}{9c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2 (a+b\cosh^{-1}(cx))}{x} + 2dex (a$$

Mathematica [A] time = 0.2309, size = 128, normalized size = 0.8

$$\frac{1}{3}\left(-\frac{3ad^2}{x} + 6adex + ae^2x^3 - \frac{be\sqrt{cx-1}\sqrt{cx+1} (c^2 (18d+ex^2)+2e)}{3c^3} + \frac{b\cosh^{-1}(cx) (-3d^2+6dex^2+e^2x^4)}{x} - 3bcd^2 \tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] ((-3*a*d^2)/x + 6*a*d*e*x + a*e^2*x^3 - (b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
2*e + c^2*(18*d + e*x^2)))/(3*c^3) + (b*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcC
osh[c*x])/x - 3*b*c*d^2*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])])/3
```

Maple [A] time = 0.018, size = 177, normalized size = 1.1

$$\frac{ax^3e^2}{3}+2\,axde-\frac{d^2a}{x}+\frac{b\mathrm{arccosh}\,(cx)\,x^3e^2}{3}+2\,b\mathrm{arccosh}\,(cx)\,xde-\frac{bd^2\mathrm{arccosh}\,(cx)}{x}-cd^2b\sqrt{cx-1}\sqrt{cx+1}\arctan\left(\frac{}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] 1/3*a*x^3*e^2+2*a*x*d*e-d^2*a/x+1/3*b*arccosh(c*x)*x^3*e^2+2*b*arccosh(c*x)
*x*d*e-d^2*b*arccosh(c*x)/x-c*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)
^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))-1/9*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*
e^2-2*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e-2/9*b/c^3*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*e^2
```

Maxima [A] time = 1.72476, size = 184, normalized size = 1.15

$$\frac{1}{3}\,ae^2x^3-\left(c\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bd^2+\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)be^2+2\,adex+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 - (c*arcsin(1/(sqrt(c^2)*abs(x))) + arccosh(c*x)/x)*b*d^2 + 1
/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)
/c^4))*b*e^2 + 2*a*d*e*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d*e/c
 - a*d^2/x
```

Fricas [A] time = 2.96235, size = 512, normalized size = 3.2

$$\frac{3\,ac^3e^2x^4+18\,bc^4d^2x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+18\,ac^3dex^2-9\,ac^3d^2+3\left(3\,bc^3d^2-6\,bc^3de-bc^3e^2\right)x\log\left(-cx+\sqrt{c^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*e^2*x^4 + 18*b*c^4*d^2*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 18
*a*c^3*d*e*x^2 - 9*a*c^3*d^2 + 3*(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*
log(-c*x + sqrt(c^2*x^2 - 1)) + 3*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^
3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x)*log(c*x + sqrt(c^2*x^2 -
 1)) - (b*c^2*e^2*x^3 + 2*(9*b*c^2*d*e + b*e^2)*x)*sqrt(c^2*x^2 - 1))/(c^3*
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{acosh}(cx))\left(d+ex^2\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2(b\operatorname{arcosh}(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^2, x)
```

3.477 $\int \frac{(d+ex^2)^2 (a+b\cosh^{-1}(cx))}{x^3} dx$

Optimal. Leaf size=321

$$-\frac{ibde\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+2de\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\cosh^{-1}\right.$$

```
[Out] (b*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*e^2*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])/(4*c) - (b*e^2*ArcCosh[c*x])/(4*c^2) - (d^2*(a + b*ArcCosh[c*x])
)/(2*x^2) + (e^2*x^2*(a + b*ArcCosh[c*x]))/2 - (I*b*d*e*Sqrt[1 - c^2*x^2]*A
rcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d*e*Sqrt[1 - c^2*x^2]*A
rcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
 2*d*e*(a + b*ArcCosh[c*x])*Log[x] - (2*b*d*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]
*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*d*e*Sqrt[1 - c^2*x^2]*PolyLo
g[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.814215, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {266, 43, 5790, 12, 6742, 95, 90, 52, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{ibde\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+2de\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\cosh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*e^2*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])/(4*c) - (b*e^2*ArcCosh[c*x])/(4*c^2) - (d^2*(a + b*ArcCosh[c*x])
)/(2*x^2) + (e^2*x^2*(a + b*ArcCosh[c*x]))/2 - (I*b*d*e*Sqrt[1 - c^2*x^2]*A
rcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d*e*Sqrt[1 - c^2*x^2]*A
rcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
 2*d*e*(a + b*ArcCosh[c*x])*Log[x] - (2*b*d*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]
*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*d*e*Sqrt[1 - c^2*x^2]*PolyLo
g[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\cosh^{-1}(cx))}{x^3}\,dx = -\frac{d^2(a+b\cosh^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\cosh^{-1}(cx)) + 2de(a+b\cosh^{-1}(cx))\log($$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\cosh^{-1}(cx)) + 2de(a+b\cosh^{-1}(cx))\log($$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\cosh^{-1}(cx)) + 2de(a+b\cosh^{-1}(cx))\log($$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\cosh^{-1}(cx)) + 2de(a+b\cosh^{-1}(cx))\log($$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{d^2(a+b\cosh^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(cx)}{4c^2} - \frac{d^2(a+b\cos}{2x^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(cx)}{4c^2} - \frac{d^2(a+b\cos}{2x^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(cx)}{4c^2} - \frac{d^2(a+b\cos}{2x^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(cx)}{4c^2} - \frac{d^2(a+b\cos}{2x^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(cx)}{4c^2} - \frac{d^2(a+b\cos}{2x^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(cx)}{4c^2} - \frac{d^2(a+b\cos}{2x^2}$$

<u>Mathematica</u> [A] time = 0.437015, size = 173, normalized size = 0.54

$$\frac{1}{4}\left(4bde\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\right)-\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)-\frac{2ad^2}{x^2}+8ade\log(x)+2ae\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 + (2*b*d^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
- ArcCosh[c*x]))/x^2 + (b*e^2*(-(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + 2*c^2*
x^2*ArcCosh[c*x] - 2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/c^2 + 8*a*d*e*Lo
g[x] + 4*b*d*e*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]
) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/4
```

Maple [A] time = 0.316, size = 198, normalized size = 0.6

$$\frac{ax^2e^2}{2}+2\,ade\ln(cx)-\frac{ad^2}{2\,x^2}-b(\operatorname{arccosh}(cx))^2de+\frac{b\operatorname{arccosh}(cx)\,x^2e^2}{2}-\frac{be^2x}{4\,c}\sqrt{cx-1}\sqrt{cx+1}-\frac{be^2\operatorname{arccosh}(cx)}{4\,c^2}+\frac{bcd}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] 1/2*a*x^2*e^2+2*a*d*e*ln(c*x)-1/2*a*d^2/x^2-b*arccosh(c*x)^2*d*e+1/2*b*arcc
osh(c*x)*x^2*e^2-1/4*b*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4*b*e^2*arccos
h(c*x)/c^2+1/2*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x-1/2*c^2*b*d^2-1/2*b*ar
ccosh(c*x)*d^2/x^2+2*b*d*e*arccosh(c*x)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2+1)+b*d*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,ae^2x^2+\frac{1}{2}\,bd^2\left(\frac{\sqrt{c^2x^2-1}c}{x}-\frac{\operatorname{arcosh}(cx)}{x^2}\right)+2\,ade\log(x)-\frac{ad^2}{2\,x^2}+\int be^2x\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\frac{2\,bde\log\left(cx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*e^2*x^2 + 1/2*b*d^2*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) + 2*a*
d*e*log(x) - 1/2*a*d^2/x^2 + integrate(b*e^2*x*log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1)) + 2*b*d*e*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\operatorname{arcosh}(cx)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccosh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))\left(d + ex^2\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2(b\,\mathrm{arcosh}\,(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^3, x)
```

3.478 $\int \frac{(d+ex^2)^2 (a+b\cosh^{-1}(cx))}{x^4} \, dx$

Optimal. Leaf size=184

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bcd\sqrt{c^2x^2-1}\left(c^2d+1\right.}{6\sqrt{cx-}}$$

```
[Out] (b*e^2*(1 - c^2*x^2))/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2*(1 - c^2*
x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*ArcCosh[c*x]))/(3*
x^3) - (2*d*e*(a + b*ArcCosh[c*x]))/x + e^2*x*(a + b*ArcCosh[c*x]) + (b*c*d
*(c^2*d + 12*e)*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.278789, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {270, 5790, 520, 1251, 897, 1157, 388, 205}

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bcd\sqrt{c^2x^2-1}\left(c^2d+1\right.}{6\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```


```
[Out] (b*e^2*(1 - c^2*x^2))/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2*(1 - c^2*
x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*ArcCosh[c*x]))/(3*
x^3) - (2*d*e*(a + b*ArcCosh[c*x]))/x + e^2*x*(a + b*ArcCosh[c*x]) + (b*c*d
*(c^2*d + 12*e)*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a+b\cosh^{-1}(cx)\right)-(bc)\int$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(bc\sqrt{\cdot\right.}}{}$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(bc\sqrt{\cdot\right.}}{}$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(b\sqrt{-\right.}}{}$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a\right.$$

$$= \frac{be^2\left(1-c^2x^2\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\,\mathrm{c}\right.}{x}$$

$$= \frac{be^2\left(1-c^2x^2\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\,\mathrm{c}\right.}{x}$$

Mathematica [A] time = 0.237185, size = 133, normalized size = 0.72

$$-\frac{ad^2}{3x^3}-\frac{2ade}{x}+ae^2x-\frac{1}{6}bcd\left(c^2d+12e\right)\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{cd^2}{6x^2}-\frac{e^2}{c}\right)-\frac{b\cosh^{-1}(cx)\left(d^2+6\right.}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -(a*d^2)/(3*x^3) - (2*a*d*e)/x + a*e^2*x + b*(-(e^2/c) + (c*d^2)/(6*x^2))*S
qrt[-1 + c*x]*Sqrt[1 + c*x] - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCosh[c*x]
)/(3*x^3) - (b*c*d*(c^2*d + 12*e)*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])])
/6
```

Maple [A] time = 0.02, size = 196, normalized size = 1.1

$$axe^2 - 2\frac{ade}{x} - \frac{ad^2}{3x^3} + b\operatorname{arccosh}(cx)\,xe^2 - 2\frac{bd\operatorname{arccosh}(cx)\,e}{x} - \frac{bd^2\operatorname{arccosh}(cx)}{3x^3} - \frac{c^3bd^2}{6}\sqrt{cx-1}\sqrt{cx+1}\arctan\left(\frac{}{\sqrt{c^2x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] a*x*e^2-2*a*d*e/x-1/3*a*d^2/x^3+b*arccosh(c*x)*x*e^2-2*b*arccosh(c*x)*d*e/x
-1/3*b*arccosh(c*x)*d^2/x^3-1/6*c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-
1)^(1/2)*d^2*arctan(1/(c^2*x^2-1)^(1/2))-2*c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))*d*e+1/6*b*c*d^2*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/x^2-1/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2
```

Maxima [A] time = 1.68929, size = 176, normalized size = 0.96

$$-\frac{1}{6}\left(\left(\left(c^2\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right) - \frac{\sqrt{c^2x^2-1}}{x^2}\right)c + \frac{2\operatorname{arcosh}(cx)}{x^3}\right)bd^2 - 2\left(c\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right) + \frac{\operatorname{arcosh}(cx)}{x}\right)bde + ae^2x + \frac{(cx\operatorname{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*arcsin(1/(sqrt(c^2)*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arcc
osh(c*x)/x^3)*b*d^2 - 2*(c*arcsin(1/(sqrt(c^2)*abs(x))) + arccosh(c*x)/x)*b
*d*e + a*e^2*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*e^2/c - 2*a*d*e/x
 - 1/3*a*d^2/x^3
```

Fricas [A] time = 3.34796, size = 487, normalized size = 2.65

$$\frac{6\,ace^2x^4 - 12\,acdex^2 + 2\left(bc^4d^2 + 12\,bc^2de\right)x^3\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + 2\left(bcd^2 + 6\,bcde - 3\,bce^2\right)x^3\log\left(-cx + \sqrt{c^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c*e^2*x^4 - 12*a*c*d*e*x^2 + 2*(b*c^4*d^2 + 12*b*c^2*d*e)*x^3*arct
an(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*log(
-c*x + sqrt(c^2*x^2 - 1)) - 2*a*c*d^2 + 2*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2 -
b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*log(c*x + sqrt(c^2*x^2 - 1
)) + (b*c^2*d^2*x - 6*b*e^2*x^3)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))\left(d + ex^2\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2(b\,\mathrm{arcosh}\,(cx) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^4, x)
```

3.479 $\int x^4 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=435

$$\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{11}e^3x^{11}\left(a+b\cosh^{-1}(cx)\right)-\frac{be(}{}$$

```
[Out] (b*(231*c^6*d^3 + 495*c^4*d^2*e + 385*c^2*d*e^2 + 105*e^3)*(1 - c^2*x^2))/(
1155*c^11*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(462*c^6*d^3 + 1485*c^4*d^2*e
+ 1540*c^2*d*e^2 + 525*e^3)*(1 - c^2*x^2)^2)/(3465*c^11*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]) + (b*(77*c^6*d^3 + 495*c^4*d^2*e + 770*c^2*d*e^2 + 350*e^3)*(1 -
 c^2*x^2)^3)/(1925*c^11*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e*(99*c^4*d^2 +
308*c^2*d*e + 210*e^2)*(1 - c^2*x^2)^4)/(1617*c^11*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (b*e^2*(11*c^2*d + 15*e)*(1 - c^2*x^2)^5)/(297*c^11*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (b*e^3*(1 - c^2*x^2)^6)/(121*c^11*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (d^3*x^5*(a + b*ArcCosh[c*x]))/5 + (3*d^2*e*x^7*(a + b*ArcCosh[c*x]
))/7 + (d*e^2*x^9*(a + b*ArcCosh[c*x]))/3 + (e^3*x^11*(a + b*ArcCosh[c*x]))
/11
```

Rubi [A] time = 0.616826, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 5790, 12, 1610, 1799, 1620}

$$\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{11}e^3x^{11}\left(a+b\cosh^{-1}(cx)\right)-\frac{be(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(231*c^6*d^3 + 495*c^4*d^2*e + 385*c^2*d*e^2 + 105*e^3)*(1 - c^2*x^2))/(
1155*c^11*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(462*c^6*d^3 + 1485*c^4*d^2*e
+ 1540*c^2*d*e^2 + 525*e^3)*(1 - c^2*x^2)^2)/(3465*c^11*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]) + (b*(77*c^6*d^3 + 495*c^4*d^2*e + 770*c^2*d*e^2 + 350*e^3)*(1 -
 c^2*x^2)^3)/(1925*c^11*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e*(99*c^4*d^2 +
308*c^2*d*e + 210*e^2)*(1 - c^2*x^2)^4)/(1617*c^11*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (b*e^2*(11*c^2*d + 15*e)*(1 - c^2*x^2)^5)/(297*c^11*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (b*e^3*(1 - c^2*x^2)^6)/(121*c^11*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (d^3*x^5*(a + b*ArcCosh[c*x]))/5 + (3*d^2*e*x^7*(a + b*ArcCosh[c*x]
))/7 + (d*e^2*x^9*(a + b*ArcCosh[c*x]))/3 + (e^3*x^11*(a + b*ArcCosh[c*x]))
```

/11

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^4 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+b\cosh^{-1}\right.$$

$$= \frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+b\cosh^{-1}\right.$$

$$= \frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+b\cosh^{-1}\right.$$

$$= \frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+b\cosh^{-1}\right.$$

$$= \frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+b\cosh^{-1}\right.$$

$$= \frac{b\left(231c^6d^3+495c^4d^2e+385c^2de^2+105e^3\right)\left(1-c^2x^2\right)}{1155c^{11}\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\left(462c^6d^3+1485c^4\right.}{3465}$$

Mathematica [A] time = 0.388668, size = 276, normalized size = 0.63

$$\frac{3465ax^5\left(495d^2ex^2+231d^3+385de^2x^4+105e^3x^6\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^{10}x^4\left(245025d^2ex^2+160083d^3+148225de^2x^4+33075e^3x^6\right)+2c^8\left(147015\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3465*a*x^5*(231*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6) - (b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(134400*e^3 + 4480*c^2*e^2*(121*d + 15*e*x^2) +
80*c^4*e*(9801*d^2 + 3388*d*e*x^2 + 630*e^2*x^4) + 24*c^6*(17787*d^3 + 1633
5*d^2*e*x^2 + 8470*d*e^2*x^4 + 1750*e^3*x^6) + c^10*x^4*(160083*d^3 + 24502
5*d^2*e*x^2 + 148225*d*e^2*x^4 + 33075*e^3*x^6) + 2*c^8*(106722*d^3*x^2 + 1
47015*d^2*e*x^4 + 84700*d*e^2*x^6 + 18375*e^3*x^8)))/c^11 + 3465*b*x^5*(231
*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6)*ArcCosh[c*x])/4002075
```

Maple [A] time = 0.013, size = 335, normalized size = 0.8

$$\frac{1}{c^5}\left(\frac{a}{c^6}\left(\frac{e^3c^{11}x^{11}}{11}+\frac{de^2c^{11}x^9}{3}+\frac{3\,c^{11}d^2ex^7}{7}+\frac{c^{11}x^5d^3}{5}\right)+\frac{b}{c^6}\left(\frac{\operatorname{arccosh}(cx)\,e^3c^{11}x^{11}}{11}+\frac{\operatorname{arccosh}(cx)\,de^2c^{11}x^9}{3}+\frac{3\operatorname{arccos}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(a/c^6*(1/11*e^3*c^11*x^11+1/3*d*e^2*c^11*x^9+3/7*c^11*d^2*e*x^7+1/5*
c^11*x^5*d^3)+b/c^6*(1/11*arccosh(c*x)*e^3*c^11*x^11+1/3*arccosh(c*x)*d*e^2
*c^11*x^9+3/7*arccosh(c*x)*c^11*d^2*e*x^7+1/5*arccosh(c*x)*c^11*x^5*d^3-1/4
002075*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(33075*c^10*e^3*x^10+148225*c^10*d*e^2*x
^8+245025*c^10*d^2*e*x^6+36750*c^8*e^3*x^8+160083*c^10*d^3*x^4+169400*c^8*d
*e^2*x^6+294030*c^8*d^2*e*x^4+42000*c^6*e^3*x^6+213444*c^8*d^3*x^2+203280*c
^6*d*e^2*x^4+392040*c^6*d^2*e*x^2+50400*c^4*e^3*x^4+426888*c^6*d^3+271040*c
^4*d*e^2*x^2+784080*c^4*d^2*e+67200*c^2*e^3*x^2+542080*c^2*d*e^2+134400*e^3
)))
```

Maxima [A] time = 1.19795, size = 609, normalized size = 1.4

$$\frac{1}{11}ae^3x^{11}+\frac{1}{3}ade^2x^9+\frac{3}{7}ad^2ex^7+\frac{1}{5}ad^3x^5+\frac{1}{75}\left(15\,x^5\,\mathrm{arcosh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/11*a*e^3*x^11 + 1/3*a*d*e^2*x^9 + 3/7*a*d^2*e*x^7 + 1/5*a*d^3*x^5 + 1/75*
(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x
^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^3 + 3/245*(35*x^7*arccosh(c*x) - (
5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2
- 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d^2*e + 1/945*(315*x^9*arccos
h(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*
sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2
 - 1)/c^10)*c)*b*d*e^2 + 1/7623*(693*x^11*arccosh(c*x) - (63*sqrt(c^2*x^2 -
 1)*x^10/c^2 + 70*sqrt(c^2*x^2 - 1)*x^8/c^4 + 80*sqrt(c^2*x^2 - 1)*x^6/c^6
+ 96*sqrt(c^2*x^2 - 1)*x^4/c^8 + 128*sqrt(c^2*x^2 - 1)*x^2/c^10 + 256*sqrt(
c^2*x^2 - 1)/c^12)*c)*b*e^3
```

Fricas [A] time = 2.58607, size = 868, normalized size = 2.

$$363825\,ac^{11}e^3x^{11}+1334025\,ac^{11}de^2x^9+1715175\,ac^{11}d^2ex^7+800415\,ac^{11}d^3x^5+3465\left(105\,bc^{11}e^3x^{11}+385\,bc^{11}de^2x^9+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4002075*(363825*a*c^11*e^3*x^11 + 1334025*a*c^11*d*e^2*x^9 + 1715175*a*c^
11*d^2*e*x^7 + 800415*a*c^11*d^3*x^5 + 3465*(105*b*c^11*e^3*x^11 + 385*b*c^
11*d*e^2*x^9 + 495*b*c^11*d^2*e*x^7 + 231*b*c^11*d^3*x^5)*log(c*x + sqrt(c^
2*x^2 - 1)) - (33075*b*c^10*e^3*x^10 + 426888*b*c^6*d^3 + 1225*(121*b*c^10*
d*e^2 + 30*b*c^8*e^3)*x^8 + 784080*b*c^4*d^2*e + 25*(9801*b*c^10*d^2*e + 67
76*b*c^8*d*e^2 + 1680*b*c^6*e^3)*x^6 + 542080*b*c^2*d*e^2 + 3*(53361*b*c^10
*d^3 + 98010*b*c^8*d^2*e + 67760*b*c^6*d*e^2 + 16800*b*c^4*e^3)*x^4 + 13440
0*b*e^3 + 4*(53361*b*c^8*d^3 + 98010*b*c^6*d^2*e + 67760*b*c^4*d*e^2 + 1680
0*b*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))/c^11
```

Sympy [A] time = 130.21, size = 638, normalized size = 1.47

$$\begin{cases} \frac{ad^3x^5}{5} + \frac{3ad^2ex^7}{7} + \frac{ade^2x^9}{3} + \frac{ae^3x^{11}}{11} + \frac{bd^3x^5\operatorname{acosh}(cx)}{5} + \frac{3bd^2ex^7\operatorname{acosh}(cx)}{7} + \frac{bde^2x^9\operatorname{acosh}(cx)}{3} + \frac{be^3x^{11}\operatorname{acosh}(cx)}{11} - \frac{bd^3x^4\sqrt{c^2x^2-1}}{25c} - \frac{3bd^2 \cdots}{} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^5}{5} + \frac{3d^2ex^7}{7} + \frac{de^2x^9}{3} + \frac{e^3x^{11}}{11}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**5/5 + 3*a*d**2*e*x**7/7 + a*d*e**2*x**9/3 + a*e**3*x**
11/11 + b*d**3*x**5*acosh(c*x)/5 + 3*b*d**2*e*x**7*acosh(c*x)/7 + b*d*e**2*
x**9*acosh(c*x)/3 + b*e**3*x**11*acosh(c*x)/11 - b*d**3*x**4*sqrt(c**2*x**2
 - 1)/(25*c) - 3*b*d**2*e*x**6*sqrt(c**2*x**2 - 1)/(49*c) - b*d*e**2*x**8*s
qrt(c**2*x**2 - 1)/(27*c) - b*e**3*x**10*sqrt(c**2*x**2 - 1)/(121*c) - 4*b*
d**3*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 18*b*d**2*e*x**4*sqrt(c**2*x**2 -
 1)/(245*c**3) - 8*b*d*e**2*x**6*sqrt(c**2*x**2 - 1)/(189*c**3) - 10*b*e**3
*x**8*sqrt(c**2*x**2 - 1)/(1089*c**3) - 8*b*d**3*sqrt(c**2*x**2 - 1)/(75*c*
*5) - 24*b*d**2*e*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*d*e**2*x**4*sq
rt(c**2*x**2 - 1)/(315*c**5) - 80*b*e**3*x**6*sqrt(c**2*x**2 - 1)/(7623*c**
5) - 48*b*d**2*e*sqrt(c**2*x**2 - 1)/(245*c**7) - 64*b*d*e**2*x**2*sqrt(c**
2*x**2 - 1)/(945*c**7) - 32*b*e**3*x**4*sqrt(c**2*x**2 - 1)/(2541*c**7) - 1
28*b*d*e**2*sqrt(c**2*x**2 - 1)/(945*c**9) - 128*b*e**3*x**2*sqrt(c**2*x**2
 - 1)/(7623*c**9) - 256*b*e**3*sqrt(c**2*x**2 - 1)/(7623*c**11), Ne(c, 0)),
 ((a + I*pi*b/2)*(d**3*x**5/5 + 3*d**2*e*x**7/7 + d*e**2*x**9/3 + e**3*x**1
1/11), True))
```

Giac [A] time = 1.46826, size = 552, normalized size = 1.27

$$\frac{1}{5}\,ad^3x^5+\frac{1}{75}\left(15\,x^5\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{3\left(c^2x^2-1\right)^{\frac{5}{2}}+10\left(c^2x^2-1\right)^{\frac{3}{2}}+15\sqrt{c^2x^2-1}}{c^5}\right)bd^3+\frac{1}{7623}\left(693\,ax^{11}+\left(693\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*a*d^3*x^5 + 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1
)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))/c^5)*b*d^3 + 1/762
3*(693*a*x^11 + (693*x^11*log(c*x + sqrt(c^2*x^2 - 1)) - (63*(c^2*x^2 - 1)^
(11/2) + 385*(c^2*x^2 - 1)^(9/2) + 990*(c^2*x^2 - 1)^(7/2) + 1386*(c^2*x^2
- 1)^(5/2) + 1155*(c^2*x^2 - 1)^(3/2) + 693*sqrt(c^2*x^2 - 1))/c^11)*b)*e^3
 + 1/945*(315*a*d*x^9 + (315*x^9*log(c*x + sqrt(c^2*x^2 - 1)) - (35*(c^2*x^
2 - 1)^(9/2) + 180*(c^2*x^2 - 1)^(7/2) + 378*(c^2*x^2 - 1)^(5/2) + 420*(c^2
*x^2 - 1)^(3/2) + 315*sqrt(c^2*x^2 - 1))/c^9)*b*d)*e^2 + 3/245*(35*a*d^2*x^
7 + (35*x^7*log(c*x + sqrt(c^2*x^2 - 1)) - (5*(c^2*x^2 - 1)^(7/2) + 21*(c^2
*x^2 - 1)^(5/2) + 35*(c^2*x^2 - 1)^(3/2) + 35*sqrt(c^2*x^2 - 1))/c^7)*b*d^2
)*e
```

3.480 $\int x^3 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=494

$$\frac{\left(d+ex^2\right)^5 \left(a+b\cosh^{-1}(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4 \left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{bx\left(1-c^2x^2\right)\left(26c^4d^2+201c^2de+126e^2\right)\left(d+ex^2\right)^2}{9600c^5e\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -(b*(1232*c^8*d^4 - 2536*c^6*d^3*e - 7758*c^4*d^2*e^2 - 6615*c^2*d*e^3 - 18
90*e^4)*x*(1 - c^2*x^2))/(76800*c^9*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(1
36*c^6*d^3 - 1096*c^4*d^2*e - 1617*c^2*d*e^2 - 630*e^3)*x*(1 - c^2*x^2)*(d
+ e*x^2))/(38400*c^7*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(26*c^4*d^2 + 201
*c^2*d*e + 126*e^2)*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(9600*c^5*e*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*(11*c^2*d + 18*e)*x*(1 - c^2*x^2)*(d + e*x^2)^3)/(16
00*c^3*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^4)/
(100*c*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(d + e*x^2)^4*(a + b*ArcCosh[c*
x]))/(8*e^2) + ((d + e*x^2)^5*(a + b*ArcCosh[c*x]))/(10*e^2) + (b*(128*c^10
*d^5 - 480*c^6*d^3*e^2 - 800*c^4*d^2*e^3 - 525*c^2*d*e^4 - 126*e^5)*Sqrt[-1
 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(5120*c^10*e^2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.647944, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {266, 43, 5790, 12, 566, 528, 388, 217, 206}

$$\frac{\left(d+ex^2\right)^5 \left(a+b\cosh^{-1}(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4 \left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{bx\left(1-c^2x^2\right)\left(26c^4d^2+201c^2de+126e^2\right)\left(d+ex^2\right)^2}{9600c^5e\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```


```
[Out] -(b*(1232*c^8*d^4 - 2536*c^6*d^3*e - 7758*c^4*d^2*e^2 - 6615*c^2*d*e^3 - 18
90*e^4)*x*(1 - c^2*x^2))/(76800*c^9*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(1
36*c^6*d^3 - 1096*c^4*d^2*e - 1617*c^2*d*e^2 - 630*e^3)*x*(1 - c^2*x^2)*(d
+ e*x^2))/(38400*c^7*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(26*c^4*d^2 + 201
*c^2*d*e + 126*e^2)*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(9600*c^5*e*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*(11*c^2*d + 18*e)*x*(1 - c^2*x^2)*(d + e*x^2)^3)/(16
00*c^3*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^4)/
(100*c*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(d + e*x^2)^4*(a + b*ArcCosh[c*
x]))/(8*e^2) + ((d + e*x^2)^5*(a + b*ArcCosh[c*x]))/(10*e^2) + (b*(128*c^10
```

```
*d^5 - 480*c^6*d^3*e^2 - 800*c^4*d^2*e^3 - 525*c^2*d*e^4 - 126*e^5)*Sqrt[-1
 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(5120*c^10*e^2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 566

```
Int[((e1_) + (f1_.)*(x_)^(n2_.))^(r_.)*((e2_) + (f2_.)*(x_)^(n2_.))^(r_.)*(
(a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :>
Dist[((e1 + f1*x^(n/2))^FracPart[r]*(e2 + f2*x^(n/2))^FracPart[r])/(e1*e2 +
 f1*f2*x^n)^FracPart[r], Int[(a + b*x^n)^p*(c + d*x^n)^q*(e1*e2 + f1*f2*x^n
)^r, x], x] /; FreeQ[{a, b, c, d, e1, f1, e2, f2, n, p, q, r}, x] && EqQ[n2
, n/2] && EqQ[e2*f1 + e1*f2, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
```

```
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{d\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-(bc)\int\frac{(d+}{40e^2}$$

$$=-\frac{d\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-\frac{(bc)\int\frac{(d+ex^2}{\sqrt{-1}}}{4}$$

$$=-\frac{d\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-\frac{\left(bc\sqrt{-1+c}\right.}{40e^2}$$

$$=\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^4}{100ce\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5\left(a+b\,\mathrm{c}\right.}{10e^2}$$

$$=\frac{b\left(11c^2d+18e\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)^3}{1600c^3e\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^4}{100ce\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d\left(d+ex^2\right)^4}{}$$

$$=\frac{b\left(26c^4d^2+201c^2de+126e^2\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)^2}{9600c^5e\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(11c^2d+18e\right)x\left(1-c^2\right.}{1600c^3e\sqrt{-1+cx}}$$

$$=-\frac{b\left(136c^6d^3-1096c^4d^2e-1617c^2de^2-630e^3\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)}{38400c^7e\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(26c^4\right.}{}$$

$$=-\frac{b\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4\right)x\left(1-c^2x^2\right)}{76800c^9e\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b}{}$$

$$=-\frac{b\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4\right)x\left(1-c^2x^2\right)}{76800c^9e\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b}{}$$

$$=-\frac{b\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4\right)x\left(1-c^2x^2\right)}{76800c^9e\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b}{}$$

Mathematica [A] time = 0.52861, size = 294, normalized size = 0.6

$$\frac{1920ax^4\left(20d^2ex^2+10d^3+15de^2x^4+4e^3x^6\right)-\frac{bx\sqrt{cx-1}\sqrt{cx+1}\left(16c^8\left(400d^2ex^4+300d^3x^2+225de^2x^6+48e^3x^8\right)+8c^6\left(1000d^2ex^2+900d^3+525de^2x^4-\right.\right.}{c^9}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (1920*a*x^4*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6) - (b*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(1890*e^3 + 315*c^2*e^2*(25*d + 4*e*x^2) + 6*c^4*e*
(2000*d^2 + 875*d*e*x^2 + 168*e^2*x^4) + 8*c^6*(900*d^3 + 1000*d^2*e*x^2 +
525*d*e^2*x^4 + 108*e^3*x^6) + 16*c^8*(300*d^3*x^2 + 400*d^2*e*x^4 + 225*d*
e^2*x^6 + 48*e^3*x^8)))/c^9 + 1920*b*x^4*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*
x^4 + 4*e^3*x^6)*ArcCosh[c*x] - (30*b*(480*c^6*d^3 + 800*c^4*d^2*e + 525*c^
2*d*e^2 + 126*e^3)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/c^10)/76800
```

Maple [A] time = 0.02, size = 659, normalized size = 1.3

$$-\frac{5\,bx^3d^2e}{48\,c^3}\sqrt{cx-1}\sqrt{cx+1}-\frac{3\,bd^3}{32\,c^4}\sqrt{cx-1}\sqrt{cx+1}\ln\left(cx+\sqrt{c^2x^2-1}\right)\frac{1}{\sqrt{c^2x^2-1}}-\frac{105\,bxde^2}{1024\,c^7}\sqrt{cx-1}\sqrt{cx+1}+\frac{d^3\mathrm{barc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] -5/48/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*d^2*e-3/32/c^4*d^3*b*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))-105/1024/c^7*
b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d*e^2+1/4*d^3*b*arccosh(c*x)*x^4-1/16/c*d^3
*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3+1/4*d^3*a*x^4+1/10*b*arccosh(c*x)*e^3*x^
10+3/8*a*d*e^2*x^8+1/2*a*d^2*e*x^6+1/10*a*e^3*x^10-105/1024/c^8*b*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d*e^2-5/32/c
^6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2)
)*d^2*e-21/1280/c^7*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^3-63/2560/c^9*b*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x-1/100/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^
9-9/800/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^7-21/1600/c^5*b*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*e^3*x^5-5/32/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d^2*e+3/8
*b*arccosh(c*x)*d*e^2*x^8+1/2*b*arccosh(c*x)*d^2*e*x^6-3/64/c*b*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*x^7*d*e^2-1/12/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5*d^2*e-7
/128/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5*d*e^2-63/2560/c^10*b*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e^3*ln(c*x+(c^2*x^2-1)^(1/2))-35/512/c^5
*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*d*e^2-3/32*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c^3
```

Maxima [A] time = 1.19067, size = 706, normalized size = 1.43

$$\frac{1}{10}\,ae^3x^{10}+\frac{3}{8}\,ade^2x^8+\frac{1}{2}\,ad^2ex^6+\frac{1}{4}\,ad^3x^4+\frac{1}{32}\left(8\,x^4\,\mathrm{arcosh}\,(cx)-\left(\frac{2\,\sqrt{c^2x^2-1}x^3}{c^2}+\frac{3\,\sqrt{c^2x^2-1}x}{c^4}+\frac{3\,\log\left(2\,c^2x+2\right.}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/10*a*e^3*x^10 + 3/8*a*d*e^2*x^8 + 1/2*a*d^2*e*x^6 + 1/4*a*d^3*x^4 + 1/32*
(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/
c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*
d^3 + 1/96*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^
2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c
^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^6))*c)*b*d^2*e + 1/1024*(384*x^8*arccos
h(c*x) - (48*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*
sqrt(c^2*x^2 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x +
 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^8))*c)*b*d*e^2 + 1/12800*(1280
*x^10*arccosh(c*x) - (128*sqrt(c^2*x^2 - 1)*x^9/c^2 + 144*sqrt(c^2*x^2 - 1)
*x^7/c^4 + 168*sqrt(c^2*x^2 - 1)*x^5/c^6 + 210*sqrt(c^2*x^2 - 1)*x^3/c^8 +
315*sqrt(c^2*x^2 - 1)*x/c^10 + 315*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c
^2))/(sqrt(c^2)*c^10))*c)*b*e^3
```

<u>Fricas</u> [A] time = 2.53091, size = 798, normalized size = 1.62

$$\frac{7680\,ac^{10}e^3x^{10} + 28800\,ac^{10}de^2x^8 + 38400\,ac^{10}d^2ex^6 + 19200\,ac^{10}d^3x^4 + 15\left(512\,bc^{10}e^3x^{10} + 1920\,bc^{10}de^2x^8 + 2560\,bc^{10}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/76800*(7680*a*c^10*e^3*x^10 + 28800*a*c^10*d*e^2*x^8 + 38400*a*c^10*d^2*e
*x^6 + 19200*a*c^10*d^3*x^4 + 15*(512*b*c^10*e^3*x^10 + 1920*b*c^10*d*e^2*x
^8 + 2560*b*c^10*d^2*e*x^6 + 1280*b*c^10*d^3*x^4 - 480*b*c^6*d^3 - 800*b*c^
4*d^2*e - 525*b*c^2*d*e^2 - 126*b*e^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (768*
b*c^9*e^3*x^9 + 144*(25*b*c^9*d*e^2 + 6*b*c^7*e^3)*x^7 + 8*(800*b*c^9*d^2*e
 + 525*b*c^7*d*e^2 + 126*b*c^5*e^3)*x^5 + 10*(480*b*c^9*d^3 + 800*b*c^7*d^2
*e + 525*b*c^5*d*e^2 + 126*b*c^3*e^3)*x^3 + 15*(480*b*c^7*d^3 + 800*b*c^5*d
^2*e + 525*b*c^3*d*e^2 + 126*b*c*e^3)*x)*sqrt(c^2*x^2 - 1))/c^10
```

Sympy [A] time = 50.6566, size = 604, normalized size = 1.22

$$\begin{cases} \frac{ad^3x^4}{4} + \frac{ad^2ex^6}{2} + \frac{3ade^2x^8}{8} + \frac{ae^3x^{10}}{10} + \frac{bd^3x^4\operatorname{acosh}(cx)}{4} + \frac{bd^2ex^6\operatorname{acosh}(cx)}{2} + \frac{3bde^2x^8\operatorname{acosh}(cx)}{8} + \frac{be^3x^{10}\operatorname{acosh}(cx)}{10} - \frac{bd^3x^3\sqrt{c^2x^2-1}}{16c} - \frac{bd^2ex}{} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^4}{4} + \frac{d^2ex^6}{2} + \frac{3de^2x^8}{8} + \frac{e^3x^{10}}{10}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**4/4 + a*d**2*e*x**6/2 + 3*a*d*e**2*x**8/8 + a*e**3*x**
10/10 + b*d**3*x**4*acosh(c*x)/4 + b*d**2*e*x**6*acosh(c*x)/2 + 3*b*d*e**2*
x**8*acosh(c*x)/8 + b*e**3*x**10*acosh(c*x)/10 - b*d**3*x**3*sqrt(c**2*x**2
 - 1)/(16*c) - b*d**2*e*x**5*sqrt(c**2*x**2 - 1)/(12*c) - 3*b*d*e**2*x**7*s
qrt(c**2*x**2 - 1)/(64*c) - b*e**3*x**9*sqrt(c**2*x**2 - 1)/(100*c) - 3*b*d
**3*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 5*b*d**2*e*x**3*sqrt(c**2*x**2 - 1)/(
48*c**3) - 7*b*d*e**2*x**5*sqrt(c**2*x**2 - 1)/(128*c**3) - 9*b*e**3*x**7*s
qrt(c**2*x**2 - 1)/(800*c**3) - 3*b*d**3*acosh(c*x)/(32*c**4) - 5*b*d**2*e*
x*sqrt(c**2*x**2 - 1)/(32*c**5) - 35*b*d*e**2*x**3*sqrt(c**2*x**2 - 1)/(512
*c**5) - 21*b*e**3*x**5*sqrt(c**2*x**2 - 1)/(1600*c**5) - 5*b*d**2*e*acosh(
c*x)/(32*c**6) - 105*b*d*e**2*x*sqrt(c**2*x**2 - 1)/(1024*c**7) - 21*b*e**3
*x**3*sqrt(c**2*x**2 - 1)/(1280*c**7) - 105*b*d*e**2*acosh(c*x)/(1024*c**8)
 - 63*b*e**3*x*sqrt(c**2*x**2 - 1)/(2560*c**9) - 63*b*e**3*acosh(c*x)/(2560
*c**10), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x**4/4 + d**2*e*x**6/2 + 3*d*e**2
*x**8/8 + e**3*x**10/10), True))
```

Giac [A] time = 1.57286, size = 612, normalized size = 1.24

$$\frac{1}{4}ad^3x^4 + \frac{1}{32}\left(8x^4\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(\sqrt{c^2x^2-1}x\left(\frac{2x^2}{c^2} + \frac{3}{c^4}\right) - \frac{3\log\left(\left|-x|c| + \sqrt{c^2x^2-1}\right|\right)}{c^4|c|}\right)c\right)bd^3 + \frac{1}{12800}\left(1280\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + 1/32*(8*x^4*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 -
1)*x*(2*x^2/c^2 + 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*a
bs(c)))*c)*b*d^3 + 1/12800*(1280*a*x^10 + (1280*x^10*log(c*x + sqrt(c^2*x^2
 - 1)) - (sqrt(c^2*x^2 - 1)*(2*(4*(2*x^2*(8*x^2/c^2 + 9/c^4) + 21/c^6)*x^2
+ 105/c^8)*x^2 + 315/c^10)*x - 315*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/
```

```
(c^10*abs(c)))*c)*b)*e^3 + 1/1024*(384*a*d*x^8 + (384*x^8*log(c*x + sqrt(c^
2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*(2*(4*x^2*(6*x^2/c^2 + 7/c^4) + 35/c^6)*x^
2 + 105/c^8)*x - 105*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^8*abs(c)))*
c)*b*d)*e^2 + 1/96*(48*a*d^2*x^6 + (48*x^6*log(c*x + sqrt(c^2*x^2 - 1)) - (
sqrt(c^2*x^2 - 1)*(2*x^2*(4*x^2/c^2 + 5/c^4) + 15/c^6)*x - 15*log(abs(-x*ab
s(c) + sqrt(c^2*x^2 - 1)))/(c^6*abs(c)))*c)*b*d^2)*e
```

3.481 $\int x^2 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=365

$$\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^3x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{be(1}{}$$

```
[Out] (b*(105*c^6*d^3 + 189*c^4*d^2*e + 135*c^2*d*e^2 + 35*e^3)*(1 - c^2*x^2))/(3
15*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(105*c^6*d^3 + 378*c^4*d^2*e + 40
5*c^2*d*e^2 + 140*e^3)*(1 - c^2*x^2)^2)/(945*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (b*e*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^3)/(525*c^9*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^2*(27*c^2*d + 28*e)*(1 - c^2*x^2)^4)/(44
1*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*(1 - c^2*x^2)^5)/(81*c^9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^3*(a + b*ArcCosh[c*x]))/3 + (3*d^2*e*x^5*
(a + b*ArcCosh[c*x]))/5 + (3*d*e^2*x^7*(a + b*ArcCosh[c*x]))/7 + (e^3*x^9*(
a + b*ArcCosh[c*x]))/9
```

Rubi [A] time = 0.541253, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 5790, 12, 1610, 1799, 1620}

$$\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^3x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{be(1}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(105*c^6*d^3 + 189*c^4*d^2*e + 135*c^2*d*e^2 + 35*e^3)*(1 - c^2*x^2))/(3
15*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(105*c^6*d^3 + 378*c^4*d^2*e + 40
5*c^2*d*e^2 + 140*e^3)*(1 - c^2*x^2)^2)/(945*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (b*e*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^3)/(525*c^9*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^2*(27*c^2*d + 28*e)*(1 - c^2*x^2)^4)/(44
1*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*(1 - c^2*x^2)^5)/(81*c^9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^3*(a + b*ArcCosh[c*x]))/3 + (3*d^2*e*x^5*
(a + b*ArcCosh[c*x]))/5 + (3*d*e^2*x^7*(a + b*ArcCosh[c*x]))/7 + (e^3*x^9*(
a + b*ArcCosh[c*x]))/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\cosh^{-1}\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\cosh^{-1}\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\cosh^{-1}\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\cosh^{-1}\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\cosh^{-1}\right.$$

$$= \frac{b\left(105c^6d^3+189c^4d^2e+135c^2de^2+35e^3\right)\left(1-c^2x^2\right)}{315c^9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\left(105c^6d^3+378c^4d^2e\right.}{945c^9}$$

Mathematica [A] time = 0.311631, size = 236, normalized size = 0.65

$$\frac{315ax^3\left(189d^2ex^2+105d^3+135de^2x^4+35e^3x^6\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^8\left(11907d^2ex^4+11025d^3x^2+6075de^2x^6+1225e^3x^8\right)+2c^6\left(7938d^2ex^2+11025\right.\right.}{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (315*a*x^3*(105*d^3 + 189*d^2*e*x^2 + 135*d*e^2*x^4 + 35*e^3*x^6) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(4480*e^3 + 80*c^2*e^2*(243*d + 28*e*x^2) + 24*c^4
*e*(1323*d^2 + 405*d*e*x^2 + 70*e^2*x^4) + 2*c^6*(11025*d^3 + 7938*d^2*e*x^
2 + 3645*d*e^2*x^4 + 700*e^3*x^6) + c^8*(11025*d^3*x^2 + 11907*d^2*e*x^4 +
6075*d*e^2*x^6 + 1225*e^3*x^8)))/c^9 + 315*b*x^3*(105*d^3 + 189*d^2*e*x^2 +
 135*d*e^2*x^4 + 35*e^3*x^6)*ArcCosh[c*x])/99225
```

Maple [A] time = 0.012, size = 289, normalized size = 0.8

$$\frac{1}{c^3}\left(\frac{a}{c^6}\left(\frac{e^3c^9x^9}{9}+\frac{3\,de^2c^9x^7}{7}+\frac{3\,c^9d^2ex^5}{5}+\frac{x^3c^9d^3}{3}\right)+\frac{b}{c^6}\left(\frac{\operatorname{arccosh}(cx)\,e^3c^9x^9}{9}+\frac{3\operatorname{arccosh}(cx)\,de^2c^9x^7}{7}+\frac{3\operatorname{arccosh}(c\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(a/c^6*(1/9*e^3*c^9*x^9+3/7*d*e^2*c^9*x^7+3/5*c^9*d^2*e*x^5+1/3*x^3*c
^9*d^3)+b/c^6*(1/9*arccosh(c*x)*e^3*c^9*x^9+3/7*arccosh(c*x)*d*e^2*c^9*x^7+
3/5*arccosh(c*x)*c^9*d^2*e*x^5+1/3*arccosh(c*x)*c^9*x^3*d^3-1/99225*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*(1225*c^8*e^3*x^8+6075*c^8*d*e^2*x^6+11907*c^8*d^2*e*x
^4+1400*c^6*e^3*x^6+11025*c^8*d^3*x^2+7290*c^6*d*e^2*x^4+15876*c^6*d^2*e*x^
2+1680*c^4*e^3*x^4+22050*c^6*d^3+9720*c^4*d*e^2*x^2+31752*c^4*d^2*e+2240*c^
2*e^3*x^2+19440*c^2*d*e^2+4480*e^3)))
```

Maxima [A] time = 1.16467, size = 505, normalized size = 1.38

$$\frac{1}{9}ae^3x^9+\frac{3}{7}ade^2x^7+\frac{3}{5}ad^2ex^5+\frac{1}{3}ad^3x^3+\frac{1}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^3+\frac{1}{25}\left(15x^5\operatorname{arcosh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*e^3*x^9 + 3/7*a*d*e^2*x^7 + 3/5*a*d^2*e*x^5 + 1/3*a*d^3*x^3 + 1/9*(3*
x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))
*b*d^3 + 1/25*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(
c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^2*e + 3/245*(35*x^7*
arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 +
 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d*e^2 + 1/283
5*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 -
 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 +
 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*e^3
```

Fricas [A] time = 2.40888, size = 701, normalized size = 1.92

$$\frac{11025\,ac^9e^3x^9+42525\,ac^9de^2x^7+59535\,ac^9d^2ex^5+33075\,ac^9d^3x^3+315\left(35\,bc^9e^3x^9+135\,bc^9de^2x^7+189\,bc^9d^2ex^5+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*e^3*x^9 + 42525*a*c^9*d*e^2*x^7 + 59535*a*c^9*d^2*e*x^
5 + 33075*a*c^9*d^3*x^3 + 315*(35*b*c^9*e^3*x^9 + 135*b*c^9*d*e^2*x^7 + 189
*b*c^9*d^2*e*x^5 + 105*b*c^9*d^3*x^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (1225*
b*c^8*e^3*x^8 + 22050*b*c^6*d^3 + 31752*b*c^4*d^2*e + 25*(243*b*c^8*d*e^2 +
 56*b*c^6*e^3)*x^6 + 19440*b*c^2*d*e^2 + 3*(3969*b*c^8*d^2*e + 2430*b*c^6*d
*e^2 + 560*b*c^4*e^3)*x^4 + 4480*b*e^3 + (11025*b*c^8*d^3 + 15876*b*c^6*d^2
*e + 9720*b*c^4*d*e^2 + 2240*b*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))/c^9
```

Sympy [A] time = 36.9676, size = 532, normalized size = 1.46

$$\begin{cases} \frac{ad^3x^3}{3} + \frac{3ad^2ex^5}{5} + \frac{3ade^2x^7}{7} + \frac{ae^3x^9}{9} + \frac{bd^3x^3\operatorname{acosh}(cx)}{3} + \frac{3bd^2ex^5\operatorname{acosh}(cx)}{5} + \frac{3bde^2x^7\operatorname{acosh}(cx)}{7} + \frac{be^3x^9\operatorname{acosh}(cx)}{9} - \frac{bd^3x^2\sqrt{c^2x^2-1}}{9c} - \frac{3bd^2 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^3}{3} + \frac{3d^2ex^5}{5} + \frac{3de^2x^7}{7} + \frac{e^3x^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**3/3 + 3*a*d**2*e*x**5/5 + 3*a*d*e**2*x**7/7 + a*e**3*x
**9/9 + b*d**3*x**3*acosh(c*x)/3 + 3*b*d**2*e*x**5*acosh(c*x)/5 + 3*b*d*e**
2*x**7*acosh(c*x)/7 + b*e**3*x**9*acosh(c*x)/9 - b*d**3*x**2*sqrt(c**2*x**2
 - 1)/(9*c) - 3*b*d**2*e*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 3*b*d*e**2*x**6*
sqrt(c**2*x**2 - 1)/(49*c) - b*e**3*x**8*sqrt(c**2*x**2 - 1)/(81*c) - 2*b*d
**3*sqrt(c**2*x**2 - 1)/(9*c**3) - 4*b*d**2*e*x**2*sqrt(c**2*x**2 - 1)/(25*
c**3) - 18*b*d*e**2*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 8*b*e**3*x**6*sqr
t(c**2*x**2 - 1)/(567*c**3) - 8*b*d**2*e*sqrt(c**2*x**2 - 1)/(25*c**5) - 24
*b*d*e**2*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**3*x**4*sqrt(c**2*x*
*2 - 1)/(945*c**5) - 48*b*d*e**2*sqrt(c**2*x**2 - 1)/(245*c**7) - 64*b*e**3
*x**2*sqrt(c**2*x**2 - 1)/(2835*c**7) - 128*b*e**3*sqrt(c**2*x**2 - 1)/(283
5*c**9), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x**3/3 + 3*d**2*e*x**5/5 + 3*d*e*
*2*x**7/7 + e**3*x**9/9), True))
```

Giac [A] time = 1.41622, size = 479, normalized size = 1.31

$$\frac{1}{3}ad^3x^3 + \frac{1}{9}\left(3x^3\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{\left(c^2x^2-1\right)^{\frac{3}{2}} + 3\sqrt{c^2x^2-1}}{c^3}\right)bd^3 + \frac{1}{2835}\left(315ax^9 + \left(315x^9\log\left(cx + \sqrt{c^2x^2-1}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^3*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3
/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*d^3 + 1/2835*(315*a*x^9 + (315*x^9*log(c*
x + sqrt(c^2*x^2 - 1)) - (35*(c^2*x^2 - 1)^(9/2) + 180*(c^2*x^2 - 1)^(7/2)
+ 378*(c^2*x^2 - 1)^(5/2) + 420*(c^2*x^2 - 1)^(3/2) + 315*sqrt(c^2*x^2 - 1)
)/c^9)*b)*e^3 + 3/245*(35*a*d*x^7 + (35*x^7*log(c*x + sqrt(c^2*x^2 - 1)) -
(5*(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5/2) + 35*(c^2*x^2 - 1)^(3/2) +
35*sqrt(c^2*x^2 - 1))/c^7)*b*d)*e^2 + 1/25*(15*a*d^2*x^5 + (15*x^5*log(c*x
+ sqrt(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15
*sqrt(c^2*x^2 - 1))/c^5)*b*d^2)*e
```

3.482 $\int x\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=358

$$\frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e}+\frac{bx\left(1-c^2x^2\right)\left(104c^4d^2+104c^2de+35e^2\right)\left(d+ex^2\right)}{1536c^5\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bx\left(1-c^2x^2\right)\left(2c^2d+e\right)\left(40c^4d^2\right.}{3072c^7\sqrt{cx-1}\sqrt{cx}}$$

[Out] (5*b*(2*c^2*d + e)*(40*c^4*d^2 + 40*c^2*d*e + 21*e^2)*x*(1 - c^2*x^2))/(3072*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(104*c^4*d^2 + 104*c^2*d*e + 35*e^2)*x*(1 - c^2*x^2)*(d + e*x^2))/(1536*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*b*(2*c^2*d + e)*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(384*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^3)/(64*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((d + e*x^2)^4*(a + b*ArcCosh[c*x]))/(8*e) - (b*(128*c^8*d^4 + 256*c^6*d^3*e + 288*c^4*d^2*e^2 + 160*c^2*d*e^3 + 35*e^4)*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(1024*c^8*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])

Rubi [A] time = 0.364305, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5788, 902, 416, 528, 388, 217, 206}

$$\frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e}+\frac{bx\left(1-c^2x^2\right)\left(104c^4d^2+104c^2de+35e^2\right)\left(d+ex^2\right)}{1536c^5\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bx\left(1-c^2x^2\right)\left(2c^2d+e\right)\left(40c^4d^2\right.}{3072c^7\sqrt{cx-1}\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[x*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]

[Out] (5*b*(2*c^2*d + e)*(40*c^4*d^2 + 40*c^2*d*e + 21*e^2)*x*(1 - c^2*x^2))/(3072*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(104*c^4*d^2 + 104*c^2*d*e + 35*e^2)*x*(1 - c^2*x^2)*(d + e*x^2))/(1536*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*b*(2*c^2*d + e)*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(384*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^3)/(64*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((d + e*x^2)^4*(a + b*ArcCosh[c*x]))/(8*e) - (b*(128*c^8*d^4 + 256*c^6*d^3*e + 288*c^4*d^2*e^2 + 160*c^2*d*e^3 + 35*e^4)*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(1024*c^8*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 902

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^FracPart[m])/(d*
f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e*f + d*g, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^4}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{8e}$$

$$=\frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{\left(d+ex^2\right)^4}{\sqrt{-1+c^2x^2}}dx}{8e\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^3}{64c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e}-\frac{\left(b\sqrt{-1+c^2x^2}\right)\int\frac{(d\cdots}{}}{64ce\sqrt{\cdots}}$$

$$=\frac{7b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)^2}{384c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^3}{64c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d+ex^2\right)^4\left(a\cdots\right.}{}$$

$$=\frac{b\left(104c^4d^2+104c^2de+35e^2\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)}{1536c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{7b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\cdots}{384c^3\sqrt{-1+cx}\sqrt{1\cdots}}$$

$$=\frac{5b\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(104c^4d^2+104c^2de+3\cdots\right.}{1536c^5\sqrt{-1\cdots}}$$

$$=\frac{5b\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(104c^4d^2+104c^2de+3\cdots\right.}{1536c^5\sqrt{-1\cdots}}$$

$$=\frac{5b\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(104c^4d^2+104c^2de+3\cdots\right.}{1536c^5\sqrt{-1\cdots}}$$

Mathematica [A] time = 0.384862, size = 256, normalized size = 0.72

$$\frac{cx\left(384ac^7x\left(6d^2ex^2+4d^3+4de^2x^4+e^3x^6\right)-b\sqrt{cx-1}\sqrt{cx+1}\left(16c^6\left(36d^2ex^2+48d^3+16de^2x^4+3e^3x^6\right)+8c^4e\left(108d\cdots\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (c*x*(384*a*c^7*x*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x^6) - b*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(105*e^3 + 10*c^2*e^2*(48*d + 7*e*x^2) + 8*c^4*e*(108
*d^2 + 40*d*e*x^2 + 7*e^2*x^4) + 16*c^6*(48*d^3 + 36*d^2*e*x^2 + 16*d*e^2*x
^4 + 3*e^3*x^6))) + 384*b*c^8*x^2*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*
x^6)*ArcCosh[c*x] - 6*b*(256*c^6*d^3 + 288*c^4*d^2*e + 160*c^2*d*e^2 + 35*e
^3)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(3072*c^8)
```

Maple [A] time = 0.017, size = 553, normalized size = 1.5

$$\frac{ae^3x^8}{8}+\frac{ade^2x^6}{2}+\frac{3\,ad^2ex^4}{4}+\frac{ax^2d^3}{2}+\frac{b\mathrm{arccosh}\,(cx)\,e^3x^8}{8}+\frac{b\mathrm{arccosh}\,(cx)\,de^2x^6}{2}+\frac{3\,b\mathrm{arccosh}\,(cx)\,d^2ex^4}{4}+\frac{b\mathrm{arccosh}\,(c}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/8*a*e^3*x^8+1/2*a*d*e^2*x^6+3/4*a*d^2*e*x^4+1/2*a*x^2*d^3+1/8*b*arccosh(c
*x)*e^3*x^8+1/2*b*arccosh(c*x)*d*e^2*x^6+3/4*b*arccosh(c*x)*d^2*e*x^4+1/2*b
*arccosh(c*x)*x^2*d^3-1/64/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^7-1/12/c*b
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5*d*e^2-3/16/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*x^3*d^2*e-1/4*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4/c^2*b*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d^3-7/384/c^3*b
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^5-5/48/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*x^3*d*e^2-9/32/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d^2*e-9/32/c^4*b*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d^2*e-35/
1536/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^3-5/32/c^5*b*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*x*d*e^2-5/32/c^6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)
*ln(c*x+(c^2*x^2-1)^(1/2))*d*e^2-35/1024/c^7*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
e^3*x-35/1024/c^8*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e^3*ln(c*
x+(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.17718, size = 601, normalized size = 1.68

$$\frac{1}{8}\,ae^3x^8+\frac{1}{2}\,ade^2x^6+\frac{3}{4}\,ad^2ex^4+\frac{1}{2}\,ad^3x^2+\frac{1}{4}\left(2\,x^2\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right)\right)bd^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^3*x^8 + 1/2*a*d*e^2*x^6 + 3/4*a*d^2*e*x^4 + 1/2*a*d^3*x^2 + 1/4*(2*
x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^
2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^2)))*b*d^3 + 3/32*(8*x^4*arccosh(c*x) - (2*s
qrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sq
rt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*d^2*e + 1/96*(48*x^6*arcco
sh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*
sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(
sqrt(c^2)*c^6))*c)*b*d*e^2 + 1/3072*(384*x^8*arccosh(c*x) - (48*sqrt(c^2*x^
2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c^
6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqr
t(c^2))/(sqrt(c^2)*c^8))*c)*b*e^3
```

Fricas [A] time = 2.45161, size = 657, normalized size = 1.84

$$\frac{384\,ac^8e^3x^8 + 1536\,ac^8de^2x^6 + 2304\,ac^8d^2ex^4 + 1536\,ac^8d^3x^2 + 3\left(128\,bc^8e^3x^8 + 512\,bc^8de^2x^6 + 768\,bc^8d^2ex^4 + 512\,bc^8\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3072*(384*a*c^8*e^3*x^8 + 1536*a*c^8*d*e^2*x^6 + 2304*a*c^8*d^2*e*x^4 + 1
536*a*c^8*d^3*x^2 + 3*(128*b*c^8*e^3*x^8 + 512*b*c^8*d*e^2*x^6 + 768*b*c^8*
d^2*e*x^4 + 512*b*c^8*d^3*x^2 - 256*b*c^6*d^3 - 288*b*c^4*d^2*e - 160*b*c^2
*d*e^2 - 35*b*e^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (48*b*c^7*e^3*x^7 + 8*(32
*b*c^7*d*e^2 + 7*b*c^5*e^3)*x^5 + 2*(288*b*c^7*d^2*e + 160*b*c^5*d*e^2 + 35
*b*c^3*e^3)*x^3 + 3*(256*b*c^7*d^3 + 288*b*c^5*d^2*e + 160*b*c^3*d*e^2 + 35
*b*c*e^3)*x)*sqrt(c^2*x^2 - 1))/c^8
```

Sympy [A] time = 19.4456, size = 490, normalized size = 1.37

$$\begin{cases} \frac{ad^3x^2}{2} + \frac{3ad^2ex^4}{4} + \frac{ade^2x^6}{2} + \frac{ae^3x^8}{8} + \frac{bd^3x^2\operatorname{acosh}(cx)}{2} + \frac{3bd^2ex^4\operatorname{acosh}(cx)}{4} + \frac{bde^2x^6\operatorname{acosh}(cx)}{2} + \frac{be^3x^8\operatorname{acosh}(cx)}{8} - \frac{bd^3x\sqrt{c^2x^2-1}}{4c} - \frac{3bd^2ex^3\ldots}{1\ldots} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^2}{2} + \frac{3d^2ex^4}{4} + \frac{de^2x^6}{2} + \frac{e^3x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**2/2 + 3*a*d**2*e*x**4/4 + a*d*e**2*x**6/2 + a*e**3*x**
8/8 + b*d**3*x**2*acosh(c*x)/2 + 3*b*d**2*e*x**4*acosh(c*x)/4 + b*d*e**2*x*
*6*acosh(c*x)/2 + b*e**3*x**8*acosh(c*x)/8 - b*d**3*x*sqrt(c**2*x**2 - 1)/(
4*c) - 3*b*d**2*e*x**3*sqrt(c**2*x**2 - 1)/(16*c) - b*d*e**2*x**5*sqrt(c**2
*x**2 - 1)/(12*c) - b*e**3*x**7*sqrt(c**2*x**2 - 1)/(64*c) - b*d**3*acosh(c
*x)/(4*c**2) - 9*b*d**2*e*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 5*b*d*e**2*x**3
*sqrt(c**2*x**2 - 1)/(48*c**3) - 7*b*e**3*x**5*sqrt(c**2*x**2 - 1)/(384*c**
3) - 9*b*d**2*e*acosh(c*x)/(32*c**4) - 5*b*d*e**2*x*sqrt(c**2*x**2 - 1)/(32
*c**5) - 35*b*e**3*x**3*sqrt(c**2*x**2 - 1)/(1536*c**5) - 5*b*d*e**2*acosh(
c*x)/(32*c**6) - 35*b*e**3*x*sqrt(c**2*x**2 - 1)/(1024*c**7) - 35*b*e**3*ac
osh(c*x)/(1024*c**8), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x**2/2 + 3*d**2*e*x*
*4/4 + d*e**2*x**6/2 + e**3*x**8/8), True))
```

Giac [A] time = 1.58357, size = 552, normalized size = 1.54

$$\frac{1}{2}ad^3x^2 + \frac{1}{4}\left(2x^2\log\left(cx + \sqrt{c^2x^2-1}\right) - c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2} - \frac{\log\left(\left|-x|c| + \sqrt{c^2x^2-1}\right|\right)}{c^2|c|}\right)\right)bd^3 + \frac{1}{3072}\left(384\,ax^8 + \left(384\,x^8\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*a*d^3*x^2 + 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 - 1)) - c*(sqrt(c^2*x^2 -
 1)*x/c^2 - log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^2*abs(c))))*b*d^3 +
1/3072*(384*a*x^8 + (384*x^8*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 -
 1)*(2*(4*x^2*(6*x^2/c^2 + 7/c^4) + 35/c^6)*x^2 + 105/c^8)*x - 105*log(abs(
-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^8*abs(c)))*c)*b)*e^3 + 1/96*(48*a*d*x^6
+ (48*x^6*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*(2*x^2*(4*x^2/c
^2 + 5/c^4) + 15/c^6)*x - 15*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^6*a
bs(c)))*c)*b*d)*e^2 + 3/32*(8*a*d^2*x^4 + (8*x^4*log(c*x + sqrt(c^2*x^2 - 1
)) - (sqrt(c^2*x^2 - 1)*x*(2*x^2/c^2 + 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(
c^2*x^2 - 1)))/(c^4*abs(c)))*c)*b*d^2)*e
```

3.483 $\int \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=287

$$d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+d^3x\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{be\left(1-c^2x\right.}{1}$$

```
[Out] (b*(35*c^6*d^3 + 35*c^4*d^2*e + 21*c^2*d*e^2 + 5*e^3)*(1 - c^2*x^2))/(35*c^
7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(
1 - c^2*x^2)^2)/(105*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*e^2*(7*c^2*d
+ 5*e)*(1 - c^2*x^2)^3)/(175*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^3*(1
- c^2*x^2)^4)/(49*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*x*(a + b*ArcCosh[
c*x]) + d^2*e*x^3*(a + b*ArcCosh[c*x]) + (3*d*e^2*x^5*(a + b*ArcCosh[c*x]))
/5 + (e^3*x^7*(a + b*ArcCosh[c*x]))/7
```

Rubi [A] time = 0.37592, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {194, 5705, 12, 1610, 1799, 1850}

$$d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+d^3x\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{be\left(1-c^2x\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(35*c^6*d^3 + 35*c^4*d^2*e + 21*c^2*d*e^2 + 5*e^3)*(1 - c^2*x^2))/(35*c^
7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(
1 - c^2*x^2)^2)/(105*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*e^2*(7*c^2*d
+ 5*e)*(1 - c^2*x^2)^3)/(175*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^3*(1
- c^2*x^2)^4)/(49*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*x*(a + b*ArcCosh[
c*x]) + d^2*e*x^3*(a + b*ArcCosh[c*x]) + (3*d*e^2*x^5*(a + b*ArcCosh[c*x]))
/5 + (e^3*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx = d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)+$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)+$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)+$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)+$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)+$$

$$= \frac{b\left(35c^6d^3+35c^4d^2e+21c^2de^2+5e^3\right)\left(1-c^2x^2\right)}{35c^7\sqrt{-1+cx}\sqrt{1+cx}}-\frac{be\left(35c^4d^2+42c^2de+15e^2\right)\left(\right.}{105c^7\sqrt{-1+cx}\sqrt{1+c}}$$

Mathematica [A] time = 0.271764, size = 193, normalized size = 0.67

$$a\left(d^2ex^3+d^3x+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^6\left(1225d^2ex^2+3675d^3+441de^2x^4+75e^3x^6\right)+2c^4e\left(1225d^2+\right.\right.}{3675c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 2
94*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 +
 75*e^3*x^6)))/(3675*c^7) + (b*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*
e^3*x^6)*ArcCosh[c*x])/35
```

Maple [A] time = 0.012, size = 235, normalized size = 0.8

$$\frac{1}{c}\left(\frac{a}{c^6}\left(\frac{e^3c^7x^7}{7}+\frac{3\,c^7de^2x^5}{5}+c^7d^2ex^3+xc^7d^3\right)+\frac{b}{c^6}\left(\frac{\mathrm{arccosh}\,(cx)\,e^3c^7x^7}{7}+\frac{3\,\mathrm{arccosh}\,(cx)\,de^2c^7x^5}{5}+\mathrm{arccosh}\,(cx)\,c^7d^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(a/c^6*(1/7*e^3*c^7*x^7+3/5*c^7*d*e^2*x^5+c^7*d^2*e*x^3+x*c^7*d^3)+b/c^
6*(1/7*arccosh(c*x)*e^3*c^7*x^7+3/5*arccosh(c*x)*d*e^2*c^7*x^5+arccosh(c*x)
*c^7*d^2*e*x^3+arccosh(c*x)*c^7*x*d^3-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(7
5*c^6*e^3*x^6+441*c^6*d*e^2*x^4+1225*c^6*d^2*e*x^2+90*c^4*e^3*x^4+3675*c^6*
d^3+588*c^4*d*e^2*x^2+2450*c^4*d^2*e+120*c^2*e^3*x^2+1176*c^2*d*e^2+240*e^3
)))
```

Maxima [A] time = 1.13003, size = 387, normalized size = 1.35

$$\frac{1}{7}ae^3x^7+\frac{3}{5}ade^2x^5+ad^2ex^3+\frac{1}{3}\left(3x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^2e+\frac{1}{25}\left(15x^5\,\mathrm{arcosh}\,(cx)-\left(\frac{3\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arccosh(c*x) - c
*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2*e + 1/25*(15*
x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c
^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arccosh(c*x) - (5*
sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 -
1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*x*arccosh(c*
x) - sqrt(c^2*x^2 - 1))*b*d^3/c
```

Fricas [A] time = 2.35321, size = 559, normalized size = 1.95

$$\frac{525\,ac^7e^3x^7+2205\,ac^7de^2x^5+3675\,ac^7d^2ex^3+3675\,ac^7d^3x+105\left(5\,bc^7e^3x^7+21\,bc^7de^2x^5+35\,bc^7d^2ex^3+35\,bc^7d^3x\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 + 3
675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^2*
e*x^3 + 35*b*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*e^3*x^6 +
```

```
3675*b*c^6*d^3 + 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^2 +
10*b*c^4*e^3)*x^4 + 240*b*e^3 + (1225*b*c^6*d^2*e + 588*b*c^4*d*e^2 + 120*b
*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] time = 13.3653, size = 396, normalized size = 1.38

$$\begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x\operatorname{acosh}(cx) + bd^2ex^3\operatorname{acosh}(cx) + \frac{3bde^2x^5\operatorname{acosh}(cx)}{5} + \frac{be^3x^7\operatorname{acosh}(cx)}{7} - \frac{bd^3\sqrt{c^2x^2-1}}{c} - \\ \left(a + \frac{i\pi b}{2}\right)\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 + b
*d**3*x*acosh(c*x) + b*d**2*e*x**3*acosh(c*x) + 3*b*d*e**2*x**5*acosh(c*x)/
5 + b*e**3*x**7*acosh(c*x)/7 - b*d**3*sqrt(c**2*x**2 - 1)/c - b*d**2*e*x**2
*sqrt(c**2*x**2 - 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(c**2*x**2 - 1)/(25*c) - b
*e**3*x**6*sqrt(c**2*x**2 - 1)/(49*c) - 2*b*d**2*e*sqrt(c**2*x**2 - 1)/(3*c
**3) - 4*b*d*e**2*x**2*sqrt(c**2*x**2 - 1)/(25*c**3) - 6*b*e**3*x**4*sqrt(c
**2*x**2 - 1)/(245*c**3) - 8*b*d*e**2*sqrt(c**2*x**2 - 1)/(25*c**5) - 8*b*e
**3*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**3*sqrt(c**2*x**2 - 1)/(24
5*c**7), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5
 + e**3*x**7/7), True))
```

Giac [A] time = 1.37013, size = 396, normalized size = 1.38

$$\left(x\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{\sqrt{c^2x^2-1}}{c}\right)bd^3 + ad^3x + \frac{1}{245}\left(35\,ax^7 + \left(35\,x^7\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{5\left(c^2x^2-1\right)^{\frac{7}{2}} + 21\left(c^2x\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d^3 + a*d^3*x + 1/
245*(35*a*x^7 + (35*x^7*log(c*x + sqrt(c^2*x^2 - 1)) - (5*(c^2*x^2 - 1)^(7/
2) + 21*(c^2*x^2 - 1)^(5/2) + 35*(c^2*x^2 - 1)^(3/2) + 35*sqrt(c^2*x^2 - 1)
)/c^7)*b)*e^3 + 1/25*(15*a*d*x^5 + (15*x^5*log(c*x + sqrt(c^2*x^2 - 1)) - (
```

```
3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))/c^5)
*b*d)*e^2 + 1/3*(3*a*d^2*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*
x^2 - 1)^(3/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*d^2)*e
```

3.484 $\int \frac{(d+ex^2)^3(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=509

$$-\frac{ibd^3\sqrt{1-c^2x^2}\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+d^3\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh\right.$$

```
[Out] (-3*b*d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (9*b*d*e^2*x*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(32*c^3) - (5*b*e^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96
*c^5) - (3*b*d*e^2*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c) - (5*b*e^3*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(144*c^3) - (b*e^3*x^5*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(36*c) - (3*b*d^2*e*ArcCosh[c*x])/(4*c^2) - (9*b*d*e^2*ArcCosh[c*x]
)/(32*c^4) - (5*b*e^3*ArcCosh[c*x])/(96*c^6) + (3*d^2*e*x^2*(a + b*ArcCosh[
c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcCosh[c*x]))/4 + (e^3*x^6*(a + b*ArcCosh[c
*x]))/6 - ((I/2)*b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[
c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*(a + b*ArcCosh[c*x])*Log[x] -
(b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 - ((I/2)*b*d^3*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.09202, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {266, 43, 5790, 12, 6742, 90, 52, 100, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{ibd^3\sqrt{1-c^2x^2}\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+d^3\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (-3*b*d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (9*b*d*e^2*x*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(32*c^3) - (5*b*e^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96
*c^5) - (3*b*d*e^2*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c) - (5*b*e^3*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(144*c^3) - (b*e^3*x^5*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(36*c) - (3*b*d^2*e*ArcCosh[c*x])/(4*c^2) - (9*b*d*e^2*ArcCosh[c*x]
)/(32*c^4) - (5*b*e^3*ArcCosh[c*x])/(96*c^6) + (3*d^2*e*x^2*(a + b*ArcCosh[
```

```
c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcCosh[c*x]))/4 + (e^3*x^6*(a + b*ArcCosh[c
*x]))/6 - ((I/2)*b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[
c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*(a + b*ArcCosh[c*x])*Log[x] -
(b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 - ((I/2)*b*d^3*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
```

```
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
```

```
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\cosh^{-1}(cx\right.$$

$$= \frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\cosh^{-1}(cx\right.$$

$$= \frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\cosh^{-1}(cx\right.$$

$$= \frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\cosh^{-1}(cx\right.$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{3bde^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{be^3x^5\sqrt{-1+cx}\sqrt{1+cx}}{36c}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{3bde^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{be^3x^5\sqrt{-1+cx}\sqrt{1+cx}}{36c}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{3bde^2x^3\sqrt{-1+cx}\sqrt{1+c}}{16c}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{3bde^2x^3\sqrt{-1+cx}\sqrt{1+c}}{16c}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{5be^3x\sqrt{-1+cx}\sqrt{1+cx}}{96c^5}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{5be^3x\sqrt{-1+cx}\sqrt{1+cx}}{96c^5}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{5be^3x\sqrt{-1+cx}\sqrt{1+cx}}{96c^5}$$

Mathematica [A] time = 0.743615, size = 314, normalized size = 0.62

$$\frac{1}{2}bd^3\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\right)-\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)+\frac{3}{2}ad^2ex^2+ad^3\log(x)+\frac{3}{4}ad$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (3*a*d^2*e*x^2)/2 + (3*a*d*e^2*x^4)/4 + (a*e^3*x^6)/6 - (3*b*d^2*e*(c*x*Sqr
t[-1 + c*x]*Sqrt[1 + c*x] - 2*c^2*x^2*ArcCosh[c*x] + 2*ArcTanh[Sqrt[(-1 + c
*x)/(1 + c*x)]]))/(4*c^2) - (3*b*d*e^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3
 + 2*c^2*x^2) - 8*c^4*x^4*ArcCosh[c*x] + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x
)]]))/(32*c^4) - (b*e^3*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(15 + 10*c^2*x^2
+ 8*c^4*x^4) - 48*c^6*x^6*ArcCosh[c*x] + 30*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*
x)]]))/(288*c^6) + a*d^3*Log[x] + (b*d^3*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Lo
g[1 + E^(-2*ArcCosh[c*x])]) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

Maple [A] time = 0.139, size = 351, normalized size = 0.7

$$\frac{ae^3x^6}{6}+\frac{3\,ade^2x^4}{4}+\frac{3\,ad^2ex^2}{2}+d^3a\ln(cx)+\frac{bd^3}{2}\mathrm{polylog}\left(2,-\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)^2\right)-\frac{d^3b\,(\mathrm{arccosh}\,(cx))^2}{2}+\frac{b\mathrm{arcco}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/6*a*e^3*x^6+3/4*a*d*e^2*x^4+3/2*a*d^2*e*x^2+d^3*a*ln(c*x)+1/2*d^3*b*polyl
og(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/2*d^3*b*arccosh(c*x)^2+1/6*b*a
rccosh(c*x)*e^3*x^6+3/4*b*arccosh(c*x)*d*e^2*x^4+3/2*b*arccosh(c*x)*d^2*e*x
^2-3/16*b*d*e^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-9/32*b*d*e^2*x*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/c^3-3/4*b*d^2*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/36*b*e^
3*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-5/144*b*e^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/c^3-5/96*b*e^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5+d^3*b*arccosh(c*x)*ln
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)-9/32*b*d*e^2*arccosh(c*x)/c^4-3/4*b
*d^2*e*arccosh(c*x)/c^2-5/96*b*e^3*arccosh(c*x)/c^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\,ae^3x^6+\frac{3}{4}\,ade^2x^4+\frac{3}{2}\,ad^2ex^2+ad^3\log(x)+\int be^3x^5\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+3\,bde^2x^3\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/6*a*e^3*x^6 + 3/4*a*d*e^2*x^4 + 3/2*a*d^2*e*x^2 + a*d^3*log(x) + integrat
e(b*e^3*x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d*e^2*x^3*log(c*x
```

```
+ sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d^2*e*x*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1)) + b*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^3x^6 + 3\,ade^2x^4 + 3\,ad^2ex^2 + ad^3 + \left(be^3x^6 + 3\,bde^2x^4 + 3\,bd^2ex^2 + bd^3\right)\text{arcosh}\,(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{acosh}\,(cx))\left(d + ex^2\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^3 (b\,\text{arcosh}\,(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x, x)
```

3.485 $\int \frac{(d+ex^2)^3(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=265

$$3d^2ex\left(a+b\cosh^{-1}(cx)\right)-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+de^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{be\left(1-c^2x^2\right)\left(\right.}{5c^5\sqrt{c}}$$

```
[Out] (b*e*(15*c^4*d^2 + 5*c^2*d*e + e^2)*(1 - c^2*x^2))/(5*c^5*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (b*e^2*(5*c^2*d + 2*e)*(1 - c^2*x^2)^2)/(15*c^5*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*e^3*(1 - c^2*x^2)^3)/(25*c^5*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (d^3*(a + b*ArcCosh[c*x]))/x + 3*d^2*e*x*(a + b*ArcCosh[c*x]) + d*
e^2*x^3*(a + b*ArcCosh[c*x]) + (e^3*x^5*(a + b*ArcCosh[c*x]))/5 + (b*c*d^3*
Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Rubi [A] time = 0.417573, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5790, 1610, 1799, 1620, 63, 205}

$$3d^2ex\left(a+b\cosh^{-1}(cx)\right)-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+de^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{be\left(1-c^2x^2\right)\left(\right.}{5c^5\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (b*e*(15*c^4*d^2 + 5*c^2*d*e + e^2)*(1 - c^2*x^2))/(5*c^5*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (b*e^2*(5*c^2*d + 2*e)*(1 - c^2*x^2)^2)/(15*c^5*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*e^3*(1 - c^2*x^2)^3)/(25*c^5*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (d^3*(a + b*ArcCosh[c*x]))/x + 3*d^2*e*x*(a + b*ArcCosh[c*x]) + d*
e^2*x^3*(a + b*ArcCosh[c*x]) + (e^3*x^5*(a + b*ArcCosh[c*x]))/5 + (b*c*d^3*
Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} + 3d^2ex\left(a+b\cosh^{-1}(cx)\right) + de^2x^3\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{5}$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} + 3d^2ex\left(a+b\cosh^{-1}(cx)\right) + de^2x^3\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{5}$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} + 3d^2ex\left(a+b\cosh^{-1}(cx)\right) + de^2x^3\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{5}$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} + 3d^2ex\left(a+b\cosh^{-1}(cx)\right) + de^2x^3\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{5}$$

$$= \frac{be\left(15c^4d^2+5c^2de+e^2\right)\left(1-c^2x^2\right)}{5c^5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^2\left(5c^2d+2e\right)\left(1-c^2x^2\right)^2}{15c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be^3\left(1-c^2\right.}{25c^5\sqrt{-1+cx}}$$

$$= \frac{be\left(15c^4d^2+5c^2de+e^2\right)\left(1-c^2x^2\right)}{5c^5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^2\left(5c^2d+2e\right)\left(1-c^2x^2\right)^2}{15c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be^3\left(1-c^2\right.}{25c^5\sqrt{-1+cx}}$$

$$= \frac{be\left(15c^4d^2+5c^2de+e^2\right)\left(1-c^2x^2\right)}{5c^5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^2\left(5c^2d+2e\right)\left(1-c^2x^2\right)^2}{15c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be^3\left(1-c^2\right.}{25c^5\sqrt{-1+cx}}$$

Mathematica [A] time = 0.337214, size = 182, normalized size = 0.69

$$3ad^2ex - \frac{ad^3}{x} + ade^2x^3 + \frac{1}{5}ae^3x^5 - \frac{be\sqrt{cx-1}\sqrt{cx+1}\left(c^4\left(225d^2+25dex^2+3e^2x^4\right)+2c^2e\left(25d+2ex^2\right)+8e^2\right)}{75c^5} + \frac{b\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*d^3)/x) + 3*a*d^2*e*x + a*d*e^2*x^3 + (a*e^3*x^5)/5 - (b*e*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(8*e^2 + 2*c^2*e*(25*d + 2*e*x^2) + c^4*(225*d^2 + 25*d*e
*x^2 + 3*e^2*x^4)))/(75*c^5) + (b*(-5*d^3 + 15*d^2*e*x^2 + 5*d*e^2*x^4 + e^
3*x^6)*ArcCosh[c*x])/(5*x) - b*c*d^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)]
```

Maple [A] time = 0.019, size = 282, normalized size = 1.1

$$\frac{ae^3x^5}{5} + ade^2x^3 + 3\,ad^2ex - \frac{ad^3}{x} + \frac{b\operatorname{arccosh}(cx)\,e^3x^5}{5} + b\operatorname{arccosh}(cx)\,de^2x^3 + 3\,b\operatorname{arccosh}(cx)\,d^2ex - \frac{bd^3\operatorname{arccosh}(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] 1/5*a*e^3*x^5+a*d*e^2*x^3+3*a*d^2*e*x-a*d^3/x+1/5*b*arccosh(c*x)*e^3*x^5+b*
arccosh(c*x)*d*e^2*x^3+3*b*arccosh(c*x)*d^2*e*x-b*arccosh(c*x)*d^3/x-c*b*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*arctan(1/(c^2*x^2-1)^(1/2))
-1/25*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4*e^3-1/3*b/c*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*x^2*d*e^2-3*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2*e-4/75*b/c^3*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*x^2*e^3-2/3*b/c^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e^2-8/
75*b/c^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3
```

Maxima [A] time = 1.68462, size = 302, normalized size = 1.14

$$\frac{1}{5}\,ae^3x^5 + ade^2x^3 - \left(c\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right) + \frac{\operatorname{arcosh}(cx)}{x}\right)bd^3 + \frac{1}{3}\left(3\,x^3\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bde^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a*e^3*x^5 + a*d*e^2*x^3 - (c*arcsin(1/(sqrt(c^2)*abs(x))) + arccosh(c*x
)/x)*b*d^3 + 1/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqr
t(c^2*x^2 - 1)/c^4))*b*d*e^2 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2
- 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*
e^3 + 3*a*d^2*e*x + 3*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^2*e/c - a*
d^3/x
```

Fricas [A] time = 3.12706, size = 716, normalized size = 2.7

$$15\,ac^5e^3x^6 + 75\,ac^5de^2x^4 + 150\,bc^6d^3x\arctan\left(-cx + \sqrt{c^2x^2-1}\right) + 225\,ac^5d^2ex^2 - 75\,ac^5d^3 + 15\left(5\,bc^5d^3 - 15\,bc^5d^2e -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/75*(15*a*c^5*e^3*x^6 + 75*a*c^5*d*e^2*x^4 + 150*b*c^6*d^3*x*arctan(-c*x +
 sqrt(c^2*x^2 - 1)) + 225*a*c^5*d^2*e*x^2 - 75*a*c^5*d^3 + 15*(5*b*c^5*d^3
- 15*b*c^5*d^2*e - 5*b*c^5*d*e^2 - b*c^5*e^3)*x*log(-c*x + sqrt(c^2*x^2 - 1
)) + 15*(b*c^5*e^3*x^6 + 5*b*c^5*d*e^2*x^4 + 15*b*c^5*d^2*e*x^2 - 5*b*c^5*d
^3 + (5*b*c^5*d^3 - 15*b*c^5*d^2*e - 5*b*c^5*d*e^2 - b*c^5*e^3)*x)*log(c*x
+ sqrt(c^2*x^2 - 1)) - (3*b*c^4*e^3*x^5 + (25*b*c^4*d*e^2 + 4*b*c^2*e^3)*x^
3 + (225*b*c^4*d^2*e + 50*b*c^2*d*e^2 + 8*b*e^3)*x)*sqrt(c^2*x^2 - 1))/(c^5
*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acosh}(cx)) \left(d + ex^2\right)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^3 (b \operatorname{arcosh}(cx) + a)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^2, x)
```

3.486 $\int \frac{(d+ex^2)^3 (a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=476

$$-\frac{3ibd^2e\sqrt{1-c^2x^2}\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+3d^2e\log(x)\left(a+b\cosh^{-1}(cx)\right)-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cos\right.$$

```
[Out] -(b*c*d^3*(1 - c^2*x^2))/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*e^2*(8*c
^2*d + e)*x*(1 - c^2*x^2))/(32*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*x
^3*(1 - c^2*x^2))/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh
[c*x]))/(2*x^2) + (3*d*e^2*x^2*(a + b*ArcCosh[c*x]))/2 + (e^3*x^4*(a + b*Ar
cCosh[c*x]))/4 - (((3*I)/2)*b*d^2*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (3*b*e^2*(8*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTan
h[(c*x)/Sqrt[-1 + c^2*x^2]])/(32*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*d
^2*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) + 3*d^2*e*(a + b*ArcCosh[c*x])*Log[x] - (3*b*d^2*e*S
qrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((3*
I)/2)*b*d^2*e*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.76026, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.905, Rules used = {266, 43, 5790, 12, 6742, 1610, 1807, 1584, 459, 321, 217, 206, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{3ibd^2e\sqrt{1-c^2x^2}\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+3d^2e\log(x)\left(a+b\cosh^{-1}(cx)\right)-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cos\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```


```
[Out] -(b*c*d^3*(1 - c^2*x^2))/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*e^2*(8*c
^2*d + e)*x*(1 - c^2*x^2))/(32*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*x
^3*(1 - c^2*x^2))/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh
[c*x]))/(2*x^2) + (3*d*e^2*x^2*(a + b*ArcCosh[c*x]))/2 + (e^3*x^4*(a + b*Ar
cCosh[c*x]))/4 - (((3*I)/2)*b*d^2*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (3*b*e^2*(8*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTan
h[(c*x)/Sqrt[-1 + c^2*x^2]])/(32*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*d
```

```
^2*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) + 3*d^2*e*(a + b*ArcCosh[c*x])*Log[x] - (3*b*d^2*e*S
qrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((3*
I)/2)*b*d^2*e*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
```

d, 0] && EqQ[m, n] && !IntegerQ[m]

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^3x^3\left(1-c^2x^2\right)}{16c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3be^2\left(8c^2d+e\right)x\left(1-c^2x^2\right)}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^3x^3\left(1-c^2x^2\right)}{16c\sqrt{-1+cx}\sqrt{1+cx}}-$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3be^2\left(8c^2d+e\right)x\left(1-c^2x^2\right)}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^3x^3\left(1-c^2x^2\right)}{16c\sqrt{-1+cx}\sqrt{1+cx}}-$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3be^2\left(8c^2d+e\right)x\left(1-c^2x^2\right)}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^3x^3\left(1-c^2x^2\right)}{16c\sqrt{-1+cx}\sqrt{1+cx}}-$$

Mathematica [A] time = 0.657731, size = 267, normalized size = 0.56

$$\frac{1}{4}\left(-6bd^2e\,\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+12ad^2e\log(x)-\frac{2ad^3}{x^2}+6ade^2x^2+ae^3x^4-\frac{3bde^2\left(cx\sqrt{cx-1}\sqrt{cx+1}+2\tanh\right.}{c^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^3)/x^2 + 6*a*d*e^2*x^2 + a*e^3*x^4 + (2*b*d^3*(c*x*Sqrt[-1 + c*x]*
Sqrt[1 + c*x] - ArcCosh[c*x]))/x^2 + 6*b*d*e^2*x^2*ArcCosh[c*x] + b*e^3*x^4
*ArcCosh[c*x] - (3*b*d*e^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sq
rt[(-1 + c*x)/(1 + c*x)]]))/c^2 - (b*e^3*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(3 + 2*c^2*x^2) + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(8*c^4) + 6*b*d^2
*e*ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) + 12*a*d^2*
e*Log[x] - 6*b*d^2*e*PolyLog[2, -E^(-2*ArcCosh[c*x])])/4
```

Maple [A] time = 0.165, size = 296, normalized size = 0.6

$$\frac{ae^3x^4}{4} + \frac{3\,ax^2de^2}{2} + 3\,ad^2e\ln(cx) - \frac{d^3a}{2\,x^2} + \frac{b\text{arccosh}\,(cx)\,e^3x^4}{4} - \frac{bd^3\text{arccosh}\,(cx)}{2\,x^2} + \frac{3\,b\text{arccosh}\,(cx)\,x^2de^2}{2} + \frac{3\,bd^2e}{2}\text{poly}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] 1/4*a*e^3*x^4+3/2*a*x^2*d*e^2+3*a*d^2*e*ln(c*x)-1/2*d^3*a/x^2+1/4*b*arccosh
(c*x)*e^3*x^4-1/2*d^3*b*arccosh(c*x)/x^2+3/2*b*arccosh(c*x)*x^2*d*e^2+3/2*b
*d^2*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/16/c*b*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*e^3*x^3-3/32/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x-1/2*d
^3*b*c^2+1/2*c*d^3*b/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)+3*b*d^2*e*arccosh(c*x)*l
n((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+1)-3/4/c^2*b*arccosh(c*x)*d*e^2-3/4/c
*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d*e^2-3/2*b*d^2*e*arccosh(c*x)^2-3/32/c^4*
b*arccosh(c*x)*e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,ae^3x^4 + \frac{3}{2}\,ade^2x^2 + \frac{1}{2}\,bd^3\left(\frac{\sqrt{c^2x^2-1}c}{x} - \frac{\text{arcosh}\,(cx)}{x^2}\right) + 3\,ad^2e\log(x) - \frac{ad^3}{2\,x^2} + \int be^3x^3\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + 3/2*a*d*e^2*x^2 + 1/2*b*d^3*(sqrt(c^2*x^2 - 1)*c/x - arccos
h(c*x)/x^2) + 3*a*d^2*e*log(x) - 1/2*a*d^3/x^2 + integrate(b*e^3*x^3*log(c*
```

```
x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d*e^2*x*log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1)) + 3*b*d^2*e*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^3x^6 + 3\,ade^2x^4 + 3\,ad^2ex^2 + ad^3 + \left(be^3x^6 + 3\,bde^2x^4 + 3\,bd^2ex^2 + bd^3\right)\operatorname{arcosh}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(cx))\left(d + ex^2\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^3 (b\operatorname{arcosh}(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^3, x)
```

3.487 $\int \frac{(d+ex^2)^3(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=260

$$-\frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+3de^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{bcd^2\sqrt{c^2x^2-}}{}$$

```
[Out] (b*e^2*(9*c^2*d + e)*(1 - c^2*x^2))/(3*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(b*c*d^3*(1 - c^2*x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^3*(1 -
c^2*x^2)^2)/(9*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh[c*x]
))/(3*x^3) - (3*d^2*e*(a + b*ArcCosh[c*x]))/x + 3*d*e^2*x*(a + b*ArcCosh[c*
x]) + (e^3*x^3*(a + b*ArcCosh[c*x]))/3 + (b*c*d^2*(c^2*d + 18*e)*Sqrt[-1 +
c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.462042, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {270, 5790, 12, 1610, 1799, 1621, 897, 1153, 205}

$$-\frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+3de^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{bcd^2\sqrt{c^2x^2-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```


```
[Out] (b*e^2*(9*c^2*d + e)*(1 - c^2*x^2))/(3*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(b*c*d^3*(1 - c^2*x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^3*(1 -
c^2*x^2)^2)/(9*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh[c*x]
))/(3*x^3) - (3*d^2*e*(a + b*ArcCosh[c*x]))/x + 3*d*e^2*x*(a + b*ArcCosh[c*
x]) + (e^3*x^3*(a + b*ArcCosh[c*x]))/3 + (b*c*d^2*(c^2*d + 18*e)*Sqrt[-1 +
c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex^2)^3 (a+b\cosh^{-1}(cx))}{x^4}\,dx = -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de^2x(a+b\cosh^{-1}(cx)) +$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de^2x(a+b\cosh^{-1}(cx)) +$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de^2x(a+b\cosh^{-1}(cx)) +$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de^2x(a+b\cosh^{-1}(cx)) +$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de$$

$$= \frac{be^2(9c^2d+e)(1-c^2x^2)}{3c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^3(1-c^2x^2)^2}{9c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3(a}{}$$

$$= \frac{be^2(9c^2d+e)(1-c^2x^2)}{3c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^3(1-c^2x^2)^2}{9c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3(a}{}$$

Mathematica [A] time = 0.375422, size = 184, normalized size = 0.71

$$\frac{1}{6}\left(-\frac{18ad^2e}{x} - \frac{2ad^3}{x^3} + 18ade^2x + 2ae^3x^3 - \frac{b\sqrt{cx-1}\sqrt{cx+1}\left(-3c^4d^3 + 2c^2e^2x^2(27d+ex^2) + 4e^3x^2\right)}{3c^3x^2} - bcd^2(c^2d+18e)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] ((-2*a*d^3)/x^3 - (18*a*d^2*e)/x + 18*a*d*e^2*x + 2*a*e^3*x^3 - (b*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(-3*c^4*d^3 + 4*e^3*x^2 + 2*c^2*e^2*x^2*(27*d + e*x^2)
))/(3*c^3*x^2) + (2*b*(-d^3 - 9*d^2*e*x^2 + 9*d*e^2*x^4 + e^3*x^6)*ArcCosh[
c*x])/x^3 - b*c*d^2*(c^2*d + 18*e)*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])]
)/6
```

Maple [A] time = 0.021, size = 278, normalized size = 1.1

$$\frac{ae^3x^3}{3} + 3\,axde^2 - 3\,\frac{ad^2e}{x} - \frac{d^3a}{3\,x^3} + \frac{b\mathrm{arccosh}\,(cx)\,e^3x^3}{3} + 3\,b\mathrm{arccosh}\,(cx)\,xde^2 - 3\,\frac{bd^2\mathrm{arccosh}\,(cx)\,e}{x} - \frac{bd^3\mathrm{arccosh}\,(cx)}{3\,x^3} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] 1/3*a*e^3*x^3+3*a*x*d*e^2-3*a*d^2*e/x-1/3*d^3*a/x^3+1/3*b*arccosh(c*x)*e^3*
x^3+3*b*arccosh(c*x)*x*d*e^2-3*b*arccosh(c*x)*d^2*e/x-1/3*d^3*b*arccosh(c*x
)/x^3-1/6*c^3*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/
(c^2*x^2-1)^(1/2))-3*c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arct
an(1/(c^2*x^2-1)^(1/2))*d^2*e+1/6*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2-1
/9/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*e^3-3/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2
)*d*e^2-2/9/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3
```

Maxima [A] time = 1.68571, size = 271, normalized size = 1.04

$$\frac{1}{3}\,ae^3x^3 - \frac{1}{6}\left(\left(\left(c^2\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right) - \frac{\sqrt{c^2x^2-1}}{x^2}\right)c + \frac{2\,\mathrm{arcosh}\,(cx)}{x^3}\right)bd^3 - 3\left(c\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right) + \frac{\mathrm{arcosh}\,(cx)}{x}\right)bd^2e + \frac{1}{9}\left(3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^3*x^3 - 1/6*((c^2*arcsin(1/(sqrt(c^2)*abs(x))) - sqrt(c^2*x^2 - 1)/
x^2)*c + 2*arccosh(c*x)/x^3)*b*d^3 - 3*(c*arcsin(1/(sqrt(c^2)*abs(x))) + ar
ccosh(c*x)/x)*b*d^2*e + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/
c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*e^3 + 3*a*d*e^2*x + 3*(c*x*arccosh(c*x) -
 sqrt(c^2*x^2 - 1))*b*d*e^2/c - 3*a*d^2*e/x - 1/3*a*d^3/x^3
```

Fricas [A] time = 4.27927, size = 683, normalized size = 2.63

$$\frac{6\,ac^3e^3x^6 + 54\,ac^3de^2x^4 - 54\,ac^3d^2ex^2 - 6\,ac^3d^3 + 6\left(bc^6d^3 + 18\,bc^4d^2e\right)x^3\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + 6\left(bc^3d^3 + 9\,bc^3\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^3*e^3*x^6 + 54*a*c^3*d*e^2*x^4 - 54*a*c^3*d^2*e*x^2 - 6*a*c^3*d
^3 + 6*(b*c^6*d^3 + 18*b*c^4*d^2*e)*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) +
6*(b*c^3*d^3 + 9*b*c^3*d^2*e - 9*b*c^3*d*e^2 - b*c^3*e^3)*x^3*log(-c*x + sq
rt(c^2*x^2 - 1)) + 6*(b*c^3*e^3*x^6 + 9*b*c^3*d*e^2*x^4 - 9*b*c^3*d^2*e*x^2
 - b*c^3*d^3 + (b*c^3*d^3 + 9*b*c^3*d^2*e - 9*b*c^3*d*e^2 - b*c^3*e^3)*x^3)
*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^2*e^3*x^5 - 3*b*c^4*d^3*x + 2*(27*b*
c^2*d*e^2 + 2*b*e^3)*x^3)*sqrt(c^2*x^2 - 1))/(c^3*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(cx))\left(d + ex^2\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^3(b\operatorname{arcosh}(cx) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^4, x)
```

3.488 $\int (d+ex^2)^4 (a+b\cosh^{-1}(cx))\,dx$

Optimal. Leaf size=395

$$\frac{6}{5}d^2e^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{7}de^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^4x^9\left(\right.$$

```
[Out] (b*(315*c^8*d^4 + 420*c^6*d^3*e + 378*c^4*d^2*e^2 + 180*c^2*d*e^3 + 35*e^4)
*(1 - c^2*x^2))/(315*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*e*(105*c^6*d^
3 + 189*c^4*d^2*e + 135*c^2*d*e^2 + 35*e^3)*(1 - c^2*x^2)^2)/(945*c^9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (2*b*e^2*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*(1 -
 c^2*x^2)^3)/(525*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*e^3*(9*c^2*d + 7
*e)*(1 - c^2*x^2)^4)/(441*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^4*(1 - c
^2*x^2)^5)/(81*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^4*x*(a + b*ArcCosh[c*x
]) + (4*d^3*e*x^3*(a + b*ArcCosh[c*x]))/3 + (6*d^2*e^2*x^5*(a + b*ArcCosh[c
*x]))/5 + (4*d*e^3*x^7*(a + b*ArcCosh[c*x]))/7 + (e^4*x^9*(a + b*ArcCosh[c*
x]))/9
```

Rubi [A] time = 0.47495, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {194, 5705, 12, 1610, 1799, 1850}

$$\frac{6}{5}d^2e^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{7}de^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^4x^9\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (b*(315*c^8*d^4 + 420*c^6*d^3*e + 378*c^4*d^2*e^2 + 180*c^2*d*e^3 + 35*e^4)
*(1 - c^2*x^2))/(315*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*e*(105*c^6*d^
3 + 189*c^4*d^2*e + 135*c^2*d*e^2 + 35*e^3)*(1 - c^2*x^2)^2)/(945*c^9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (2*b*e^2*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*(1 -
 c^2*x^2)^3)/(525*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*e^3*(9*c^2*d + 7
*e)*(1 - c^2*x^2)^4)/(441*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^4*(1 - c
^2*x^2)^5)/(81*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^4*x*(a + b*ArcCosh[c*x
]) + (4*d^3*e*x^3*(a + b*ArcCosh[c*x]))/3 + (6*d^2*e^2*x^5*(a + b*ArcCosh[c
*x]))/5 + (4*d*e^3*x^7*(a + b*ArcCosh[c*x]))/7 + (e^4*x^9*(a + b*ArcCosh[c*
x]))/9
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(d+ex^2\right)^4 \left(a+b\cosh^{-1}(cx)\right)\,dx = d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh^{-1}(cx)\right)$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh^{-1}(cx)\right)$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh^{-1}(cx)\right)$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh^{-1}(cx)\right)$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh^{-1}(cx)\right)$$

$$= \frac{b\left(315c^8d^4+420c^6d^3e+378c^4d^2e^2+180c^2de^3+35e^4\right)\left(1-c^2x^2\right)}{315c^9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{4be\left(105c^6d^3\right.}{}$$

Mathematica [A] time = 0.390997, size = 265, normalized size = 0.67

$$\frac{315ax\left(378d^2e^2x^4+420d^3ex^2+315d^4+180de^3x^6+35e^4x^8\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^8\left(23814d^2e^2x^4+44100d^3ex^2+99225d^4+8100de^3x^6+1225e^4x\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (315*a*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^
4*x^8) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4480*e^4 + 320*c^2*e^3*(81*d + 7*
e*x^2) + 48*c^4*e^2*(1323*d^2 + 270*d*e*x^2 + 35*e^2*x^4) + 8*c^6*e*(11025*
d^3 + 3969*d^2*e*x^2 + 1215*d*e^2*x^4 + 175*e^3*x^6) + c^8*(99225*d^4 + 441
00*d^3*e*x^2 + 23814*d^2*e^2*x^4 + 8100*d*e^3*x^6 + 1225*e^4*x^8)))/c^9 + 3
15*b*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^4*
x^8)*ArcCosh[c*x])/99225
```

Maple [A] time = 0.013, size = 331, normalized size = 0.8

$$\frac{1}{c}\left(\frac{a}{c^8}\left(\frac{e^4c^9x^9}{9}+\frac{4\,c^9de^3x^7}{7}+\frac{6\,c^9d^2e^2x^5}{5}+\frac{4\,c^9d^3ex^3}{3}+c^9d^4x\right)+\frac{b}{c^8}\left(\frac{\operatorname{arccosh}(cx)\,e^4c^9x^9}{9}+\frac{4\operatorname{arccosh}(cx)\,c^9de^3x^7}{7}+\frac{6\,\mathrm{ar}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(a/c^8*(1/9*e^4*c^9*x^9+4/7*c^9*d*e^3*x^7+6/5*c^9*d^2*e^2*x^5+4/3*c^9*d
^3*e*x^3+c^9*d^4*x)+b/c^8*(1/9*arccosh(c*x)*e^4*c^9*x^9+4/7*arccosh(c*x)*c^
9*d*e^3*x^7+6/5*arccosh(c*x)*c^9*d^2*e^2*x^5+4/3*arccosh(c*x)*c^9*d^3*e*x^3
+arccosh(c*x)*c^9*d^4*x-1/99225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*e^4*x
^8+8100*c^8*d*e^3*x^6+23814*c^8*d^2*e^2*x^4+1400*c^6*e^4*x^6+44100*c^8*d^3*
e*x^2+9720*c^6*d*e^3*x^4+99225*c^8*d^4+31752*c^6*d^2*e^2*x^2+1680*c^4*e^4*x
^4+88200*c^6*d^3*e+12960*c^4*d*e^3*x^2+63504*c^4*d^2*e^2+2240*c^2*e^4*x^2+2
5920*c^2*d*e^3+4480*e^4)))
```

Maxima [A] time = 1.11512, size = 560, normalized size = 1.42

$$\frac{1}{9}ae^4x^9+\frac{4}{7}ade^3x^7+\frac{6}{5}ad^2e^2x^5+\frac{4}{3}ad^3ex^3+\frac{4}{9}\left(3\,x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^3e+\frac{2}{25}\left(15\,x^5\operatorname{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*e^4*x^9 + 4/7*a*d*e^3*x^7 + 6/5*a*d^2*e^2*x^5 + 4/3*a*d^3*e*x^3 + 4/9
*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c
^4))*b*d^3*e + 2/25*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4
*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^2*e^2 + 4/245*
(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x
^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d*e^3
 + 1/2835*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c
^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x
^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*e^4 + a*d^4*x + (c*x*arccosh(c*x)
 - sqrt(c^2*x^2 - 1))*b*d^4/c
```

Fricas [A] time = 2.69421, size = 803, normalized size = 2.03

$$\frac{11025\,ac^9e^4x^9+56700\,ac^9de^3x^7+119070\,ac^9d^2e^2x^5+132300\,ac^9d^3ex^3+99225\,ac^9d^4x+315\left(35\,bc^9e^4x^9+180\,bc^9de^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*e^4*x^9 + 56700*a*c^9*d*e^3*x^7 + 119070*a*c^9*d^2*e^2
*x^5 + 132300*a*c^9*d^3*e*x^3 + 99225*a*c^9*d^4*x + 315*(35*b*c^9*e^4*x^9 +
 180*b*c^9*d*e^3*x^7 + 378*b*c^9*d^2*e^2*x^5 + 420*b*c^9*d^3*e*x^3 + 315*b*
c^9*d^4*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (1225*b*c^8*e^4*x^8 + 99225*b*c^8
*d^4 + 88200*b*c^6*d^3*e + 63504*b*c^4*d^2*e^2 + 25920*b*c^2*d*e^3 + 100*(8
1*b*c^8*d*e^3 + 14*b*c^6*e^4)*x^6 + 4480*b*e^4 + 6*(3969*b*c^8*d^2*e^2 + 16
20*b*c^6*d*e^3 + 280*b*c^4*e^4)*x^4 + 4*(11025*b*c^8*d^3*e + 7938*b*c^6*d^2
*e^2 + 3240*b*c^4*d*e^3 + 560*b*c^2*e^4)*x^2)*sqrt(c^2*x^2 - 1))/c^9
```

Sympy [A] time = 41.6936, size = 600, normalized size = 1.52

$$\begin{cases} ad^4x + \frac{4ad^3ex^3}{3} + \frac{6ad^2e^2x^5}{5} + \frac{4ade^3x^7}{7} + \frac{ae^4x^9}{9} + bd^4x\operatorname{acosh}(cx) + \frac{4bd^3ex^3\operatorname{acosh}(cx)}{3} + \frac{6bd^2e^2x^5\operatorname{acosh}(cx)}{5} + \frac{4bde^3x^7\operatorname{acosh}(cx)}{7} + \frac{be^4x^9 \\ \left(a + \frac{i\pi b}{2}\right)\left(d^4x + \frac{4d^3ex^3}{3} + \frac{6d^2e^2x^5}{5} + \frac{4de^3x^7}{7} + \frac{e^4x^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**4*x + 4*a*d**3*e*x**3/3 + 6*a*d**2*e**2*x**5/5 + 4*a*d*e**3
*x**7/7 + a*e**4*x**9/9 + b*d**4*x*acosh(c*x) + 4*b*d**3*e*x**3*acosh(c*x)/
3 + 6*b*d**2*e**2*x**5*acosh(c*x)/5 + 4*b*d*e**3*x**7*acosh(c*x)/7 + b*e**4
*x**9*acosh(c*x)/9 - b*d**4*sqrt(c**2*x**2 - 1)/c - 4*b*d**3*e*x**2*sqrt(c*
*2*x**2 - 1)/(9*c) - 6*b*d**2*e**2*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 4*b*d*
e**3*x**6*sqrt(c**2*x**2 - 1)/(49*c) - b*e**4*x**8*sqrt(c**2*x**2 - 1)/(81*
c) - 8*b*d**3*e*sqrt(c**2*x**2 - 1)/(9*c**3) - 8*b*d**2*e**2*x**2*sqrt(c**2
*x**2 - 1)/(25*c**3) - 24*b*d*e**3*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 8*
b*e**4*x**6*sqrt(c**2*x**2 - 1)/(567*c**3) - 16*b*d**2*e**2*sqrt(c**2*x**2
- 1)/(25*c**5) - 32*b*d*e**3*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**
4*x**4*sqrt(c**2*x**2 - 1)/(945*c**5) - 64*b*d*e**3*sqrt(c**2*x**2 - 1)/(24
5*c**7) - 64*b*e**4*x**2*sqrt(c**2*x**2 - 1)/(2835*c**7) - 128*b*e**4*sqrt(
c**2*x**2 - 1)/(2835*c**9), Ne(c, 0)), ((a + I*pi*b/2)*(d**4*x + 4*d**3*e*x
**3/3 + 6*d**2*e**2*x**5/5 + 4*d*e**3*x**7/7 + e**4*x**9/9), True))
```

Giac [A] time = 1.4616, size = 547, normalized size = 1.38

$$\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{\sqrt{c^2x^2-1}}{c}\right)bd^4+ad^4x+\frac{1}{2835}\left(315\,ax^9+\left(315\,x^9\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{35\left(c^2x^2-1\right)^{\frac{9}{2}}+18\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d^4 + a*d^4*x + 1/
2835*(315*a*x^9 + (315*x^9*log(c*x + sqrt(c^2*x^2 - 1)) - (35*(c^2*x^2 - 1)
^(9/2) + 180*(c^2*x^2 - 1)^(7/2) + 378*(c^2*x^2 - 1)^(5/2) + 420*(c^2*x^2 -
 1)^(3/2) + 315*sqrt(c^2*x^2 - 1))/c^9)*b)*e^4 + 4/245*(35*a*d*x^7 + (35*x^
7*log(c*x + sqrt(c^2*x^2 - 1)) - (5*(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^
(5/2) + 35*(c^2*x^2 - 1)^(3/2) + 35*sqrt(c^2*x^2 - 1))/c^7)*b*d)*e^3 + 2/25
*(15*a*d^2*x^5 + (15*x^5*log(c*x + sqrt(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1)^(5
/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))/c^5)*b*d^2)*e^2 + 4/9*
(3*a*d^3*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3/2) +
 3*sqrt(c^2*x^2 - 1))/c^3)*b*d^3)*e
```

3.489 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=627

$$-\frac{b(-d)^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{5/2}}+\frac{b(-d)^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{5/2}}-\frac{b(-d)^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^{5/2}}$$

```
[Out] -((a*d*x)/e^2) + (b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e^2) - (2*b*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/(9*c^3*e) - (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c
*e) - (b*d*x*ArcCosh[c*x])/e^2 + (x^3*(a + b*ArcCosh[c*x]))/(3*e) + ((-d)^(
3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e])])/(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 +
(Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) +
((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-
d] + Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCosh[c*x])*L
og[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(5
/2)) - (b*(-d)^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e]))])/(2*e^(5/2)) + (b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - (b*(-d)^(3/2)*P
olyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(
2*e^(5/2)) + (b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
+ Sqrt[-(c^2*d) - e])])/(2*e^(5/2))
```

Rubi [A] time = 1.05156, antiderivative size = 627, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5792, 5654, 74, 5662, 100, 12, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b(-d)^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{5/2}}+\frac{b(-d)^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{5/2}}-\frac{b(-d)^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -((a*d*x)/e^2) + (b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e^2) - (2*b*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/(9*c^3*e) - (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c
*e) - (b*d*x*ArcCosh[c*x])/e^2 + (x^3*(a + b*ArcCosh[c*x]))/(3*e) + ((-d)^(
3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e])])/(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 +
```

```
(Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) +
((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-
d] + Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCosh[c*x])*L
og[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(5
/2)) - (b*(-d)^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e]))])/(2*e^(5/2)) + (b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - (b*(-d)^(3/2)*P
olyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(
2*e^(5/2)) + (b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
+ Sqrt[-(c^2*d) - e])])/(2*e^(5/2))
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx=\int\left(-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^2}+\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{e}+\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)}\right)dx$$

$$=-\frac{d\int\left(a+b\cosh^{-1}(cx)\right)dx}{e^2}+\frac{d^2\int\frac{a+b\cosh^{-1}(cx)}{d+ex^2}dx}{e^2}+\frac{\int x^2\left(a+b\cosh^{-1}(cx)\right)dx}{e}$$

$$=-\frac{adx}{e^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{(bd)\int\cosh^{-1}(cx)\,dx}{e^2}+\frac{d^2\int\left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+}{2d(}\right.}{e^2}$$

$$=-\frac{adx}{e^2}-\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9ce}-\frac{bdx\cosh^{-1}(cx)}{e^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{(-d)^{3/2}\int\frac{a+b}{\sqrt{}}}{2e^{}}$$

$$=-\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}-\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9ce}-\frac{bdx\cosh^{-1}(cx)}{e^2}+\frac{x^3\left(a+b\cosh\right.}{3e}$$

$$=-\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}-\frac{2b\sqrt{-1+cx}\sqrt{1+cx}}{9c^3e}-\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9ce}-\frac{bdx\cosh^{}}{e^2}$$

$$=-\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}-\frac{2b\sqrt{-1+cx}\sqrt{1+cx}}{9c^3e}-\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9ce}-\frac{bdx\cosh^{}}{e^2}$$

$$=-\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}-\frac{2b\sqrt{-1+cx}\sqrt{1+cx}}{9c^3e}-\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9ce}-\frac{bdx\cosh^{}}{e^2}$$

$$=-\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}-\frac{2b\sqrt{-1+cx}\sqrt{1+cx}}{9c^3e}-\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9ce}-\frac{bdx\cosh^{}}{e^2}$$

Mathematica [C] time = 1.38052, size = 524, normalized size = 0.84

$$b\left(-id^{3/2}\left(2\text{PolyLog}\left(2,\frac{i\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2d+e}-c\sqrt{d}}\right)+2\text{PolyLog}\left(2,-\frac{i\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2d+e}+c\sqrt{d}}\right)+\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{i\sqrt{e}e^{\cosh^{-1}}}{c\sqrt{d}-\sqrt{c^2d}}\right.\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -((a*d*x)/e^2) + (a*x^3)/(3*e) + (a*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^
(5/2) + (b*((4*d*Sqrt[e]*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - c*x*ArcCos
h[c*x]))/c - (4*e^(3/2)*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c^2*x^2) - 3*c^3
*x^3*ArcCosh[c*x]))/(9*c^3) - I*d^(3/2)*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(L
og[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] - Sqrt[c^2*d + e])] + Log[1 +
(I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])) + 2*PolyLog[2,
(I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + 2*PolyLog[2,
 ((-I)*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])]) + I*d^(3/2)*
(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*S
qrt[d]) + Sqrt[c^2*d + e])] + Log[1 - (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d]
 + Sqrt[c^2*d + e])])) + 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d]
 - Sqrt[c^2*d + e])] + 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] +
 Sqrt[c^2*d + e])])))/(4*e^(5/2))
```

Maple [C] time = 3.506, size = 364, normalized size = 0.6

$$\frac{x^3 a}{3e} - \frac{adx}{e^2} + \frac{ad^2}{e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} - \frac{bdx\operatorname{arccosh}(cx)}{e^2} + \frac{bd}{ce^2}\sqrt{cx-1}\sqrt{cx+1} + \frac{cbd^2}{2e^2}\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] 1/3*a*x^3/e-a*d*x/e^2+a*d^2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-b*d*x*a
rccosh(c*x)/e^2+b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e^2+1/2*c*b*d^2/e^2*sum(_
R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4
+(4*c^2*d+2*e)*_Z^2+e))-1/2*c*b*d^2/e^2*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arcc
osh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-
1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/9*
b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e+1/3*b/e*arccosh(c*x)*x^3-2/9*b*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c^3/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4 \operatorname{arcosh}(cx) + ax^4}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b \operatorname{acosh}(cx))}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^4}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d), x)
```

3.490 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=521

$$-\frac{bd\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}-\frac{bd\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}-\frac{bd\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^2}-\frac{bd\text{PolyLog}\left(2,\right.}{2e}$$

```
[Out] -(b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*e) - (b*ArcCosh[c*x])/(4*c^2*e) +
(x^2*(a + b*ArcCosh[c*x]))/(2*e) + (d*(a + b*ArcCosh[c*x])^2)/(2*b*e^2) - (
d*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/(2*e^2) - (d*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) - (d*(a + b*ArcCosh[c*
x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2
*e^2) - (d*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] + Sqrt[-(c^2*d) - e])])/(2*e^2) - (b*d*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^2) - (b*d*PolyLog[2, (Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) - (b*d*PolyLog
[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^2)
 - (b*d*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e
])])/(2*e^2)
```

Rubi [A] time = 0.911154, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5792, 5662, 90, 52, 5800, 5562, 2190, 2279, 2391}

$$-\frac{bd\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}-\frac{bd\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}-\frac{bd\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^2}-\frac{bd\text{PolyLog}\left(2,\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -(b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*e) - (b*ArcCosh[c*x])/(4*c^2*e) +
(x^2*(a + b*ArcCosh[c*x]))/(2*e) + (d*(a + b*ArcCosh[c*x])^2)/(2*b*e^2) - (
d*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/(2*e^2) - (d*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) - (d*(a + b*ArcCosh[c*
x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2
*e^2) - (d*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
```

```
] + Sqrt[-(c^2*d) - e])])/(2*e^2) - (b*d*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^2) - (b*d*PolyLog[2, (Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) - (b*d*PolyLog
[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^2)
 - (b*d*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e
])])/(2*e^2)
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int \left(\frac{x\left(a+b\cosh^{-1}(cx)\right)}{e} - \frac{dx\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int x\left(a+b\cosh^{-1}(cx)\right)dx}{e} - \frac{d\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{e}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} - \frac{(bc)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2e} - \frac{d\int \left(-\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{e}$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2e^{3/2}} - \frac{d\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,d}{2e^{3/2}}$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\,\text{Subst}\left(\int \frac{(a+bx)\sinh(x}{c\sqrt{-d}-\sqrt{e}\cosh}\right.}{2e^{3/}}$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} +$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} -$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} -$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} -$$

Mathematica [A] time = 0.521905, size = 512, normalized size = 0.98

$$-\frac{2bc^2d\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)+2bc^2d\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-c\sqrt{-d}}\right)+2bc^2d\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)+2bc^2d\text{I}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -(-2*a*c^2*e*x^2 + b*c*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*c^2*e*x^2*Arc
Cosh[c*x] - 2*b*c^2*d*ArcCosh[c*x]^2 + 2*b*e*ArcTanh[Sqrt[(-1 + c*x)/(1 + c
*x)]] + 2*b*c^2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
```

```
 - Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh
[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*ArcCosh[c*x]*Log[1
 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*
ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
 e])] + 2*a*c^2*d*Log[d + e*x^2] + 2*b*c^2*d*PolyLog[2, (Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*PolyLog[2, (Sqrt[e]*E^
ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*PolyLog[2,
-((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))] + 2*b*c^2*d*
PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*
c^2*e^2)
```

Maple [C] time = 0.29, size = 2912, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] 1/2*a/e*x^2-1/4*b*arccosh(c*x)/c^2/e-1/2*a*d/e^2*ln(c^2*e*x^2+c^2*d)+b*arcc
osh(c*x)^2*d/e^2+1/2*b*arccosh(c*x)/e*x^2-1/2*b*d/e^2*sum((_R1^2*e+4*c^2*d+
2*e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1
/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z
^4+(4*c^2*d+2*e)*_Z^2+e))-1/4*b/e^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d-2*c^4*b/e^4*d^3*ln(1-e*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*
arccosh(c*x)-2*c^2*b/e^3*d^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2
*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)+1/2*b/e/(c^2*d+e)*ln(1-e*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))
*arccosh(c*x)*d+b/e^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-
2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d*(c^2*d*(c^2*d+e))^(1/2)-3/4*b/
e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(
c^2*d*(c^2*d+e))^(1/2)-e))*d*(c^2*d*(c^2*d+e))^(1/2)+b*(c^2*d*(c^2*d+e))^(1
/2)*d/e^2/(c^2*d+e)*arccosh(c*x)^2+1/4*b*(c^2*d*(c^2*d+e))^(1/2)*d/e^2/(c^2
*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)-e))+1/8/c^2*b*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*d+e)*polylog(2,
e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e
))-2*c^6*b*d^4/e^4/(c^2*d+e)*arccosh(c*x)^2-4*c^4*b*d^3/e^3/(c^2*d+e)*arcco
sh(c*x)^2+c^6*b*d^4/e^4/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))+5/4*c^2*b/e^2/(c^2*d+e)*poly
log(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(
1/2)-e))*d^2-5/2*c^2*b/e^2/(c^2*d+e)*arccosh(c*x)^2*d^2+2*c^4*b*d^3/e^3/(c^
```

2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(
c^2*d+e))^(1/2)-e))+c^2*b/e^4*d^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)-2*c^
2*b/e^4*d^2*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)-1/8/c^2*b/e/(c^2*d+e)*po
lylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))
^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)-2*c^4*b*d^3/e^4/(c^2*d+e)*ln(1-e*(c*x+(c
x-1)^(1/2)(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccos
h(c*x)*(c^2*d*(c^2*d+e))^(1/2)-3*c^2*b/e^3*d^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*
x)*(c^2*d*(c^2*d+e))^(1/2)+1/2*b*(c^2*d*(c^2*d+e))^(1/2)*d/e^2/(c^2*d+e)*ar
ccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)-e))-3/2*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d*(c^2*d*(c^2*d
+e))^(1/2)+5/2*c^2*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d^2+4*c^4*b/e^3*d^3
/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c
^2*d+e))^(1/2)-e))*arccosh(c*x)+2*c^6*b*d^4/e^4/(c^2*d+e)*ln(1-e*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c
*x)+2*c^2*b/e^4*d^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*
(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)-1/4/c^2*b/
e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(
c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)+1/4/c^2*b*(c^2*d*(
c^2*d+e))^(1/2)/e/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))+2*c^4*b*d^3/e^4/(c^2*d+e)*
arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)+3*c^2*b*d^2/e^3/(c^2*d+e)*arccosh(c*
x)^2*(c^2*d*(c^2*d+e))^(1/2)-c^4*b*d^3/e^4/(c^2*d+e)*polylog(2,e*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c
^2*d+e))^(1/2)-3/2*c^2*b*d^2/e^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(
1/2)+2*c^2*b/e^3*d^2*arccosh(c*x)^2-1/2*b/e/(c^2*d+e)*arccosh(c*x)^2*d+1/2*
b/e^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c
^2*d+e))^(1/2)-e))*d*(c^2*d*(c^2*d+e))^(1/2)+1/4*b/e/(c^2*d+e)*polylog(2,e*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))
*d-b/e^3*arccosh(c*x)^2*d*(c^2*d*(c^2*d+e))^(1/2)-1/2*b/e^2*ln(1-e*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh
(c*x)*d-c^2*b/e^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d
-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d^2-c^4*b/e^4*d^3*polylog(2,e*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))+2*c^4*b/e^4*d
^3*arccosh(c*x)^2-1/4*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a\left(\frac{x^2}{e}-\frac{d\log\left(ex^2+d\right)}{e^2}\right)+b\int\frac{x^3\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(x^2/e - d*log(e*x^2 + d)/e^2) + b*integrate(x^3*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/(e*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^3\operatorname{arcosh}(cx)+ax^3}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccosh(c*x) + a*x^3)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\left(a+b\operatorname{acosh}(cx)\right)}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\operatorname{arcosh}(cx)+a\right)x^3}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^3/(e*x^2 + d), x)
```

3.491 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=544

$$-\frac{b\sqrt{-d}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^{3/2}}+\frac{b\sqrt{-}}{}$$

```
[Out] (a*x)/e - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e) + (b*x*ArcCosh[c*x])/e + (
Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
- Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1
+ (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2))
+ (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-
d] + Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(3/2
)) - (b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2
, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^(3/2
)) + (b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c
^2*d) - e])])/(2*e^(3/2))
```

Rubi [A] time = 0.901976, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5792, 5654, 74, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\sqrt{-d}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^{3/2}}+\frac{b\sqrt{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```



```
[Out] (a*x)/e - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e) + (b*x*ArcCosh[c*x])/e + (
Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
- Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1
+ (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2))
+ (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-
d] + Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(3/2
```

```
)) - (b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2
, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^(3/2
)) + (b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c
^2*d) - e])])/(2*e^(3/2))
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
```

```
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx=\int\left(\frac{a+b\cosh^{-1}(cx)}{e}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int\left(a+b\cosh^{-1}(cx)\right)dx}{e}-\frac{d\int\frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e}$$

$$=\frac{ax}{e}+\frac{b\int\cosh^{-1}(cx)\,dx}{e}-\frac{d\int\left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{e}$$

$$=\frac{ax}{e}+\frac{bx\cosh^{-1}(cx)}{e}-\frac{(bc)\int\frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e}-\frac{\sqrt{-d}\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2e}-\frac{\sqrt{-d}\int\frac{a+b\cosh^{-1}}{\sqrt{-d}+\sqrt{e}}}{2e}$$

$$=\frac{ax}{e}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}-\frac{\sqrt{-d}\,\text{Subst}\left(\int\frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx,x,\cosh^{-1}(\right.}{2e}$$

$$=\frac{ax}{e}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}-\frac{\sqrt{-d}\,\text{Subst}\left(\int\frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}e^x}\,dx,x,\cosh\right.}{2e}$$

$$=\frac{ax}{e}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-}}\right.}{2e^{3/2}}$$

$$=\frac{ax}{e}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-}}\right.}{2e^{3/2}}$$

$$=\frac{ax}{e}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-}}\right.}{2e^{3/2}}$$

Mathematica [C] time = 0.698351, size = 457, normalized size = 0.84

$$\frac{ib\left(-c\sqrt{d}\left(-2\text{PolyLog}\left(2,\frac{i\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2d+e}-c\sqrt{d}}\right)-2\text{PolyLog}\left(2,-\frac{i\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2d+e}+c\sqrt{d}}\right)+\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-2\left(\log\left(1+\frac{i\sqrt{e}e^{\cosh^{-1}}}{c\sqrt{d}-\sqrt{c^2d}}\right.\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] (4*a*c*Sqrt[e]*x - 4*a*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + I*b*((4*I)*S
qrt[e]*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - c*x*ArcCosh[c*x]) - c*Sqrt[d
]*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
```

```
rt[d] - Sqrt[c^2*d + e])] + Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] +
 Sqrt[c^2*d + e])])) - 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]
) + Sqrt[c^2*d + e])] - 2*PolyLog[2, ((-I)*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
d] + Sqrt[c^2*d + e])]) + c*Sqrt[d]*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1
+ (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + Log[1 - (I
*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])) - 2*PolyLog[2, (I
*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] - Sqrt[c^2*d + e])] - 2*PolyLog[2, (I*S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])))/(4*c*e^(3/2))
```

Maple [C] time = 0.617, size = 284, normalized size = 0.5

$$\frac{ax}{e}-\frac{ad}{e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{bx\mathrm{arccosh}\,(cx)}{e}-\frac{b}{ce}\sqrt{cx-1}\sqrt{cx+1}-\frac{cbd}{2e}\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1}{\_R1^2e+2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] a*x/e-a*d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+b*x*arccosh(c*x)/e-b*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c/e-1/2*c*b*d/e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(
c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(
1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/2*c*b*
d/e*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(
c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=Ro
otOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2\,\text{arcosh}\left(cx\right)+ax^2}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\left(a+b\,\text{acosh}\left(cx\right)\right)}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\text{arcosh}\left(cx\right)+a\right)x^2}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d), x)
```

3.492 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=449

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{co}\cdots}}{\sqrt{c^2(-d\cdots}}\right.}{2e}$$

```
[Out] -(a + b*ArcCosh[c*x])^2/(2*b*e) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCosh[c
*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(
2*e) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
 Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^Arc
Cosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e) + (b*PolyLog[2, -((Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e) + (b*PolyLo
g[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (
b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))]
)/(2*e) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d
) - e])])/(2*e)
```

Rubi [A] time = 0.737832, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5792, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{co}\cdots}}{\sqrt{c^2(-d\cdots}}\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -(a + b*ArcCosh[c*x])^2/(2*b*e) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCosh[c
*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(
2*e) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
 Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^Arc
Cosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e) + (b*PolyLog[2, -((Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e) + (b*PolyLo
g[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (
b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))]
```

```
)/(2*e) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d
) - e])])/(2*e)
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int\left(-\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{e}}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{e}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}+\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{e}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}e^x}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{e}}-\frac{\text{Subst}\left(\int \frac{e^x(a}{c\sqrt{-d}+\sqrt{-}}\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log}{2e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log}{2e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log}{2e}$$

Mathematica [A] time = 0.129602, size = 447, normalized size = 1.

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{c}}{\sqrt{c^2(-d}}\right.}{2e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -(b*ArcCosh[c*x]^2)/(2*e) + (b*ArcCosh[c*x]*Log[1 - (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*ArcCosh[c*x]*Log[1 + (Sqrt
[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*ArcCosh[
c*x]*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(
2*e) + (b*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(2*e) + (a*Log[d + e*x^2])/(2*e) + (b*PolyLog[2, -((Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e) + (b*PolyLog[2
, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*P
```

```
olyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(
2*e) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
 e])])/(2*e)
```

Maple [C] time = 0.194, size = 2805, normalized size = 6.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] c^4*b*d^2/e^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2
*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)+3/2*c^2*b/e^2/
(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*
d*(c^2*d+e))^(1/2)-e))*d*(c^2*d*(c^2*d+e))^(1/2)-3*c^2*b*(c^2*d*(c^2*d+e))^
(1/2)*d/e^2/(c^2*d+e)*arccosh(c*x)^2-4*c^4*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(
c*x)*d^2-5/2*c^2*b/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(
-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d-2*c^6*b/e^3*d^3/(c^2*
d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e
))^(1/2)-e))*arccosh(c*x)-2*c^2*b/e^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d*(c^2*d*(c^2*d+
e))^(1/2)-1/4/c^2*b*(c^2*d*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arccosh(c*x)*ln(1-e
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)
)+1/4/c^2*b/d/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*
d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)-2*c^2*
b*arccosh(c*x)^2*d/e^2+c^4*b/e^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d^2+c^2*b/e^2*polylog(2,e*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d-
2*c^4*b/e^3*d^2*arccosh(c*x)^2-1/4*b*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*d+e)*po
lylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))
^(1/2)-e))+3/4*b/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^
2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)-b/e^2*ln(
1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)-b*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*
d+e)*arccosh(c*x)^2+2*c^4*b/e^3*d^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(
c^2*d+e))^(1/2)+3*c^2*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d*(c^2*d*(c^2*d+
e))^(1/2)+b/e^2*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)-1/2*b/e^2*polylog(2,
e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e
```

```
))*(c^2*d*(c^2*d+e))^(1/2)+1/2*b/e*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)-1/2*b/(c^2*d+e)*ln(
1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
-e))*arccosh(c*x)+1/2*b/(c^2*d+e)*arccosh(c*x)^2-1/4*b/(c^2*d+e)*polylog(2,
e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e
))+1/4*b/e*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2
*d*(c^2*d+e))^(1/2)-e))+1/2*b/e*sum((_R1^2*e+4*c^2*d+2*e)/(_R1^2*e+2*c^2*d+
e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c
*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+
e))-b/e*arccosh(c*x)^2+1/2*a/e*ln(c^2*e*x^2+c^2*d)+3/2*b/e/(c^2*d+e)*ln(1-e
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e)
)*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)+2*c^2*b/e^3*arccosh(c*x)^2*d*(c^2*d*
(c^2*d+e))^(1/2)-1/8/c^2*b*(c^2*d*(c^2*d+e))^(1/2)/d/(c^2*d+e)*polylog(2,e*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))
-5/4*c^2*b/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*
c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d+4*c^4*b/e^2/(c^2*d+e)*arccosh(c*x)^2*
d^2+5/2*c^2*b/e/(c^2*d+e)*arccosh(c*x)^2*d+2*c^6*b*d^3/e^3/(c^2*d+e)*arccos
h(c*x)^2-c^2*b/e^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*
d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*b/e^3*d^2*l
n(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/
2)-e))*arccosh(c*x)-c^6*b*d^3/e^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))-2*c^4*b/e^2/(c^2*d
+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2
*d+e))^(1/2)-e))*d^2+1/8/c^2*b/d/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(
1/2)-2*c^4*b*d^2/e^3/(c^2*d+e)*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^2
*b/e^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d
+e))^(1/2)-e))*arccosh(c*x)*d-1/2*b*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*d+e)*arc
cosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^
2*d+e))^(1/2)-e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int \frac{x \log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)}{ex^2 + d}\, dx + \frac{a \log\left(ex^2 + d\right)}{2\, e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + 1/2*
a*log(e*x^2 + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx \operatorname{arcosh}(cx) + ax}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccosh(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(a + b \operatorname{acosh}(cx))}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d), x)
```

3.493 $\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=501

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-}\right.}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqr
t[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 + (Sq
rt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
- e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rubi [A] time = 0.734363, antiderivative size = 501, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-}\right.}{2\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqr
t[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 + (Sq
rt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
- e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*S
```

```
qrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}}-\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx,x,\cosh^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}+\sqrt{e}\cosh(x)}\,dx,x,\cosh^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}e^x}\,dx,x,\cosh^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}+\sqrt{-c^2d-e}-\sqrt{e}e^x}\,dx,x,\cosh^{-1}(cx\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b}{}$$

Mathematica [A] time = 0.320305, size = 397, normalized size = 0.79

$$\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)-b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-c\sqrt{-d}}\right)-b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)+b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}}}{\sqrt{c^2(-d)-e+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e])]) + (a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(
-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + (a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] - (a + b*ArcCosh[c*x])
*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] + b*Po
```

```
lyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])] - b*Po
lyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] - b
*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))]
+ b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(2*Sqrt[-d]*Sqrt[e])
```

Maple [C] time = 0.065, size = 232, normalized size = 0.5

$$a\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{bc}{2}\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1}{\_R1^2e+2c^2d+e}\left(\mathrm{arccosh}\,(cx)\ln\left(\frac{1}{\_R1}\left(\_R1-cx-\sqrt{cx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] a/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/2*c*b*sum(_R1/(_R1^2*e+2*c^2*d+e)*(
arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(
c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-
1/2*c*b*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/
2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R
1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arcosh}\,(cx)+a}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{acosh}(cx)}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(cx) + a}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d), x)
```

3.494 $\int \frac{a+b\cosh^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal. Leaf size=489

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{}}{\sqrt{c^2}}\right.}{2d}$$

```
[Out] (a + b*ArcCosh[c*x])^2/(b*d) + ((a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[
c*x])])/d - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(2*d) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d) - ((a + b*ArcCosh
[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(2*d) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*d) - (b*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(2*d)
 - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e
]))])/(2*d) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) - e])])/(2*d) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
+ Sqrt[-(c^2*d) - e]))])/(2*d) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*
Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d)
```

Rubi [A] time = 0.92487, antiderivative size = 472, normalized size of antiderivative = 0.97, number of steps used = 25, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5792, 5660, 3718, 2190, 2279, 2391, 5800, 5562}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{}}{\sqrt{c^2}}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/(2*d) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d) - ((a + b*ArcCosh[c*x])*Log[
1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d) - ((
a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c
^2*d) - e])])/(2*d) + ((a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/d
- (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]
))])/(2*d) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^
```

```
2*d) - e])])/(2*d) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
 Sqrt[-(c^2*d) - e]))])/(2*d) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d) + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])
/(2*d)
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{dx} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d}$$

$$= \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(cx)\right)}{d} - \frac{e\int \left(-\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{d}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{d} + \frac{\sqrt{e}\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2d} - \frac{\sqrt{e}\int$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)}{d} - \frac{b\,\text{Subst}\left(\int \log\left(1+e^{2x}\right)dx,\right.}{d}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh^{-1}(cx)}\right)}{2d} + \frac{\sqrt{e}\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{(a+}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{(a+}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{(a+}{}$$

Mathematica [C] time = 0.775332, size = 418, normalized size = 0.85

$$\frac{b\left(\text{PolyLog}\left(2,-\frac{\left(-2\sqrt{c^2d\left(c^2d+e\right)}+2c^2d+e\right)e^{-2\cosh^{-1}(cx)}}{e}\right)+\text{PolyLog}\left(2,-\frac{\left(2\sqrt{c^2d\left(c^2d+e\right)}+2c^2d+e\right)e^{-2\cosh^{-1}(cx)}}{e}\right)-2\text{PolyLog}\left(2,-e^{-2\cos}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (4*a*Log[x] - 2*a*Log[d + e*x^2] + b*((-4*I)*ArcSin[Sqrt[1 + (c^2*d)/e]]*Ar
cTanh[(c*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[c^2*d*(c^2*d + e)]*x
```

```
)] + 4*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - 2*ArcCosh[c*x]*Log[1 + (
2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] + (2*I)*Ar
cSin[Sqrt[1 + (c^2*d)/e]]*Log[1 + (2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])
/(e*E^(2*ArcCosh[c*x]))] - 2*ArcCosh[c*x]*Log[1 + (2*c^2*d + e + 2*Sqrt[c^2
*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] - (2*I)*ArcSin[Sqrt[1 + (c^2*d)/e]
]*Log[1 + (2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))]
 - 2*PolyLog[2, -E^(-2*ArcCosh[c*x])] + PolyLog[2, -((2*c^2*d + e - 2*Sqrt[
c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x])))] + PolyLog[2, -((2*c^2*d + e +
2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x])))]))/(4*d)
```

Maple [C] time = 0.159, size = 393, normalized size = 0.8

$$-\frac{a\ln\left(x^2c^2e+c^2d\right)}{2d}+\frac{a\ln(cx)}{d}-\frac{be}{4d}\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1^2+1}{\_R1^2e+2c^2d+e}\left(\operatorname{arccosh}(cx)\ln\left(\frac{1}{\_R1}\left(\_R1-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(e*x^2+d),x)
```

```
[Out] -1/2*a/d*ln(c^2*e*x^2+c^2*d)+a/d*ln(c*x)-1/4*b*e/d*sum((_R1^2+1)/(_R1^2*e+2
*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog
((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e
)*_Z^2+e))+b/d*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d*a
rccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d*dilog(1+I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)-1/4*b/d*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a\left(\frac{\log\left(ex^2+d\right)}{d}-\frac{2\log(x)}{d}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{ex^3+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1))/(e*x^3 + d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{ex^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\text{acosh}\,(cx)}{x\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(ex^2+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x), x)
```

3.495 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal. Leaf size=543

$$-\frac{b\sqrt{e}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\text{Poly}}{}$$

```
[Out] -((a + b*ArcCosh[c*x])/(d*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCosh[c*x])*
Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d
)^(3/2)) + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCosh
[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (b*S
qrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2))
```

Rubi [A] time = 0.906472, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5792, 5662, 92, 205, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\sqrt{e}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCosh[c*x])*
Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d
)^(3/2)) + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCosh
[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
```

```
/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (b*S
qrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2))
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
```

```
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2(d+ex^2)}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{dx^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)}\right) dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{(bc)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d} - \frac{e\int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right) dx}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{\left(bc^2\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right)}{d} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2(-d)^{3/2}} - \frac{e\int$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} - \frac{e\,\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} - \frac{e\,\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}e^x}\,dx, x, \cosh^{-1}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx}}{c\sqrt{-d}-\sqrt{-c^2d}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx}}{c\sqrt{-d}-\sqrt{-c^2d}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx}}{c\sqrt{-d}-\sqrt{-c^2d}}$$

Mathematica [A] time = 1.39716, size = 549, normalized size = 1.01

$$\frac{1}{2}\left(\frac{b\sqrt{e}\text{PolyLog}\left(2, \frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{(-d)^{3/2}} + \frac{bd\sqrt{e}\text{PolyLog}\left(2, \frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-c\sqrt{-d}}\right)}{(-d)^{5/2}} + \frac{bd\sqrt{e}\text{PolyLog}\left(2, -\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{(-d)^{5/2}} + \frac{b\sqrt{e}\text{Pol}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] ((-2*(a + b*ArcCosh[c*x]))/(d*x) + (2*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1
 + c^2*x^2]])/(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d*Sqrt[e]*(a + b*ArcCosh[
c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/
(-d)^(5/2) + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])
/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])])/(-d)^(3/2) + (Sqrt[e]*(a + b*ArcCos
h[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(-d)^(3/2) + (d*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (b*Sqrt[e]*PolyLog[2,
 (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(-d)^(3/2) +
(b*d*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^
2*d) - e])])/(-d)^(5/2) + (b*d*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(-d)^(5/2) + (b*Sqrt[e]*PolyLog[2, (
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(3/2))/2
```

Maple [C] time = 0.774, size = 329, normalized size = 0.6

$$-\frac{a}{dx}-\frac{ae}{d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{b\mathrm{arccosh}\,(cx)}{dx}-\frac{be}{8\,cd^2}\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4\,c^2d+2\,e\right)\_Z^2+e\right)}\frac{4\_R1^2c^2d+\_R1^2e+e}{\_R1\,\left(\_R1^2e+2\,c^2d+e\right)}\left(\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(e*x^2+d),x)
```

```
[Out] -a/d/x-a*e/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-b*arccosh(c*x)/d/x-1/8*b/c
/d^2*e*sum((4*_R1^2*c^2*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*
ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/8*b/c/d^2*e
*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-
(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+2*c*b/d*arctan(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx) + a}{ex^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\text{acosh}\,(cx)}{x^2\,(d + ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**2*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arcosh}\,(cx) + a}{(ex^2 + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^2), x)
```

3.496 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=550

$$\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}+\frac{be\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}+\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^2}+\frac{be\text{PolyLog}\left(2,\right.}{2d}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x) - (a + b*ArcCosh[c*x])/(2*d*x^2)
 - (e*(a + b*ArcCosh[c*x])^2)/(b*d^2) - (e*(a + b*ArcCosh[c*x])*Log[1 + E^(
-2*ArcCosh[c*x])])/d^2 + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) + (e*(a + b*ArcCosh[c*x]
)*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d
^2) + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
+ Sqrt[-(c^2*d) - e])])/(2*d^2) + (e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) + (b*e*PolyLog[
2, -E^(-2*ArcCosh[c*x])])/(2*d^2) + (b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^2) + (b*e*PolyLog[2, (Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) + (b*e*PolyLog
[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^2)
 + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e
])])/(2*d^2)
```

Rubi [A] time = 0.954864, antiderivative size = 531, normalized size of antiderivative = 0.97, number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5792, 5662, 95, 5660, 3718, 2190, 2279, 2391, 5800, 5562}

$$\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}+\frac{be\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}+\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^2}+\frac{be\text{PolyLog}\left(2,\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x) - (a + b*ArcCosh[c*x])/(2*d*x^2)
 + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) - e])])/(2*d^2) + (e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) + (e*(a + b*ArcCos
h[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(2*d^2) + (e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
```

```
t[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) - (e*(a + b*ArcCosh[c*x])*Log[1 + E^(
2*ArcCosh[c*x])])/d^2 + (b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e]))])/(2*d^2) + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) + (b*e*PolyLog[2, -((Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^2) + (b*e*Poly
Log[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2)
 - (b*e*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^2)
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{dx^3} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x} + \frac{e^2x\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right) dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^2} + \frac{e^2\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2d} - \frac{e\,\mathrm{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(cx)\right)}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2} - \frac{(2e)\,\mathrm{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x,\right.}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\right.}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)}{d^2} + \frac{(be)\,\mathrm{Sub}}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{e\left(a+\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{e\left(a+\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d^2} + \frac{e\left(a+\right.}{}$$

Mathematica [C] time = 1.33231, size = 479, normalized size = 0.87

$$\frac{b\left(-e\mathrm{PolyLog}\left(2, -\frac{\left(-2\sqrt{c^2d\left(c^2d+e\right)}+2c^2d+e\right)e^{-2\cosh^{-1}(cx)}}{e}\right) - e\mathrm{PolyLog}\left(2, -\frac{\left(2\sqrt{c^2d\left(c^2d+e\right)}+2c^2d+e\right)e^{-2\cosh^{-1}(cx)}}{e}\right) + 2e\mathrm{PolyLog}\left(2, -e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)),x]
```

```
[Out] ((-2*a*d)/x^2 - 4*a*e*Log[x] + 2*a*e*Log[d + e*x^2] + b*((2*c*d*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x))/x - (2*d*ArcCosh[c*x])/x^2 + (4*I)*e*ArcSin[Sqrt
```

```
[1 + (c^2*d)/e]]*ArcTanh[(c*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[c
^2*d*(c^2*d + e)]*x)] - 4*e*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] + 2*e
*ArcCosh[c*x]*Log[1 + (2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*Arc
Cosh[c*x]))] - (2*I)*e*ArcSin[Sqrt[1 + (c^2*d)/e]]*Log[1 + (2*c^2*d + e - 2
*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] + 2*e*ArcCosh[c*x]*Log[1
+ (2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] + (2*I)
*e*ArcSin[Sqrt[1 + (c^2*d)/e]]*Log[1 + (2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d +
 e)])/(e*E^(2*ArcCosh[c*x]))] + 2*e*PolyLog[2, -E^(-2*ArcCosh[c*x])] - e*Po
lyLog[2, -((2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))
)] - e*PolyLog[2, -((2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCo
sh[c*x]))])]))/(4*d^2)
```

Maple [C] time = 0.202, size = 462, normalized size = 0.8

$$\frac{ae\ln\left(x^2c^2e+c^2d\right)}{2\,d^2}-\frac{a}{2\,dx^2}-\frac{ae\ln(cx)}{d^2}+\frac{bc}{2\,dx}\sqrt{cx-1}\sqrt{cx+1}-\frac{bc^2}{2\,d}-\frac{b\operatorname{arccosh}(cx)}{2\,dx^2}+\frac{be^2}{4\,d^2}\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4\,c^2d+2\,e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(e*x^2+d),x)
```

```
[Out] 1/2*a*e/d^2*ln(c^2*e*x^2+c^2*d)-1/2*a/d/x^2-a/d^2*e*ln(c*x)+1/2*b*c*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d/x-1/2*c^2*b/d-1/2*b/d*arccosh(c*x)/x^2+1/4*b/d^2*e^2
*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=R
ootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-b/d^2*e*arccosh(c*x)*ln(1+I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))-b/d^2*e*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2)))-b/d^2*e*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-b/d^2*e*di
log(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/4*b/d^2*e*sum((_R1^2*e+4*c^2*d
+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^
4+(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{e\log\left(ex^2+d\right)}{d^2}-\frac{2\,e\log(x)}{d^2}-\frac{1}{dx^2}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{ex^5+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + b*integrate(log
(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^5 + d*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{ex^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\text{acosh}\,(cx)}{x^3\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**3*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(ex^2+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^3), x)
```

3.497 $\int \frac{a+b\cosh^{-1}(cx)}{x^4(d+ex^2)}\,dx$

Optimal. Leaf size=624

$$-\frac{be^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{5/2}}+\frac{be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{5/2}}-\frac{be^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2(-d)^{5/2}}+\frac{be^{3/2}\text{Po}}{}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2) - (a + b*ArcCosh[c*x])/(3*d*x^
3) + (e*(a + b*ArcCosh[c*x]))/(d^2*x) + (b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1
 + c*x]])/(6*d) - (b*c*e*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 + (e^(3/
2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) +
 (e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(
5/2)) - (b*e^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e]))])/(2*(-d)^(5/2)) + (b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (b*e^(3/2)*Pol
yLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*
(-d)^(5/2)) + (b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2))
```

Rubi [A] time = 0.981366, antiderivative size = 624, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5792, 5662, 103, 12, 92, 205, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{be^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{5/2}}+\frac{be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{5/2}}-\frac{be^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2(-d)^{5/2}}+\frac{be^{3/2}\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d + e*x^2)),x]
```


```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2) - (a + b*ArcCosh[c*x])/(3*d*x^
3) + (e*(a + b*ArcCosh[c*x]))/(d^2*x) + (b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1
 + c*x]])/(6*d) - (b*c*e*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 + (e^(3/
2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (S
```

```
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) +
 (e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(
5/2)) - (b*e^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e]))])/(2*(-d)^(5/2)) + (b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (b*e^(3/2)*Pol
yLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*
(-d)^(5/2)) + (b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2))
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d+ex^2\right)}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{dx^4}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x^2}+\frac{e^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^4}\,dx}{d}-\frac{e\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{d^2}+\frac{e^2\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{(bc)\int \frac{1}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3d}-\frac{(bce)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{(bc)\int \frac{c^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{6d}-\frac{(bc}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}-\frac{bce\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}-\frac{bce\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{bc^3\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{6d}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{bc^3\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{6d}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{bc^3\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{6d}$$

Mathematica [A] time = 1.48439, size = 641, normalized size = 1.03

$$\frac{1}{6}\left(\frac{3be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{(-d)^{5/2}}-\frac{3be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-c\sqrt{-d}}\right)}{(-d)^{5/2}}-\frac{3be^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{(-d)^{5/2}}+\frac{3be^{3/}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d + e*x^2)),x]
```

```
[Out] ((-2*(a + b*ArcCosh[c*x]))/(d*x^3) + (6*e*(a + b*ArcCosh[c*x]))/(d^2*x) - (
6*b*c*e*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(d^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (b*c*(-1 + c^2*x^2 + c^2*x^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqr
t[-1 + c^2*x^2]]))/(d*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*e^(3/2)*(a + b
*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
 - e])])/(-d)^(5/2) + (3*e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^Ar
cCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (3*e^(3/2)*
(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(
c^2*d) - e])])/(-d)^(5/2) - (3*e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (3*b*e^(
3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]
)/(-d)^(5/2) - (3*b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-
d]) + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) - (3*b*e^(3/2)*PolyLog[2, -((Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(-d)^(5/2) + (3*b*e^(
3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(-d)^(5/2))/6
```

Maple [C] time = 0.811, size = 410, normalized size = 0.7

$$\frac{ae^2}{d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{a}{3\,dx^3}+\frac{ae}{d^2x}+\frac{bc}{6\,dx^2}\sqrt{cx-1}\sqrt{cx+1}+\frac{b\mathrm{arccosh}\,(cx)\,e}{d^2x}-\frac{b\mathrm{arccosh}\,(cx)}{3\,dx^3}-2\,\frac{bce\arctan\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(e*x^2+d),x)
```

```
[Out] a*e^2/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/3*a/d/x^3+a/d^2*e/x+1/6*b*c
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2+b*arccosh(c*x)/d^2*e/x-1/3*b*arccosh(c*x
)/d/x^3-2*c*b/d^2*e*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/8/c*b/d^3*e^2
*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-
(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/3*c^3*b/d*arctan(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))+1/8/c*b/d^3*e^2*sum((4*_R1^2*c^2*d+_R1^2*e+e)/_R
1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2)
)/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+
(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{ex^6+dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x^6 + d*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{acosh}\,(cx)}{x^4\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**4*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{arcosh}\,(cx)+a}{\left(ex^2+d\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^4), x)
```

3.498 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=562

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e}{\sqrt{c^2(-d}}\right.}{2e^2}$$

```
[Out] (d*(a + b*ArcCosh[c*x]))/(2*e^2*(d + e*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*
e^2) - (b*c*Sqrt[d]*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]
*Sqrt[-1 + c^2*x^2])])/(2*e^2*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) - e])])/(2*e^2) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) + ((a + b*ArcCosh[c*x]
)*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e
^2) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*e^2) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c
*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^2) + (b*PolyLog[2, (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) + (b*PolyLog[2, -((Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^2) + (b*Poly
Log[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^2)
```

Rubi [A] time = 0.990744, antiderivative size = 562, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5792, 5788, 519, 377, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{co}}{\sqrt{c^2(-d}}\right.}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (d*(a + b*ArcCosh[c*x]))/(2*e^2*(d + e*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*
e^2) - (b*c*Sqrt[d]*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]
*Sqrt[-1 + c^2*x^2])])/(2*e^2*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) - e])])/(2*e^2) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCo
```

```
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) + ((a + b*ArcCosh[c*x]
)*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e
^2) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*e^2) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c
*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^2) + (b*PolyLog[2, (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) + (b*PolyLog[2, -((Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^2) + (b*Poly
Log[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^2)
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int\left(-\frac{dx\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{e} - \frac{d\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{e}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{(bcd)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}\,dx}{2e^2} + \frac{\int\left(-\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right.}{e}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2e^{3/2}} + \frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2e^{3/2}} - \frac{\left(bcd\sqrt{-1+c^2x^2}\right)\int \frac{}{\sqrt{-1+}}}{2e^2\sqrt{-1+cx}\sqrt{1}}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2e^{3/2}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\sin}{c\sqrt{-d}+\sqrt{e}\,c}\right.}{2}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} - \frac{bc\sqrt{d}\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2e^2\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\text{S}}{}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} - \frac{bc\sqrt{d}\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2e^2\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(}{}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} - \frac{bc\sqrt{d}\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2e^2\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(}{}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} - \frac{bc\sqrt{d}\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2e^2\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(}{}$$

Mathematica [C] time = 1.91497, size = 693, normalized size = 1.23

$$b\left(2\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-ic\sqrt{d}}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-ic\sqrt{d}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+ic\sqrt{d}}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cdots}}{\sqrt{c^2(-\cdots}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((2*a*d)/(d + e*x^2) + 2*a*Log[d + e*x^2] + b*(-2*ArcCosh[c*x]^2 + 2*ArcCos
h[c*x]*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]
) + 2*ArcCosh[c*x]*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(
c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])]) - I*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2
*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e
]) - I*Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqr
t[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/
(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + 2*Po
lyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]
 + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])]))/(4*e^2)
```

Maple [C] time = 0.338, size = 2964, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/8/c^2*b*(c^2*d*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))+1/8/c^2*b/d/e
/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2
*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)+3/4*b/e^2/(c^2*d+e)*polylog
(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2
)-e))*(c^2*d*(c^2*d+e))^(1/2)-b/e^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^
(1/2)-1/2*b/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*
d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)-1/4*b*(c^2*d*(c^2*d+e))^(1/2)/
e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)-e))-b*(c^2*d*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*arccosh(
c*x)^2+1/2*c^2*a/e^2*d/(c^2*e*x^2+c^2*d)+c^2*b/e^3*polylog(2,e*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d+c^4*b/e^4
```

*d^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))-2*c^2*b/e^3*arccosh(c*x)^2*d-2*c^4*b/e^4*d^2*arccosh(c*x)^2+2*c^4*b*d^2/e^4/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)-1/4/c^2*b*(c^2*d*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))+1/4/c^2*b/d/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)+3*c^2*b/e^3/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d*(c^2*d*(c^2*d+e))^(1/2)+1/2*a/e^2*ln(c^2*e*x^2+c^2*d)-b*arccosh(c*x)^2/e^2+1/2*b/e^2*sum((_R1^2*e+4*c^2*d+2*e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/4*b/e^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))+1/2*b/e/(c^2*d+e)*arccosh(c*x)^2-1/4*b/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))+1/2*b/e^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)-1/2*b/e^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)+b/e^3*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)+3/2*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)-5/4*c^2*b*d/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))-c^2*b/e^4*d*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)+4*c^4*b/e^3/(c^2*d+e)*arccosh(c*x)^2*d^2+1/2*c^2*b*arccosh(c*x)/e^2*d/(c^2*e*x^2+c^2*d)+2*c^2*b/e^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d+2*c^4*b/e^4*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d^2+5/2*c^2*b/e^2/(c^2*d+e)*arccosh(c*x)^2*d+2*c^2*b/e^4*d*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)-2*c^4*b/e^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d^2+2*c^6*b*d^3/e^4/(c^2*d+e)*arccosh(c*x)^2-c^6*b*d^3/e^4/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))-1/2*b*(c^2*d*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))+1/2*b*(c^2*d*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*arctanh(1/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e)/(c^4*d^2+c^2*d*e)^(1/2))-4*c^4*b/e^3/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d^2-5/2*c^2*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d-2*c^6*b*d^3/e^4/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)-2*c^2*b/e^4*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d*(c^2*d*(c^2*d+e))^(1/2)-3*c^2*b/e^3/(c^2*d+e)*arccosh(c*x)^2*d*(c^2*d*(c^2*d

```
+e))^(1/2)+3/2*c^2*b/e^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d*(c^2*d*(c^2*d+e))^(1/2)-2
*c^4*b*d^2/e^4/(c^2*d+e)*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)+c^4*b*d^2/e
^4/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c
^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{d}{e^3x^2+de^2}+\frac{\log\left(ex^2+d\right)}{e^2}\right)+b\int\frac{x^3\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{e^2x^4+2\,dex^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bx^3\operatorname{arcosh}(cx)+ax^3}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccosh(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\left(a+b\operatorname{acosh}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^3}{(ex^2 + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^3/(e*x^2 + d)^2, x)
```

3.499 $\int \frac{x(a+b\cosh^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=113

$$\frac{bc\sqrt{c^2x^2-1}\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2\sqrt{d}e\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}-\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}$$

```
[Out] -(a + b*ArcCosh[c*x])/(2*e*(d + e*x^2)) + (b*c*Sqrt[-1 + c^2*x^2]*ArcTanh[(
Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*Sqrt[d]*e*Sqrt[c^2*d +
 e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.0941429, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5788, 519, 377, 208}

$$\frac{bc\sqrt{c^2x^2-1}\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2\sqrt{d}e\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}-\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcCosh[c*x])/(2*e*(d + e*x^2)) + (b*c*Sqrt[-1 + c^2*x^2]*ArcTanh[(
Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*Sqrt[d]*e*Sqrt[c^2*d +
 e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
```

```
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx &= -\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}\,dx}{2e}\\
&= -\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\left(d+ex^2\right)}\,dx}{2e\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{\left(bc\sqrt{-1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{d-\left(c^2d+e\right)x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{2e\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{bc\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2\sqrt{d}e\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

Mathematica [A] time = 0.307635, size = 123, normalized size = 1.09

$$-\frac{\frac{a}{d+ex^2}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\tan^{-1}\left(\frac{x\sqrt{c^2(-d)-e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}\sqrt{c^2x^2-1}\sqrt{c^2(-d)-e}}+\frac{b\cosh^{-1}(cx)}{d+ex^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(a/(d + e*x^2) + (b*ArcCosh[c*x])/(d + e*x^2) - (b*c*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*ArcTan[(Sqrt[-(c^2*d) - e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[
d]*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c^2*x^2]))/(2*e)
```

Maple [B] time = 0.047, size = 638, normalized size = 5.7

$$-\frac{c^2a}{2\,e\left(x^2c^2e+c^2d\right)}-\frac{c^2b\mathrm{arccosh}\,(cx)}{2\,e\left(x^2c^2e+c^2d\right)}-\frac{bc^4d}{4}\sqrt{cx-1}\sqrt{cx+1}\ln\left(2\,\frac{1}{cxe-\sqrt{-c^2de}}\left(\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}e+\sqrt{-c^2de}cx-e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*arccosh(c*x)-1
/4*c^4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/((-c^2*d*e)^(1/2)+e)
/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)*ln(2*((c^2*x^2-
1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(
1/2)))*d+1/4*c^4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/((-c^2*d*e
)^(1/2)+e)/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)*ln(-2
*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+
(-c^2*d*e)^(1/2)))*d-1/4*c^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2
)/((-c^2*d*e)^(1/2)+e)/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)
^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-
e)/(c*x*e-(-c^2*d*e)^(1/2)))*e+1/4*c^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x
^2-1)^(1/2)/((-c^2*d*e)^(1/2)+e)/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c
^2*d+e)/e)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e
)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.33244, size = 1122, normalized size = 9.93

$$\left[-\frac{2\,ac^2d^2 - 2\left(bc^2de + be^2\right)x^2\log\left(cx + \sqrt{c^2x^2-1}\right) + 2\,ade - \left(bcex^2 + bcd\right)\sqrt{c^2d^2+de}\,\log\left(-\frac{2\,c^2d^2 - \left(4\,c^4d^2 + 4\,c^2de + e^2\right)x^2 + de - 2}{} \right.}{4\left(c^2d^3e + d^2e^2 + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 - 2*(b*c^2*d*e + b*e^2)*x^2*log(c*x + sqrt(c^2*x^2 - 1))
 + 2*a*d*e - (b*c*e*x^2 + b*c*d)*sqrt(c^2*d^2 + d*e)*log(-(2*c^2*d^2 - (4*c
^4*d^2 + 4*c^2*d*e + e^2)*x^2 + d*e - 2*sqrt(c^2*d^2 + d*e)*((2*c^3*d + c*e
)*x^2 - c*d) - 2*sqrt(c^2*x^2 - 1)*(sqrt(c^2*d^2 + d*e)*(2*c^2*d + e)*x + 2
*(c^3*d^2 + c*d*e)*x))/(e*x^2 + d)) - 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b
*e^2)*x^2)*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e
^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 - (b*c^2*d*e + b*e^2)*x^2*log(c*x + sqrt(
c^2*x^2 - 1)) + a*d*e - (b*c*e*x^2 + b*c*d)*sqrt(-c^2*d^2 - d*e)*arctan((sq
rt(-c^2*d^2 - d*e)*sqrt(c^2*x^2 - 1)*e*x - sqrt(-c^2*d^2 - d*e)*(c*e*x^2 +
c*d))/(c^2*d^2 + d*e)) - (b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*log(
-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(a + b\,\mathrm{acosh}\,(cx))}{\left(d + ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x}{(ex^2 + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d)^2, x)
```

3.500 $\int \frac{a+b\cosh^{-1}(cx)}{x(d+ex^2)^2}\,dx$

Optimal. Leaf size=598

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{}}{\sqrt{c^2}}\right.}{2d^2}$$

```
[Out] (a + b*ArcCosh[c*x])/(2*d*(d + e*x^2)) + (a + b*ArcCosh[c*x])^2/(b*d^2) - (
b*c*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x
^2])])/(2*d^(3/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((a + b*A
rcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^2 - ((a + b*ArcCosh[c*x])*Log[
1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) -
((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e])])/(2*d^2) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) - ((a + b*ArcCosh[c*x])*L
og[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2)
 - (b*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(2*d^2) - (b*PolyLog[2, -((Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^2) - (b*PolyLog[2
, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) - (b
*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])
/(2*d^2) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) - e])])/(2*d^2)
```

Rubi [A] time = 1.06196, antiderivative size = 581, normalized size of antiderivative = 0.97, number of steps used = 29, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5792, 5660, 3718, 2190, 2279, 2391, 5788, 519, 377, 208, 5800, 5562}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{}}{\sqrt{c^2}}\right.}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] (a + b*ArcCosh[c*x])/(2*d*(d + e*x^2)) - (b*c*Sqrt[-1 + c^2*x^2]*ArcTanh[(S
qrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(3/2)*Sqrt[c^2*d + e]
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) - ((a + b*ArcCosh[
```

```
c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/
(2*d^2) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] + Sqrt[-(c^2*d) - e])])/(2*d^2) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) + ((a + b*ArcCo
sh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/d^2 - (b*PolyLog[2, -((Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^2) - (b*PolyLog[2, (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) - (b*PolyL
og[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^
2) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e
])])/(2*d^2) + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^2)
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^2x} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)^2} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{\mathrm{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(cx)\right)}{d^2} - \frac{(bc)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}\,dx}{2d}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2} + \frac{2\,\mathrm{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{d^2} + \frac{\sqrt{e}\int \frac{a+b\cos}{\sqrt{-d}}}{2d}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)}{d^2} - \frac{b\,\mathrm{Subst}\left(\right.}{}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{bc\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(c}\right.}{d^2}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{bc\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(}}{c\sqrt{-d}-\sqrt{-c^2}}\right.}{2d^2}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{bc\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(}}{c\sqrt{-d}-\sqrt{-c^2}}\right.}{2d^2}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{bc\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(}}{c\sqrt{-d}-\sqrt{-c^2}}\right.}{2d^2}$$

Mathematica [F] time = 4.8704, size = 0, normalized size = 0.

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2), x]
```

Maple [C] time = 0.203, size = 529, normalized size = 0.9

$$\frac{ac^2}{2\,d\left(x^2c^2e+c^2d\right)}-\frac{a\ln\left(x^2c^2e+c^2d\right)}{2\,d^2}+\frac{a\ln\left(cx\right)}{d^2}+\frac{bc^2\mathrm{arccosh}\left(cx\right)}{2\,d\left(x^2c^2e+c^2d\right)}+\frac{b}{2\,d^2\left(c^2d+e\right)}\sqrt{c^2d\left(c^2d+e\right)}\mathrm{Artanh}\left(\frac{1}{4}\left(2\left(cx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x)
```

```
[Out] 1/2*a*c^2/d/(c^2*e*x^2+c^2*d)-1/2*a/d^2*ln(c^2*e*x^2+c^2*d)+a/d^2*ln(c*x)+1
/2*b*c^2*arccosh(c*x)/d/(c^2*e*x^2+c^2*d)+1/2*b*(c^2*d*(c^2*d+e))^(1/2)/d^2
/(c^2*d+e)*arctanh(1/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e
)/(c^4*d^2+c^2*d*e)^(1/2))-1/4*b/d^2*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2
*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R
1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z
^2+e))+b/d^2*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^2*a
rccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^2*dilog(1+I*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^2*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))-1/4*b/d^2*e*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c
*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(
1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{1}{dex^2+d^2}-\frac{\log\left(ex^2+d\right)}{d^2}+\frac{2\,\log\left(x\right)}{d^2}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{e^2x^5+2\,dex^3+d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrate
(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arcosh}(cx) + a}{e^2x^5 + 2\,dex^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(ex^2 + d\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x), x)
```

3.501 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d+ex^2)^2}\,dx$

Optimal. Leaf size=634

$$\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{d^3}+\frac{be\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{d^3}+\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{d^3}+\frac{be\text{PolyLog}\left(2,\right.}{d^3}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d^2*x) - (a + b*ArcCosh[c*x])/(2*d^2*
x^2) - (e*(a + b*ArcCosh[c*x]))/(2*d^2*(d + e*x^2)) - (2*e*(a + b*ArcCosh[c
*x])^2)/(b*d^3) + (b*c*e*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sq
rt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) - (2*e*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^3 + (
e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/d^3 + (e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/d^3 + (e*(a + b*ArcCosh[c*x])*Log[
1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/d^3 + (e*(
a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c
^2*d) - e])])/d^3 + (b*e*PolyLog[2, -E^(-2*ArcCosh[c*x])])/d^3 + (b*e*PolyL
og[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/d^3 +
 (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])
])/d^3 + (b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^
2*d) - e]))])/d^3 + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/d^3
```

Rubi [A] time = 1.08718, antiderivative size = 616, normalized size of antiderivative = 0.97, number of steps used = 31, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5792, 5662, 95, 5660, 3718, 2190, 2279, 2391, 5788, 519, 377, 208, 5800, 5562}

$$\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{d^3}+\frac{be\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{d^3}+\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{d^3}+\frac{be\text{PolyLog}\left(2,\right.}{d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d^2*x) - (a + b*ArcCosh[c*x])/(2*d^2*
x^2) - (e*(a + b*ArcCosh[c*x]))/(2*d^2*(d + e*x^2)) + (b*c*e*Sqrt[-1 + c^2*
```

```
x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(5/2)*
Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (e*(a + b*ArcCosh[c*x])*Log
[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/d^3 + (e*
(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(
c^2*d) - e])])/d^3 + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x
])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/d^3 + (e*(a + b*ArcCosh[c*x])*Log[1
+ (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/d^3 - (2*e*(
a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/d^3 + (b*e*PolyLog[2, -((S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/d^3 + (b*e*Poly
Log[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/d^3 + (
b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
)])/d^3 + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2
*d) - e])])/d^3 - (b*e*PolyLog[2, -E^(2*ArcCosh[c*x])])/d^3
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^2x^3} - \frac{2e\left(a+b\cosh^{-1}(cx)\right)}{d^3x} + \frac{e^2x\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^2} + \frac{2e^2x\left(a+b\cosh^{-1}(\right.}{d^3\left(d+ex^2\right)}\right.$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{d^2} - \frac{(2e)\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^3} + \frac{\left(2e^2\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^3} + \frac{e^2\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2d^2} - \frac{(2e)\,\text{Subst}\left(\int(a+bx)\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{bd^3} - \frac{(4e}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{bd^3} - \frac{2e\,(}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{d}\sqrt{}}\right.}{2d^{5/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{d}\sqrt{}}\right.}{2d^{5/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{d}\sqrt{}}\right.}{2d^{5/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{d}\sqrt{}}\right.}{2d^{5/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{}}$$

Mathematica [F] time = 5.71047, size = 0, normalized size = 0.

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2), x]
```

Maple [C] time = 0.24, size = 723, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a*e/d^2/(c^2*e*x^2+c^2*d)+a*e/d^3*ln(c^2*e*x^2+c^2*d)-1/2*a/d^2/x^
2-2*a/d^3*e*ln(c*x)+1/2*c^3*b*x/d^2/(c^2*e*x^2+c^2*d)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*e+1/2*c^3*b/x/d/(c^2*e*x^2+c^2*d)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/2*c^
4*b*x^2/d^2/(c^2*e*x^2+c^2*d)*e-1/2*c^4*b/d/(c^2*e*x^2+c^2*d)-c^2*b*arccosh
(c*x)*e/d^2/(c^2*e*x^2+c^2*d)-1/2*c^2*b/x^2/d/(c^2*e*x^2+c^2*d)*arccosh(c*x
)-1/2*b*(c^2*d*(c^2*d+e))^(1/2)/d^3/(c^2*d+e)*e*arctanh(1/4*(2*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e)/(c^4*d^2+c^2*d*e)^(1/2))+1/2*b/d^3*e
*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x
-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_
R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-2*b/d^3*e*arccosh(c*x)*ln(1+I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-2*b/d^3*e*arccosh(c*x)*ln(1-I*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2)))-2*b/d^3*e*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))-2*b/d^3*e*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/2*b/d^3*e^2*
sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(
c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=Ro
otOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a\left(\frac{2\,ex^2+d}{d^2ex^4+d^3x^2}-\frac{2\,e\log\left(ex^2+d\right)}{d^3}+\frac{4\,e\log\left(x\right)}{d^3}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{e^2x^7+2\,dex^5+d^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4*e*
log(x)/d^3) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^7 +
 2*d*e*x^5 + d^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{e^{2}x^{7}+2\,dex^{5}+d^{2}x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{arcosh}\,(cx)+a}{\left(ex^{2}+d\right)^{2}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x^3), x)
```

3.502 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=839

$$\frac{x\cosh^{-1}(cx)b}{e^2}+\frac{cd\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)b}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{5/2}}-\frac{cd\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)b}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{5/2}}-\frac{3\sqrt{-d}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4e^{5/2}}$$

```
[Out] (a*x)/e^2 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e^2) + (b*x*ArcCosh[c*x])/e
^2 - (d*(a + b*ArcCosh[c*x]))/(4*e^(5/2)*(Sqrt[-d] - Sqrt[e]*x)) + (d*(a +
b*ArcCosh[c*x]))/(4*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*d*ArcTanh[(Sqrt[
c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 +
c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) -
 (b*c*d*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d]
 - Sqrt[e]]*Sqrt[-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d]
 + Sqrt[e]]*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*Sqrt[-d]
*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*S
qrt[-d]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
 Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*e^(5/2)) + (3*b*Sqrt[
-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])
/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) - e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*e^(5/2))
```

Rubi [A] time = 2.18778, antiderivative size = 839, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5792, 5654, 74, 5707, 5802, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{x\cosh^{-1}(cx)b}{e^2}+\frac{cd\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)b}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{5/2}}-\frac{cd\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)b}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{5/2}}-\frac{3\sqrt{-d}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/e^2 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e^2) + (b*x*ArcCosh[c*x])/e
^2 - (d*(a + b*ArcCosh[c*x]))/(4*e^(5/2)*(Sqrt[-d] - Sqrt[e]*x)) + (d*(a +
b*ArcCosh[c*x]))/(4*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*d*ArcTanh[(Sqrt[
c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 +
c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) -
 (b*c*d*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d]
 - Sqrt[e]]*Sqrt[-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d]
 + Sqrt[e]]*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*Sqrt[-d]
*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*S
qrt[-d]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
 Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*e^(5/2)) + (3*b*Sqrt[
-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])
/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) - e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*e^(5/2))
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int\left(\frac{a+b\cosh^{-1}(cx)}{e^2}+\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^2}-\frac{2d\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int\left(a+b\cosh^{-1}(cx)\right)dx}{e^2}-\frac{(2d)\int\frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e^2}+\frac{d^2\int\frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e^2}$$

$$=\frac{ax}{e^2}+\frac{b\int\cosh^{-1}(cx)\,dx}{e^2}-\frac{(2d)\int\left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{e^2}+\frac{d^2\int\left(-\frac{e(a+}{4d(}\right.}{}$$

$$=\frac{ax}{e^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{(bc)\int\frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e^2}-\frac{\sqrt{-d}\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{e^2}-\frac{\sqrt{-d}\int\frac{a+b\cosh^{-1}(c}{\sqrt{-d}+\sqrt{e}x}}{e^2}$$

$$=\frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)}-$$

$$=\frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+$$

$$=\frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+$$

$$=\frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+$$

$$=\frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+$$

$$=\frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+$$

$$=\frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+$$

Mathematica [C] time = 2.22501, size = 776, normalized size = 0.92

$$b\left(-3i\sqrt{d}\left(2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-ic\sqrt{d}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+ic\sqrt{d}}\right)+\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}}{-\sqrt{c^2}}\right.\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (8*a*Sqrt[e]*x + (4*a*d*Sqrt[e]*x)/(d + e*x^2) - 12*a*Sqrt[d]*ArcTan[(Sqrt[
e]*x)/Sqrt[d]] + b*((8*Sqrt[e]*(-(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + c
*x*ArcCosh[c*x]))/c + 2*d*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log
[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d)
- e]) + 2*d*(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(-Sqrt[e] -
 I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqr
t[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - (3*I)*Sqrt
[d]*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c
*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqr
t[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)
*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])
/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]) + (3*I)*Sqrt[d]*(ArcCosh[c*x]*(-ArcC
osh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2
*d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
- e])])) + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-
(c^2*d) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt
[-(c^2*d) - e])])))/(8*e^(5/2))
```

Maple [C] time = 2.26, size = 1749, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] a*x/e^2+1/2*c^2*a/e^2*d*x/(c^2*e*x^2+c^2*d)-3/2*a/e^2*d/(d*e)^(1/2)*arctan(
x*e/(d*e)^(1/2))+c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^3*
```

```
arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1
/2)+e)*e)^(1/2))/e^5/(c^2*d+e)+c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)
*e)^(1/2)*d^2*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d
*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)+c*b*((2*c^2*d+2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+
2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*d/e^5*(c^2*d*(c^2*d+e))^(1/2)-b*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/c/e^2+1/2*c^2*b*arccosh(c*x)/e^2*d*x/(c^2*e*x^2+c^2
*d)+b*x*arccosh(c*x)/e^2+c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(
1/2)*d^3*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)-e)*e)^(1/2))/e^5/(c^2*d+e)+c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)-c*b*(-(2*c^2*d-2
*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*d/e^5*(c^2*d*(c^2*d
+e))^(1/2)-c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*
e)^(1/2))*d^2/e^5-1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*
arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^
(1/2)-e)*e)^(1/2))*d/e^4-c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1
/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e)
)^(1/2)+e)*e)^(1/2))*d^2/e^5-1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)
*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2))*d/e^4+3/4*c*b/e^2*d*sum(1/_R1/(_R1^2*e+2*c^2*d+e
)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e
))-3/4*c*b/e^2*d*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x
-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_
R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+c^3*b*(-(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/(
(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^5/(c^2*d+e)*(c^2*d*(c^2*
d+e))^(1/2)+1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d*arct
anh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2
)-e)*e)^(1/2))/e^4/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)-c^3*b*((2*c^2*d+2*(c^2
*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^2*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^5/(c^2*d+e)*(c^2*d*(
c^2*d+e))^(1/2)-1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d*a
rctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/
2)+e)*e)^(1/2))/e^4/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4 \operatorname{arcosh}(cx) + ax^4}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b \operatorname{acosh}(cx))}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^4}{(ex^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

3.503 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=792

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{\ }}{\sqrt{c^2}}\right.}{4\sqrt{-de}}$$

```
[Out] (a + b*ArcCosh[c*x])/(4*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcCosh[c*
x])/(4*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sq
rt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(2*Sqrt
[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/2)) + (b*c*ArcTanh[(
Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[
-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/
2)) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 +
(Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^
(3/2)) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1
 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]
*e^(3/2)) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) - e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2,
-((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(4*Sqrt[-d]
*e^(3/2)) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2
*d) - e])])/(4*Sqrt[-d]*e^(3/2))
```

Rubi [A] time = 1.9687, antiderivative size = 792, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5792, 5707, 5802, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{\ }}{\sqrt{c^2}}\right.}{4\sqrt{-de}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcCosh[c*x])/(4*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcCosh[c*
x])/(4*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sq
rt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(2*Sqrt
[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/2)) + (b*c*ArcTanh[(
Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[
-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/
2)) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 +
(Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^
(3/2)) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1
 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]
*e^(3/2)) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) - e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2,
-((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(4*Sqrt[-d]
*e^(3/2)) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2
*d) - e])])/(4*Sqrt[-d]*e^(3/2))
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \left(-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)^2}+\frac{a+b\cosh^{-1}(cx)}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e} - \frac{d\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e}$$

$$= \frac{\int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{e} - \frac{d\int \left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e(\ldots}{\ldots}\right.}{e}$$

$$= \frac{1}{4}\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx + \frac{1}{4}\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\,dx + \frac{1}{2}\int \frac{a+b\cosh^{-1}(cx)}{-de-e^2x^2}\,dx - \frac{\int \frac{a+b\,\mathrm{c}\ldots}{\sqrt{-}\ldots}}{2\ldots}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{1}{2}\int \left(-\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2de\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{-d}\left(a+b\,\mathrm{c}\ldots\right.}{2de\left(\sqrt{-d}\ldots\right.}\right.$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{(bc)\,\mathrm{Subst}\left(\int \frac{1}{c\sqrt{-d}\sqrt{e}+e-\left(c\sqrt{-d}\sqrt{e}-e\right)x^2}\,dx, x, \frac{\sqrt{1+c\ldots}}{\sqrt{-1+\ldots}}\right.}{2e}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{3/2}} + \frac{bc\tanh^{-1}\ldots}{2\sqrt{c\sqrt{-d}-\ldots}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{3/2}} + \frac{bc\tanh^{-1}\ldots}{2\sqrt{c\sqrt{-d}-\ldots}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{3/2}} + \frac{bc\tanh^{-1}\ldots}{2\sqrt{c\sqrt{-d}-\ldots}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{3/2}} + \frac{bc\tanh^{-1}\ldots}{2\sqrt{c\sqrt{-d}-\ldots}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}e^{3/2}} + \frac{bc\tanh^{-1}\ldots}{2\sqrt{c\sqrt{-d}-\ldots}}$$

Mathematica [C] time = 1.79727, size = 719, normalized size = 0.91

$$b\left(\frac{i\left(2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-ic\sqrt{d}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+ic\sqrt{d}}\right)+\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{-\sqrt{c^2(-d)-e}+ic\sqrt{d}}\right)+\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+ic\sqrt{d}}\right)\right)\right)\right)}{\sqrt{d}}+\frac{i\left(-2\cdots\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 b*((-2*ArcCosh[c*x])/(I*Sqrt[d] + Sqrt[e]*x) - 2*(ArcCosh[c*x]/((-I)*Sqrt[
d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d)
- e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt
[e]*x))])/Sqrt[-(c^2*d) - e]) - (2*c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x +
 Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I
*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + (I*(ArcCosh[c*x]*(-ArcCosh[c*
x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])
] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))
+ 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e
])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
- e]))]))/Sqrt[d] + (I*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1 + (Sqrt[e]*E^
ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqrt[e]*E^A
rcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) - 2*PolyLog[2, -((Sqrt[e
]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))] - 2*PolyLog[2, (S
qrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))/Sqrt[d]))/(8*e
^(3/2))
```

Maple [C] time = 0.957, size = 1689, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e*x/(c^2*e*x^2+c^2*d)+1/2*a/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)-1/2*c^2*b*arccosh(c*x)/e*x/(c^2*e*x^2+c^2*d)-c^5*b*(-(2*c^2*d-2*(c^2*d*(c
^2*d+e))^(1/2)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/
```

```
((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)-c^3*b*(-(2*
c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d*arctanh((c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d
+e)*(c^2*d*(c^2*d+e))^(1/2)-c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e
)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^
2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)*d-1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2
*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c
^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)*(c^2*d*(c^2*d+e))
^(1/2)+c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(
1/2))*d/e^4+c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e
)^(1/2))/e^4*(c^2*d*(c^2*d+e))^(1/2)+1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))
^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2
*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3-c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2)*d^2*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2
*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)+c^3*b*((2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)*(c^2*d*
(c^2*d+e))^(1/2)-c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arct
an((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+
e)*e)^(1/2))/e^3/(c^2*d+e)*d+1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)
*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+c^3*b*((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*d/e^4-c*b*((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4*(c^2*d*(c^
2*d+e))^(1/2)+1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arcta
n((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e
)*e)^(1/2))/e^3+1/4*c*b/e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/4*c*b/e*sum(1/_R1/
(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/
_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4
*c^2*d+2*e)*_Z^2+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \operatorname{arcosh}(cx) + ax^2}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,(a + b\operatorname{acosh}(cx))}{\left(d + ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)x^2}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

3.504 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=804

$$-\frac{\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)\Big(a+b\,c}{4(-d)^{3/2}\sqrt{e}}$$

```
[Out] -(a + b*ArcCosh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcCosh
[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*ArcTanh[(Sqrt[c*Sqrt[-d]
 - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(2
*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) - (b*c*Ar
cTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]
]*Sqrt[-1 + c*x])]])/(2*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[
e]]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
rt[-d] - Sqrt[-(c^2*d) - e])]])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCosh[c*x
])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/(4*
(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcC
osh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]
)]])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
rt[-d] - Sqrt[-(c^2*d) - e]))]])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/(4*(-d)^(3/2)*Sqrt
[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d)
 - e]))]])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(4*(-d)^(3/2)*Sqrt[e])
```

Rubi [A] time = 1.02423, antiderivative size = 804, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5707, 5802, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)\Big(a+b\,c}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcCosh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcCosh
[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*ArcTanh[(Sqrt[c*Sqrt[-d]
 - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(2
*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) - (b*c*Ar
cTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]
]*Sqrt[-1 + c*x])])/(2*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[
e]]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
rt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCosh[c*x
])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*
(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcC
osh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]
)])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
rt[-d] - Sqrt[-(c^2*d) - e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt
[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d)
 - e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e])
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \int \left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d\left(-de-e^2x^2\right)}\right)dx$$

$$= -\frac{e\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{4d}-\frac{e\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\,dx}{4d}-\frac{e\int \frac{a+b\cosh^{-1}(cx)}{-de-e^2x^2}\,dx}{2d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(\sqrt{-d}\sqrt{e}-ex\right)}\,dx}{4d}-\frac{(bc)\int \frac{\cdots}{\sqrt{-1+cx}\cdots}}{\cdots}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{4(-d)^{3/2}}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{4(-d)^{3/2}}+\frac{(bc)\,\mathrm{S}\cdots}{\cdots}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}-\frac{bc\tanh^{-1}\cdots}{2d\sqrt{c\sqrt{-d}\cdots}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}-\frac{bc\tanh^{-1}\cdots}{2d\sqrt{c\sqrt{-d}\cdots}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}-\frac{bc\tanh^{-1}\cdots}{2d\sqrt{c\sqrt{-d}\cdots}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}-\frac{bc\tanh^{-1}\cdots}{2d\sqrt{c\sqrt{-d}\cdots}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}-\frac{bc\tanh^{-1}\cdots}{2d\sqrt{c\sqrt{-d}\cdots}}$$

Mathematica [C] time = 1.89263, size = 733, normalized size = 0.91

$$\frac{1}{2}\left(\frac{b\left(i\left(2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-ic\sqrt{d}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+ic\sqrt{d}}\right)+\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\cosh}}{-\sqrt{c^2(-d)-}}\right.\right.\right.\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*(2*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*S
qrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
)/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - 2*
Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] -
I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt
[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + I*(ArcCosh[
c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sq
rt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-
(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] +
Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d]
 + Sqrt[-(c^2*d) - e]))]) - I*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sq
rt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqr
t[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, -
((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))] + 2*PolyL
og[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])))/(4*d^
(3/2)*Sqrt[e]))/2
```

Maple [C] time = 0.884, size = 1695, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*c^2*a*x/d/(c^2*e*x^2+c^2*d)+1/2*a/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
+1/2*c^2*b*arccosh(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4*c*b/d*sum(_R1/(_R1^2*e+2*
c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog(
(_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)
*_Z^2+e))+c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e
)^(1/2))/e^3/(c^2*d+e)*d+c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d
+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+c^3*b*(-(2*c^
2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)/e^2+
1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))
/d/(c^2*d+e)/e^2*(c^2*d*(c^2*d+e))^(1/2)-c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d
+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3-c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d/e^3*(c^2*d*(c^2*d+e))^(1/2)-1/2*
c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d/e
^2+c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e
^3/(c^2*d+e)*d-c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)
*e)^(1/2))/e^3/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+c^3*b*((2*c^2*d+2*(c^2*d*(
c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)/e^2-1/2*c*b*((2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)/e^2*(c
^2*d*(c^2*d+e))^(1/2)-c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)
*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(
1/2)+e)*e)^(1/2))/e^3+c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*a
rctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/
2)+e)*e)^(1/2))/d/e^3*(c^2*d*(c^2*d+e))^(1/2)-1/2*c*b*((2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c
^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/e^2-1/4*c*b/d*sum(1/_R1/(_R1^
2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+
dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*
d+2*e)*_Z^2+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{acosh}\,(cx)}{\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{arcosh}\,(cx)+a}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^2, x)
```

3.505 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

Optimal. Leaf size=846

$$-\frac{3\sqrt{e}\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}}}{\sqrt{-d}c+\sqrt{-}}\right.}{4(}$$

```
[Out] -((a + b*ArcCosh[c*x])/(d^2*x)) + (Sqrt[e]*(a + b*ArcCosh[c*x]))/(4*d^2*(Sq
rt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcCosh[c*x]))/(4*d^2*(Sqrt[-d] + Sq
rt[e]*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 - (b*c*Sqrt[e]*A
rcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e
]]*Sqrt[-1 + c*x])])/(2*d^2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sq
rt[e]]) + (b*c*Sqrt[e]*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(
Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(2*d^2*Sqrt[c*Sqrt[-d] - Sqrt[
e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]) - (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2))
+ (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCosh[c*x]
)*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(
-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x
])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*(-d
)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(4*(-d)^(5/2)) - (3*b*Sq
rt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
])/(4*(-d)^(5/2))
```

Rubi [A] time = 2.02192, antiderivative size = 846, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {5792, 5662, 92, 205, 5707, 5802, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{3\sqrt{e}\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}}}{\sqrt{-d}c+\sqrt{-}}\right.}{4(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d^2*x)) + (Sqrt[e]*(a + b*ArcCosh[c*x]))/(4*d^2*(Sq
rt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcCosh[c*x]))/(4*d^2*(Sqrt[-d] + Sq
rt[e]*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 - (b*c*Sqrt[e]*A
rcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e
]]*Sqrt[-1 + c*x])])/(2*d^2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sq
rt[e]]) + (b*c*Sqrt[e]*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(
Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(2*d^2*Sqrt[c*Sqrt[-d] - Sqrt[
e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]) - (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2))
+ (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCosh[c*x]
)*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(
-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x
])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*(-d
)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(4*(-d)^(5/2)) - (3*b*Sq
rt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
])/(4*(-d)^(5/2))
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
```

```
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{(bc)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d^2} - \frac{e\int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{d^2} -$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\left(bc^2\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right)}{d^2} + \frac{e\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2(-d)^{5/2}} + \frac{e\int}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1\right.}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1\right.}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1\right.}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1\right.}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1\right.}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1\right.}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1\right.}{d^2}$$

Mathematica [C] time = 2.52744, size = 820, normalized size = 0.97

$$-12\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)a-\frac{8\sqrt{d}a}{x}-\frac{4\sqrt{d}exa}{ex^2+d}+b\left(8\sqrt{d}\left(\frac{c\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\cosh^{-1}(cx)}{x}\right)-2\sqrt{d}\sqrt{e}\left(\frac{\cosh^{-1}(cx)}{\sqrt{e}x-i\sqrt{d}}+\frac{c\log\left(\frac{2e\left(\sqrt{d}xc^2+i\sqrt{e}\cdots\right.}{c\sqrt{-d}\cdots}\right)}{\sqrt{-}\cdots}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(8*Sqrt[d]*(-(ArcCosh[c*x]/x) + (c*Sqrt[-1 + c^2*x^2]
*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - 2*Sqrt[d]*Sq
rt[e]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c
^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[
-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + 2*Sqrt[d]*Sq
rt[e]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*
c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-
(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - (3*I)*Sqrt[e]
*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sq
rt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d
] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*
Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I
*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]) + (3*I)*Sqrt[e]*(ArcCosh[c*x]*(-ArcCosh
[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e
])])) + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^
2*d) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(
c^2*d) - e])])))/(8*d^(5/2))
```

Maple [C] time = 2.61, size = 1821, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x)
```

```
[Out] -1/2*a/d^2*e*c^2*x/(c^2*e*x^2+c^2*d)-3/2*a/d^2*e/(d*e)^(1/2)*arctan(x*e/(d*
e)^(1/2))-a/d^2/x-3/2*b*arccosh(c*x)/d^2*e*c^2*x/(c^2*e*x^2+c^2*d)-b*c^2/x*
```

```
arccosh(c*x)/(c^2*e*x^2+c^2*d)/d-b*c^5*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*
d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)/e^2-b*c^3*(-(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d/(c^2*d+e)/e^2*(c^2*d*(c^2*d+
e))^(1/2)-b*c^3*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e
)^(1/2))/d/(c^2*d+e)/e-1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d
+e))^(1/2)-e)*e)^(1/2))/d^2/(c^2*d+e)/e*(c^2*d*(c^2*d+e))^(1/2)+b*c^3*(-(2*
c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d/e^2+c*b*(-
(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d^2/e^2*(
c^2*d*(c^2*d+e))^(1/2)+1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d
+e))^(1/2)-e)*e)^(1/2))/d^2/e-b*c^5*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*
e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2
*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)/e^2+b*c^3*((2*c^2*d+2*(c^2*d*(c^2*d+e))
^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)/e^2*(c^2*d*(c^2*d+e))^(1/2)
-b*c^3*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/(
c^2*d+e)/e+1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e
)^(1/2))/d^2/(c^2*d+e)/e*(c^2*d*(c^2*d+e))^(1/2)+b*c^3*((2*c^2*d+2*(c^2*d*(
c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/e^2-c*b*((2*c^2*d+2*(c^2*d*(
c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d^2/e^2*(c^2*d*(c^2*d+e))^(1/2
)+1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/
d^2/e+3/16*b/c/d^3*e*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arcco
sh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+2*c*b
/d^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-3/16*b/c/d^3*e*sum((4*_R1^2*c^
2*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1
/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_
R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{e^2x^6+2\,dex^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

3.506 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=737

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{co}}}{\sqrt{c^2(-d}}\right.}{2e^3}$$

```
[Out] (b*c*d*x*(1 - c^2*x^2))/(8*e^2*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d
+ e*x^2)) - (d^2*(a + b*ArcCosh[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*Ar
cCosh[c*x]))/(e^3*(d + e*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*e^3) - (b*c*Sq
rt[d]*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2
*x^2])])/(e^3*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d]*
(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[
-1 + c^2*x^2])])/(8*e^3*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(
c^2*d) - e])])/(2*e^3) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^3) + ((a + b*ArcCosh[c*x])*Lo
g[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^3)
+ ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt
[-(c^2*d) - e])])/(2*e^3) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^3) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^3) + (b*PolyLog[2, -((Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^3) + (b*PolyLog[
2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^3)
```

Rubi [A] time = 1.18224, antiderivative size = 737, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5792, 5788, 519, 382, 377, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\text{co}}}{\sqrt{c^2(-d}}\right.}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*d*x*(1 - c^2*x^2))/(8*e^2*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d
+ e*x^2)) - (d^2*(a + b*ArcCosh[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*Ar
cCosh[c*x]))/(e^3*(d + e*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*e^3) - (b*c*Sq
rt[d]*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2
*x^2])])/(e^3*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d]*
(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[
-1 + c^2*x^2])])/(8*e^3*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(
c^2*d) - e])])/(2*e^3) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^3) + ((a + b*ArcCosh[c*x])*Lo
g[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^3)
+ ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt
[-(c^2*d) - e])])/(2*e^3) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^3) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^3) + (b*PolyLog[2, -((Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^3) + (b*PolyLog[
2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^3)
```

<u>Rule 5792</u>

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

<u>Rule 5788</u>

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

<u>Rule 519</u>

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

<u>Rule 382</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
```

```
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int\left(\frac{d^2x\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^3}-\frac{2dx\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{e^2}-\frac{(2d)\int\frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{e^2}+\frac{d^2\int\frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx}{e^2}$$

$$=-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}-\frac{(bcd)\int\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}\,dx}{e^3}+\frac{\left(bcd^2\right)}{}$$

$$=-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}-\frac{\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2e^{5/2}}+\frac{\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2e^{5/2}}$$

$$=\frac{bcdx\left(1-c^2x^2\right)}{8e^2\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}$$

$$=\frac{bcdx\left(1-c^2x^2\right)}{8e^2\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}$$

$$=\frac{bcdx\left(1-c^2x^2\right)}{8e^2\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}$$

$$=\frac{bcdx\left(1-c^2x^2\right)}{8e^2\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}$$

$$=\frac{bcdx\left(1-c^2x^2\right)}{8e^2\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}$$

Mathematica [C] time = 7.1105, size = 1155, normalized size = 1.57

$$-\frac{ad^2}{4e^3\left(ex^2+d\right)^2}+\frac{ad}{e^3\left(ex^2+d\right)}+\frac{a\log\left(ex^2+d\right)}{2e^3}+b\left(-\frac{7i\sqrt{d}\left(\frac{\cosh^{-1}(cx)}{\sqrt{e}x-i\sqrt{d}}+\frac{c\log\left(\frac{2e\left(\sqrt{d}xc^2+i\sqrt{e}-i\sqrt{-dc^2-e}\sqrt{cx-1}\sqrt{cx+1}\right)}{c\sqrt{-dc^2-e}\left(i\sqrt{e}x+\sqrt{d}\right)}\right)}{\sqrt{-dc^2-e}}\right)}{16e^3}-\frac{7i\sqrt{d}\left(-\frac{\cos}{\sqrt{e}}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(a*d^2)/(4*e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x^2
])/(2*e^3) + b*((((-7*I)/16)*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*
x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqr
t[-(c^2*d) - e]))/e^3 - (((7*I)/16)*Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sq
rt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))
])/Sqrt[-(c^2*d) - e]))/e^3 - (d*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d
+ e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sq
rt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] -
 c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d + I
*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/(16*e^(5/2)) - (d*((c
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcC
osh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (c^3*Sqrt[d]*(Log[4] + Log[(
e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d +
e)^(3/2))))/(16*e^(5/2)) + (ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[
e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E
^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((
Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))])/(4*e^3) + (Ar
cCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqr
t[d] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d]
 + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c
*Sqrt[d] + Sqrt[-(c^2*d) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I
*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])/(4*e^3))
```

Maple [C] time = 0.79, size = 5196, normalized size = 7.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}a\left(\frac{4\,dex^2+3\,d^2}{e^5x^4+2\,de^4x^2+d^2e^3}+\frac{2\,\log\left(ex^2+d\right)}{e^3}\right)+b\int\frac{x^5\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + b*integrate(x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3*x^6
 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^5\,\text{arcosh}\left(cx\right)+ax^5}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccosh(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^5}{(ex^2 + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^5/(e*x^2 + d)^3, x)
```

3.507 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=231

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b\sqrt{1-c^2x^2}\sin^{-1}(cx)}{4de^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{1-c^2x^2}\left(2c^2d+3e\right)\tan^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8\sqrt{d}e^2\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)^{3/2}}-\frac{bcx\left(1-c^2x^2\right)}{8e\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+\right.}$$

```
[Out] -(b*c*x*(1 - c^2*x^2))/(8*e*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e
*x^2)) + (x^4*(a + b*ArcCosh[c*x]))/(4*d*(d + e*x^2)^2) - (b*Sqrt[1 - c^2*x
^2]*ArcSin[c*x])/(4*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*(2*c^2*d + 3
*e)*Sqrt[1 - c^2*x^2]*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2]
)])/(8*Sqrt[d]*e^2*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.361612, antiderivative size = 241, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {264, 5790, 12, 519, 470, 523, 217, 206, 377, 208}

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b\sqrt{c^2x^2-1}\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{4de^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{c^2x^2-1}\left(2c^2d+3e\right)\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{8\sqrt{d}e^2\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)^{3/2}}-\frac{bcx}{8e\sqrt{cx-1}\sqrt{cx+}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*x*(1 - c^2*x^2))/(8*e*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e
*x^2)) + (x^4*(a + b*ArcCosh[c*x]))/(4*d*(d + e*x^2)^2) - (b*Sqrt[-1 + c^2*
x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(4*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (b*c*(2*c^2*d + 3*e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(
Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8*Sqrt[d]*e^2*(c^2*d + e)^(3/2)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - (bc)\int \frac{x^4}{4d\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^2}\,dx$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{(bc)\int \frac{x^4}{\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^2}\,dx}{4d}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{x^4}{\sqrt{-1+c^2x^2}\left(d+ex^2\right)^2}\,dx}{4d\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{bcx\left(1-c^2x^2\right)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{-d}{\sqrt{-}}}{8de\left(c^2d+e\right)\sqrt{-1}}$$

$$= -\frac{bcx\left(1-c^2x^2\right)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{}{\sqrt{-}}}{4de^2\sqrt{-1+cx}\sqrt{1}}$$

$$= -\frac{bcx\left(1-c^2x^2\right)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\text{Subs}}{4de^2\sqrt{-1}}$$

$$= -\frac{bcx\left(1-c^2x^2\right)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{b\sqrt{-1+c^2x^2}\tanh^{-1}\left(\right.}{4de^2\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A] time = 0.890434, size = 192, normalized size = 0.83

$$\frac{\frac{\frac{bcex\sqrt{cx-1}\sqrt{cx+1}\left(d+ex^2\right)}{c^2d+e}-2a\left(d+2ex^2\right)}{\left(d+ex^2\right)^2} - \frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(2c^2d+3e\right)\tan^{-1}\left(\frac{x\sqrt{c^2(-d)-e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}\sqrt{c^2x^2-1}\left(c^2(-d)-e\right)^{3/2}} - \frac{2b\cosh^{-1}(cx)\left(d+2ex^2\right)}{\left(d+ex^2\right)^2}}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (((b*c*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(c^2*d + e) - 2*a*(d +
 2*e*x^2))/(d + e*x^2)^2 - (2*b*(d + 2*e*x^2)*ArcCosh[c*x])/(d + e*x^2)^2 -
 (b*c*(2*c^2*d + 3*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d) -
e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*(-(c^2*d) - e)^(3/2)*Sqrt[-1
```

```
+ c^2*x^2]))/(8*e^2)
```

Maple [B] time = 0.038, size = 2499, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/2*c^2*a/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4*a/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^
2*b*arccosh(c*x)/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4*b*arccosh(c*x)/e^2*d/(c^2*e*
x^2+c^2*d)^2+1/8*c^8*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1
/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^
2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1
)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1
/2)))*x^2*d^2+1/8*c^8*b*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/
2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2
*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)
^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/
2)))*d^3-1/8*c^8*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))
/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*
e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2
)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*
x^2*d^2-1/8*c^8*b*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-
c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^
(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(
-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*d^3
+1/8*c^5*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(c*x*e-
(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2*x*d+5/16*c^
6*b*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/
2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-
c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)
/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*x^2*d+5/16*c^
6*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/
2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-
c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)
/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*d^2-5/16*c^6*
b*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)
/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^
2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^
(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*x^2*d-5/16*c^6*b*
```

```
e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(
-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*
d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1
/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*d^2+1/8*c^3*b*e^3*(
c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(c*x*e-(-c^2*d*e)^(1/2)
)/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2*x+3/16*c^4*b*e^4*(c*x+1)^(1
/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^
(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^
2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*
d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*x^2+3/16*c^4*b*e^3*(c*x+1)^(1/2
)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1
/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/
(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*
e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*d-3/16*c^4*b*e^4*(c*x+1)^(1/2)*(c
*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/
(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2
*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/
2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*x^2-3/16*c^4*b*e^3*(c*x+1)^(1/2)*(c*x-1
)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x
*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2
-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c
*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\,b\left(\frac{\left(c^4d+2\,c^2e\right)\log\left(ex^2+d\right)}{c^4d^2e^2+2\,c^2de^3+e^4}+\frac{c^4d^3+c^2d^2e+\left(c^4d^2e+c^2de^2\right)x^2+2\left(c^4d^3+2\,c^2d^2e+de^2+2\left(c^4d^2e+2\,c^2de^2+e^3\right)x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b*((c^4*d + 2*c^2*e)*log(e*x^2 + d)/(c^4*d^2*e^2 + 2*c^2*d*e^3 + e^4)
+ (c^4*d^3 + c^2*d^2*e + (c^4*d^2*e + c^2*d*e^2)*x^2 + 2*(c^4*d^3 + 2*c^2*d
^2*e + d*e^2 + 2*(c^4*d^2*e + 2*c^2*d*e^2 + e^3)*x^2)*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1)) - (c^4*d^3 + 2*c^2*d^2*e + (c^4*d*e^2 + 2*c^2*e^3)*x^4 +
2*(c^4*d^2*e + 2*c^2*d*e^2)*x^2)*log(c*x + 1) - (c^4*d^3 + 2*c^2*d^2*e + (c
^4*d*e^2 + 2*c^2*e^3)*x^4 + 2*(c^4*d^2*e + 2*c^2*d*e^2)*x^2)*log(c*x - 1))/
(c^4*d^4*e^2 + 2*c^2*d^3*e^3 + d^2*e^4 + (c^4*d^2*e^4 + 2*c^2*d*e^5 + e^6)*
x^4 + 2*(c^4*d^3*e^3 + 2*c^2*d^2*e^4 + d*e^5)*x^2) + 8*integrate(1/4*(2*c*e
```

```
*x^2 + c*d)/(c^3*e^4*x^7 - c*d^2*e^2*x + (2*c^3*d*e^3 - c*e^4)*x^5 + (c^3*d
^2*e^2 - 2*c*d*e^3)*x^3 + (c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2
+ (c^2*d^2*e^2 - 2*d*e^3)*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)
) - 1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)
```

Fricas [B] time = 3.33259, size = 2473, normalized size = 10.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(2*a - b)*c^4*d^4 + 2*(4*a - b)*c^2*d^3*e - 4*(b*c^4*d^2*e^2 + 2*
b*c^2*d*e^3 + b*e^4)*x^4*log(c*x + sqrt(c^2*x^2 - 1)) + 4*a*d^2*e^2 - 2*(b*
c^4*d^2*e^2 + b*c^2*d*e^3)*x^4 + 4*((2*a - b)*c^4*d^3*e + (4*a - b)*c^2*d^2
*e^2 + 2*a*d*e^3)*x^2 - (2*b*c^3*d^3 + 3*b*c*d^2*e + (2*b*c^3*d*e^2 + 3*b*c
*e^3)*x^4 + 2*(2*b*c^3*d^2*e + 3*b*c*d*e^2)*x^2)*sqrt(c^2*d^2 + d*e)*log(-(
2*c^2*d^2 - (4*c^4*d^2 + 4*c^2*d*e + e^2)*x^2 + d*e - 2*sqrt(c^2*d^2 + d*e)
*((2*c^3*d + c*e)*x^2 - c*d) - 2*sqrt(c^2*x^2 - 1)*(sqrt(c^2*d^2 + d*e)*(2*
c^2*d + e)*x + 2*(c^3*d^2 + c*d*e)*x))/(e*x^2 + d)) - 4*(b*c^4*d^4 + 2*b*c^
2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^
4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - 2
*sqrt(c^2*x^2 - 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^
2*e^2)*x))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^
2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(
(2*a - b)*c^4*d^4 + (4*a - b)*c^2*d^3*e - 2*(b*c^4*d^2*e^2 + 2*b*c^2*d*e^3
+ b*e^4)*x^4*log(c*x + sqrt(c^2*x^2 - 1)) + 2*a*d^2*e^2 - (b*c^4*d^2*e^2 +
b*c^2*d*e^3)*x^4 + 2*((2*a - b)*c^4*d^3*e + (4*a - b)*c^2*d^2*e^2 + 2*a*d*e
^3)*x^2 - (2*b*c^3*d^3 + 3*b*c*d^2*e + (2*b*c^3*d*e^2 + 3*b*c*e^3)*x^4 + 2*
(2*b*c^3*d^2*e + 3*b*c*d*e^2)*x^2)*sqrt(-c^2*d^2 - d*e)*arctan((sqrt(-c^2*d
^2 - d*e)*sqrt(c^2*x^2 - 1)*e*x - sqrt(-c^2*d^2 - d*e)*(c*e*x^2 + c*d))/(c^
2*d^2 + d*e)) - 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 +
 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x
^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)*((b*c^3*d^2*e^2 + b*c
*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3
+ d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*
c^2*d^3*e^4 + d^2*e^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^3}{(ex^2 + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^3/(e*x^2 + d)^3, x)
```

3.508 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=177

$$-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bc\sqrt{c^2x^2-1}\left(2c^2d+e\right)\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{8d^{3/2}e\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)^{3/2}}+\frac{bcx\left(1-c^2x^2\right)}{8d\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)\left(d+ex^2\right)}$$

```
[Out] (b*c*x*(1 - c^2*x^2))/(8*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*
x^2)) - (a + b*ArcCosh[c*x])/(4*e*(d + e*x^2)^2) + (b*c*(2*c^2*d + e)*Sqrt[
-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8
*d^(3/2)*e*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.135168, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5788, 519, 382, 377, 208}

$$-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bc\sqrt{c^2x^2-1}\left(2c^2d+e\right)\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{8d^{3/2}e\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)^{3/2}}+\frac{bcx\left(1-c^2x^2\right)}{8d\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```


```
[Out] (b*c*x*(1 - c^2*x^2))/(8*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*
x^2)) - (a + b*ArcCosh[c*x])/(4*e*(d + e*x^2)^2) + (b*c*(2*c^2*d + e)*Sqrt[
-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8
*d^(3/2)*e*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{(bc)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^2}\,dx}{4e}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\left(d+ex^2\right)^2}\,dx}{4e\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{bcx\left(1-c^2x^2\right)}{8d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{\left(bc\left(2c^2d+e\right)\sqrt{-1+c^2x^2}\right)}{8de\left(c^2d+e\right)\sqrt{-1}}$$

$$= \frac{bcx\left(1-c^2x^2\right)}{8d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{\left(bc\left(2c^2d+e\right)\sqrt{-1+c^2x^2}\right)}{8de\left(c^2d+\right.}$$

$$= \frac{bcx\left(1-c^2x^2\right)}{8d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{bc\left(2c^2d+e\right)\sqrt{-1+c^2x^2}\,\mathrm{ta}}{8d^{3/2}e\left(c^2d+e\right)^{3/2}\sqrt{-}}$$

Mathematica [A] time = 0.980429, size = 183, normalized size = 1.03

$$\frac{1}{8}\left(-\frac{\frac{2a}{e}+\frac{bcx\sqrt{cx-1}\sqrt{cx+1}\left(d+ex^2\right)}{d\left(c^2d+e\right)}}{\left(d+ex^2\right)^2} - \frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(2c^2d+e\right)\tan^{-1}\left(\frac{x\sqrt{c^2(-d)-e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{d^{3/2}e\sqrt{c^2x^2-1}\left(c^2(-d)-e\right)^{3/2}} - \frac{2b\cosh^{-1}(cx)}{e\left(d+ex^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (-(((2*a)/e + (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(d*(c^2*d +
e)))/(d + e*x^2)^2) - (2*b*ArcCosh[c*x])/(e*(d + e*x^2)^2) - (b*c*(2*c^2*d
+ e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d) - e]*x)/(Sqrt[d]*Sq
rt[-1 + c^2*x^2])])/(d^(3/2)*(-(c^2*d) - e)^(3/2)*e*Sqrt[-1 + c^2*x^2]))/8
```

Maple [B] time = 0.03, size = 2443, normalized size = 13.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/4*c^4*a/e/(c^2*e*x^2+c^2*d)^2-1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2*arccosh(c*
x)-1/8*c^8*b*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2
*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/
2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c
^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*x^2*d-
1/8*c^8*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*
e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))
^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*
d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*d^2+1/8*c
^8*b*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1
/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((
-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e
)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*x^2*d+1/8*c^
8*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/
2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-
c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)
/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*d^2-1/8*c^5*b
*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(c*x*e-(-c^2*d*e)
^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2*x-3/16*c^6*b*e^4*(c*x
+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+
e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/
2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-
c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*x^2-3/16*c^6*b*e^3*(c*x+1)^
(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e
)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e
)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*
d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*d+3/16*c^6*b*e^4*(c*x+1)^(1/2)*
(c*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2
)/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c
^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)
^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*x^2+3/16*c^6*b*e^3*(c*x+1)^(1/2)*(c
*x-1)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/
(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2
*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(
1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*d-1/8*c^3*b*e^4*(c*x+1)^(1/2)*(c*x-1)
^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(c*x*e-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2
))^2/((-c^2*d*e)^(1/2)+e)^2/d*x-1/16*c^4*b*e^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)/
(c*x*e+(-c^2*d*e)^(1/2))/(c*x*e-(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(e-(
-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*d*e)^(1/2)/d/(c^2*x^2-1)^(1
/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/
```

```
(c*x*e-(-c^2*d*e)^(1/2)))*x^2-1/16*c^4*b*e^4*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c
*x*e+(-c^2*d*e)^(1/2))/(c*x*e-(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(e-(-c
^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*d*e)^(1/2)/(c^2*x^2-1)^(1/2)*
ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x
*e-(-c^2*d*e)^(1/2)))+1/16*c^4*b*e^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c
^2*d*e)^(1/2))/(c*x*e-(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(e-(-c^2*d*e)^
(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*d*e)^(1/2)/d/(c^2*x^2-1)^(1/2)*ln(-2*
(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(
-c^2*d*e)^(1/2)))*x^2+1/16*c^4*b*e^4*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e+(-c
^2*d*e)^(1/2))/(c*x*e-(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(e-(-c^2*d*e)^
(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*d*e)^(1/2)/(c^2*x^2-1)^(1/2)*ln(-2*(-
(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c
^2*d*e)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\left(\frac{c^4\log\left(ex^2+d\right)}{c^4d^2e+2\,c^2de^2+e^3}+8\,c\int\frac{1}{4\left(c^3e^3x^7+\left(2\,c^3de^2-ce^3\right)x^5-cd^2ex+\left(c^3d^2e-2\,cde^2\right)x^3+\left(c^2e^3x^6+\left(2\,c^2de^2-e^3\right)x\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(e*x^2 + d)/(c^4*d^2*e + 2*c^2*d*e^2 + e^3) + 8*c*integrate(1/
4/(c^3*e^3*x^7 + (2*c^3*d*e^2 - c*e^3)*x^5 - c*d^2*e*x + (c^3*d^2*e - 2*c*d
*e^2)*x^3 + (c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 - d^2*e + (c^2*d^2*e - 2
*d*e^2)*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x) - (c^4*d^2 + c^2*
d*e + (c^4*d*e + c^2*e^2)*x^2 - 2*(c^4*d^2 + 2*c^2*d*e + e^2)*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) + (c^4*e^2*x^4 + 2*c^4*d*e*x^2 + c^4*d^2)*log(c*x
 + 1) + (c^4*e^2*x^4 + 2*c^4*d*e*x^2 + c^4*d^2)*log(c*x - 1))/(c^4*d^4*e +
2*c^2*d^3*e^2 + d^2*e^3 + (c^4*d^2*e^3 + 2*c^2*d*e^4 + e^5)*x^4 + 2*(c^4*d^
3*e^2 + 2*c^2*d^2*e^3 + d*e^4)*x^2))*b - 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2
*e)
```

Fricas [B] time = 3.39367, size = 2522, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(2*a + b)*c^4*d^4 + 2*(4*a + b)*c^2*d^3*e + 4*a*d^2*e^2 + 2*(b*c^
4*d^2*e^2 + b*c^2*d*e^3)*x^4 + 4*(b*c^4*d^3*e + b*c^2*d^2*e^2)*x^2 - (2*b*c
^3*d^3 + b*c*d^2*e + (2*b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e + b*c
*d*e^2)*x^2)*sqrt(c^2*d^2 + d*e)*log(-(2*c^2*d^2 - (4*c^4*d^2 + 4*c^2*d*e +
 e^2)*x^2 + d*e - 2*sqrt(c^2*d^2 + d*e)*((2*c^3*d + c*e)*x^2 - c*d) - 2*sqr
t(c^2*x^2 - 1)*(sqrt(c^2*d^2 + d*e)*(2*c^2*d + e)*x + 2*(c^3*d^2 + c*d*e)*x
))/(e*x^2 + d)) - 4*((b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4
*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x + sqrt(c^2*x^2 - 1)) - 4*(
b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*
e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(-c*x + sqrt
(c^2*x^2 - 1)) + 2*sqrt(c^2*x^2 - 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*
c^3*d^3*e + b*c*d^2*e^2)*x))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^
4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3
*e^4)*x^2), -1/8*((2*a + b)*c^4*d^4 + (4*a + b)*c^2*d^3*e + 2*a*d^2*e^2 + (
b*c^4*d^2*e^2 + b*c^2*d*e^3)*x^4 + 2*(b*c^4*d^3*e + b*c^2*d^2*e^2)*x^2 - (2
*b*c^3*d^3 + b*c*d^2*e + (2*b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e +
 b*c*d*e^2)*x^2)*sqrt(-c^2*d^2 - d*e)*arctan((sqrt(-c^2*d^2 - d*e)*sqrt(c^2
*x^2 - 1)*e*x - sqrt(-c^2*d^2 - d*e)*(c*e*x^2 + c*d))/(c^2*d^2 + d*e)) - 2*
((b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2
*e^2 + b*d*e^3)*x^2)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^4*d^4 + 2*b*c^2*
d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*
d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + sqr
t(c^2*x^2 - 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^
2)*x))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4
+ d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)x}{\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d)^3, x)
```

3.509 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=772

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^3}$$

```
[Out] -(b*c*e*x*(1 - c^2*x^2))/(8*d^2*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d
 + e*x^2)) + (a + b*ArcCosh[c*x])/(4*d*(d + e*x^2)^2) + (a + b*ArcCosh[c*x]
)/(2*d^2*(d + e*x^2)) + (a + b*ArcCosh[c*x])^2/(b*d^3) - (b*c*Sqrt[-1 + c^2
*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(5/2)
*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(2*c^2*d + e)*Sqrt[-1
 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8*d
^(5/2)*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((a + b*ArcCosh[c*
x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^3 - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt
[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3) - ((a + b*A
rcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) -
 e])])/(2*d^3) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*
Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^3) - ((a + b*ArcCosh[c*x])*Log[1 + (S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^3) - (b*Pol
yLog[2, -E^(-2*ArcCosh[c*x])])/(2*d^3) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^3) - (b*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3) - (b*PolyLog[
2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^3)
- (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(2*d^3)
```

Rubi [A] time = 1.24614, antiderivative size = 755, normalized size of antiderivative = 0.98, number of steps used = 34, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {5792, 5660, 3718, 2190, 2279, 2391, 5788, 519, 382, 377, 208, 5800, 5562}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] -(b*c*e*x*(1 - c^2*x^2))/(8*d^2*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d
 + e*x^2)) + (a + b*ArcCosh[c*x])/(4*d*(d + e*x^2)^2) + (a + b*ArcCosh[c*x]
)/(2*d^2*(d + e*x^2)) - (b*c*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)
/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (b*c*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d +
 e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8*d^(5/2)*(c^2*d + e)^(3/2)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3) - ((a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3) -
 ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(2*d^3) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^3) + ((a + b*ArcCosh[c*x])*
Log[1 + E^(2*ArcCosh[c*x])])/d^3 - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^3) - (b*PolyLog[2, (Sqrt[e]*E^Ar
cCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3) - (b*PolyLog[2, -((
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^3) - (b*P
olyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d
^3) + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^3)
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^3x} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)^3} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^2} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d^3\left(d+ex^2\right)}\right)$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^3} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^3} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx}{d}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)} + \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(cx)\right)}{d^3} - \frac{(bc)\int \cdots}{\cdots}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^3} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}\cdots\right.}{d^3}$$

$$= -\frac{bcex\left(1-c^2x^2\right)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} + \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)} - \frac{\left(a+b\cos\cdots\right.}{2b\cdots}$$

$$= -\frac{bcex\left(1-c^2x^2\right)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} + \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)} - \frac{bc\sqrt{-1+\cdots}}{2d^{5/2}\sqrt{c\cdots}}$$

$$= -\frac{bcex\left(1-c^2x^2\right)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} + \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)} - \frac{bc\sqrt{-1+\cdots}}{2d^{5/2}\sqrt{c\cdots}}$$

$$= -\frac{bcex\left(1-c^2x^2\right)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} + \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)} - \frac{bc\sqrt{-1+\cdots}}{2d^{5/2}\sqrt{c\cdots}}$$

$$= -\frac{bcex\left(1-c^2x^2\right)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} + \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)} - \frac{bc\sqrt{-1+\cdots}}{2d^{5/2}\sqrt{c\cdots}}$$

Mathematica [F] time = 8.1311, size = 0, normalized size = 0.

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3), x]
```

Maple [C] time = 0.26, size = 1478, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x)
```

```
[Out] 3/4*b*c^4/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*e-1/8*b*c^6/d^2/(c^2
*d+e)/(c^2*e*x^2+c^2*d)^2*e^2*x^4-1/4*b*c^6/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2
*x^2*e+a/d^3*ln(c*x)-1/2*a/d^3*ln(c^2*e*x^2+c^2*d)+1/8*b*c^5/d^2/(c^2*d+e)/
(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*e^2+1/8*b*c^5/d/(c^2*d+
e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*e+1/2*b*c^6/d/(c^2*d+e
)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x^2*e+1/2*b*c^4/d^2/(c^2*d+e)/(c^2*e*x^2
+c^2*d)^2*arccosh(c*x)*x^2*e^2+1/2*a*c^2/d^2/(c^2*e*x^2+c^2*d)+1/4*a*c^4/d/
(c^2*e*x^2+c^2*d)^2-1/8*b*c^6/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2+5/8*b*(c^2*d*(c
^2*d+e))^(1/2)/d^3/(c^2*d+e)^2*e*arctanh(1/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2*e+4*c^2*d+2*e)/(c^4*d^2+c^2*d*e)^(1/2))+b/d^3/(c^2*d+e)*e*arccosh(
c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^3/(c^2*d+e)*e*arccosh(c*
x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+3/4*b*c^2*(c^2*d*(c^2*d+e))^(1
/2)/d^2/(c^2*d+e)^2*arctanh(1/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+4*
c^2*d+2*e)/(c^4*d^2+c^2*d*e)^(1/2))+b*c^2/d^2/(c^2*d+e)*arccosh(c*x)*ln(1+I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b*c^2/d^2/(c^2*d+e)*arccosh(c*x)*ln(1-I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-1/4*b*c^2/d^2/(c^2*d+e)*e*sum((_R1^2+1)
/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))
/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(
4*c^2*d+2*e)*_Z^2+e))+3/4*b*c^6/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)-
1/4*b/d^3/(c^2*d+e)*e*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(
c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(
1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+b/d^3/(c
^2*d+e)*e*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^3/(c^2*d+e)*e*di
log(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-1/4*b/d^3/(c^2*d+e)*e^2*sum((_R1
^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(
1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_
Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/4*b*c^2/d^2/(c^2*d+e)*sum((_R1^2*e+4*c^2*d+e)/
(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/
_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4
*c^2*d+2*e)*_Z^2+e))+b*c^2/d^2/(c^2*d+e)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+
```

```
1)^(1/2)))+b*c^2/d^2/(c^2*d+e)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}a\left(\frac{2ex^2+3d}{d^2e^2x^4+2d^3ex^2+d^4}-\frac{2\log\left(ex^2+d\right)}{d^3}+\frac{4\log\left(x\right)}{d^3}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{e^3x^7+3de^2x^5+3d^2ex^3+d^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)
/d^3 + 4*log(x)/d^3) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(
e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\left(cx\right)+a}{e^3x^7+3de^2x^5+3d^2ex^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(ex^2 + d\right)^3 x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^3*x), x)
```

3.510 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=834

$$\frac{bcx\left(1-c^2x^2\right)e^2}{8d^3\left(dc^2+e\right)\sqrt{cx-1}\sqrt{cx+1}\left(ex^2+d\right)}-\frac{3\left(a+b\cosh^{-1}(cx)\right)^2e}{bd^4}-\frac{\left(a+b\cosh^{-1}(cx)\right)e}{d^3\left(ex^2+d\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)e}{4d^2\left(ex^2+d\right)^2}+\frac{bc\left(2dc\right.}{8}$$

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d^3*x) + (b*c*e^2*x*(1 - c^2*x^2))/(8
*d^3*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2)) - (a + b*ArcCosh
[c*x])/(2*d^3*x^2) - (e*(a + b*ArcCosh[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a
 + b*ArcCosh[c*x]))/(d^3*(d + e*x^2)) - (3*e*(a + b*ArcCosh[c*x])^2)/(b*d^4
) + (b*c*e*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1
+ c^2*x^2])])/(d^(7/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c
*e*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sq
rt[-1 + c^2*x^2])])/(8*d^(7/2)*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - (3*e*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^4 + (3*e*(a
 + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^
2*d) - e])])/(2*d^4) + (3*e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*e*(a + b*ArcCosh[c*
x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2
*d^4) + (3*e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*b*e*PolyLog[2, -E^(-2*ArcCosh[c*x]
)])/(2*d^4) + (3*b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e]))])/(2*d^4) + (3*b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*b*e*PolyLog[2, -((Sqrt[e]*E
^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^4) + (3*b*e*PolyLo
g[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^4)
```

Rubi [A] time = 1.29537, antiderivative size = 815, normalized size of antiderivative = 0.98, number of steps used = 36, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5792, 5662, 95, 5660, 3718, 2190, 2279, 2391, 5788, 519, 382, 377, 208, 5800, 5562}

$$\frac{bcx\left(1-c^2x^2\right)e^2}{8d^3\left(dc^2+e\right)\sqrt{cx-1}\sqrt{cx+1}\left(ex^2+d\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)e}{d^3\left(ex^2+d\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)e}{4d^2\left(ex^2+d\right)^2}+\frac{bc\left(2dc^2+e\right)\sqrt{c^2x^2-1}\tanh^{-1}\left(\right.}{8d^{7/2}\left(dc^2+e\right)^{3/2}\sqrt{cx-1}\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d^3*x) + (b*c*e^2*x*(1 - c^2*x^2))/(8
*d^3*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2)) - (a + b*ArcCosh
[c*x])/(2*d^3*x^2) - (e*(a + b*ArcCosh[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a
 + b*ArcCosh[c*x]))/(d^3*(d + e*x^2)) + (b*c*e*Sqrt[-1 + c^2*x^2]*ArcTanh[(
Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(d^(7/2)*Sqrt[c^2*d + e]*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*e*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*Arc
Tanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8*d^(7/2)*(c^2*d +
 e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*e*(a + b*ArcCosh[c*x])*Log[1 -
 (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*
e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/(2*d^4) + (3*e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^Arc
Cosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*e*(a + b*ArcCos
h[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(2*d^4) - (3*e*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/d^4 + (3
*b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]
))])/(2*d^4) + (3*b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) - e])])/(2*d^4) + (3*b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^4) + (3*b*e*PolyLog[2, (Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^4) - (3*b*e*PolyLog[
2, -E^(2*ArcCosh[c*x])])/(2*d^4)
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
```

```
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^3}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^3x^3} - \frac{3e\left(a+b\cosh^{-1}(cx)\right)}{d^4x} + \frac{e^2x\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^3} + \frac{2e^2x\left(a+b\cosh^{-1}(cx\right.}{d^3\left(d+ex^2\right)^2}\right.$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{d^3} - \frac{(3e)\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^4} + \frac{\left(3e^2\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^4} + \frac{\left(2e^2\right)\int \frac{x\left(a+b\cosh^{-1}(cx\right.}{\left(d+ex^2\right)^2}}{d^3}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^3\left(d+ex^2\right)} + \frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2d^3} - (\ldots$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^3x} - \frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^3\left(d+ex^2\right)} + \frac{3e\left(a\ldots\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^3x} + \frac{bce^2x\left(1-c^2x^2\right)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\ldots\right.}{4d^2\left(\ldots\right.}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^3x} + \frac{bce^2x\left(1-c^2x^2\right)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\ldots\right.}{4d^2\left(\ldots\right.}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^3x} + \frac{bce^2x\left(1-c^2x^2\right)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\ldots\right.}{4d^2\left(\ldots\right.}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^3x} + \frac{bce^2x\left(1-c^2x^2\right)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\ldots\right.}{4d^2\left(\ldots\right.}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^3x} + \frac{bce^2x\left(1-c^2x^2\right)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\ldots\right.}{4d^2\left(\ldots\right.}$$

Mathematica [F] time = 12.0105, size = 0, normalized size = 0.

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3), x]
```

Maple [C] time = 0.403, size = 1928, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x)
```

```
[Out] -1/4*c^4*a*e/d^2/(c^2*e*x^2+c^2*d)^2-3*b/d^4*e^2/(c^2*d+e)*dilog(1+I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))-3*b/d^4*e^2/(c^2*d+e)*dilog(1-I*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))+3/4*b/d^4*e^3/(c^2*d+e)*sum((_R1^2+1)/(_R1^2*e+2*c^2*d
+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-
c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2
+e))+3/4*b/d^4*e^2/(c^2*d+e)*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(a
rccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c
*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+3
/8*c^5*b*x^3/d^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
e^3+1/2*c^7*b/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*x^3*e^2+c^7*b/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
x*e+7/8*c^5*b*x/d^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)*e^2+1/2*c^5*b/x/d/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*e-1/2*a/d^3/x^2-3/2*c^6*b/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x
)*x^2*e^2-3/2*c^4*b*x^2/d^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*e^3-
1/2*c^4*b/x^2/d/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*e-9/4*c^4*b/d^2/
(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*e^2-1/2*c^8*b/d^2/(c^2*d+e)/(c^2
*e*x^2+c^2*d)^2*e^2*x^4-c^8*b/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*x^2*e-3/4*c^6
*b*x^2/d^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*e^2-3/8*c^6*b*x^4/d^3/(c^2*e*x^2+c
^2*d)^2/(c^2*d+e)*e^3-3*c^2*b/d^3/(c^2*d+e)*e*arccosh(c*x)*ln(1+I*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2)))-3*c^2*b/d^3/(c^2*d+e)*e*arccosh(c*x)*ln(1-I*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2)))-5/4*c^2*b*(c^2*d*(c^2*d+e))^(1/2)/d^3/(c^2*d+
e)^2*e*arctanh(1/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e)/(c
^4*d^2+c^2*d*e)^(1/2))-9/4*c^6*b/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*
x)*e+1/2*c^7*b/x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+
3/2*a*e/d^4*ln(c^2*e*x^2+c^2*d)-3*a/d^4*e*ln(c*x)-3/8*c^6*b/d/(c^2*e*x^2+c^
2*d)^2/(c^2*d+e)*e-1/2*c^6*b/x^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)
-1/2*c^8*b/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2+3/4*c^2*b/d^3/(c^2*d+e)*e*sum((_R1
^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)
*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=
```

```
RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3*c^2*b/d^3/(c^2*d+e)*e*dilog(1+I*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-3*c^2*b/d^3/(c^2*d+e)*e*dilog(1-I*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2)))+3/4*c^2*b/d^3/(c^2*d+e)*e^2*sum((_R1^2+1)/(_R1^2*e
+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dil
og((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2
*e)*_Z^2+e))-9/8*b*(c^2*d*(c^2*d+e))^(1/2)/d^4/(c^2*d+e)^2*e^2*arctanh(1/4*
(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e)/(c^4*d^2+c^2*d*e)^(1/
2))-3*b/d^4*e^2/(c^2*d+e)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))-3*b/d^4*e^2/(c^2*d+e)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2)))-c^2*a*e/d^3/(c^2*e*x^2+c^2*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}a\left(\frac{6e^2x^4+9dex^2+2d^2}{d^3e^2x^6+2d^4ex^4+d^5x^2}-\frac{6e\log\left(ex^2+d\right)}{d^4}+\frac{12e\log(x)}{d^4}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{e^3x^9+3de^2x^7+3d^2ex^5+d^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*e^2*x^4 + 9*d*e*x^2 + 2*d^2)/(d^3*e^2*x^6 + 2*d^4*e*x^4 + d^5*x^
2) - 6*e*log(e*x^2 + d)/d^4 + 12*e*log(x)/d^4) + b*integrate(log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{e^3x^9+3de^2x^7+3d^2ex^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^
3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(ex^2 + d\right)^3 x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^3*x^3), x)
```

3.511 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1224

result too large to display

```
[Out] -(b*c*Sqrt[-d]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*e^2*(c^2*d + e)*(Sqrt[-d]
- Sqrt[e]*x)) - (b*c*Sqrt[-d]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*e^2*(c^2*d
+ e)*(Sqrt[-d] + Sqrt[e]*x)) - (Sqrt[-d]*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*
(Sqrt[-d] - Sqrt[e]*x)^2) + (5*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d]
- Sqrt[e]*x)) + (Sqrt[-d]*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d] + Sqr
t[e]*x)^2) - (5*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) -
 (b*c^3*d*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-
d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d]
 + Sqrt[e])^(3/2)*e^(5/2)) - (5*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqr
t[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*Sqrt[c*Sqrt[-d
] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) + (b*c^3*d*ArcTanh[(Sqrt[c
*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c
*x])]])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)*e^(5/2)
) + (5*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[
-d] - Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[
-d] + Sqrt[e]]*e^(5/2)) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2)) - (3*(a +
 b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*
d) - e])]])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2))
- (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sq
rt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2, -((Sqrt[e]*E^Ar
cCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))]])/(16*Sqrt[-d]*e^(5/2)) + (3
*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/
(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) - e]))]])/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2)
)
```

Rubi [A] time = 3.9617, antiderivative size = 1224, normalized size of antiderivative = 1., number of steps used = 80, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.524, Rules used = {5792, 5707, 5802, 96, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{bd\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}e^{5/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}e^{5/2}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c}{8\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{5/2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[-d]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*e^2*(c^2*d + e)*(Sqrt[-d]
- Sqrt[e]*x)) - (b*c*Sqrt[-d]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*e^2*(c^2*d
+ e)*(Sqrt[-d] + Sqrt[e]*x)) - (Sqrt[-d]*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*
(Sqrt[-d] - Sqrt[e]*x)^2) + (5*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d]
- Sqrt[e]*x)) + (Sqrt[-d]*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d] + Sqr
t[e]*x)^2) - (5*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) -
 (b*c^3*d*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-
d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d]
 + Sqrt[e])^(3/2)*e^(5/2)) - (5*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqr
t[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*Sqrt[c*Sqrt[-d
] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) + (b*c^3*d*ArcTanh[(Sqrt[c
*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c
*x])]])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)*e^(5/2)
) + (5*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[
-d] - Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[
-d] + Sqrt[e]]*e^(5/2)) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a +
 b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*
d) - e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*Sqrt[-d]*e^(5/2))
- (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sq
rt[-(c^2*d) - e])])/(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2, -((Sqrt[e]*E^Ar
cCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(16*Sqrt[-d]*e^(5/2)) + (3
*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/
(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) - e]))])/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*Sqrt[-d]*e^(5/2)
)
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
```

```
+ e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
```

```
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int\left(\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^3} - \frac{2d\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{e^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e^2} - \frac{(2d)\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e^2} + \frac{d^2\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx}{e^2}$$

$$= \frac{\int\left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{e^2} - \frac{(2d)\int\left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2} -\right.}{e^2}$$

$$= -\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}e^2} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}e^2} - \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{16e} - \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\,dx}{16e} - \frac{3\int}{}$$

$$= -\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2} + \frac{5\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)^2} - \frac{5\left(a+b\cos\right.}{16e^{5/2}\left(\sqrt{-d}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2} +$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2} +$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2} +$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2} +$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2} +$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2} +$$

Mathematica [C] time = 6.90056, size = 1185, normalized size = 0.97

$$-\frac{5ax}{8e^2\left(ex^2+d\right)}+\frac{adx}{4e^2\left(ex^2+d\right)^2}+\frac{3a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{5/2}}+b\left(-\frac{5\left(\frac{\cosh^{-1}(cx)}{\sqrt{e}x-i\sqrt{d}}+\frac{c\log\left(\frac{2e\left(\sqrt{d}xc^2+i\sqrt{e}-i\sqrt{-dc^2-e}\sqrt{cx-1}\sqrt{cx+1}\right)}{c\sqrt{-dc^2-e}\left(i\sqrt{e}x+\sqrt{d}\right)}\right)}{\sqrt{-dc^2-e}}\right)}{16e^{5/2}}+\frac{5\left(-\frac{\cosh^{-1}(c}{\sqrt{e}x+i\sqrt{}}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((-5*(ArcCosh[c*x]/((-I)*Sqrt[
d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d)
- e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt
[e]*x))])/Sqrt[-(c^2*d) - e]))/(16*e^(5/2)) + (5*(-(ArcCosh[c*x]/(I*Sqrt[d]
 + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) -
e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e
]*x))])/Sqrt[-(c^2*d) - e]))/(16*e^(5/2)) + ((I/16)*Sqrt[d]*((c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/
(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt
[c^2*d + e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/
2))))/e^2 - ((I/16)*Sqrt[d]*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*
(I*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2)
 - (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d
]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sq
rt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/e^2 + (((3*I)/32)*(ArcCosh[c*x]*
(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(
c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[
-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sq
rt[-(c^2*d) - e]))]))/(Sqrt[d]*e^(5/2)) - (((3*I)/32)*(ArcCosh[c*x]*(-ArcCo
sh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) -
 e])])) + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[
-(c^2*d) - e])]))/(Sqrt[d]*e^(5/2)))
```

Maple [C] time = 2.02, size = 3125, normalized size = 2.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 7/4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2
))/e^4/(c^2*d+e)*d-9/4*c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2
)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^
(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)^2*d^2-5/8*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d
+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(
1/2)+5/4*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(
1/2))/e^4/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+5/8*c*b*((2*c^2*d+2*(c^2*d*(c^2
*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2
*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*(c^2*d*(c^2*d+e))
^(1/2)-5/4*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1
/2))/e^4/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)-5/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^
2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*d+7/4*c^3*b*((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)*
d-c^7*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^3*arctanh((c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1
/2))/e^5/(c^2*d+e)^2+c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)
*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))
^(1/2)-e)*e)^(1/2))*d^2/e^5/(c^2*d+e)-c^7*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(
1/2)+e)*e)^(1/2)*d^3*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2
*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^5/(c^2*d+e)^2+c^5*b*((2*c^2*d+2*(c^
2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e
/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*d^2/e^5/(c^2*d+e)-9/4*c^5
*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/
(c^2*d+e)^2*d^2-5/4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*
arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^
(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*d-1/8*c^5*b/e/(c^2*d+e)/(c^2*e*x^2+c^2*d
)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*d-1/8*c^5*b/e^2/(c^2*d+e)/(c^2*e*x^2+c^
2*d)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*d^2+c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))
^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2
```

```
*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*d/e^5/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/
2)+c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^2*arctan((c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2
))/e^5/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(1/2)-c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+
2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*d/e^5/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1
/2)-7/4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^
(1/2))/e^4/(c^2*d+e)^2*d*(c^2*d*(c^2*d+e))^(1/2)+3/8*a/e^2/(d*e)^(1/2)*arct
an(x*e/(d*e)^(1/2))-3/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e^2*d*x+5/8*c*b*(-(2*c^2*
d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)+7/
4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^
4/(c^2*d+e)^2*d*(c^2*d*(c^2*d+e))^(1/2)-c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e)
)^(1/2)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^
2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^5/(c^2*d+e)^2*(c^2*d*(c^2*d+e)
)^(1/2)-5/8*c^6*b/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x^3*d-3/8*c^
6*b/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x*d^2-3/8*c^4*b/e/(c^2*d
+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x*d+5/8*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+
2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)+3/16*c^3*b/e^2/(c^2*d+
e)*d*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c
*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=Roo
tOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3/16*c^3*b/e^2/(c^2*d+e)*d*sum(1/_R1/(_R1
^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)
+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2
*d+2*e)*_Z^2+e))-5/8*c^4*b/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x^3-3
/16*c*b/e/(c^2*d+e)*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-5/8*c^4*a/(c^2*e*x^2+c^2
*d)^2*x^3/e+3/16*c*b/e/(c^2*d+e)*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*
ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4\operatorname{arcosh}(cx)+ax^4}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)x^4}{\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

3.512 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1234

result too large to display

```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*Sqrt[-d]*e*(c^2*d + e)*(Sqrt[-d] -
Sqrt[e]*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*Sqrt[-d]*e*(c^2*d + e)
*(Sqrt[-d] + Sqrt[e]*x)) - (a + b*ArcCosh[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[
-d] - Sqrt[e]*x)^2) - (a + b*ArcCosh[c*x])/(16*d*e^(3/2)*(Sqrt[-d] - Sqrt[e
]*x)) + (a + b*ArcCosh[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)^2)
 + (a + b*ArcCosh[c*x])/(16*d*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c^3*ArcT
anh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*
Sqrt[-1 + c*x])])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3
/2)*e^(3/2)) + (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqr
t[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(8*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*S
qrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/2)) - (b*c^3*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt
[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(8*(c*Sqr
t[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)*e^(3/2)) - (b*c*ArcTanh
[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqr
t[-1 + c*x])])/(8*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e
^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCosh[c*x])*L
og[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d
)^(3/2)*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCos
h[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2, (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(
3/2)) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d
) - e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2))
```

Rubi [A] time = 2.91835, antiderivative size = 1234, normalized size of antiderivative = 1., number of steps used = 62, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.524, Rules used = {5792, 5707, 5802, 96, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}e^{3/2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}e^{3/2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c}{8d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}e^{3/2}}-\frac{b}{8d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*Sqrt[-d]*e*(c^2*d + e)*(Sqrt[-d] -
Sqrt[e]*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*Sqrt[-d]*e*(c^2*d + e)
*(Sqrt[-d] + Sqrt[e]*x)) - (a + b*ArcCosh[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[
-d] - Sqrt[e]*x)^2) - (a + b*ArcCosh[c*x])/(16*d*e^(3/2)*(Sqrt[-d] - Sqrt[e
]*x)) + (a + b*ArcCosh[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)^2)
 + (a + b*ArcCosh[c*x])/(16*d*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c^3*ArcT
anh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*
Sqrt[-1 + c*x])])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3
/2)*e^(3/2)) + (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqr
t[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(8*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*S
qrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/2)) - (b*c^3*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt
[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(8*(c*Sqr
t[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)*e^(3/2)) - (b*c*ArcTanh
[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqr
t[-1 + c*x])])/(8*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e
^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCosh[c*x])*L
og[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d
)^(3/2)*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCos
h[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2, (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(
3/2)) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d
) - e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2))
```

<u>Rule 5792</u>

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
```

```
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
```

```
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \left(-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)^3}+\frac{a+b\cosh^{-1}(cx)}{e\left(d+ex^2\right)^2}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e} - \frac{d\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx}{e}$$

$$= \frac{\int \left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d\left(-de-e^2x^2\right)}\right)dx}{e} - \frac{d\int \left(-\frac{e^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{8(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-ex\right)^3} - \cdots\right.}{}$$

$$= \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{16d} + \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\,dx}{16d} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{4d} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\,dx}{4d} + \cdots$$

$$= -\frac{a+b\cosh^{-1}(cx)}{16\sqrt{-d}e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2} - \frac{a+b\cosh^{-1}(cx)}{16de^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\cosh^{-1}(cx)}{16\sqrt{-d}e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)^2} + \cdots$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \cdots$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \cdots$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \cdots$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \cdots$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \cdots$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16\sqrt{-d}e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)} - \cdots$$

Mathematica [C] time = 6.7376, size = 1193, normalized size = 0.97

$$\frac{ax}{8de\left(ex^2+d\right)}-\frac{ax}{4e\left(ex^2+d\right)^2}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}+b\left(\frac{\frac{\cosh^{-1}(cx)}{\sqrt{e}x-i\sqrt{d}}+\frac{c\log\left(\frac{2e\left(\sqrt{d}xc^2+i\sqrt{e}-i\sqrt{-dc^2-e}\sqrt{cx-1}\sqrt{cx+1}\right)}{c\sqrt{-dc^2-e}\left(i\sqrt{e}x+\sqrt{d}\right)}\right)}{\sqrt{-dc^2-e}}}{16de^{3/2}}-\frac{-\frac{\cosh^{-1}(cx)}{\sqrt{e}x+i\sqrt{d}}-\frac{c\log\left(\frac{2e\left(-i\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}}{16d\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(a*x)/(4*e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*((ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e
]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/S
qrt[-(c^2*d) - e])/(16*d*e^(3/2)) - (-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)
) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[
-(c^2*d) - e])/(16*d*e^(3/2)) - ((I/16)*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(
(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*((-I)*Sqrt[
d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sq
rt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3
*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) +
((I/16)*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]
*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (c^3*Sqrt[d]*(Log
[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqrt[c^2*d + e
]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e
]*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + ((I/32)*(ArcCosh[c*x]*(-ArcCosh[c*x] +
 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] +
Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*
PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]
+ 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]
))]))/(d^(3/2)*e^(3/2)) - ((I/32)*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 +
 (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 -
(Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[
2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))] + 2*P
olyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))/(d
^(3/2)*e^(3/2)))
```

Maple [C] time = 1.105, size = 2269, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)

[Out] -1/8*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2)
)/e^2/d/(c^2*d+e)-1/8*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*a
rctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/
2)+e)*e)^(1/2))/e^2/d/(c^2*d+e)+1/4*c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/
2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*
d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*d+1/4*c^5*b*(-(2*c^2*d-2*(c
^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*d+1/4*
c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e
^3/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(1/2)-1/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+
2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(1
/2)+1/8*c^5*b/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2
+1/8*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2)
)/(c^2*d+e)^2/d/e^2*(c^2*d*(c^2*d+e))^(1/2)-1/4*c*b*(-(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/d/(c^2*d+e)*(c^2*d*(c^2*d+
e))^(1/2)-1/8*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)
^(1/2))/(c^2*d+e)^2/d/e^2*(c^2*d*(c^2*d+e))^(1/2)+1/4*c*b*((2*c^2*d+2*(c^2*
d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/(
(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/d/(c^2*d+e)*(c^2*d*(c^2
*d+e))^(1/2)-1/8*c^6*b/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x*d+1/8
*c^5*b/e*d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)+1/8*c^
4*b*e/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x^3-1/4*c^3*b*(-(2*c^2*d
-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)-1/4
*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3
/(c^2*d+e)+1/4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arcta
nh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)
-e)*e)^(1/2))/(c^2*d+e)^2/e^2+1/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)
+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*
```

```
(c^2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/e^2+1/16*c^3*b/e/(c^2*d+e)*sum(_R
1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2)
)/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+
(4*c^2*d+2*e)*_Z^2+e))-1/16*c^3*b/e/(c^2*d+e)*sum(1/_R1/(_R1^2*e+2*c^2*d+e)
*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)
)+1/8*c^4*a/(c^2*e*x^2+c^2*d)^2/d*x^3-1/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e*x+1/8
*a/d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/16*c*b/d/(c^2*d+e)*sum(_R1/(_R
1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1
)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^
2*d+2*e)*_Z^2+e))-1/16*c*b/d/(c^2*d+e)*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arcco
sh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/8*c
^4*b/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x+1/8*c^6*b/(c^2*d+e)/(c^2*
e*x^2+c^2*d)^2*arccosh(c*x)*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2\,\text{arcosh}\,(cx) + ax^2}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^2}{(ex^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

3.513 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=1234

result too large to display

```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d] -
Sqrt[e]*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*(-d)^(3/2)*(c^2*d + e)
*(Sqrt[-d] + Sqrt[e]*x)) - (a + b*ArcCosh[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqr
t[-d] - Sqrt[e]*x)^2) - (3*(a + b*ArcCosh[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d]
- Sqrt[e]*x)) + (a + b*ArcCosh[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqrt[-d] + Sqr
t[e]*x)^2) + (3*(a + b*ArcCosh[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x
)) - (b*c^3*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt
[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(8*d*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt
[-d] + Sqrt[e])^(3/2)*Sqrt[e]) + (3*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]
*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(8*d^2*Sqrt[c
*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) + (b*c^3*ArcTanh[(
Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[
-1 + c*x])])/(8*d*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)
*Sqrt[e]) - (3*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt
[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(8*d^2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*
Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt
[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(5/2)*Sqrt
[e]) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 - Sqrt[-(c^2*d) - e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCosh[c*x])*
Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-
d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(16*(-
d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
Sqrt[-(c^2*d) - e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -((Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(16*(-d)^(5/2)*Sqrt[e]
) + (3*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])])/(16*(-d)^(5/2)*Sqrt[e])
```

Rubi [A] time = 1.46034, antiderivative size = 1234, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.556, Rules used = {5707, 5802, 96, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8d\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}\sqrt{e}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)c^3}{8d\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}c+\sqrt{e}\right)^{3/2}\sqrt{e}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{cx-1}}\right)c}{8d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}c+\sqrt{e}}\sqrt{e}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d] -
Sqrt[e]*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*(-d)^(3/2)*(c^2*d + e)
*(Sqrt[-d] + Sqrt[e]*x)) - (a + b*ArcCosh[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqr
t[-d] - Sqrt[e]*x)^2) - (3*(a + b*ArcCosh[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d]
- Sqrt[e]*x)) + (a + b*ArcCosh[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqrt[-d] + Sqr
t[e]*x)^2) + (3*(a + b*ArcCosh[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x
)) - (b*c^3*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt
[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt
[-d] + Sqrt[e])^(3/2)*Sqrt[e]) + (3*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]
*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d^2*Sqrt[c
*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) + (b*c^3*ArcTanh[(
Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[
-1 + c*x])]])/(8*d*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)
*Sqrt[e]) - (3*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt
[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d^2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*
Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt
[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/(16*(-d)^(5/2)*Sqrt
[e]) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 - Sqrt[-(c^2*d) - e])]])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCosh[c*x])*
Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(16*(-
d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))]])/(16*(-
d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
Sqrt[-(c^2*d) - e])]])/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -((Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))]])/(16*(-d)^(5/2)*Sqrt[e]
) + (3*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])]])/(16*(-d)^(5/2)*Sqrt[e])
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
```

```
(p > 0 || IGtQ[n, 0])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx = \int\left(-\frac{e^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{8(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-ex\right)^3}-\frac{3e\left(a+b\cosh^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{8(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+ex\right)^3}-\frac{3e\left(a+b\cosh^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\right.}\right.$$

$$= -\frac{(3e)\int\frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{16d^2}-\frac{(3e)\int\frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\,dx}{16d^2}-\frac{(3e)\int\frac{a+b\cosh^{-1}(cx)}{-de-e^2x^2}\,dx}{8d^2}-\frac{e^{3/2}\int\frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^3}}{8(-d)^{3/2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)^2}-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{16d^2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)^2}+\frac{3\left(a+b\right.}{16d^2\sqrt{e}\left(\right.}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\right.}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\right.}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\right.}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\right.}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\right.}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\right.}$$

Mathematica [C] time = 6.3874, size = 1184, normalized size = 0.96

$$\frac{3ax}{8d^2\left(ex^2+d\right)}+\frac{ax}{4d\left(ex^2+d\right)^2}+\frac{3a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}}+b\left(\frac{3\left(\frac{\cosh^{-1}(cx)}{\sqrt{e}x-i\sqrt{d}}+\frac{c\log\left(\frac{2e\left(\sqrt{d}xc^2+i\sqrt{e}-i\sqrt{-dc^2-e}\sqrt{cx-1}\sqrt{cx+1}\right)}{c\sqrt{-dc^2-e}\left(i\sqrt{e}x+\sqrt{d}\right)}\right)}{\sqrt{-dc^2-e}}\right)}{16d^2\sqrt{e}}-\frac{3\left(-\frac{\cosh^{-1}(cx)}{\sqrt{e}x+i\sqrt{d}}-\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*((3*(ArcCosh[c*x]/((-I)*Sqrt[d] +
Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x
))])/Sqrt[-(c^2*d) - e]))/(16*d^2*Sqrt[e]) - (3*(-(ArcCosh[c*x]/(I*Sqrt[d]
+ Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e
]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]
*x))])/Sqrt[-(c^2*d) - e]))/(16*d^2*Sqrt[e]) + ((I/16)*((c*Sqrt[-1 + c*x]*S
qrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt
[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*
d + e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))
/d^(3/2) - ((I/16)*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I*Sqrt[d
] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (c^3*S
qrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqr
t[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))
]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/d^(3/2) + (((3*I)/32)*(ArcCosh[c*x]*(-Arc
Cosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d
) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) -
e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2
*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]))]))/(d^(5/2)*Sqrt[e]) - (((3*I)/32)*(ArcCosh[c*x]*(-ArcCosh[c*
x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]
)) + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d
) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2
*d) - e])]))/(d^(5/2)*Sqrt[e]))
```

Maple [C] time = 1.283, size = 3128, normalized size = 2.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 3/8*a/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+5/8*c^6*b/(c^2*d+e)/(c^2*e*x^
2+c^2*d)^2*arccosh(c*x)*x-c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^
(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*
d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)-c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^
(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c
^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)+3/16*c*b/d^2/(c^2*d+e)*e*s
um(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^
(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*
_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3/16*c*b/d^2/(c^2*d+e)*e*sum(1/_R1/(_R1^2*e+2*c
^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((
_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*
_Z^2+e))+7/4*c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e
)*e)^(1/2))/(c^2*d+e)^2/e^2+7/4*c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e
)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/e^2-1/8*c^5*b/(c^2*e*x^2+c^2*d)^2/(
c^2*d+e)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+3/8*c^4*b/d^2/(c^2*e*x^2+c^2*d)^2/(c^2
*d+e)*arccosh(c*x)*x^3*e^2+5/8*c^4*b/d/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccos
h(c*x)*x*e+3/8*c^6*b*e/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x^3+3/8
*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d^
2/(c^2*d+e)^2/e*(c^2*d*(c^2*d+e))^(1/2)-3/4*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^2/d^2/(c^2*d+e)*(c^2*d*(c^2*d+e)
)^(1/2)+5/4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh(
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)
*e)^(1/2))/(c^2*d+e)^2/d/e^2*(c^2*d*(c^2*d+e))^(1/2)-c^3*b*(-(2*c^2*d-2*(c^
2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/d/(c^2*d+e)*(c^2*d*
(c^2*d+e))^(1/2)-5/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*
arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1
/2)+e)*e)^(1/2))/(c^2*d+e)^2/d/e^2*(c^2*d*(c^2*d+e))^(1/2)+c^3*b*((2*c^2*d+
2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/d/(c^2*d+e)*(c^2
*d*(c^2*d+e))^(1/2)-3/8*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)
*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(
```

```
1/2)+e)*e)^(1/2))/d^2/(c^2*d+e)^2/e*(c^2*d*(c^2*d+e))^(1/2)+3/4*c*b*((2*c^2
*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^2/d^2/(c^2*d+e)
*(c^2*d*(c^2*d+e))^(1/2)-5/4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*
e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)-e)*e)^(1/2))/e^2/d/(c^2*d+e)-5/4*c^3*b*((2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c
^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^2/d/(c^2*d+e)+c^7*b*((2*c^2*d
+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*d+c
^7*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^
3/(c^2*d+e)^2*d+c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arct
anh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2
)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(1/2)-c^5*b*((2*c^2*d+2*(c
^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*(c^2*d*(
c^2*d+e))^(1/2)+3/4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*
arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^
(1/2)-e)*e)^(1/2))/d/(c^2*d+e)^2/e-3/8*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(
1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e/d^2/(c^2*d+e)+3/4*c^3*b*((2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)^2/e-3/8*c*b
*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e/d^2/(c^
2*d+e)-1/8*c^5*b/d/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^2*e-3/16*c^3*b/d/(c^2*d+e)*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*
ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+3/16*c^3*b/d/
(c^2*d+e)*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/
2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R
1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/4*c^4*a*x/d/(c^2*e*x^2+c^2*d)^2+3/
8*c^2*a/d^2*x/(c^2*e*x^2+c^2*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arcosh}(cx) + a}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(ex^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^3, x)
```

3.514 $\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]), x]
```

Rubi [A] time = 0.0330496, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$$

Mathematica [A] time = 5.76263, size = 0, normalized size = 0.

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]), x]
```

Maple [A] time = 0.862, size = 0, normalized size = 0.

$$\int (a + b\text{arccosh}\,(cx))\,\sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex^2 + d}(b\,\text{arcosh}\,(cx) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\text{acosh}\,(cx))\,\sqrt{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arcosh}(cx) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a), x)
```

3.515 $\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2], x]
```

Rubi [A] time = 0.0302334, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 3.74189, size = 0, normalized size = 0.

$$\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2], x]
```

Maple [A] time = 0.434, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(cx)) \frac{1}{\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b \operatorname{arcosh}(cx) + a}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{acosh}(cx)}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(cx) + a}{\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.516 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\sqrt{c^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (x*(a + b*ArcCosh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*Sqrt[-1 + c^2*x^2]*ArcTan
h[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.191738, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {191, 5705, 12, 519, 444, 63, 217, 206}

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\sqrt{c^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*Sqrt[-1 + c^2*x^2]*ArcTan
h[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
```

```
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx &= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-(bc)\int\frac{x}{d\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d+ex^2}}\,dx\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{(bc)\int\frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d+ex^2}}\,dx}{d}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{x}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2d\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{\left(b\sqrt{-1+c^2x^2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{cd\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{\left(b\sqrt{-1+c^2x^2}\right)\mathrm{Subst}\left(\int\frac{1}{1-\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{cd\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}
$$

Mathematica [C] time = 3.11816, size = 556, normalized size = 5.5

$$
\frac{\frac{2b(cx-1)^{3/2}\sqrt{\frac{(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)}{(cx-1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(\frac{c\left(\sqrt{e}-ic\sqrt{d}\right)\left(\sqrt{e}x+i\sqrt{d}\right)\sqrt{\frac{\frac{ic\sqrt{d}}{\sqrt{e}}+c(-x)+\frac{i\sqrt{e}x}{\sqrt{d}}+1}{1-cx}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}x}{\sqrt{d}}-1}{2-2cx}}\right),\frac{4ic\sqrt{d}\sqrt{e}}{\left(c\sqrt{d}+i\sqrt{e}\right)^2}\right)}{cx-1}+c\sqrt{d}\left(-c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{\left(c^2d+e\right)\left(d+ex^2\right)}{de(cx-1)^2}}\sqrt{-\cdots}\right.}{c\sqrt{cx+1}\left(c^2d+e\right)\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}x}{\sqrt{d}}-1}{1-cx}}}\cdots}{d\sqrt{d+ex^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (a*x + b*x*ArcCosh[c*x] + (2*b*(-1 + c*x)^(3/2)*Sqrt[((c*Sqrt[d] - I*Sqrt[e
])*(1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(-1 + c*x))]*((c*((-I)*c*Sqrt[d] + S
```

```
qrt[e])*(I*Sqrt[d] + Sqrt[e]*x)*Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*
Sqrt[e]*x)/Sqrt[d])/(1 - c*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/
Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqr
t[e])/(c*Sqrt[d] + I*Sqrt[e])^2])/(-1 + c*x) + c*Sqrt[d]*(-(c*Sqrt[d]) + I*
Sqrt[e])*Sqrt[((c^2*d + e)*(d + e*x^2))/(d*e*(-1 + c*x)^2)]*Sqrt[-((-1 + (I
*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticPi[(
2*c*Sqrt[d])/(c*Sqrt[d] + I*Sqrt[e]), ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqr
t[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e
])/(c*Sqrt[d] + I*Sqrt[e])^2]))/(c*(c^2*d + e)*Sqrt[1 + c*x]*Sqrt[-((-1 + (
I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]))/(d*Sqrt[d
 + e*x^2])
```

Maple [F] time = 0.536, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(cx)) \left(ex^2 + d\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94761, size = 736, normalized size = 7.29

$$\left[\frac{4\sqrt{ex^2 + d}bex\log\left(cx + \sqrt{c^2x^2 - 1}\right) + 4\sqrt{ex^2 + d}aex + \left(bex^2 + bd\right)\sqrt{e}\log\left(8c^4e^2x^4 + c^4d^2 - 6c^2de + 8\left(c^4de - c^2e^2\right)x^2 - \right.}{4\left(de^2x^2 + d^2e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(e*x^2 + d)*b*e*x*log(c*x + sqrt(c^2*x^2 - 1)) + 4*sqrt(e*x^2 +
 d)*a*e*x + (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e
 + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 -
 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2))/(d*e^2*x^2 + d^2*e), 1/2*(2*sqrt(e*x^2
+ d)*b*e*x*log(c*x + sqrt(c^2*x^2 - 1)) + 2*sqrt(e*x^2 + d)*a*e*x + (b*e*x^
2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sq
rt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)))/(d*e
^2*x^2 + d^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acosh}(cx)}{\left(d + ex^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(ex^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

3.517 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2b\sqrt{1-c^2x^2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}}$$

```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*d*(c^2*d + e)*Sqrt[d + e*x^2]) + (x*
(a + b*ArcCosh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCosh[c*x]))/
(3*d^2*Sqrt[d + e*x^2]) + (2*b*Sqrt[1 - c^2*x^2]*ArcTan[(Sqrt[e]*Sqrt[1 - c
^2*x^2])/(c*Sqrt[d + e*x^2])])/(3*d^2*Sqrt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.182742, antiderivative size = 190, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {192, 191, 5705, 12, 519, 571, 78, 63, 217, 206}

$$\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{2b\sqrt{c^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\left(1-c^2x^2\right)}{3d\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*(1 - c^2*x^2))/(3*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[d +
e*x^2]) + (x*(a + b*ArcCosh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*Ar
cCosh[c*x]))/(3*d^2*Sqrt[d + e*x^2]) - (2*b*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqr
t[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(3*d^2*Sqrt[e]*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - (bc)\int \frac{x\left(3d+2ex^2\right)}{3d^2\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^{3/2}}\,dx$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x\left(3d+2ex^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^{3/2}}\,dx}{3d^2}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{x\left(3d+2ex^2\right)}{\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3d^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\text{Subst}\left(\int \frac{3d+2ex}{\sqrt{-1+c^2x}(d+ex)^{3/2}}\,dx, \ldots\right.}{6d^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{bc\left(1-c^2x^2\right)}{3d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d+ex^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bc \ldots}{\ldots}$$

$$= \frac{bc\left(1-c^2x^2\right)}{3d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d+ex^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(2 \ldots}{\ldots}$$

$$= \frac{bc\left(1-c^2x^2\right)}{3d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d+ex^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(2 \ldots}{\ldots}$$

$$= \frac{bc\left(1-c^2x^2\right)}{3d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d+ex^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{2b \ldots}{\ldots}$$

Mathematica [C] time = 2.24151, size = 633, normalized size = 3.48

$$\frac{\frac{4b(cx-1)^{3/2}\left(d+ex^2\right)\sqrt{\frac{(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)}{(cx-1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(\frac{c\left(\sqrt{e}-ic\sqrt{d}\right)\left(\sqrt{e}x+i\sqrt{d}\right)\sqrt{\frac{\frac{ic\sqrt{d}}{\sqrt{e}}+c(-x)+\frac{i\sqrt{e}x}{\sqrt{d}}+1}{1-cx}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}x}{\sqrt{d}}-1}{2-2cx}}\right),\frac{4ic\sqrt{d}\sqrt{e}}{\left(c\sqrt{d}+i\sqrt{e}\right)^2}\right)}{cx-1}+c\sqrt{d}\left(-c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{\left(c^2d+e\right)(d+\ldots}{de(cx-1)^{\ldots}}}\ldots\right)}{cd^2\sqrt{cx+1}\left(c^2d+e\right)\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}x}{\sqrt{d}}-1}{1-cx}}}\ldots}{3\left(d+ex\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (-((b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(d*(c^2*d + e))) + (a*x*(
3*d + 2*e*x^2))/d^2 + (b*x*(3*d + 2*e*x^2)*ArcCosh[c*x])/d^2 + (4*b*(-1 + c
*x)^(3/2)*Sqrt[((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])
*(-1 + c*x))]*(d + e*x^2)*((c*((-I)*c*Sqrt[d] + Sqrt[e])*(I*Sqrt[d] + Sqrt[
e]*x)*Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(1 - c
*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/S
qrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] + I*Sqrt[
e])^2])/(-1 + c*x) + c*Sqrt[d]*(-(c*Sqrt[d]) + I*Sqrt[e])*Sqrt[((c^2*d + e)
*(d + e*x^2))/(d*e*(-1 + c*x)^2)]*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((
I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticPi[(2*c*Sqrt[d])/(c*Sqrt[d] +
I*Sqrt[e]), ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/Sqrt
[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] + I*Sqrt[e])
^2]))/(c*d^2*(c^2*d + e)*Sqrt[1 + c*x]*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] +
 c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]))/(3*(d + e*x^2)^(3/2))
```

Maple [F] time = 0.806, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(cx)) \left(ex^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} a \left(\frac{2x}{\sqrt{ex^2 + d}d^2} + \frac{x}{\left(ex^2 + d\right)^{\frac{3}{2}} d} \right) + b \int \frac{\log\left(cx + \sqrt{cx + 1}\sqrt{cx - 1}\right)}{\left(ex^2 + d\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d)^(5/2), x)
```

Fricas [B] time = 2.52482, size = 1503, normalized size = 8.26

$$\left[\frac{\left(bc^2d^3 + \left(bc^2de^2 + be^3\right)x^4 + bd^2e + 2\left(bc^2d^2e + bde^2\right)x^2\right)\sqrt{e}\log\left(8c^4e^2x^4 + c^4d^2 - 6c^2de + 8\left(c^4de - c^2e^2\right)x^2 - 4\left(2c^3ex\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b
*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e -
 c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2
+ d)*sqrt(e) + e^2) + 2*(2*(b*c^2*d*e^2 + b*e^3)*x^3 + 3*(b*c^2*d^2*e + b*d
*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 2*(2*(a*c^2*d*e^2 +
 a*e^3)*x^3 + 3*(a*c^2*d^2*e + a*d*e^2)*x - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqr
t(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*
e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2), 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 +
 b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*
(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e
^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + (2*(b*c^2*d*e^2 + b*e^3)*x^3 + 3
*(b*c^2*d^2*e + b*d*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) +
(2*(a*c^2*d*e^2 + a*e^3)*x^3 + 3*(a*c^2*d^2*e + a*d*e^2)*x - (b*c*d*e^2*x^2
 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c
^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

3.518 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex^2)^{7/2}}\,dx$

Optimal. Leaf size=284

$$\frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}+\frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{2bc\left(1-c^2x^2\right)\left(3c^2d+2e\right)}{15d^2\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)^2\sqrt{d+ex^2}}-\frac{8b}{}$$

```
[Out] (b*c*(1 - c^2*x^2))/(15*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x
^2)^(3/2)) + (2*b*c*(3*c^2*d + 2*e)*(1 - c^2*x^2))/(15*d^2*(c^2*d + e)^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]) + (x*(a + b*ArcCosh[c*x]))/(5*d
*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcCosh[c*x]))/(15*d^2*(d + e*x^2)^(3/2))
 + (8*x*(a + b*ArcCosh[c*x]))/(15*d^3*Sqrt[d + e*x^2]) - (8*b*Sqrt[-1 + c^2
*x^2]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(15*d^3*Sq
rt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.801408, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {192, 191, 5705, 12, 519, 6715, 949, 78, 63, 217, 206}

$$\frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}+\frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{2bc\left(1-c^2x^2\right)\left(3c^2d+2e\right)}{15d^2\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)^2\sqrt{d+ex^2}}-\frac{8b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^(7/2),x]
```


```
[Out] (b*c*(1 - c^2*x^2))/(15*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x
^2)^(3/2)) + (2*b*c*(3*c^2*d + 2*e)*(1 - c^2*x^2))/(15*d^2*(c^2*d + e)^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]) + (x*(a + b*ArcCosh[c*x]))/(5*d
*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcCosh[c*x]))/(15*d^2*(d + e*x^2)^(3/2))
 + (8*x*(a + b*ArcCosh[c*x]))/(15*d^3*Sqrt[d + e*x^2]) - (8*b*Sqrt[-1 + c^2
*x^2]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(15*d^3*Sq
rt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
```

```
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{7/2}}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}-(bc)\int\frac{x\left(15d^2+2\right.}{15d^3\sqrt{-1+cx}}$$

$$=\frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}-\frac{(bc)\int\frac{x\left(15d^2+20dex^2+8e\right.}{\sqrt{-1+cx}\sqrt{1+cx}\left(d+e\right.}}{15d^3}$$

$$=\frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{x\left(15\right.}{\sqrt{-}}}{15d^3\sqrt{-1+c}}$$

$$=\frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\mathrm{Subs}}{30d^3\sqrt{-}}$$

$$=\frac{bc\left(1-c^2x^2\right)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+$$

$$=\frac{bc\left(1-c^2x^2\right)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^{3/2}}+\frac{2bc\left(3c^2d+2e\right)\left(1-c^2x^2\right)}{15d^2\left(c^2d+e\right)^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d+ex^2}}+\frac{x\left(\right.}{}$$

$$=\frac{bc\left(1-c^2x^2\right)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^{3/2}}+\frac{2bc\left(3c^2d+2e\right)\left(1-c^2x^2\right)}{15d^2\left(c^2d+e\right)^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d+ex^2}}+\frac{x\left(\right.}{}$$

$$=\frac{bc\left(1-c^2x^2\right)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^{3/2}}+\frac{2bc\left(3c^2d+2e\right)\left(1-c^2x^2\right)}{15d^2\left(c^2d+e\right)^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d+ex^2}}+\frac{x\left(\right.}{}$$

$$=\frac{bc\left(1-c^2x^2\right)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^{3/2}}+\frac{2bc\left(3c^2d+2e\right)\left(1-c^2x^2\right)}{15d^2\left(c^2d+e\right)^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{d+ex^2}}+\frac{x\left(\right.}{}$$

Mathematica [C] time = 3.79011, size = 685, normalized size = 2.41

$$\frac{16b(cx-1)^{3/2}\left(d+ex^2\right)^2\sqrt{\frac{(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)}{(cx-1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(\frac{c\left(\sqrt{e}-ic\sqrt{d}\right)\left(\sqrt{e}x+i\sqrt{d}\right)\sqrt{\frac{\frac{ic\sqrt{d}}{\sqrt{e}}+c(-x)+\frac{i\sqrt{e}x}{\sqrt{d}}+1}{1-cx}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}x}{\sqrt{d}}-1}{2-2cx}}\right),\frac{4ic\sqrt{d}\sqrt{e}}{\left(c\sqrt{d}+i\sqrt{e}\right)^2}\right)}{cx-1}+c\sqrt{d}\left(-c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{\left(c^2d+e\right)(\cdots}{de(cx-\cdots}}\cdots\right.}{cd^3\sqrt{cx+1}\left(c^2d+e\right)\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}x}{\sqrt{d}}-1}{1-cx}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] ((a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4))/d^3 - (b*c*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(d + e*x^2)*(e*(5*d + 4*e*x^2) + c^2*d*(7*d + 6*e*x^2)))/(d^2*(c^2*d
+ e)^2) + (b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcCosh[c*x])/d^3 + (16*b*
(-1 + c*x)^(3/2)*Sqrt[((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))/((c*Sqrt[d] + I*S
qrt[e])*(-1 + c*x))]*(d + e*x^2)^2*((c*((-I)*c*Sqrt[d] + Sqrt[e])*(I*Sqrt[d
] + Sqrt[e]*x)*Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d
])/(1 - c*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*S
qrt[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d]
+ I*Sqrt[e])^2])/(-1 + c*x) + c*Sqrt[d]*(-(c*Sqrt[d]) + I*Sqrt[e])*Sqrt[((c
^2*d + e)*(d + e*x^2))/(d*e*(-1 + c*x)^2)]*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[
d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticPi[(2*c*Sqrt[d])/(c*S
qrt[d] + I*Sqrt[e]), ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt
[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] + I
*Sqrt[e])^2]))/(c*d^3*(c^2*d + e)*Sqrt[1 + c*x]*Sqrt[-((-1 + (I*Sqrt[e]*x)/
Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]))/(15*(d + e*x^2)^(5/2))
```

Maple [F] time = 1.055, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(cx))\left(ex^2+d\right)^{-\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{15}a\left(\frac{8x}{\sqrt{ex^2+d}d^3}+\frac{4x}{\left(ex^2+d\right)^{\frac{3}{2}}d^2}+\frac{3x}{\left(ex^2+d\right)^{\frac{5}{2}}d}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\left(ex^2+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*a*(8*x/(sqrt(e*x^2 + d)*d^3) + 4*x/((e*x^2 + d)^(3/2)*d^2) + 3*x/((e*x
^2 + d)^(5/2)*d)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x
^2 + d)^(7/2), x)
```

Fricas [B] time = 3.40401, size = 2782, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(2*(b*c^4*d^5 + 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*e^
5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*
(b*c^4*d^4*e + 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4
+ c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d
- c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + (8*(b*c^4*d^2*e^3
 + 2*b*c^2*d*e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e
^4)*x^3 + 15*(b*c^4*d^4*e + 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1)) + (8*(a*c^4*d^2*e^3 + 2*a*c^2*d*e^4 + a*e^5)*
x^5 + 20*(a*c^4*d^3*e^2 + 2*a*c^2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e
+ 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x - (7*b*c^3*d^4*e + 5*b*c*d^3*e^2 + 2*(3*b*
c^3*d^2*e^3 + 2*b*c*d*e^4)*x^4 + (13*b*c^3*d^3*e^2 + 9*b*c*d^2*e^3)*x^2)*sq
rt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^4*d^8*e + 2*c^2*d^7*e^2 + d^6*e^3 + (c
^4*d^5*e^4 + 2*c^2*d^4*e^5 + d^3*e^6)*x^6 + 3*(c^4*d^6*e^3 + 2*c^2*d^5*e^4
+ d^4*e^5)*x^4 + 3*(c^4*d^7*e^2 + 2*c^2*d^6*e^3 + d^5*e^4)*x^2), 1/15*(4*(b
*c^4*d^5 + 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*e^5)*x^6 + b*
```

```
d^3*e^2 + 3*(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*(b*c^4*d^4*
e + 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^
2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e +
(c^3*d*e - c*e^2)*x^2)) + (8*(b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*e^5)*x^5 +
20*(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e^4)*x^3 + 15*(b*c^4*d^4*e + 2*b*
c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) +
(8*(a*c^4*d^2*e^3 + 2*a*c^2*d*e^4 + a*e^5)*x^5 + 20*(a*c^4*d^3*e^2 + 2*a*c^
2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e + 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x
 - (7*b*c^3*d^4*e + 5*b*c*d^3*e^2 + 2*(3*b*c^3*d^2*e^3 + 2*b*c*d*e^4)*x^4 +
 (13*b*c^3*d^3*e^2 + 9*b*c*d^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d)
)/(c^4*d^8*e + 2*c^2*d^7*e^2 + d^6*e^3 + (c^4*d^5*e^4 + 2*c^2*d^4*e^5 + d^3
*e^6)*x^6 + 3*(c^4*d^6*e^3 + 2*c^2*d^5*e^4 + d^4*e^5)*x^4 + 3*(c^4*d^7*e^2
+ 2*c^2*d^6*e^3 + d^5*e^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcosh}\,(cx)+a}{\left(ex^2+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^(7/2), x)
```

3.519 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=558

$$-\frac{b\sqrt{1-c^2x^2}(fx)^{m+2}\left(\frac{e(m+2)\left(3c^4d^2\left(m^2+12m+35\right)^2+3c^2de(m+7)^2\left(m^2+7m+12\right)+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)}{(m+3)(m+5)(m+7)}+\frac{c^6d^3(m+3)(m+5)(m+7)}{m+1}\right)\text{Hyp}}{c^5f^2(m+2)(m+3)(m+5)(m+7)\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*e*(3*c^2*d*e*(7 + m)^2*(12 + 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2
+ e^2*(360 + 342*m + 119*m^2 + 18*m^3 + m^4))*(f*x)^(2 + m)*(1 - c^2*x^2))/
(c^5*f^2*(3 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e
^2*(3*c^2*d*(7 + m)^2 + e*(30 + 11*m + m^2))*(f*x)^(4 + m)*(1 - c^2*x^2))/(
c^3*f^4*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*(f*x)^(6
 + m)*(1 - c^2*x^2))/(c*f^6*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*
(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 + m)*(a
 + b*ArcCosh[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcCosh[c
*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCosh[c*x]))/(f^7*(7 + m)
) - (b*((c^6*d^3*(3 + m)*(5 + m)*(7 + m))/(1 + m) + (e*(2 + m)*(3*c^2*d*e*(
7 + m)^2*(12 + 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2 + e^2*(360 + 342*
m + 119*m^2 + 18*m^3 + m^4)))/((3 + m)*(5 + m)*(7 + m)))*(f*x)^(2 + m)*Sqrt
[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(c^5*f
^2*(2 + m)*(3 + m)*(5 + m)*(7 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 2.80914, antiderivative size = 529, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {270, 5790, 12, 1610, 1809, 1267, 459, 365, 364}

$$\frac{3d^2e(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d^3(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{3de^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\cosh^{-1}(cx)\right)}{f^7(m+7}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*e*(3*c^2*d*e*(7 + m)^2*(12 + 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2
+ e^2*(360 + 342*m + 119*m^2 + 18*m^3 + m^4))*(f*x)^(2 + m)*(1 - c^2*x^2))/
(c^5*f^2*(3 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e
^2*(3*c^2*d*(7 + m)^2 + e*(30 + 11*m + m^2))*(f*x)^(4 + m)*(1 - c^2*x^2))/(
c^3*f^4*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*(f*x)^(6
```

```
 + m)*(1 - c^2*x^2))/(c*f^6*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*
(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 + m)*(a
 + b*ArcCosh[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcCosh[c
*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCosh[c*x]))/(f^7*(7 + m)
) - (b*c*(d^3/(2 + 3*m + m^2) + (e*(3*c^2*d*e*(7 + m)^2*(12 + 7*m + m^2) +
3*c^4*d^2*(35 + 12*m + m^2)^2 + e^2*(360 + 342*m + 119*m^2 + 18*m^3 + m^4))
)/(c^6*(3 + m)^2*(5 + m)^2*(7 + m)^2))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hype
rgeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
```

```
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{3de^2(fx$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{3de^2(fx$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{3de^2(fx$$

$$= \frac{be^3(fx)^{6+m}\left(1-c^2x^2\right)}{cf^6(7+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^3$$

$$= \frac{be^2\left(3c^2d(7+m)^2+e\left(30+11m+m^2\right)\right)(fx)^{4+m}\left(1-c^2x^2\right)}{c^3f^4(5+m)^2(7+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be^3(fx)^6}{cf^6(7+m)^2}$$

$$= \frac{be\left(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\left(360+3\right.\right.}{c^5f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+}}$$

$$= \frac{be\left(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\left(360+3\right.\right.}{c^5f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+}}$$

$$= \frac{be\left(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\left(360+3\right.\right.}{c^5f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+}}$$

Mathematica [A] time = 1.38727, size = 397, normalized size = 0.71

$$x(fx)^m\left(-\frac{3bcd^2ex^3\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+4}{2},\frac{m+6}{2},c^2x^2\right)}{\left(m^2+7m+12\right)\sqrt{cx-1}\sqrt{cx+1}} - \frac{bcd^3x\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+}{2}\right.}{\left(m^2+3m+2\right)\sqrt{cx-1}\sqrt{cx+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] x*(f*x)^m*((d^3*(a + b*ArcCosh[c*x]))/(1 + m) + (3*d^2*e*x^2*(a + b*ArcCosh
[c*x]))/(3 + m) + (3*d*e^2*x^4*(a + b*ArcCosh[c*x]))/(5 + m) + (e^3*x^6*(a
+ b*ArcCosh[c*x]))/(7 + m) - (b*c*e^3*x^7*Sqrt[1 - c^2*x^2]*Hypergeometric2
F1[1/2, 4 + m/2, 5 + m/2, c^2*x^2])/((7 + m)*(8 + m)*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) - (b*c*d^3*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4
 + m)/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c*
d^2*e*x^3*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^
2*x^2])/((12 + 7*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c*d*e^2*x^5*
Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (6 + m)/2, (8 + m)/2, c^2*x^2])/((
5 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Maple [F] time = 5.19, size = 0, normalized size = 0.

$$\int (fx)^m \left(ex^2+d\right)^3 (a+b\operatorname{arccosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ae^3x^6+3\,ade^2x^4+3\,ad^2ex^2+ad^3+\left(be^3x^6+3\,bde^2x^4+3\,bd^2ex^2+bd^3\right)\operatorname{arcosh}(cx)\right)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.520 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=353

$$-\frac{b\sqrt{1-c^2x^2}(fx)^{m+2}\left(\frac{c^4d^2(m+3)(m+5)}{m+1}+\frac{e(m+2)\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)}{(m+3)(m+5)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{c^3f^2(m+2)(m+3)(m+5)\sqrt{cx-1}\sqrt{cx+1}}+\frac{d^2(fx)^{}}{}$$

```
[Out] (b*e*(2*c^2*d*(5 + m)^2 + e*(12 + 7*m + m^2))*(f*x)^(2 + m)*(1 - c^2*x^2))/
(c^3*f^2*(3 + m)^2*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^2*(f*x)^(
4 + m)*(1 - c^2*x^2))/(c*f^4*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2
*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a
+ b*ArcCosh[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))
/(f^5*(5 + m)) - (b*((c^4*d^2*(3 + m)*(5 + m))/(1 + m) + (e*(2 + m)*(2*c^2*
d*(5 + m)^2 + e*(12 + 7*m + m^2)))/((3 + m)*(5 + m)))*(f*x)^(2 + m)*Sqrt[1
- c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(c^3*f^2*
(2 + m)*(3 + m)*(5 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.560145, antiderivative size = 332, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {270, 5790, 12, 520, 1267, 459, 365, 364}

$$\frac{d^2(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}-\frac{bc\sqrt{1-c^2x^2}(fx)^{m+2}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*e*(2*c^2*d*(5 + m)^2 + e*(12 + 7*m + m^2))*(f*x)^(2 + m)*(1 - c^2*x^2))/
(c^3*f^2*(3 + m)^2*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^2*(f*x)^(
4 + m)*(1 - c^2*x^2))/(c*f^4*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2
*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a
+ b*ArcCosh[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))
/(f^5*(5 + m)) - (b*c*(d^2/(2 + 3*m + m^2) + (e*(2*c^2*d*(5 + m)^2 + e*(12
+ 7*m + m^2)))/(c^4*(3 + m)^2*(5 + m)^2))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*H
ypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}}{f}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}}{f}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}}{f}$$

$$= \frac{be^2(fx)^{4+m}\left(1-c^2x^2\right)}{cf^4(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}}{f}$$

$$= \frac{be\left(2c^2d(5+m)^2+e\left(12+7m+m^2\right)\right)(fx)^{2+m}\left(1-c^2x^2\right)}{c^3f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be^2(fx)^{4+m}\left(}{cf^4(5+m)^2\sqrt{-1}}$$

$$= \frac{be\left(2c^2d(5+m)^2+e\left(12+7m+m^2\right)\right)(fx)^{2+m}\left(1-c^2x^2\right)}{c^3f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be^2(fx)^{4+m}\left(}{cf^4(5+m)^2\sqrt{-1}}$$

$$= \frac{be\left(2c^2d(5+m)^2+e\left(12+7m+m^2\right)\right)(fx)^{2+m}\left(1-c^2x^2\right)}{c^3f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be^2(fx)^{4+m}\left(}{cf^4(5+m)^2\sqrt{-1}}$$

Mathematica [A] time = 0.49686, size = 293, normalized size = 0.83

$$x(fx)^m\left(-\frac{bcd^2x\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{(m^2+3m+2)\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcdex^3\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+4}{2}\right.}{(m^2+7m+12)\sqrt{cx-1}\sqrt{cx+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] x*(f*x)^m*((d^2*(a + b*ArcCosh[c*x]))/(1 + m) + (2*d*e*x^2*(a + b*ArcCosh[c
*x]))/(3 + m) + (e^2*x^4*(a + b*ArcCosh[c*x]))/(5 + m) - (b*c*d^2*x*Sqrt[1
- c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((2 + 3*m
 + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*e*x^3*Sqrt[1 - c^2*x^2]*Hy
pergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((12 + 7*m + m^2)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*e^2*x^5*Sqrt[1 - c^2*x^2]*Hypergeometric2F
1[1/2, (6 + m)/2, (8 + m)/2, c^2*x^2])/((5 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]))
```

Maple [F] time = 4.069, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2+d)^2 (a+b\,\mathrm{arccosh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\text{arcosh}\,(cx)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.521 $\int (fx)^m \left(d+ex^2\right) \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=198

$$-\frac{b\sqrt{1-c^2x^2}(fx)^{m+2}\left(c^2d(m+3)^2+e(m+1)(m+2)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{cf^2(m+1)(m+2)(m+3)^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{d(fx)^{m+1}\left(a+b\cos\right.}{f(m+1)}$$

```
[Out] -((b*e*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*f^2*(3 + m)^2)) + (d*
(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a + b*A
rcCosh[c*x]))/(f^3*(3 + m)) - (b*(e*(1 + m)*(2 + m) + c^2*d*(3 + m)^2)*(f*x
)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^
2*x^2])/(c*f^2*(1 + m)*(2 + m)*(3 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.20317, antiderivative size = 187, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5786, 460, 126, 365, 364}

$$\frac{d(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}-\frac{b\sqrt{1-c^2x^2}(fx)^{m+2}\left(\frac{c^2d}{m^2+3m+2}+\frac{e}{(m+3)^2}\right){}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2}\right.}{cf^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*e*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*f^2*(3 + m)^2)) + (d*
(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a + b*A
rcCosh[c*x]))/(f^3*(3 + m)) - (b*(e/(3 + m)^2 + (c^2*d)/(2 + 3*m + m^2))*(f
*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
c^2*x^2])/(c*f^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} - \frac{(bc)\int \frac{(fx)^{1+m}(}{\sqrt{}}}{f(3+}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a\right.}{f^3(}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a\right.}{f^3(}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a\right.}{f^3(}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a\right.}{f^3(}$$

Mathematica [A] time = 0.618508, size = 186, normalized size = 0.94

$$x(fx)^m\left(\frac{\frac{\left(d(m+3)+e(m+1)x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{m+1} - \frac{bcex^3\sqrt{1-c^2x^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+4}{2},\frac{m+6}{2},c^2x^2\right)}{(m+4)\sqrt{cx-1}\sqrt{cx+1}}}{m+3} - \frac{bcdx\sqrt{1-c^2x^2}\text{Hypergeometric2}}{\left(m^2+3m+2\right)\sqrt{cx-}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] x*(f*x)^m*(-((b*c*d*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (
4 + m)/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (((d*
(3 + m) + e*(1 + m)*x^2)*(a + b*ArcCosh[c*x]))/(1 + m) - (b*c*e*x^3*Sqrt[1
- c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((4 + m)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(3 + m))
```

Maple [F] time = 3.49, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m}\left(ex^2+d\right)\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(aex^2 + ad + \left(bex^2 + bd\right)\mathrm{arcosh}\left(cx\right)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^m \left(a + b\,\mathrm{acosh}\left(cx\right)\right)\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))*(d + e*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")

[Out] Timed out
```

3.522 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2), x]
```

Rubi [A] time = 0.0635233, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$$

Mathematica [A] time = 9.65127, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2), x]
```

Maple [A] time = 0.619, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b\operatorname{arccosh}(cx))}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{acosh}(cx))}{d + ex^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.523 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2, x]
```

Rubi [A] time = 0.0620624, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 8.43931, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2, x]
```

Maple [A] time = 0.583, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b\operatorname{arccosh}(cx))}{(ex^2 + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{(ex^2 + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{(ex^2 + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

3.524 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3, x]
```

Rubi [A] time = 0.0616717, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

Mathematica [A] time = 15.8221, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3, x]
```

Maple [A] time = 0.589, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b\,\mathrm{arccosh}\,(cx))}{(ex^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)(fx)^m}{(ex^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\,\mathrm{arcosh}\,(cx) + a)(fx)^m}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)(fx)^m}{(ex^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^3, x)
```

3.525 $\int (d+ex^2)^3 (a+b\cosh^{-1}(cx))^2\,dx$

Optimal. Leaf size=609

$$-\frac{4bd^2e\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3c^3}-\frac{8bde^2x^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{25c^3}-\frac{16bde^2\sqrt{cx-1}\sqrt{cx+1}\left(a+\right.}{25c^5}$$

```
[Out] 2*b^2*d^3*x + (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) + (32*b^2
*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 + (8*b^2*d*e^2*x^3)/(75*c^2) + (16*
b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 + (12*b^2*e^3*x^5)/(1225*c^2
) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x]))/c - (4*b*d^2*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])
)/(3*c^3) - (16*b*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(25*c^5) - (32*b*e^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(24
5*c^7) - (2*b*d^2*e*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(3*c) - (8*b*d*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(
25*c^3) - (16*b*e^3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(245*c^5) - (6*b*d*e^2*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
))/(25*c) - (12*b*e^3*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])
)/(245*c^3) - (2*b*e^3*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
))/(49*c) + d^3*x*(a + b*ArcCosh[c*x])^2 + d^2*e*x^3*(a + b*ArcCosh[c*x])^2
 + (3*d*e^2*x^5*(a + b*ArcCosh[c*x])^2)/5 + (e^3*x^7*(a + b*ArcCosh[c*x])^2
)/7
```

Rubi [A] time = 2.09912, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5707, 5654, 5718, 8, 5662, 5759, 30}

$$-\frac{4bd^2e\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3c^3}-\frac{8bde^2x^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{25c^3}-\frac{16bde^2\sqrt{cx-1}\sqrt{cx+1}\left(a+\right.}{25c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcCosh[c*x])^2,x]
```


```
[Out] 2*b^2*d^3*x + (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) + (32*b^2
*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 + (8*b^2*d*e^2*x^3)/(75*c^2) + (16*
b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 + (12*b^2*e^3*x^5)/(1225*c^2
) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x]))/c - (4*b*d^2*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])
)/(3*c^3) - (16*b*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
```

```
(25*c^5) - (32*b*e^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(24
5*c^7) - (2*b*d^2*e*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(3*c) - (8*b*d*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(
25*c^3) - (16*b*e^3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(245*c^5) - (6*b*d*e^2*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
))/(25*c) - (12*b*e^3*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])
)/(245*c^3) - (2*b*e^3*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
))/(49*c) + d^3*x*(a + b*ArcCosh[c*x])^2 + d^2*e*x^3*(a + b*ArcCosh[c*x])^2
 + (3*d*e^2*x^5*(a + b*ArcCosh[c*x])^2)/5 + (e^3*x^7*(a + b*ArcCosh[c*x])^2
)/7
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = \int \left(d^3\left(a+b\cosh^{-1}(cx)\right)^2 + 3d^2ex^2\left(a+b\cosh^{-1}(cx)\right)^2 + 3de^2x^4\left(a+b\cosh^{-1}(\right.\right.$$

$$= d^3\int \left(a+b\cosh^{-1}(cx)\right)^2 \, dx + \left(3d^2e\right)\int x^2\left(a+b\cosh^{-1}(cx)\right)^2 \, dx + \left(3de^2\right)\int x$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)^2 + d^2ex^3\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right.$$

$$= -\frac{2bd^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{2bd^2ex^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\mathrm{c}\right.}{3c}$$

$$= 2b^2d^3x + \frac{2}{9}b^2d^2ex^3 + \frac{6}{125}b^2de^2x^5 + \frac{2}{343}b^2e^3x^7 - \frac{2bd^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\mathrm{c}\right.}{c}$$

$$= 2b^2d^3x + \frac{4b^2d^2ex}{3c^2} + \frac{2}{9}b^2d^2ex^3 + \frac{8b^2de^2x^3}{75c^2} + \frac{6}{125}b^2de^2x^5 + \frac{12b^2e^3x^5}{1225c^2} + \frac{2}{343}b^2e^3$$

$$= 2b^2d^3x + \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} + \frac{2}{9}b^2d^2ex^3 + \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4} + \frac{6}{125}b^2de^2x$$

$$= 2b^2d^3x + \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} + \frac{32b^2e^3x}{245c^6} + \frac{2}{9}b^2d^2ex^3 + \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4} +$$

Mathematica [A] time = 0.842717, size = 453, normalized size = 0.74

$$\frac{11025a^2c^7x\left(35d^2ex^2+35d^3+21de^2x^4+5e^3x^6\right)-210ab\sqrt{cx-1}\sqrt{cx+1}\left(c^6\left(1225d^2ex^2+3675d^3+441de^2x^4+75e^3x^6\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*a
*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*
c^4*e*(1225*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^
2 + 441*d*e^2*x^4 + 75*e^3*x^6)) + 2*b^2*c*x*(25200*e^3 + 840*c^2*e^2*(147*
d + 5*e*x^2) + 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*
d^3 + 42875*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7
*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 29
4*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 +
75*e^3*x^6)))*ArcCosh[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*
e^2*x^4 + 5*e^3*x^6)*ArcCosh[c*x]^2)/(385875*c^7)
```

Maple [A] time = 0.085, size = 632, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^6*(1/7*e^3*c^7*x^7+3/5*c^7*d*e^2*x^5+c^7*d^2*e*x^3+x*c^7*d^3)+b^
2/c^6*(1/25725*e^3*(3675*arccosh(c*x)^2*c^7*x^7-1050*arccosh(c*x)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*c^6*x^6-1260*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^
4*x^4+150*c^7*x^7-1680*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+252
*c^5*x^5-3360*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+560*c^3*x^3+3360*c*x
)+1/375*d*e^2*c^2*(225*arccosh(c*x)^2*c^5*x^5-90*arccosh(c*x)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*c^4*x^4-120*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2
+18*c^5*x^5-240*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+40*c^3*x^3+240*c*x
)+1/9*c^4*d^2*e*(9*arccosh(c*x)^2*c^3*x^3-6*arccosh(c*x)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*c^2*x^2-12*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c^3*x^3+12*
c*x)+d^3*c^6*(arccosh(c*x)^2*c*x-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
+2*c*x))+2*a*b/c^6*(1/7*arccosh(c*x)*e^3*c^7*x^7+3/5*arccosh(c*x)*d*e^2*c^7
```

```
*x^5+arccosh(c*x)*c^7*d^2*e*x^3+arccosh(c*x)*c^7*x*d^3-1/3675*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*(75*c^6*e^3*x^6+441*c^6*d*e^2*x^4+1225*c^6*d^2*e*x^2+90*c^4*
e^3*x^4+3675*c^6*d^3+588*c^4*d*e^2*x^2+2450*c^4*d^2*e+120*c^2*e^3*x^2+1176*
c^2*d*e^2+240*e^3)))
```

Maxima [A] time = 1.15874, size = 923, normalized size = 1.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*e^3*x^7*arccosh(c*x)^2 + 1/7*a^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arccos
h(c*x)^2 + 3/5*a^2*d*e^2*x^5 + b^2*d^2*e*x^3*arccosh(c*x)^2 + a^2*d^2*e*x^3
 + b^2*d^3*x*arccosh(c*x)^2 + 2/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1
)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(c^2*x^2 -
1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4)*arccosh(c*x) - (c^2*x^3 + 6*x)/c^2)*b
^2*d^2*e + 2/25*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqr
t(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*a*b*d*e^2 - 2/375*(15*
(3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2
 - 1)/c^6)*c*arccosh(c*x) - (9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^4)*b^2*d*e^2
 + 2/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x
^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c
)*a*b*e^3 - 2/25725*(105*(5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)
*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c*arccos
h(c*x) - (75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*e^3 + 2
*b^2*d^3*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2*d^3*x + 2*(c*x*arccos
h(c*x) - sqrt(c^2*x^2 - 1))*a*b*d^3/c
```

Fricas [A] time = 1.93084, size = 1338, normalized size = 2.2

$$\frac{1125\left(49\,a^{2}+2\,b^{2}\right)c^{7}e^{3}x^{7}+189\left(49\left(25\,a^{2}+2\,b^{2}\right)c^{7}de^{2}+20\,b^{2}c^{5}e^{3}\right)x^{5}+35\left(1225\left(9\,a^{2}+2\,b^{2}\right)c^{7}d^{2}e+1176\,b^{2}c^{5}de^{2}+2\cdots\right.}{\quad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*e^3*x^7 + 189*(49*(25*a^2 + 2*b^2)*c^7*
d*e^2 + 20*b^2*c^5*e^3)*x^5 + 35*(1225*(9*a^2 + 2*b^2)*c^7*d^2*e + 1176*b^2
*c^5*d*e^2 + 240*b^2*c^3*e^3)*x^3 + 11025*(5*b^2*c^7*e^3*x^7 + 21*b^2*c^7*d
*e^2*x^5 + 35*b^2*c^7*d^2*e*x^3 + 35*b^2*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2
- 1))^2 + 105*(3675*(a^2 + 2*b^2)*c^7*d^3 + 4900*b^2*c^5*d^2*e + 2352*b^2*c
^3*d*e^2 + 480*b^2*c*e^3)*x + 210*(525*a*b*c^7*e^3*x^7 + 2205*a*b*c^7*d*e^2
*x^5 + 3675*a*b*c^7*d^2*e*x^3 + 3675*a*b*c^7*d^3*x - (75*b^2*c^6*e^3*x^6 +
3675*b^2*c^6*d^3 + 2450*b^2*c^4*d^2*e + 1176*b^2*c^2*d*e^2 + 240*b^2*e^3 +
9*(49*b^2*c^6*d*e^2 + 10*b^2*c^4*e^3)*x^4 + (1225*b^2*c^6*d^2*e + 588*b^2*c
^4*d*e^2 + 120*b^2*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2
- 1)) - 210*(75*a*b*c^6*e^3*x^6 + 3675*a*b*c^6*d^3 + 2450*a*b*c^4*d^2*e + 1
176*a*b*c^2*d*e^2 + 240*a*b*e^3 + 9*(49*a*b*c^6*d*e^2 + 10*a*b*c^4*e^3)*x^4
 + (1225*a*b*c^6*d^2*e + 588*a*b*c^4*d*e^2 + 120*a*b*c^2*e^3)*x^2)*sqrt(c^2
*x^2 - 1))/c^7
```

Sympy [A] time = 20.429, size = 996, normalized size = 1.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e**
3*x**7/7 + 2*a*b*d**3*x*acosh(c*x) + 2*a*b*d**2*e*x**3*acosh(c*x) + 6*a*b*d
*e**2*x**5*acosh(c*x)/5 + 2*a*b*e**3*x**7*acosh(c*x)/7 - 2*a*b*d**3*sqrt(c*
*2*x**2 - 1)/c - 2*a*b*d**2*e*x**2*sqrt(c**2*x**2 - 1)/(3*c) - 6*a*b*d*e**2
*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(c**2*x**2 - 1)/(49*
c) - 4*a*b*d**2*e*sqrt(c**2*x**2 - 1)/(3*c**3) - 8*a*b*d*e**2*x**2*sqrt(c**
2*x**2 - 1)/(25*c**3) - 12*a*b*e**3*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 1
6*a*b*d*e**2*sqrt(c**2*x**2 - 1)/(25*c**5) - 16*a*b*e**3*x**2*sqrt(c**2*x**
2 - 1)/(245*c**5) - 32*a*b*e**3*sqrt(c**2*x**2 - 1)/(245*c**7) + b**2*d**3*
x*acosh(c*x)**2 + 2*b**2*d**3*x + b**2*d**2*e*x**3*acosh(c*x)**2 + 2*b**2*d
**2*e*x**3/9 + 3*b**2*d*e**2*x**5*acosh(c*x)**2/5 + 6*b**2*d*e**2*x**5/125
+ b**2*e**3*x**7*acosh(c*x)**2/7 + 2*b**2*e**3*x**7/343 - 2*b**2*d**3*sqrt(
c**2*x**2 - 1)*acosh(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(c**2*x**2 - 1)*acosh(
c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(c**2*x**2 - 1)*acosh(c*x)/(25*c) - 2*b
**2*e**3*x**6*sqrt(c**2*x**2 - 1)*acosh(c*x)/(49*c) + 4*b**2*d**2*e*x/(3*c*
*2) + 8*b**2*d*e**2*x**3/(75*c**2) + 12*b**2*e**3*x**5/(1225*c**2) - 4*b**2
*d**2*e*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c**3) - 8*b**2*d*e**2*x**2*sqrt(c
**2*x**2 - 1)*acosh(c*x)/(25*c**3) - 12*b**2*e**3*x**4*sqrt(c**2*x**2 - 1)*
acosh(c*x)/(245*c**3) + 16*b**2*d*e**2*x/(25*c**4) + 16*b**2*e**3*x**3/(735
```

```
*c**4) - 16*b**2*d*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(25*c**5) - 16*b**2*
e**3*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(245*c**5) + 32*b**2*e**3*x/(245*c
**6) - 32*b**2*e**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/(245*c**7), Ne(c, 0)), (
(a + I*pi*b/2)**2*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/7), T
rue))
```

Giac [A] time = 2.57134, size = 986, normalized size = 1.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*a*b*d^3 + (x*log(c
*x + sqrt(c^2*x^2 - 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 - 1)*log(c*x + sqrt(c^2
*x^2 - 1))/c^2))*b^2*d^3 + a^2*d^3*x + 1/25725*(3675*a^2*x^7 + 210*(35*x^7*
log(c*x + sqrt(c^2*x^2 - 1)) - (5*(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5
/2) + 35*(c^2*x^2 - 1)^(3/2) + 35*sqrt(c^2*x^2 - 1))/c^7)*a*b + (3675*x^7*l
og(c*x + sqrt(c^2*x^2 - 1))^2 + 2*c*((75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^
3 + 1680*x)/c^7 - 105*(5*(c^2*x^2 - 1)^(7/2) + 21*(c^2*x^2 - 1)^(5/2) + 35*
(c^2*x^2 - 1)^(3/2) + 35*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1))/c^
8))*b^2)*e^3 + 1/375*(225*a^2*d*x^5 + 30*(15*x^5*log(c*x + sqrt(c^2*x^2 - 1
)) - (3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1)
)/c^5)*a*b*d + (225*x^5*log(c*x + sqrt(c^2*x^2 - 1))^2 + 2*c*((9*c^4*x^5 +
20*c^2*x^3 + 120*x)/c^5 - 15*(3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2
) + 15*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1))/c^6))*b^2*d)*e^2 + 1
/9*(9*a^2*d^2*x^3 + 6*(3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)^
(3/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*a*b*d^2 + (9*x^3*log(c*x + sqrt(c^2*x^2 -
 1))^2 + 2*c*((c^2*x^3 + 6*x)/c^3 - 3*((c^2*x^2 - 1)^(3/2) + 3*sqrt(c^2*x^2
 - 1))*log(c*x + sqrt(c^2*x^2 - 1))/c^4))*b^2*d^2)*e
```

3.526 $\int \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=359

$$-\frac{8bde\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3}-\frac{8be^2x^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{75c^3}-\frac{16be^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cos\right.}{75c^5}$$

```
[Out] 2*b^2*d^2*x + (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e*
x^3)/27 + (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (8*b*d*e*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(75*c^5) - (4*b*d*e*x^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(9*c) - (8*b*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(25*c) + d^2*x*(a + b*ArcCosh[c*x])^2 + (2*d*e*x
^3*(a + b*ArcCosh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcCosh[c*x])^2)/5
```

Rubi [A] time = 1.19862, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5707, 5654, 5718, 8, 5662, 5759, 30}

$$-\frac{8bde\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3}-\frac{8be^2x^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{75c^3}-\frac{16be^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cos\right.}{75c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCosh[c*x])^2,x]
```


```
[Out] 2*b^2*d^2*x + (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e*
x^3)/27 + (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (8*b*d*e*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(75*c^5) - (4*b*d*e*x^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(9*c) - (8*b*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(25*c) + d^2*x*(a + b*ArcCosh[c*x])^2 + (2*d*e*x
^3*(a + b*ArcCosh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcCosh[c*x])^2)/5
```


Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^2 \,dx = \int \left(d^2 \left(a+b\cosh^{-1}(cx)\right)^2 + 2dex^2 \left(a+b\cosh^{-1}(cx)\right)^2 + e^2x^4 \left(a+b\cosh^{-1}(cx)\right)^2\right.$$

$$= d^2 \int \left(a+b\cosh^{-1}(cx)\right)^2 \,dx + (2de) \int x^2 \left(a+b\cosh^{-1}(cx)\right)^2 \,dx + e^2 \int x^4 \left(a+b\right.$$

$$= d^2x \left(a+b\cosh^{-1}(cx)\right)^2 + \frac{2}{3}dex^3 \left(a+b\cosh^{-1}(cx)\right)^2 + \frac{1}{5}e^2x^5 \left(a+b\cosh^{-1}(cx)\right)^2$$

$$= -\frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{4bdex^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cos\right.}{9c}$$

$$= 2b^2d^2x + \frac{4}{27}b^2dex^3 + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{8}{}$$

$$= 2b^2d^2x + \frac{8b^2dex}{9c^2} + \frac{4}{27}b^2dex^3 + \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a\right.}{c}$$

$$= 2b^2d^2x + \frac{8b^2dex}{9c^2} + \frac{16b^2e^2x}{75c^4} + \frac{4}{27}b^2dex^3 + \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{-1+c}}{}$$

Mathematica [A] time = 0.535749, size = 299, normalized size = 0.83

$$\frac{225a^2c^5x\left(15d^2+10dex^2+3e^2x^4\right) - 30ab\sqrt{cx-1}\sqrt{cx+1}\left(c^4\left(225d^2+50dex^2+9e^2x^4\right)+4c^2e\left(25d+3ex^2\right)+24e^2\right) - 30}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2
+ 9*e^2*x^4)) + 2*b^2*c*x*(360*e^2 + 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^
2 + 250*d*e*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3
*e^2*x^4) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^
2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))*ArcCosh[c*x] + 225*b^2*c^5*x*
(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x]^2)/(3375*c^5)
```

Maple [A] time = 0.067, size = 402, normalized size = 1.1

$$\frac{1}{c}\left(\frac{a^2}{c^4}\left(\frac{e^2c^5x^5}{5}+\frac{2\,c^5dex^3}{3}+xc^5d^2\right)+\frac{b^2}{c^4}\left(\frac{e^2}{1125}\left(225\,\left(\operatorname{arccosh}\left(cx\right)\right)^2c^5x^5-90\operatorname{arccosh}\left(cx\right)\sqrt{cx-1}\sqrt{cx+1}c^4x^4-120\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b^2/c^4*(1/1125*e^
2*(225*arccosh(c*x)^2*c^5*x^5-90*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c
^4*x^4-120*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+18*c^5*x^5-240*
arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+40*c^3*x^3+240*c*x)+2/27*c^2*d*e*(
9*arccosh(c*x)^2*c^3*x^3-6*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2
-12*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c^3*x^3+12*c*x)+d^2*c^4*(arc
cosh(c*x)^2*c*x-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x))+2*a*b/c^
4*(1/5*arccosh(c*x)*e^2*c^5*x^5+2/3*arccosh(c*x)*c^5*d*e*x^3+arccosh(c*x)*c
^5*x*d^2-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*e^2*x^4+50*c^4*d*e*x^2+22
5*c^4*d^2+12*c^2*e^2*x^2+100*c^2*d*e+24*e^2)))
```

Maxima [A] time = 1.11498, size = 579, normalized size = 1.61

$$\frac{1}{5}\,b^2e^2x^5\operatorname{arcosh}\left(cx\right)^2+\frac{1}{5}\,a^2e^2x^5+\frac{2}{3}\,b^2dex^3\operatorname{arcosh}\left(cx\right)^2+\frac{2}{3}\,a^2dex^3+b^2d^2x\operatorname{arcosh}\left(cx\right)^2+\frac{4}{9}\left(3\,x^3\operatorname{arcosh}\left(cx\right)-c\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*e^2*x^5*arccosh(c*x)^2 + 1/5*a^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arccosh(
c*x)^2 + 2/3*a^2*d*e*x^3 + b^2*d^2*x*arccosh(c*x)^2 + 4/9*(3*x^3*arccosh(c*
x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*a*b*d*e - 4/2
7*(3*c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4)*arccosh(c*x) -
 (c^2*x^3 + 6*x)/c^2)*b^2*d*e + 2/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2
 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*a
*b*e^2 - 2/1125*(15*(3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/
c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c*arccosh(c*x) - (9*c^4*x^5 + 20*c^2*x^3 + 1
20*x)/c^4)*b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2
```

```
*d^2*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d^2/c
```

Fricas [A] time = 1.89801, size = 845, normalized size = 2.35

$$\frac{27\left(25a^2+2b^2\right)c^5e^2x^5+10\left(25\left(9a^2+2b^2\right)c^5de+12b^2c^3e^2\right)x^3+225\left(3b^2c^5e^2x^5+10b^2c^5dex^3+15b^2c^5d^2x\right)\log\left(cx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*e^2*x^5 + 10*(25*(9*a^2 + 2*b^2)*c^5*d*e +
12*b^2*c^3*e^2)*x^3 + 225*(3*b^2*c^5*e^2*x^5 + 10*b^2*c^5*d*e*x^3 + 15*b^2*
c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 - 1))^2 + 15*(225*(a^2 + 2*b^2)*c^5*d^2 +
 200*b^2*c^3*d*e + 48*b^2*c*e^2)*x + 30*(45*a*b*c^5*e^2*x^5 + 150*a*b*c^5*d
*e*x^3 + 225*a*b*c^5*d^2*x - (9*b^2*c^4*e^2*x^4 + 225*b^2*c^4*d^2 + 100*b^2
*c^2*d*e + 24*b^2*e^2 + 2*(25*b^2*c^4*d*e + 6*b^2*c^2*e^2)*x^2)*sqrt(c^2*x^
2 - 1))*log(c*x + sqrt(c^2*x^2 - 1)) - 30*(9*a*b*c^4*e^2*x^4 + 225*a*b*c^4*
d^2 + 100*a*b*c^2*d*e + 24*a*b*e^2 + 2*(25*a*b*c^4*d*e + 6*a*b*c^2*e^2)*x^2
)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] time = 7.0095, size = 602, normalized size = 1.68

$$\begin{cases} a^2d^2x+\frac{2a^2dex^3}{3}+\frac{a^2e^2x^5}{5}+2abd^2x\operatorname{acosh}(cx)+\frac{4abdex^3\operatorname{acosh}(cx)}{3}+\frac{2abe^2x^5\operatorname{acosh}(cx)}{5}-\frac{2abd^2\sqrt{c^2x^2-1}}{c}-\frac{4abdex^2\sqrt{c^2x^2-1}}{9c}-\frac{2abe^2x^4\sqrt{}}{25c} \\ \left(a+\frac{i\pi b}{2}\right)^2\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2*
x*acosh(c*x) + 4*a*b*d*e*x**3*acosh(c*x)/3 + 2*a*b*e**2*x**5*acosh(c*x)/5 -
 2*a*b*d**2*sqrt(c**2*x**2 - 1)/c - 4*a*b*d*e*x**2*sqrt(c**2*x**2 - 1)/(9*c
) - 2*a*b*e**2*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 8*a*b*d*e*sqrt(c**2*x**2 -
 1)/(9*c**3) - 8*a*b*e**2*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 16*a*b*e**2*
sqrt(c**2*x**2 - 1)/(75*c**5) + b**2*d**2*x*acosh(c*x)**2 + 2*b**2*d**2*x +
 2*b**2*d*e*x**3*acosh(c*x)**2/3 + 4*b**2*d*e*x**3/27 + b**2*e**2*x**5*acos
h(c*x)**2/5 + 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(c**2*x**2 - 1)*acosh(
```

```
c*x)/c - 4*b**2*d*e*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c) - 2*b**2*e**2
*x**4*sqrt(c**2*x**2 - 1)*acosh(c*x)/(25*c) + 8*b**2*d*e*x/(9*c**2) + 8*b**
2*e**2*x**3/(225*c**2) - 8*b**2*d*e*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c**3)
 - 8*b**2*e**2*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(75*c**3) + 16*b**2*e**2
*x/(75*c**4) - 16*b**2*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(75*c**5), Ne(c,
 0)), ((a + I*pi*b/2)**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Giac [A] time = 2.15997, size = 656, normalized size = 1.83

$$2\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{\sqrt{c^2x^2-1}}{c}\right)abd^2+\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)^2+2\,c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2-1}\log\left(cx+\sqrt{c^2x^2-1}\right)}{c^2}\right)\right)b^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*a*b*d^2 + (x*log(c
*x + sqrt(c^2*x^2 - 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 - 1)*log(c*x + sqrt(c^2
*x^2 - 1))/c^2))*b^2*d^2 + a^2*d^2*x + 1/1125*(225*a^2*x^5 + 30*(15*x^5*log
(c*x + sqrt(c^2*x^2 - 1)) - (3*(c^2*x^2 - 1)^(5/2) + 10*(c^2*x^2 - 1)^(3/2)
 + 15*sqrt(c^2*x^2 - 1))/c^5)*a*b + (225*x^5*log(c*x + sqrt(c^2*x^2 - 1))^2
 + 2*c*((9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^5 - 15*(3*(c^2*x^2 - 1)^(5/2) +
10*(c^2*x^2 - 1)^(3/2) + 15*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1))
/c^6))*b^2)*e^2 + 2/27*(9*a^2*d*x^3 + 6*(3*x^3*log(c*x + sqrt(c^2*x^2 - 1))
 - ((c^2*x^2 - 1)^(3/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*a*b*d + (9*x^3*log(c*x
+ sqrt(c^2*x^2 - 1))^2 + 2*c*((c^2*x^3 + 6*x)/c^3 - 3*((c^2*x^2 - 1)^(3/2)
+ 3*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1))/c^4))*b^2*d)*e
```

3.527 $\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=168

$$-\frac{4be\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3}+dx\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2bd\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c}+\frac{1}{3}ex^3\left(a+b\right.$$

```
[Out] 2*b^2*d*x + (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x]))/(9*c^3) - (2*b*e*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(9*c) + d*x*(a + b*ArcCosh[c*x])^2 + (e*x^3*(a + b*ArcC
osh[c*x])^2)/3
```

Rubi [A] time = 0.573427, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5707, 5654, 5718, 8, 5662, 5759, 30}

$$-\frac{4be\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3}+dx\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2bd\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c}+\frac{1}{3}ex^3\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCosh[c*x])^2,x]
```


```
[Out] 2*b^2*d*x + (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x]))/(9*c^3) - (2*b*e*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(9*c) + d*x*(a + b*ArcCosh[c*x])^2 + (e*x^3*(a + b*ArcC
osh[c*x])^2)/3
```


Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
```

```
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \int \left(d\left(a+b\cosh^{-1}(cx)\right)^2+ex^2\left(a+b\cosh^{-1}(cx)\right)^2\right)dx$$

$$= d\int \left(a+b\cosh^{-1}(cx)\right)^2\,dx + e\int x^2\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

$$= dx\left(a+b\cosh^{-1}(cx)\right)^2+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)^2-(2bcd)\int \frac{x\left(a+b\cosh^{-1}(cx)\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{2bd\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{2bex^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}\right.}{9c}$$

$$= 2b^2dx+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{4be\sqrt{-1+cx}\sqrt{1+}}{9}$$

$$= 2b^2dx+\frac{4b^2ex}{9c^2}+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{4be\sqrt{-1+}}{}$$

Mathematica [A] time = 0.277127, size = 174, normalized size = 1.04

$$\frac{9a^2c^3x\left(3d+ex^2\right)-6ab\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(9d+ex^2\right)+2e\right)-6b\cosh^{-1}(cx)\left(b\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(9d+ex^2\right)+2e\right)-3a\right.}{27c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*e + c^2*
(9*d + e*x^2)) + 2*b^2*c*x*(6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*
d + e*x^2) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*e + c^2*(9*d + e*x^2)))*ArcC
osh[c*x] + 9*b^2*c^3*x*(3*d + e*x^2)*ArcCosh[c*x]^2)/(27*c^3)
```

Maple [A] time = 0.047, size = 217, normalized size = 1.3

$$\frac{1}{c}\left(\frac{a^2}{c^2}\left(\frac{c^3x^3e}{3}+c^3dx\right)+\frac{b^2}{c^2}\left(\frac{e}{27}\left(9\left(\operatorname{arccosh}(cx)\right)^2c^3x^3-6\operatorname{arccosh}(cx)\sqrt{cx-1}\sqrt{cx+1}c^2x^2-12\operatorname{arccosh}(cx)\sqrt{cx-1}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^2*(1/3*c^3*x^3*e+c^3*d*x)+b^2/c^2*(1/27*e*(9*arccosh(c*x)^2*c^3*
x^3-6*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-12*arccosh(c*x)*(c*x
-1)^(1/2)*(c*x+1)^(1/2)+2*c^3*x^3+12*c*x)+c^2*d*(arccosh(c*x)^2*c*x-2*arcco
sh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x))+2*a*b/c^2*(1/3*arccosh(c*x)*c^3
*x^3*e+arccosh(c*x)*c^3*d*x-1/9*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*e*x^2+9*c^
2*d+2*e)))
```

Maxima [A] time = 1.09262, size = 294, normalized size = 1.75

$$\frac{1}{3}\,b^2ex^3\,\mathrm{arcosh}\,(cx)^2+\frac{1}{3}\,a^2ex^3+b^2dx\,\mathrm{arcosh}\,(cx)^2+\frac{2}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)abe-\frac{2}{27}\Big(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e*x^3*arccosh(c*x)^2 + 1/3*a^2*e*x^3 + b^2*d*x*arccosh(c*x)^2 + 2/9
*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c
^4))*a*b*e - 2/27*(3*c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4
)*arccosh(c*x) - (c^2*x^3 + 6*x)/c^2)*b^2*e + 2*b^2*d*(x - sqrt(c^2*x^2 - 1
)*arccosh(c*x)/c) + a^2*d*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*
d/c
```

Fricas [A] time = 1.86982, size = 454, normalized size = 2.7

$$\frac{\left(9\,a^2+2\,b^2\right)c^3ex^3+9\left(b^2c^3ex^3+3\,b^2c^3dx\right)\log\left(cx+\sqrt{c^2x^2-1}\right)^2+3\left(9\left(a^2+2\,b^2\right)c^3d+4\,b^2ce\right)x+6\Big(3\,abc^3ex^3+9\,abc}{27\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*c^3*e*x^3 + 9*(b^2*c^3*e*x^3 + 3*b^2*c^3*d*x)*log(c*x
 + sqrt(c^2*x^2 - 1))^2 + 3*(9*(a^2 + 2*b^2)*c^3*d + 4*b^2*c*e)*x + 6*(3*a*
b*c^3*e*x^3 + 9*a*b*c^3*d*x - (b^2*c^2*e*x^2 + 9*b^2*c^2*d + 2*b^2*e)*sqrt(
c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1)) - 6*(a*b*c^2*e*x^2 + 9*a*b*c^2*d
 + 2*a*b*e)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [A] time = 1.8622, size = 286, normalized size = 1.7

$$\begin{cases} a^2dx + \frac{a^2ex^3}{3} + 2abdx\operatorname{acosh}(cx) + \frac{2abex^3\operatorname{acosh}(cx)}{3} - \frac{2abd\sqrt{c^2x^2-1}}{c} - \frac{2abex^2\sqrt{c^2x^2-1}}{9c} - \frac{4abe\sqrt{c^2x^2-1}}{9c^3} + b^2dx\operatorname{acosh}^2(cx) + 2b^2dx \\ \left(a + \frac{i\pi b}{2}\right)^2\left(dx + \frac{ex^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*acosh(c*x) + 2*a*b*e*x**3*a
cosh(c*x)/3 - 2*a*b*d*sqrt(c**2*x**2 - 1)/c - 2*a*b*e*x**2*sqrt(c**2*x**2 -
 1)/(9*c) - 4*a*b*e*sqrt(c**2*x**2 - 1)/(9*c**3) + b**2*d*x*acosh(c*x)**2 +
 2*b**2*d*x + b**2*e*x**3*acosh(c*x)**2/3 + 2*b**2*e*x**3/27 - 2*b**2*d*sqr
t(c**2*x**2 - 1)*acosh(c*x)/c - 2*b**2*e*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x
)/(9*c) + 4*b**2*e*x/(9*c**2) - 4*b**2*e*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*
c**3), Ne(c, 0)), ((a + I*pi*b/2)**2*(d*x + e*x**3/3), True))
```

Giac [A] time = 1.77944, size = 373, normalized size = 2.22

$$2\left(x\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{\sqrt{c^2x^2-1}}{c}\right)abd + \left(x\log\left(cx + \sqrt{c^2x^2-1}\right)^2 + 2c\left(\frac{x}{c} - \frac{\sqrt{c^2x^2-1}\log\left(cx + \sqrt{c^2x^2-1}\right)}{c^2}\right)\right)b^2d +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*a*b*d + (x*log(c*x
 + sqrt(c^2*x^2 - 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 - 1)*log(c*x + sqrt(c^2*x
^2 - 1))/c^2))*b^2*d + a^2*d*x + 1/27*(9*a^2*x^3 + 6*(3*x^3*log(c*x + sqrt(
c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*a*b + (9*x
^3*log(c*x + sqrt(c^2*x^2 - 1))^2 + 2*c*((c^2*x^3 + 6*x)/c^3 - 3*((c^2*x^2
- 1)^(3/2) + 3*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1))/c^4))*b^2)*e
```

3.528 $\int \left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=51

$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c}+x\left(a+b\cosh^{-1}(cx)\right)^2+2b^2x$$

```
[Out] 2*b^2*x - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c + x*(a
+ b*ArcCosh[c*x])^2
```

Rubi [A] time = 0.156815, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5654, 5718, 8}

$$-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c}+x\left(a+b\cosh^{-1}(cx)\right)^2+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c + x*(a
+ b*ArcCosh[c*x])^2
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cosh^{-1}(cx)\right)^2\,dx &= x\left(a+b\cosh^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= -\frac{2b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c}+x\left(a+b\cosh^{-1}(cx)\right)^2+\left(2b^2\right)\int 1\,dx\\ &= 2b^2x-\frac{2b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c}+x\left(a+b\cosh^{-1}(cx)\right)^2\end{aligned}$$

Mathematica [A] time = 0.0870446, size = 84, normalized size = 1.65

$$x\left(a^2+2b^2\right)-\frac{2ab\sqrt{cx-1}\sqrt{cx+1}}{c}+\frac{2b\cosh^{-1}(cx)\left(acx-b\sqrt{cx-1}\sqrt{cx+1}\right)}{c}+b^2x\cosh^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + (2*b*(a*c*x - b*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])*ArcCosh[c*x])/c + b^2*x*ArcCosh[c*x]^2
```

Maple [A] time = 0.04, size = 78, normalized size = 1.5

$$\frac{1}{c}\left(cxa^2+b^2\left((\operatorname{arccosh}(cx))^2cx-2\operatorname{arccosh}(cx)\sqrt{cx-1}\sqrt{cx+1}+2cx\right)+2ab\left(cx\operatorname{arccosh}(cx)-\sqrt{cx-1}\sqrt{cx+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(c*x*a^2+b^2*(arccosh(c*x)^2*c*x-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)+2*c*x)+2*a*b*(c*x*arccosh(c*x)-(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [A] time = 1.17358, size = 97, normalized size = 1.9

$$b^2 x \operatorname{arcosh}(cx)^2 + 2\,b^2\left(x - \frac{\sqrt{c^2x^2-1}\operatorname{arcosh}(cx)}{c}\right) + a^2x + \frac{2\left(cx\operatorname{arcosh}(cx) - \sqrt{c^2x^2-1}\right)ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arccosh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2*x
 + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b/c
```

Fricas [B] time = 1.71647, size = 212, normalized size = 4.16

$$\frac{b^2cx\log\left(cx + \sqrt{c^2x^2-1}\right)^2 + \left(a^2 + 2\,b^2\right)cx - 2\,\sqrt{c^2x^2-1}ab + 2\left(abcx - \sqrt{c^2x^2-1}b^2\right)\log\left(cx + \sqrt{c^2x^2-1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 - 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 - 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 - 1)*b^2)*log(c*x + sqrt(c^2*x^2 - 1
)))/c
```

Sympy [A] time = 0.363591, size = 88, normalized size = 1.73

$$\begin{cases} a^2x + 2abx\operatorname{acosh}(cx) - \frac{2ab\sqrt{c^2x^2-1}}{c} + b^2x\operatorname{acosh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} & \text{for } c \neq 0 \\ x\left(a + \frac{i\pi b}{2}\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*acosh(c*x) - 2*a*b*sqrt(c**2*x**2 - 1)/c + b**2
*x*acosh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/c, Ne(c
```

```
, 0)), (x*(a + I*pi*b/2)**2, True))
```

Giac [B] time = 1.32592, size = 150, normalized size = 2.94

$$2\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{\sqrt{c^2x^2-1}}{c}\right)ab+\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)^2+2\,c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2-1}\log\left(cx+\sqrt{c^2x^2-1}\right)}{c^2}\right)\right)b^2+a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 - 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 - 1)*log(c*x + sqrt(c^2*x^2
 - 1))/c^2))*b^2 + a^2*x
```

3.529 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex^2}\,dx$

Optimal. Leaf size=763

$$-\frac{b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b\left(a+b\cosh\right.}{}$$

```
[Out] ((a + b*ArcCosh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])^2*Log[1 + (S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt
[e]) + ((a + b*ArcCosh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])^2*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-
d]*Sqrt[e]) - (b*(a + b*ArcCosh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a + b*ArcCos
h[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e
])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcCosh[c*x])*PolyLog[2, -((Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(
a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLo
g[3, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(Sqrt[-d]
*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(Sqrt[-d]*Sqrt[e])
```

Rubi [A] time = 1.30796, antiderivative size = 763, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5707, 5800, 5562, 2190, 2531, 2282, 6589}

$$-\frac{b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b\left(a+b\cosh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcCosh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])^2*Log[1 + (S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt
[e]) + ((a + b*ArcCosh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])^2*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-
d]*Sqrt[e]) - (b*(a + b*ArcCosh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a + b*ArcCos
h[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e
])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcCosh[c*x])*PolyLog[2, -((Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(
a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLo
g[3, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(Sqrt[-d]
*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^2\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{(a+bx)^2\sinh(x)}{c\sqrt{-d}+\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}e^x}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}+\sqrt{-c^2d-e}-\sqrt{e}e^x}\,dx, x, \cosh^{-1}(\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} + \cdots$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} + \cdots$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} + \cdots$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} + \cdots$$

Mathematica [A] time = 0.570427, size = 623, normalized size = 0.82

$$\frac{2b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right) - 2b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-c\sqrt{-d}}\right) - 2b\left(a+b\cosh^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])^2*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e])]) + (a + b*ArcCosh[c*x])^2*Log[1 + (Sqrt[e]*E^ArcCosh[c*x
```

```
])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + (a + b*ArcCosh[c*x])^2*Log[1 - (
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] - (a + b*ArcCosh
[c*x])^2*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
] + 2*b*(a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] - Sqrt[-(c^2*d) - e])] - 2*b*(a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^A
rcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] - 2*b*(a + b*ArcCosh[c*x
])*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))
] + 2*b*(a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] + Sqrt[-(c^2*d) - e])] - 2*b^2*PolyLog[3, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e])] + 2*b^2*PolyLog[3, (Sqrt[e]*E^ArcCosh[c*x])/(-
(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))] - 2*b^2*PolyLog[3, (Sqrt[e]*E^Ar
cCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e])
```

Maple [F] time = 0.402, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{arccosh}(cx))^2}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \operatorname{arcosh}(cx)^2 + 2\,ab \operatorname{arcosh}(cx) + a^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{acosh}(cx))^2}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x^2 + d), x)
```

3.530 $\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

Rubi [A] time = 0.0405683, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Mathematica [A] time = 15.3512, size = 0, normalized size = 0.

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

Maple [A] time = 0.299, size = 0, normalized size = 0.

$$\int (a + b\mathrm{arccosh}\,(cx))^2 \sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2 \operatorname{arcosh}\,(cx)^2 + 2\,ab \operatorname{arcosh}\,(cx) + a^2\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(e*x^2 + d), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{acosh}\,(cx))^2 \sqrt{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2*sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arcosh}(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)^2, x)
```

3.531 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi [A] time = 0.0431376, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2],x]

[Out] Defer[Int][(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx = \int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 11.0387, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2], x]
```

Maple [A] time = 0.289, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(cx))^2 \frac{1}{\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arcosh}(cx)^2 + 2\,ab \operatorname{arcosh}(cx) + a^2}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/sqrt(e*x^2 + d), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))^2}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)^2}{\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/sqrt(e*x^2 + d), x)
```

3.532 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi [A] time = 0.0471121, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 3.59507, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Maple [A] time = 0.24, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(cx))^2 \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b^2 \operatorname{arcosh}(cx)^2 + 2\, ab \operatorname{arcosh}(cx) + a^2\right)\sqrt{ex^2 + d}}{e^2x^4 + 2\, dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(e*x^2 + d)/(e
^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))^2}{\left(d + ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x**2)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)^2}{\left(ex^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x^2 + d)^(3/2), x)
```

3.533 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi [A] time = 0.0469609, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2),x]

[Out] Defer[Int][(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 7.25814, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Maple [A] time = 0.24, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(cx))^2 \left(ex^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2\left(\frac{2x}{\sqrt{ex^2+d}d^2} + \frac{x}{\left(ex^2+d\right)^{\frac{3}{2}}d}\right) + \int \frac{b^2\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)^2}{\left(ex^2+d\right)^{\frac{5}{2}}} + \frac{2ab\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + integrate(b
^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(e*x^2 + d)^(5/2) + 2*a*b*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b^2\operatorname{arcosh}(cx)^2 + 2ab\operatorname{arcosh}(cx) + a^2\right)\sqrt{ex^2+d}}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(e*x^2 + d)/(e
^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x^2 + d)^(5/2), x)
```

3.534 $\int \frac{\left(d+ex^2\right)^2}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=388

$$-\frac{de\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2bc^3}-\frac{de\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc^3}-\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^5}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\cdots}{}$$

```
[Out] -((d^2*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b*c)) - (d*e*CoshIn
tegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(2*b*c^3) - (e^2*CoshIntegral[(a
+ b*ArcCosh[c*x])/b]*Sinh[a/b])/(8*b*c^5) - (d*e*CoshIntegral[(3*(a + b*Arc
Cosh[c*x]))/b]*Sinh[(3*a)/b])/(2*b*c^3) - (3*e^2*CoshIntegral[(3*(a + b*Arc
Cosh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^5) - (e^2*CoshIntegral[(5*(a + b*ArcC
osh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^5) + (d^2*Cosh[a/b]*SinhIntegral[(a +
b*ArcCosh[c*x])/b])/(b*c) + (d*e*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x]
)/b])/(2*b*c^3) + (e^2*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b
*c^5) + (d*e*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(2*b*c
^3) + (3*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(16*b*
c^5) + (e^2*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b*c
^5)
```

Rubi [A] time = 0.792853, antiderivative size = 380, normalized size of antiderivative = 0.98, number of steps used = 27, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5707, 5658, 3303, 3298, 3301, 5670, 5448}

$$-\frac{de\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{2bc^3}-\frac{de\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{2bc^3}-\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^5}-\frac{3e^2\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d*e*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(2*b*c^3) - (e^2*CoshInte
gral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(8*b*c^5) - (d^2*CoshIntegral[(a + b*Ar
cCosh[c*x])/b]*Sinh[a/b])/(b*c) - (d*e*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x
]]*Sinh[(3*a)/b])/(2*b*c^3) - (3*e^2*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]]
*Sinh[(3*a)/b])/(16*b*c^5) - (e^2*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]]*Si
nh[(5*a)/b])/(16*b*c^5) + (d*e*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/
(2*b*c^3) + (e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b*c^5) + (d
*e*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(2*b*c^3) + (3*e^2
```

```
*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*c^5) + (e^2*Co
sh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b*c^5) + (d^2*Cosh[
a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(b*c)
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{a+b\cosh^{-1}(cx)}\,dx = \int \left(\frac{d^2}{a+b\cosh^{-1}(cx)}+\frac{2dex^2}{a+b\cosh^{-1}(cx)}+\frac{e^2x^4}{a+b\cosh^{-1}(cx)}\right)dx$$

$$= d^2\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx+(2de)\int \frac{x^2}{a+b\cosh^{-1}(cx)}\,dx+e^2\int \frac{x^4}{a+b\cosh^{-1}(cx)}\,dx$$

$$= -\frac{d^2\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\cosh^{-1}(cx)\right)}{bc}+\frac{(2de)\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}\right.}{c^3}$$

$$= \frac{(2de)\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx,x,\cosh^{-1}(cx)\right)}{c^3}+\frac{e^2\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)}+\frac{3\sinh(3x)}{16(a+bx)}+\frac{\sinh}{16(}\right.\right.}{c^5}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}+\frac{(de)\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,\right.}{2c^3}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}+\frac{\left(de\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\right.}{2}$$

$$= -\frac{de\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{2bc^3}-\frac{e^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^5}-\frac{d^2\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right.}{bc}$$

Mathematica [A] time = 0.50615, size = 254, normalized size = 0.65

$$\frac{-2\sinh\left(\frac{a}{b}\right)\left(8c^4d^2+4c^2de+e^2\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-e\sinh\left(\frac{3a}{b}\right)\left(8c^2d+3e\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+16c^4d^2\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*(8*c^4*d^2 + 4*c^2*d*e + e^2)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b
] - e*(8*c^2*d + 3*e)*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] -
e^2*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 16*c^4*d^2*Cosh[a/
```

```
b]*SinhIntegral[a/b + ArcCosh[c*x]] + 8*c^2*d*e*Cosh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]] + 2*e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 8*c^2*
d*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*e^2*Cosh[(3*a)/b
]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + e^2*Cosh[(5*a)/b]*SinhIntegral[5*(
a/b + ArcCosh[c*x])])/(16*b*c^5)
```

Maple [A] time = 0.123, size = 380, normalized size = 1.

$$\frac{1}{c}\left(-\frac{e^2}{32\,c^4b}\mathrm{e}^{-5\,\frac{a}{b}}\mathrm{Ei}\left(1,-5\,\mathrm{arccosh}\,(cx)-5\,\frac{a}{b}\right)+\frac{e^2}{32\,c^4b}\mathrm{e}^{5\,\frac{a}{b}}\mathrm{Ei}\left(1,5\,\mathrm{arccosh}\,(cx)+5\,\frac{a}{b}\right)+\frac{d^2}{2\,b}\mathrm{e}^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{arccosh}\,(cx)+\frac{a}{b}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-1/32/c^4*e^2/b*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)+1/32/c^4*e^2/b
*exp(5*a/b)*Ei(1,5*arccosh(c*x)+5*a/b)+1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b
)*d^2+1/4/c^2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d*e+1/16/c^4/b*exp(a/b)*Ei(
1,arccosh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d^2-1/4/c^2
/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d*e-1/16/c^4/b*exp(-a/b)*Ei(1,-arccosh
(c*x)-a/b)*e^2+1/4/c^2*e/b*exp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)*d+3/32/c^4
*e^2/b*exp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)-1/4/c^2*e/b*exp(-3*a/b)*Ei(1,-
3*arccosh(c*x)-3*a/b)*d-3/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a
/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)^2}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4 + 2\,dex^2 + d^2}{b\,\text{arcosh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)^2}{a + b\,\text{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{b\,\text{arcosh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arccosh(c*x) + a), x)
```

3.535 $\int \frac{d+ex^2}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=139

$$-\frac{\sinh\left(\frac{a}{b}\right)\left(4c^2d+e\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^3}-\frac{e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4bc^3}+\frac{\cosh\left(\frac{a}{b}\right)\left(4c^2d+e\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^3}+$$

```
[Out] -((4*c^2*d + e)*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(4*b*c^3) -
 (e*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[(3*a)/b])/(4*b*c^3) + ((4
*c^2*d + e)*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(4*b*c^3) + (e*
Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b*c^3)
```

Rubi [A] time = 0.381753, antiderivative size = 176, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5707, 5658, 3303, 3298, 3301, 5670, 5448}

$$-\frac{e\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^3}+\frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(e*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(4*b*c^3) - (d*CoshIntegral
[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b*c) - (e*CoshIntegral[(3*a)/b + 3*Arc
Cosh[c*x]]*Sinh[(3*a)/b])/(4*b*c^3) + (e*Cosh[a/b]*SinhIntegral[a/b + ArcCo
sh[c*x]])/(4*b*c^3) + (e*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x
]])/(4*b*c^3) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(b*c)
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
```

```
, b, c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{a+b\cosh^{-1}(cx)}\,dx = \int \left(\frac{d}{a+b\cosh^{-1}(cx)}+\frac{ex^2}{a+b\cosh^{-1}(cx)}\right)dx$$

$$= d\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx + e\int \frac{x^2}{a+b\cosh^{-1}(cx)}\,dx$$

$$= -\frac{d\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3}$$

$$= \frac{e\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\,\text{c}\right.}{bc}$$

$$= -\frac{d\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh\right.}{4c^3}$$

$$= -\frac{d\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc} + \frac{\left(e\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}\right.}{a+b}\right.}{4c^3}$$

$$= -\frac{e\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3} - \frac{d\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{e\,\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)\text{s}}{4bc^3}$$

Mathematica [A] time = 0.221541, size = 125, normalized size = 0.9

$$\frac{-\sinh\left(\frac{a}{b}\right)\left(4c^2d+e\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+4c^2d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-((4*c^2*d + e)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b]) - e*CoshIntegr
al[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] + 4*c^2*d*Cosh[a/b]*SinhIntegral[a
/b + ArcCosh[c*x]] + e*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + e*Cosh[
(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])])/(4*b*c^3)
```

Maple [A] time = 0.096, size = 178, normalized size = 1.3

$$\frac{1}{c}\left(-\frac{e}{8c^2b}e^{-3\frac{a}{b}}\text{Ei}\left(1,-3\,\text{arccosh}(cx)-3\frac{a}{b}\right)+\frac{e}{8c^2b}e^{3\frac{a}{b}}\text{Ei}\left(1,3\,\text{arccosh}(cx)+3\frac{a}{b}\right)+\frac{d}{2b}e^{\frac{a}{b}}\text{Ei}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-1/8/c^2*e/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/8/c^2*e/b*exp(3
*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)+1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d+1/
8/c^2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*e-1/2/b*exp(-a/b)*Ei(1,-arccosh(c*x
)-a/b)*d-1/8/c^2/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{b \operatorname{arcosh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{b \operatorname{arcosh}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{a + b \operatorname{acosh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{b \operatorname{arcosh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

3.536 $\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=54

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

```
[Out] -((CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b*c)) + (Cosh[a/b]*Sinh
Integral[(a + b*ArcCosh[c*x])/b])/(b*c)
```

Rubi [A] time = 0.0704804, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5658, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-1),x]
```

```
[Out] -((CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b*c)) + (Cosh[a/b]*Sinh
Integral[(a + b*ArcCosh[c*x])/b])/(b*c)
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\cos$$
$$= -\frac{\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

Mathematica [A] time = 0.0633497, size = 46, normalized size = 0.85

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-1),x]
```

```
[Out] -((CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]])/(b*c))
```

Maple [A] time = 0.032, size = 56, normalized size = 1.

$$\frac{1}{c}\left(\frac{1}{2b}e^{\frac{a}{b}}\text{Ei}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)-\frac{1}{2b}e^{-\frac{a}{b}}\text{Ei}\left(1,-\text{arccosh}(cx)-\frac{a}{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arccosh(c*
x)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arcosh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b \operatorname{arcosh}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \operatorname{acosh}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arcosh}(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccosh(c*x) + a), x)
```

3.537 $\int \frac{1}{(d+ex^2)(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(d+ex^2)(a+b\cosh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.0390261, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex^2)(a+b\cosh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+b\cosh^{-1}(cx))}\,dx = \int \frac{1}{(d+ex^2)(a+b\cosh^{-1}(cx))}\,dx$$

Mathematica [A] time = 0.568413, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)(a+b\cosh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.359, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^{2}+d\right)\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^{2}+d\right)\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{aex^{2}+ad+\left(bex^{2}+bd\right)\mathrm{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{acosh}(cx))\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)), x)
```

3.538 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.0377451, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 3.13581, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.361, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2 \left(a+b\operatorname{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2 \left(b\operatorname{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\operatorname{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*
d^2)*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2\left(b\operatorname{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)), x)
```

3.539 $\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]), x]
```

Rubi [A] time = 0.0461833, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Mathematica [A] time = 1.09629, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]),x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]), x]
```

Maple [A] time = 0.281, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\operatorname{arccosh}(cx)}\sqrt{e x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e x^2+d}}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{e x^2+d}}{b\operatorname{arcosh}(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*acosh(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

3.540 $\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.0479128, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.04994, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.256, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\,\mathrm{arccosh}\,(cx)}\frac{1}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex^2+d}(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}}{aex^2+ad+\left(bex^2+bd\right)\mathrm{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{acosh}(cx))\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*sqrt(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex^2+d}(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)), x)
```

3.541 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.0513969, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.48805, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.217, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\,\mathrm{arccosh}\,(cx)}\left(ex^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}}{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\mathrm{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*
b*d*e*x^2 + b*d^2)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\,\mathrm{acosh}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x**2)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)), x)
```

3.542 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.0509137, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]

[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 3.73987, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.222, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\mathrm{arccosh}\left(cx\right)}\left(ex^{2}+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^{2}+d\right)^{\frac{5}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^{2}+d}}{ae^{3}x^{6}+3\,ade^{2}x^{4}+3\,ad^{2}ex^{2}+ad^{3}+\left(be^{3}x^{6}+3\,bde^{2}x^{4}+3\,bd^{2}ex^{2}+bd^{3}\right)\operatorname{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3
 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\,\mathrm{acosh}\,(cx))\left(d + ex^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x**2)**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2 + d\right)^{\frac{5}{2}}(b\,\mathrm{arcosh}\,(cx) + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)), x)
```

3.543 $\int \frac{(d+ex^2)^2}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=510

$$\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2b^2c^3}+\frac{3de\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{2b^2c^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^5}+\frac{9e^2\cosh\left(\frac{3a}{b}\right.}{}$$

```
[Out] -((d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (2*d*e*x
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) - (e^2*x^4*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d^2*Cosh[a/b]*CoshI
ntegral[(a + b*ArcCosh[c*x])/b])/(b^2*c) + (d*e*Cosh[a/b]*CoshIntegral[(a +
 b*ArcCosh[c*x])/b])/(2*b^2*c^3) + (e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcCo
sh[c*x])/b])/(8*b^2*c^5) + (3*d*e*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcC
osh[c*x]))/b])/(2*b^2*c^3) + (9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Ar
cCosh[c*x]))/b])/(16*b^2*c^5) + (5*e^2*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b
*ArcCosh[c*x]))/b])/(16*b^2*c^5) - (d^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCo
sh[c*x])/b])/(b^2*c) - (d*e*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])
/(2*b^2*c^3) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b^2*
c^5) - (3*d*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(2*b^
2*c^3) - (9*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(16
*b^2*c^5) - (5*e^2*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/
(16*b^2*c^5)
```

Rubi [A] time = 0.950991, antiderivative size = 498, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5707, 5656, 5781, 3303, 3298, 3301, 5666}

$$\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{2b^2c^3}+\frac{3de\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{2b^2c^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b^2c^5}+\frac{9e^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (2*d*e*x
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) - (e^2*x^4*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d^2*Cosh[a/b]*CoshI
```

```
ntegral[a/b + ArcCosh[c*x]])/(b^2*c) + (d*e*Cosh[a/b]*CoshIntegral[a/b + Ar
cCosh[c*x]])/(2*b^2*c^3) + (e^2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])
/(8*b^2*c^5) + (3*d*e*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]])
/(2*b^2*c^3) + (9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]])
/(16*b^2*c^5) + (5*e^2*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]]
)/(16*b^2*c^5) - (d^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(b^2*c) -
 (d*e*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(2*b^2*c^3) - (e^2*Sinh[a
/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b^2*c^5) - (3*d*e*Sinh[(3*a)/b]*Si
nhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(2*b^2*c^3) - (9*e^2*Sinh[(3*a)/b]*Si
nhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(16*b^2*c^5) - (5*e^2*Sinh[(5*a)/b]*S
inhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b^2*c^5)
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
```

```
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \int \left(\frac{d^2}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{2dex^2}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{e^2x^4}{\left(a+b\cosh^{-1}(cx)\right)^2}\right)dx \\
&= d^2\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx+(2de)\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx+e^2\int \frac{x^4}{\left(a+b\cosh^{-1}(cx)\right)^2} \\
&= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(cd^2\right)\int \frac{}{\sqrt{-1+}}}{} \\
&= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d^2\,\mathrm{Subst}\left(\int\right.}{} \\
&= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right.}{} \\
&= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d^2\cosh\left(\frac{a}{b}\right)\mathrm{C}}{}
\end{aligned}
$$

Mathematica [A] time = 3.0612, size = 456, normalized size = 0.89

$$16c^4d^2\left(\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-64c^2de\left(\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\mathrm{sinl}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((-16*b*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)^2)/(a + b*ArcC
osh[c*x]) + 16*c^4*d^2*(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] - Sinh[a
/b]*SinhIntegral[a/b + ArcCosh[c*x]]) - 64*c^2*d*e*(Cosh[a/b]*CoshIntegral[
a/b + ArcCosh[c*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]) + 24*c^2*
d*e*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + Cosh[(3*a)/b]*CoshInteg
ral[3*(a/b + ArcCosh[c*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
- Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]) - 16*e^2*(3*Cosh[a/b]
*CoshIntegral[a/b + ArcCosh[c*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + Arc
Cosh[c*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - Sinh[(3*a)/b]*
SinhIntegral[3*(a/b + ArcCosh[c*x])]) + 5*e^2*(10*Cosh[a/b]*CoshIntegral[a/
b + ArcCosh[c*x]] + 5*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] +
Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcCosh[c*x])] - 10*Sinh[a/b]*SinhInteg
ral[a/b + ArcCosh[c*x]] - 5*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x
])] - Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])]))/(16*b^2*c^5)
```

Maple [B] time = 0.226, size = 1102, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(1/32*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+12*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2)+16*c^5*x^5-20*c^3*x^3+5*c*x)*e^2
/c^4/b/(a+b*arccosh(c*x))-5/32/c^4*e^2/b^2*exp(5*a/b)*Ei(1,5*arccosh(c*x)+5
*a/b)-1/32/c^4*e^2/b*(16*c^5*x^5-20*c^3*x^3+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^4*c^4+5*c*x-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^
(1/2))/(a+b*arccosh(c*x))-5/32/c^4*e^2/b^2*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)
-5*a/b)+1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*d^2/b/(a+b*arccosh(c*x))-1/2
*d^2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)+1/4*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+
c*x)*d*e/c^2/b/(a+b*arccosh(c*x))-1/4/c^2*d*e/b^2*exp(a/b)*Ei(1,arccosh(c*x
```

```
)+a/b)+1/16*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*e^2/c^4/b/(a+b*arccosh(c*x))
-1/16/c^4*e^2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*d^2*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arccosh(c
*x)-a/b)-1/4/c^2/b*d*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))
-1/4/c^2/b^2*d*e*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)-1/16/c^4/b*e^2*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-1/16/c^4/b^2*e^2*exp(-a/b)*Ei(1
,-arccosh(c*x)-a/b)+1/4*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/
2)*(c*x+1)^(1/2)+4*c^3*x^3-3*c*x)*d*e/c^2/b/(a+b*arccosh(c*x))+3/32*(-4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c^3*x^3-3*c*
x)*e^2/c^4/b/(a+b*arccosh(c*x))-3/4/c^2*e/b^2*exp(3*a/b)*Ei(1,3*arccosh(c*x
)+3*a/b)*d-9/32/c^4*e^2/b^2*exp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)-1/4/c^2*e
/b*(4*c^3*x^3-3*c*x+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*
x+1)^(1/2))/(a+b*arccosh(c*x))*d-3/32/c^4*e^2/b*(4*c^3*x^3-3*c*x+4*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))
-3/4/c^2*e/b^2*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)*d-9/32/c^4*e^2/b^2*e
xp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3e^2x^7 + \left(2\,c^3de - ce^2\right)x^5 - cd^2x + \left(c^3d^2 - 2\,cde\right)x^3 + \left(c^2e^2x^6 + \left(2\,c^2de - e^2\right)x^4 + \left(c^2d^2 - 2\,de\right)x^2 - d^2\right)\sqrt{cx+1}\sqrt{cx-1}}{abc^3x^2 + \sqrt{cx+1}\sqrt{cx-1}abc^2x - abc + \left(b^2c^3x^2 + \sqrt{cx+1}\sqrt{cx-1}b^2c^2x - b^2c\right)\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e^2*x^7 + (2*c^3*d*e - c*e^2)*x^5 - c*d^2*x + (c^3*d^2 - 2*c*d*e)*x^3
 + (c^2*e^2*x^6 + (2*c^2*d*e - e^2)*x^4 + (c^2*d^2 - 2*d*e)*x^2 - d^2)*sqrt
(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2
*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((5*c^5*e^2*x^8 + 2*(3*c
^5*d*e - 5*c^3*e^2)*x^6 + (c^5*d^2 - 12*c^3*d*e + 5*c*e^2)*x^4 + (5*c^3*e^2
*x^6 + 3*(2*c^3*d*e - c*e^2)*x^4 + c*d^2 + (c^3*d^2 - 2*c*d*e)*x^2)*(c*x +
1)*(c*x - 1) + c*d^2 - 2*(c^3*d^2 - 3*c*d*e)*x^2 + (10*c^4*e^2*x^7 + (12*c^
4*d*e - 13*c^2*e^2)*x^5 + 2*(c^4*d^2 - 7*c^2*d*e + 2*e^2)*x^3 - (c^2*d^2 -
4*d*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a
*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x +
 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*
c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4 + 2\,dex^2 + d^2}{b^2\,\text{arcosh}\,(cx)^2 + 2\,ab\,\text{arcosh}\,(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*
x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)^2}{\left(a + b\,\text{acosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*acosh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{\left(b\,\text{arcosh}\,(cx) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arccosh(c*x) + a)^2, x)
```

3.544 $\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=257

$$\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\right.}{4b^2}$$

```
[Out] -((d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (e*x^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d*Cosh[a/b]*CoshIn
tegral[(a + b*ArcCosh[c*x])/b])/(b^2*c) + (e*Cosh[a/b]*CoshIntegral[(a + b*
ArcCosh[c*x])/b])/(4*b^2*c^3) + (3*e*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*A
rcCosh[c*x]))/b])/(4*b^2*c^3) - (d*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*
x])/b])/(b^2*c) - (e*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(4*b^2
*c^3) - (3*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b^2
*c^3)
```

Rubi [A] time = 0.595345, antiderivative size = 249, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5707, 5656, 5781, 3303, 3298, 3301, 5666}

$$\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4b^2c^3}-\frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b^2c^3}-\frac{3e\sinh\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (e*x^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d*Cosh[a/b]*CoshIn
tegral[a/b + ArcCosh[c*x]])/(b^2*c) + (e*Cosh[a/b]*CoshIntegral[a/b + ArcCo
sh[c*x]])/(4*b^2*c^3) + (3*e*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh
[c*x]])/(4*b^2*c^3) - (d*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(b^2*c
) - (e*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b^2*c^3) - (3*e*Sinh[
(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b^2*c^3)
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
```

```
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{ex^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\right)dx$$

$$= d\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx + e\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

$$= -\frac{d\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{(cd)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{b} - \frac{e\,S}{}$$

$$= -\frac{d\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{d\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc} +$$

$$= -\frac{d\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, c\right.}{bc}$$

$$= -\frac{d\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c} + \frac{e\cos}{}$$

Mathematica [A] time = 1.52872, size = 225, normalized size = 0.88

$$\frac{4c^2d\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\frac{4bc^2\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(d+ex^2\right)}{a+b\cosh^{-1}(cx)}+3e\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((-4*b*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2))/(a + b*ArcCosh
[c*x]) - 8*e*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 8*e*Sinh[a/b]*Sin
hIntegral[a/b + ArcCosh[c*x]] + 4*c^2*d*(Cosh[a/b]*CoshIntegral[a/b + ArcCo
sh[c*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]) + 3*e*(3*Cosh[a/b]*C
oshIntegral[a/b + ArcCosh[c*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCo
sh[c*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - Sinh[(3*a)/b]*Si
```

```
nhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*b^2*c^3)
```

Maple [A] time = 0.144, size = 465, normalized size = 1.8

$$\frac{1}{c}\left(\frac{e}{8\,bc^{2}\,(a+b\mathrm{arccosh}\,(cx))}\left(-4\,\sqrt{cx+1}\sqrt{cx-1}x^{2}c^{2}+\sqrt{cx-1}\sqrt{cx+1}+4\,c^{3}x^{3}-3\,cx\right)-\frac{3\,e}{8\,c^{2}b^{2}}\mathrm{e}^{3\,\frac{a}{b}}\mathrm{Ei}\left(1,3\,\mathrm{arccosh}\,(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(1/8*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c^3*x^3-3*c*x)*e/c^2/b/(a+b*arccosh(c*x))-3/8/c^2*e/b^2*exp(3*a/b)*Ei(1
,3*arccosh(c*x)+3*a/b)-1/8/c^2*e/b*(4*c^3*x^3-3*c*x+4*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-3/8/c^2*e/b^
2*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)
+c*x)*d/b/(a+b*arccosh(c*x))+1/8*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*e/c^2/b
/(a+b*arccosh(c*x))-1/2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d-1/8/c^2/b^2*e
xp(a/b)*Ei(1,arccosh(c*x)+a/b)*e-1/2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a
+b*arccosh(c*x))*d-1/8/c^2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh
(c*x))*e-1/2/b^2*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d-1/8/c^2/b^2*exp(-a/b)*
Ei(1,-arccosh(c*x)-a/b)*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^{3}ex^{5}+\left(c^{3}d-ce\right)x^{3}-cdx+\left(c^{2}ex^{4}+\left(c^{2}d-e\right)x^{2}-d\right)\sqrt{cx+1}\sqrt{cx-1}}{abc^{3}x^{2}+\sqrt{cx+1}\sqrt{cx-1}abc^{2}x-abc+\left(b^{2}c^{3}x^{2}+\sqrt{cx+1}\sqrt{cx-1}b^{2}c^{2}x-b^{2}c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{}{abc^{5}x^{4}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e*x^5 + (c^3*d - c*e)*x^3 - c*d*x + (c^2*e*x^4 + (c^2*d - e)*x^2 - d)
*sqrt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*
b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^
2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*c^5*e*x^6 + (c^
5*d - 6*c^3*e)*x^4 + (3*c^3*e*x^4 + (c^3*d - c*e)*x^2 + c*d)*(c*x + 1)*(c*x
 - 1) - (2*c^3*d - 3*c*e)*x^2 + (6*c^4*e*x^5 + (2*c^4*d - 7*c^2*e)*x^3 - (c
^2*d - 2*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/(a*b*c^5*x^4 + (c*x + 1)*
```

```
(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)
*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x
^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt
(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+d}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex^2}{\left(a+b\,\text{acosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x**2)/(a + b*acosh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^2+d}{\left(b\,\text{arcosh}\,(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a)^2, x)
```

3.545 $\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) + (Cosh[a/b]*C
oshIntegral[(a + b*ArcCosh[c*x])/b])/(b^2*c) - (Sinh[a/b]*SinhIntegral[(a +
 b*ArcCosh[c*x])/b])/(b^2*c)
```

Rubi [A] time = 0.323234, antiderivative size = 86, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5656, 5781, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c}-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) + (Cosh[a/b]*C
oshIntegral[a/b + ArcCosh[c*x]])/(b^2*c) - (Sinh[a/b]*SinhIntegral[a/b + Ar
cCosh[c*x]])/(b^2*c)
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
```

```
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{c\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{b}$$

$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subs}}{}$$

$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c} - \frac{\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c}$$

Mathematica [A] time = 0.321973, size = 80, normalized size = 0.89

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\frac{b\sqrt{\frac{cx-1}{cx+1}}(cx+1)}{a+b\cosh^{-1}(cx)}}{b^2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(a + b*ArcCosh[c*x])) + Cosh[a/
b]*CoshIntegral[a/b + ArcCosh[c*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[
c*x]])/(b^2*c)
```

Maple [A] time = 0.049, size = 125, normalized size = 1.4

$$\frac{1}{c}\left(\frac{1}{2b(a+b\text{arccosh}(cx))}\left(-\sqrt{cx-1}\sqrt{cx+1}+cx\right)-\frac{1}{2b^2}e^{\frac{a}{b}}\text{Ei}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)-\frac{1}{2b(a+b\text{arccosh}(cx))}\left(cx+\sqrt{c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)/b/(a+b*arccosh(c*x))-1/2/b^2*ex
p(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*
arccosh(c*x))-1/2/b^2*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^3+\left(c^2x^2-1\right)\sqrt{cx+1}\sqrt{cx-1}-cx}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{}{abc^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*x^2 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x +
```

```
1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
 + integrate((c^4*x^4 - 2*c^2*x^2 + (c^2*x^2 + 1)*(c*x + 1)*(c*x - 1) + (2*
c^3*x^3 - c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c
*x - 1)*a*b*c^2*x^2 - 2*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x +
1)*sqrt(c*x - 1) + a*b + (b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2
*b^2*c^2*x^2 + 2*(b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2\operatorname{arcosh}(cx)^2 + 2\,ab\operatorname{arcosh}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{arcosh}(cx) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(-2), x)
```

3.546 $\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.0377301, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 172.089, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.33, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)\left(a+b\operatorname{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*e*x^4
 + (c^3*d - c*e)*a*b*x^2 - a*b*c*d + (a*b*c^2*e*x^3 + a*b*c^2*d*x)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^3*e*x^4 + (c^3*d - c*e)*b^2*x^2 - b^2*c*d + (b
^2*c^2*e*x^3 + b^2*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))) - integrate((c^5*e*x^6 - (c^5*d + 2*c^3*e)*x^4 + (c^3
*e*x^4 - (c^3*d + 3*c*e)*x^2 - c*d)*(c*x + 1)*(c*x - 1) + (2*c^3*d + c*e)*x
^2 + (2*c^4*e*x^5 - (2*c^4*d + 5*c^2*e)*x^3 + (c^2*d + 2*e)*x)*sqrt(c*x + 1
)*sqrt(c*x - 1) - c*d)/(a*b*c^5*e^2*x^8 + 2*(c^5*d*e - c^3*e^2)*a*b*x^6 + (
c^5*d^2 - 4*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 - 2*(c^3*d^2 - c*d*e)*a*b*
x^2 + (a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*d^2*x^2)*(c*x + 1)*(c*
x - 1) + 2*(a*b*c^4*e^2*x^7 + (2*c^4*d*e - c^2*e^2)*a*b*x^5 - a*b*c^2*d^2*x
 + (c^4*d^2 - 2*c^2*d*e)*a*b*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e^
2*x^8 + 2*(c^5*d*e - c^3*e^2)*b^2*x^6 + (c^5*d^2 - 4*c^3*d*e + c*e^2)*b^2*x
^4 + b^2*c*d^2 - 2*(c^3*d^2 - c*d*e)*b^2*x^2 + (b^2*c^3*e^2*x^6 + 2*b^2*c^3
*d*e*x^4 + b^2*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^2*x^7 + (2*c
^4*d*e - c^2*e^2)*b^2*x^5 - b^2*c^2*d^2*x + (c^4*d^2 - 2*c^2*d*e)*b^2*x^3)*
sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2ex^2+a^2d+\left(b^2ex^2+b^2d\right)\operatorname{arcosh}(cx)^2+2\left(abex^2+abd\right)\operatorname{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arccosh(c*x)^2 + 2*(a*b
*e*x^2 + a*b*d)*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)\left(b\operatorname{arcosh}(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)^2), x)
```

3.547 $\int \frac{1}{(d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.0367158, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \int \frac{1}{(d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [F] time = 180.005, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.422, size = 0, normalized size = 0.

$$\int \frac{1}{\left(e x^2+d\right)^2 \left(a+b \operatorname{arccosh}\left(c x\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*e^2*x
^6 + (2*c^3*d*e - c*e^2)*a*b*x^4 - a*b*c*d^2 + (c^3*d^2 - 2*c*d*e)*a*b*x^2
+ (a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^3 + a*b*c^2*d^2*x)*sqrt(c*x + 1)*sqrt(
c*x - 1) + (b^2*c^3*e^2*x^6 + (2*c^3*d*e - c*e^2)*b^2*x^4 - b^2*c*d^2 + (c^
3*d^2 - 2*c*d*e)*b^2*x^2 + (b^2*c^2*e^2*x^5 + 2*b^2*c^2*d*e*x^3 + b^2*c^2*d
^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
- integrate((3*c^5*e*x^6 - (c^5*d + 6*c^3*e)*x^4 + (3*c^3*e*x^4 - (c^3*d +
5*c*e)*x^2 - c*d)*(c*x + 1)*(c*x - 1) + (2*c^3*d + 3*c*e)*x^2 + (6*c^4*e*x^
5 - (2*c^4*d + 11*c^2*e)*x^3 + (c^2*d + 4*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1)
 - c*d)/(a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 - 2*c^3*e^3)*a*b*x^8 + (3*c^5*d^2*
e - 6*c^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 - 6*c^3*d^2*e + 3*c*d*e^2)*a*b*
x^4 + a*b*c*d^3 - (2*c^3*d^3 - 3*c*d^2*e)*a*b*x^2 + (a*b*c^3*e^3*x^8 + 3*a*
b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1
) + 2*(a*b*c^4*e^3*x^9 + (3*c^4*d*e^2 - c^2*e^3)*a*b*x^7 - a*b*c^2*d^3*x +
3*(c^4*d^2*e - c^2*d*e^2)*a*b*x^5 + (c^4*d^3 - 3*c^2*d^2*e)*a*b*x^3)*sqrt(c
*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e^3*x^10 + (3*c^5*d*e^2 - 2*c^3*e^3)*b^2*x
^8 + (3*c^5*d^2*e - 6*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d^3 - 6*c^3*d^2*e +
 3*c*d*e^2)*b^2*x^4 + b^2*c*d^3 - (2*c^3*d^3 - 3*c*d^2*e)*b^2*x^2 + (b^2*c^
```

```
3*e^3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2)*(c
*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^3*x^9 + (3*c^4*d*e^2 - c^2*e^3)*b^2*x^7 -
b^2*c^2*d^3*x + 3*(c^4*d^2*e - c^2*d*e^2)*b^2*x^5 + (c^4*d^3 - 3*c^2*d^2*e)
*b^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2e^2x^4 + 2\,a^2dex^2 + a^2d^2 + \left(b^2e^2x^4 + 2\,b^2dex^2 + b^2d^2\right)\operatorname{arcosh}\left(cx\right)^2 + 2\left(abe^2x^4 + 2\,abdex^2 + abd^2\right)\operatorname{arcosh}\left(cx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*
e*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)
*arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2 + d\right)^2 \left(b\operatorname{arcosh}\left(cx\right) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)^2), x)
```

3.548 $\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2, x]
```

Rubi [A] time = 0.0450141, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2,x]

[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 26.4411, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2, x]
```

Maple [A] time = 0.29, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}}\sqrt{ex^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{3}x^{3}+\left(c^{2}x^{2}-1\right)\sqrt{cx+1}\sqrt{cx-1}-cx\right)\sqrt{ex^{2}+d}}{abc^{3}x^{2}+\sqrt{cx+1}\sqrt{cx-1}abc^{2}x-abc+\left(b^{2}c^{3}x^{2}+\sqrt{cx+1}\sqrt{cx-1}b^{2}c^{2}x-b^{2}c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int \frac{}{abc^{5}ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)*sqrt(e*x^2 + d
)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x
^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))) + integrate((2*c^5*e*x^6 + (c^5*d - 4*c^3*e)*x^4 + (2*c^3*
e*x^4 + c^3*d*x^2 + c*d)*(c*x + 1)*(c*x - 1) - 2*(c^3*d - c*e)*x^2 + (4*c^4
*e*x^5 + 2*(c^4*d - 2*c^2*e)*x^3 - (c^2*d - e)*x)*sqrt(c*x + 1)*sqrt(c*x -
1) + c*d)*sqrt(e*x^2 + d)/(a*b*c^5*e*x^6 + (c^5*d - 2*c^3*e)*a*b*x^4 - (2*c
^3*d - c*e)*a*b*x^2 + a*b*c*d + (a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c*x + 1)*(
c*x - 1) + 2*(a*b*c^4*e*x^5 - a*b*c^2*d*x + (c^4*d - c^2*e)*a*b*x^3)*sqrt(c
*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e*x^6 + (c^5*d - 2*c^3*e)*b^2*x^4 - (2*c^3
*d - c*e)*b^2*x^2 + b^2*c*d + (b^2*c^3*e*x^4 + b^2*c^3*d*x^2)*(c*x + 1)*(c*
x - 1) + 2*(b^2*c^4*e*x^5 - b^2*c^2*d*x + (c^4*d - c^2*e)*b^2*x^3)*sqrt(c*x
 + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d+ex^2}}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*acosh(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex^2+d}}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arccosh(c*x) + a)^2, x)
```

3.549 $\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.046372, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 23.6287, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.258, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\mathrm{arccosh}\left(cx\right)\right)^{2}}\frac{1}{\sqrt{ex^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{c^{3}x^{3}+\left(c^{2}x^{2}-1\right)\sqrt{cx+1}\sqrt{cx-1}-cx}{\left(b^{2}c^{3}x^{2}+\sqrt{cx+1}\sqrt{cx-1}b^{2}c^{2}x-b^{2}c\right)\sqrt{ex^{2}+d}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left(abc^{3}x^{2}+\sqrt{cx+1}\sqrt{cx-1}abc^{2}x-abc\right)\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((b^2*c^3*x^2
+ sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*sqrt(e*x^2 + d)*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a
*b*c^2*x - a*b*c)*sqrt(e*x^2 + d)) + integrate((c^5*d*x^4 - 2*c^3*d*x^2 + (
(c^3*d + 2*c*e)*x^2 + c*d)*(c*x + 1)*(c*x - 1) + (2*(c^4*d + c^2*e)*x^3 - (
c^2*d + e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/((b^2*c^5*e*x^6 + (c^5*d -
 2*c^3*e)*b^2*x^4 - (2*c^3*d - c*e)*b^2*x^2 + b^2*c*d + (b^2*c^3*e*x^4 + b^
2*c^3*d*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e*x^5 - b^2*c^2*d*x + (c^4*d
- c^2*e)*b^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^5*e*x^6 + (c^5*d - 2*c^3*e)*a*b*x^4 - (
2*c^3*d - c*e)*a*b*x^2 + a*b*c*d + (a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c*x + 1
)*(c*x - 1) + 2*(a*b*c^4*e*x^5 - a*b*c^2*d*x + (c^4*d - c^2*e)*a*b*x^3)*sqr
t(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2ex^2+a^2d+\left(b^2ex^2+b^2d\right)\operatorname{arcosh}(cx)^2+2\left(abex^2+abd\right)\operatorname{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arccosh(c
*x)^2 + 2*(a*b*e*x^2 + a*b*d)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+b\operatorname{acosh}(cx))^2\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*sqrt(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{ex^2+d}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)^2), x)
```

3.550 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.0503267, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.223, size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{arccosh}(cx))^2}\left(ex^2+d\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((b^2*c^3*e*x^
4 + (c^3*d - c*e)*b^2*x^2 - b^2*c*d + (b^2*c^2*e*x^3 + b^2*c^2*d*x)*sqrt(c*
x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)
) + (a*b*c^3*e*x^4 + (c^3*d - c*e)*a*b*x^2 - a*b*c*d + (a*b*c^2*e*x^3 + a*b
*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)) - integrate((2*c^5*
e*x^6 - (c^5*d + 4*c^3*e)*x^4 + (2*c^3*e*x^4 - (c^3*d + 4*c*e)*x^2 - c*d)*(
c*x + 1)*(c*x - 1) + 2*(c^3*d + c*e)*x^2 + (4*c^4*e*x^5 - 2*(c^4*d + 4*c^2*
e)*x^3 + (c^2*d + 3*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*d)/((b^2*c^5*e^2*
x^8 + 2*(c^5*d*e - c^3*e^2)*b^2*x^6 + (c^5*d^2 - 4*c^3*d*e + c*e^2)*b^2*x^4
 + b^2*c*d^2 - 2*(c^3*d^2 - c*d*e)*b^2*x^2 + (b^2*c^3*e^2*x^6 + 2*b^2*c^3*d
*e*x^4 + b^2*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^2*x^7 + (2*c^4
*d*e - c^2*e^2)*b^2*x^5 - b^2*c^2*d^2*x + (c^4*d^2 - 2*c^2*d*e)*b^2*x^3)*sq
rt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)) + (a*b*c^5*e^2*x^8 + 2*(c^5*d*e - c^3*e^2)*a*b*x^6 + (c^5*d^2 - 4*c^
3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 - 2*(c^3*d^2 - c*d*e)*a*b*x^2 + (a*b*c^3
*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) + 2*(a*
b*c^4*e^2*x^7 + (2*c^4*d*e - c^2*e^2)*a*b*x^5 - a*b*c^2*d^2*x + (c^4*d^2 -
2*c^2*d*e)*a*b*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}}{a^2e^2x^4+2\,a^2dex^2+a^2d^2+\left(b^2e^2x^4+2\,b^2dex^2+b^2d^2\right)\operatorname{arcosh}\left(cx\right)^2+2\left(abe^2x^4+2\,abdex^2+abd^2\right)\operatorname{arcosh}\left(cx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*
x^4 + 2*b^2*d*e*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*
x^2 + a*b*d^2)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^2\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x**2)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

3.551 $\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.0503068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [F] time = 180.004, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.219, size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{arccosh}(cx))^2} \left(ex^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((b^2*c^3*e^2*
x^6 + (2*c^3*d*e - c*e^2)*b^2*x^4 - b^2*c*d^2 + (c^3*d^2 - 2*c*d*e)*b^2*x^2
 + (b^2*c^2*e^2*x^5 + 2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c*x + 1)*sqrt
(c*x - 1))*sqrt(e*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^
3*e^2*x^6 + (2*c^3*d*e - c*e^2)*a*b*x^4 - a*b*c*d^2 + (c^3*d^2 - 2*c*d*e)*a
*b*x^2 + (a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^3 + a*b*c^2*d^2*x)*sqrt(c*x + 1
)*sqrt(c*x - 1))*sqrt(e*x^2 + d)) - integrate((4*c^5*e*x^6 - (c^5*d + 8*c^3
*e)*x^4 + (4*c^3*e*x^4 - (c^3*d + 6*c*e)*x^2 - c*d)*(c*x + 1)*(c*x - 1) + 2
*(c^3*d + 2*c*e)*x^2 + (8*c^4*e*x^5 - 2*(c^4*d + 7*c^2*e)*x^3 + (c^2*d + 5*
e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*d)/((b^2*c^5*e^3*x^10 + (3*c^5*d*e^2
- 2*c^3*e^3)*b^2*x^8 + (3*c^5*d^2*e - 6*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d
^3 - 6*c^3*d^2*e + 3*c*d*e^2)*b^2*x^4 + b^2*c*d^3 - (2*c^3*d^3 - 3*c*d^2*e)
*b^2*x^2 + (b^2*c^3*e^3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b
^2*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^3*x^9 + (3*c^4*d*e^2 - c
^2*e^3)*b^2*x^7 - b^2*c^2*d^3*x + 3*(c^4*d^2*e - c^2*d*e^2)*b^2*x^5 + (c^4*
d^3 - 3*c^2*d^2*e)*b^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)*lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 - 2
*c^3*e^3)*a*b*x^8 + (3*c^5*d^2*e - 6*c^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3
```

```
- 6*c^3*d^2*e + 3*c*d*e^2)*a*b*x^4 + a*b*c*d^3 - (2*c^3*d^3 - 3*c*d^2*e)*a*
b*x^2 + (a*b*c^3*e^3*x^8 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*
c^3*d^3*x^2)*(c*x + 1)*(c*x - 1) + 2*(a*b*c^4*e^3*x^9 + (3*c^4*d*e^2 - c^2*
e^3)*a*b*x^7 - a*b*c^2*d^3*x + 3*(c^4*d^2*e - c^2*d*e^2)*a*b*x^5 + (c^4*d^3
 - 3*c^2*d^2*e)*a*b*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2e^3x^6+3\,a^2de^2x^4+3\,a^2d^2ex^2+a^2d^3+\left(b^2e^3x^6+3\,b^2de^2x^4+3\,b^2d^2ex^2+b^2d^3\right)\operatorname{arcosh}\left(cx\right)^2+2\left(abe^3x^6+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a^2*e^3*x^6 + 3*a^2*d*e^2*x^4 + 3*a^2*d^2*e*x^2 +
 a^2*d^3 + (b^2*e^3*x^6 + 3*b^2*d*e^2*x^4 + 3*b^2*d^2*e*x^2 + b^2*d^3)*arcc
osh(c*x)^2 + 2*(a*b*e^3*x^6 + 3*a*b*d*e^2*x^4 + 3*a*b*d^2*e*x^2 + a*b*d^3)*
arccosh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{5}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

3.552 $\int \left(d+ex^2\right)^2 \sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=672

$$-\frac{\sqrt{\pi}\sqrt{b}dee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3}-\frac{\sqrt{\pi}\sqrt{b}dee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}de\cdots}{}$$

```
[Out] d^2*x*Sqrt[a + b*ArcCosh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcCosh[c*x]])/3 + (
e^2*x^5*Sqrt[a + b*ArcCosh[c*x]])/5 - (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*c^3) - (Sqrt[b]*e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^5) - (Sqrt[b]*d*e*E^((3*a)/b)*S
qrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(24*c^3) - (Sqrt
[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
b]])/(64*c^5) - (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b
*ArcCosh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcCosh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi]*Erfi[Sqrt[a
+ b*ArcCosh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(24*c^3*E^((3*a)/b)) - (
Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(6
4*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCo
sh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

Rubi [A] time = 2.40214, antiderivative size = 672, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5707, 5654, 5781, 3307, 2180, 2204, 2205, 5664, 3312}

$$-\frac{\sqrt{\pi}\sqrt{b}dee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3}-\frac{\sqrt{\pi}\sqrt{b}dee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}de\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] d^2*x*Sqrt[a + b*ArcCosh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcCosh[c*x]])/3 + (
e^2*x^5*Sqrt[a + b*ArcCosh[c*x]])/5 - (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Erf[S
```

```
qrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*c^3) - (Sqrt[b]*e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^5) - (Sqrt[b]*d*e*E^((3*a)/b)*S
qrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(24*c^3) - (Sqrt
[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
b]])/(64*c^5) - (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b
*ArcCosh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcCosh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi]*Erfi[Sqrt[a
+ b*ArcCosh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(24*c^3*E^((3*a)/b)) - (
Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(6
4*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCo
sh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\cosh^{-1}(cx)}\,dx = \int \left(d^2\sqrt{a+b\cosh^{-1}(cx)}+2dex^2\sqrt{a+b\cosh^{-1}(cx)}+e^2x^4\sqrt{a+b\cosh^{-1}(cx)}\right) dx$$

$$= d^2\int \sqrt{a+b\cosh^{-1}(cx)}\,dx+(2de)\int x^2\sqrt{a+b\cosh^{-1}(cx)}\,dx+e^2\int x^4\sqrt{a+b\,\text{co}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}(cx)}-\frac{1}{2}($$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}(cx)}-\frac{(b}{}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}(cx)}-\frac{(b}{}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}(cx)}-\frac{d^2}{}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}}{}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}}{}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}}{}$$

Mathematica [A] time = 6.36767, size = 536, normalized size = 0.8

$$\frac{be^{-\frac{5a}{b}}\left(450e^{\frac{6a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)\left(-be\left(4c^2d+e\right)\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\sqrt{-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}}+8ac^4d^2\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (b*(450*E^((6*a)/b)*(8*a*c^4*d^2*Sqrt[a/b + ArcCosh[c*x]] + 8*b*c^4*d^2*Arc
Cosh[c*x]*Sqrt[a/b + ArcCosh[c*x]] - b*e*(4*c^2*d + e)*Sqrt[-((a + b*ArcCos
h[c*x])/b)]*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)])*Gamma[3/2, a/b + ArcCosh[c
```

```
*x]] - 9*Sqrt[5]*b*e^2*Sqrt[a/b + ArcCosh[c*x]]*Sqrt[-((a + b*ArcCosh[c*x])
^2/b^2)]*Gamma[3/2, (-5*(a + b*ArcCosh[c*x]))/b] - E^((2*a)/b)*(25*Sqrt[3]*
b*e*(8*c^2*d + 3*e)*Sqrt[a/b + ArcCosh[c*x]]*Sqrt[-((a + b*ArcCosh[c*x])^2/
b^2)]*Gamma[3/2, (-3*(a + b*ArcCosh[c*x]))/b] + 450*E^((2*a)/b)*(8*a*c^4*d^
2*Sqrt[-((a + b*ArcCosh[c*x])/b)] + 8*b*c^4*d^2*ArcCosh[c*x]*Sqrt[-((a + b*
ArcCosh[c*x])/b)] + b*e*(4*c^2*d + e)*Sqrt[a/b + ArcCosh[c*x]]*Sqrt[-((a +
b*ArcCosh[c*x])^2/b^2)])*Gamma[3/2, -((a + b*ArcCosh[c*x])/b)] + b*e*E^((6*
a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)]*(
25*Sqrt[3]*(8*c^2*d + 3*e)*Gamma[3/2, (3*(a + b*ArcCosh[c*x]))/b] + 9*Sqrt[
5]*e*E^((2*a)/b)*Gamma[3/2, (5*(a + b*ArcCosh[c*x]))/b]))))/(7200*c^5*E^((5
*a)/b)*(a + b*ArcCosh[c*x])^(3/2))
```

Maple [F] time = 0.256, size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^2 \sqrt{a+b\operatorname{arccosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^2 \sqrt{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2*sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{acosh}(cx)} \left(d + ex^2\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))*(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.553 $\int (d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=322

$$-\frac{\sqrt{\pi}\sqrt{b}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}-\frac{\sqrt{\pi}\sqrt{b}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{-\frac{3a}{b}}\text{E}\cdots}{}$$

```
[Out] d*x*Sqrt[a + b*ArcCosh[c*x]] + (e*x^3*Sqrt[a + b*ArcCosh[c*x]])/3 - (Sqrt[b
]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b
]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*c^3) - (Sqr
t[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/
(4*c*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rubi [A] time = 1.28662, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5707, 5654, 5781, 3307, 2180, 2204, 2205, 5664, 3312}

$$-\frac{\sqrt{\pi}\sqrt{b}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}-\frac{\sqrt{\pi}\sqrt{b}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{-\frac{3a}{b}}\text{E}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] d*x*Sqrt[a + b*ArcCosh[c*x]] + (e*x^3*Sqrt[a + b*ArcCosh[c*x]])/3 - (Sqrt[b
]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b
]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*c^3) - (Sqr
t[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/
(4*c*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \left(d+ex^2\right)\sqrt{a+b\cosh^{-1}(cx)}\,dx = \int \left(d\sqrt{a+b\cosh^{-1}(cx)}+ex^2\sqrt{a+b\cosh^{-1}(cx)}\right)dx$$

$$= d\int \sqrt{a+b\cosh^{-1}(cx)}\,dx + e\int x^2\sqrt{a+b\cosh^{-1}(cx)}\,dx$$

$$= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{1}{2}(bcd)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a}}$$

$$= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{(bd)\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,x,\cosh\right.}{2c}$$

$$= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{(bd)\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\cosh\right.}{4c}$$

$$= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{d\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b}\right.}{2c}$$

$$= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}$$

$$= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}$$

$$= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}$$

Mathematica [A] time = 2.6717, size = 317, normalized size = 0.98

$$\frac{ee^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\text{Gamma}\left(\frac{3}{2},-\frac{3(a+b\cos}{b}\right.\right.}{72c^3\sqrt{-\frac{(a+b\,\mathrm{co}}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (d*Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/S
qrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a +
 b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b)) + (e*Sqrt[a + b*ArcCosh[c*x]]*(9*E^((4
*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, a/b + ArcCosh[c*x]] + Sqr
t[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c*x]))/b] + 9*E
^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, -((a + b*ArcCosh[c*x])/b)] +
 Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, (3*(a + b*A
rcCosh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)])
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)\sqrt{a+b\,\mathrm{arccosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)\sqrt{b\,\mathrm{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\operatorname{acosh}(cx)}\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.554 $\int \sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\cosh^{-1}(cx)}$$

```
[Out] x*Sqrt[a + b*ArcCosh[c*x]] - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCo
sh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rubi [A] time = 0.420343, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5654, 5781, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\cosh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcCosh[c*x]] - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCo
sh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
```

```
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}(cx)}\,dx = x\sqrt{a+b\cosh^{-1}(cx)} - \frac{1}{2}(bc)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{2c}$$

$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4c} - \frac{b\,\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(\right.}{4c}$$

$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{2c} - \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.}{2c}$$

$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}$$

Mathematica [A] time = 0.187994, size = 100, normalized size = 0.98

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\,\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}}+\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/Sqr
t[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a + b
*ArcCosh[c*x])/b)]))/(2*c*E^(a/b))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\text{arccosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arcosh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{acosh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.555 $\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2), x]
```

Rubi [A] time = 0.0573845, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2),x]
```

```
[Out] Defer[Int][Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$$

Mathematica [A] time = 4.6627, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2),x]
```

```
[Out] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2), x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int \frac{1}{e x^{2}+d}\sqrt{a +b \,\mathrm{arccosh}\left(c x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x)
```

```
[Out] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b \operatorname{acosh}{\left(c x \right)}}}{d + e x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.556 $\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2, x]
```

Rubi [A] time = 0.0571904, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2,x]

[Out] Defer[Int][Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 20.744, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2,x]
```

```
[Out] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2, x]
```

Maple [A] time = 0.396, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2 + d\right)^2} \sqrt{a + b\operatorname{arccosh}\left(cx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\operatorname{arcosh}\left(cx\right) + a}}{\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)/(e*x^2 + d)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b \operatorname{acosh}(cx)}}{\left(d + ex^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(1/2)/(e*x**2+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.557 $\int (d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=442

$$-\frac{3\sqrt{\pi}b^{3/2}ee^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}b^{3/2}ee^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e}{}$$

```
[Out] (-3*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(2*c) - (b*e
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(3*c^3) - (b*e*x^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(6*c) + d*x*(a + b*A
rcCosh[c*x])^(3/2) + (e*x^3*(a + b*ArcCosh[c*x])^(3/2))/3 - (3*b^(3/2)*d*E^
(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*
E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^3) - (b^(3/2)
*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(96*c^3) + (3*b^(3/2)*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8
*c*E^(a/b)) + (3*b^(3/2)*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
/(32*c^3*E^(a/b)) + (b^(3/2)*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)/b))
```

Rubi [A] time = 1.73169, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5707, 5654, 5718, 5658, 3308, 2180, 2205, 2204, 5664, 5759, 5670, 5448}

$$-\frac{3\sqrt{\pi}b^{3/2}ee^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}b^{3/2}ee^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-3*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(2*c) - (b*e
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(3*c^3) - (b*e*x^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(6*c) + d*x*(a + b*A
rcCosh[c*x])^(3/2) + (e*x^3*(a + b*ArcCosh[c*x])^(3/2))/3 - (3*b^(3/2)*d*E^
(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*
E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^3) - (b^(3/2)
*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(96*c^3) + (3*b^(3/2)*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8
*c*E^(a/b)) + (3*b^(3/2)*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
```

```
/(32*c^3*E^(a/b)) + (b^(3/2)*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)/b))
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx=\int \left(d\left(a+b\cosh^{-1}(cx)\right)^{3/2}+ex^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}\right)dx$$

$$=d\int \left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx+e\int x^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$$

$$=dx\left(a+b\cosh^{-1}(cx)\right)^{3/2}+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{1}{2}(3bcd)\int \frac{x\sqrt{a+b\cosh}}{\sqrt{-1+cx}\sqrt{1}}$$

$$=-\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{bex^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-}}}{6c}$$

$$=-\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(c}}{3c^3}$$

$$=-\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(c}}{3c^3}$$

$$=-\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(c}}{3c^3}$$

$$=-\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(c}}{3c^3}$$

$$=-\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(c}}{3c^3}$$

$$=-\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(c}}{3c^3}$$

Mathematica [A] time = 3.32939, size = 812, normalized size = 1.84

$$\frac{ade^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}}+\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right)}{2c}+\frac{aee^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cos}{b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (a*d*Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])
/Sqrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a
 + b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b)) + (a*e*Sqrt[a + b*ArcCosh[c*x]]*(9*E
^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, a/b + ArcCosh[c*x]] +
 Sqrt[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c*x]))/b] +
 9*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, -((a + b*ArcCosh[c*x])/b
)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, (3*(a +
 b*ArcCosh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])^2/b^
2)]) + (b*d*(-12*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*
x]] + 8*c*x*ArcCosh[c*x]*Sqrt[a + b*ArcCosh[c*x]] + ((2*a + 3*b)*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]))/Sqrt[b] + ((
2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh
[a/b]))/Sqrt[b]))/(8*c) + (Sqrt[b]*e*(9*(-12*Sqrt[b]*Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*Sqrt[b]*c*x*ArcCosh[c*x]*Sqrt[a
 + b*ArcCosh[c*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqr
t[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh
[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[
3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (2*
a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*
a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcCosh[c*x]]*(2*ArcCosh[c*x
]*Cosh[3*ArcCosh[c*x]] - Sinh[3*ArcCosh[c*x]])))/(288*c^3)
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int \left(e x^{2}+d\right) \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e x^{2}+d\right) \left(b \operatorname{arcosh}\left(c x\right) + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acosh}(cx))^{\frac{3}{2}} \left(d + ex^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(3/2)*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.558 $\int \left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\right.$$

```
[Out] (-3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(2*c) + x*(a +
 b*ArcCosh[c*x])^(3/2) - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh
[c*x]]/Sqrt[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(8*c*E^(a/b))
```

Rubi [A] time = 0.401951, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5654, 5718, 5658, 3308, 2180, 2205, 2204}

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(3/2),x]
```


```
[Out] (-3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(2*c) + x*(a +
 b*ArcCosh[c*x])^(3/2) - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh
[c*x]]/Sqrt[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(8*c*E^(a/b))
```


Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
```

```
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx = x\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= -\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{3/2} + \frac{1}{4}\left(3b^2\right)\int \frac{\ldots}{\sqrt{a+b\,\mathrm{c}\ldots}}$$

$$= -\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{(3b)\,\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}\ldots\right)}{\sqrt{x}\ldots}\ldots\right.}{\ldots}$$

$$= -\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{(3b)\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\ldots\right)}}{\sqrt{x}\ldots}\ldots\right.}{\ldots}$$

$$= -\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{(3b)\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\ldots\right.}{\ldots}$$

$$= -\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x\left(a+b\cosh^{-1}(cx)\right)^{3/2} - \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+\ldots}}{\ldots}\right.}{8c}$$

Mathematica [A] time = 0.643033, size = 269, normalized size = 1.92

$$\frac{ae^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\,\mathrm{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}}+\frac{\mathrm{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right)}{2c} + \frac{b\left(\frac{\sqrt{\pi}(2a-3b)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c\ldots}}{\sqrt{b}}\right.}{\sqrt{b}}\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/S
qrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a +
 b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b)) + (b*(-12*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*c*x*ArcCosh[c*x]*Sqrt[a + b*ArcCosh[c
*x]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a
/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*
x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(8*c)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcosh}(cx) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acosh}(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.559 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2), x]
```

Rubi [A] time = 0.0665648, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx=\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Mathematica [A] time = 2.06754, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2), x]
```

Maple [A] time = 0.245, size = 0, normalized size = 0.

$$\int \frac{1}{ex^2+d}\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x)
```

```
[Out] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{acosh}\left(cx\right)\right)^{\frac{3}{2}}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(3/2)/(d + e*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.560 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Rubi [A] time = 0.0613161, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 12.7228, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Maple [A] time = 0.426, size = 0, normalized size = 0.

$$\int \frac{1}{\left(e x^{2}+d\right)^{2}} \left(a +b \operatorname{arccosh}\left(c x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{\frac{3}{2}}}{{\left(e x^{2} + d\right)}^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(3/2)/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.561 $\int \frac{(d+ex^2)^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=608

$$-\frac{\sqrt{\pi}dee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{3}}dee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\pi}dee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{\cdots}}{\cdots}\right)}{4\sqrt{b}c^3}$$

```
[Out] -(d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c)
 - (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*
c^3) - (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*Sqr
t[b]*c^5) - (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sq
rt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) - (e^2*E^((5*a)/b)*Sqrt[
Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (
d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
+ (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c^3*E^(a
/b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*Sqrt[b]*c^
5*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]])/(4*Sqrt[b]*c^3*E^((3*a)/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*
ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sqrt[Pi/5]*Erf
i[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((5*a)/b))
```

Rubi [A] time = 1.14779, antiderivative size = 608, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5707, 5658, 3308, 2180, 2205, 2204, 5670, 5448}

$$-\frac{\sqrt{\pi}dee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{3}}dee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\pi}dee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{\cdots}}{\cdots}\right)}{4\sqrt{b}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -(d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c)
 - (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*
c^3) - (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*Sqr
t[b]*c^5) - (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x
```

```
]])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sq
rt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) - (e^2*E^((5*a)/b)*Sqrt[
Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (
d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
+ (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c^3*E^(a
/b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*Sqrt[b]*c^
5*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]])/(4*Sqrt[b]*c^3*E^((3*a)/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*
ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sqrt[Pi/5]*Erf
i[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((5*a)/b))
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \int \left(\frac{d^2}{\sqrt{a+b\cosh^{-1}(cx)}}+\frac{2dex^2}{\sqrt{a+b\cosh^{-1}(cx)}}+\frac{e^2x^4}{\sqrt{a+b\cosh^{-1}(cx)}}\right)dx$$

$$= d^2\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx+(2de)\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx+e^2\int \frac{x^4}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

$$= -\frac{d^2\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\cosh^{-1}(cx)\right)}{bc}+\frac{(2de)\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx,x,\cosh\right.}{c^3}$$

$$= -\frac{d^2\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\cosh^{-1}(cx)\right)}{2bc}+\frac{d^2\,\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\cosh^{-1}(c\right.}{2bc}$$

$$= -\frac{d^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(cx)}\right)}{bc}+\frac{d^2\,\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(\right.}{bc}$$

$$= -\frac{d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}-\frac{(de)\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx,x\right.}{4c^3}$$

$$= -\frac{d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}-\frac{(de)\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,\right.}{2bc^3}$$

$$= -\frac{d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}-\frac{dee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}-\frac{e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx}}{\sqrt{b}}\right.}{16\sqrt{b}c^5}$$

Mathematica [A] time = 1.09197, size = 530, normalized size = 0.87

$$\frac{e^{-\frac{5a}{b}}\left(30e^{\frac{6a}{b}}\left(8c^4d^2+4c^2de+e^2\right)\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+240c^4d^2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\text{Gamma}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (30*(8*c^4*d^2 + 4*c^2*d*e + e^2)*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, a/b + ArcCosh[c*x]] + 3*Sqrt[5]*e^2*Sqrt[-((a + b*ArcCosh[c*x])/b)]*
```

```
Gamma[1/2, (-5*(a + b*ArcCosh[c*x]))/b] + 40*Sqrt[3]*c^2*d*e*E^((2*a)/b)*Sq
rt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] + 15*
Sqrt[3]*e^2*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a +
 b*ArcCosh[c*x]))/b] + 240*c^4*d^2*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/
b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] + 120*c^2*d*e*E^((4*a)/b)*Sqrt[-(
(a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] + 30*e^2*E^(
(4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/
b)] + 40*Sqrt[3]*c^2*d*e*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3
*(a + b*ArcCosh[c*x]))/b] + 15*Sqrt[3]*e^2*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c
*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] + 3*Sqrt[5]*e^2*E^((10*a)/b)*Sq
rt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a + b*ArcCosh[c*x]))/b])/(480*c^5*E^(
(5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.246, size = 0, normalized size = 0.

$$\int \left(ex^{2}+d\right)^{2} \frac{1}{\sqrt{a+b\operatorname{arccosh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^{2}+d\right)^{2}}{\sqrt{b\,\operatorname{arcosh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)^2}{\sqrt{a + b \operatorname{acosh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.562 $\int \frac{d+ex^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=287

$$-\frac{\sqrt{\pi}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{3}}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}ee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

```
[Out] -(d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) -
 (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3)
 - (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
])/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2
*Sqrt[b]*c*E^(a/b)) + (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(
8*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.534771, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5707, 5658, 3308, 2180, 2205, 2204, 5670, 5448}

$$-\frac{\sqrt{\pi}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{3}}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}ee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*ArcCosh[c*x]],x]
```


```
[Out] -(d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) -
 (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3)
 - (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
])/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2
*Sqrt[b]*c*E^(a/b)) + (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(
8*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```


Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \int \left(\frac{d}{\sqrt{a+b\cosh^{-1}(cx)}}+\frac{ex^2}{\sqrt{a+b\cosh^{-1}(cx)}}\right)dx$$

$$= d\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx + e\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

$$= -\frac{d\,\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} + \frac{e\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{c^3}$$

$$= -\frac{d\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{2bc} + \frac{d\,\mathrm{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{2bc}$$

$$= -\frac{d\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{bc} + \frac{d\,\mathrm{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{bc}$$

$$= -\frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{e\,\mathrm{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}\right.}{8c^3}$$

$$= -\frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{e\,\mathrm{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a\cdots}\right.}{4bc^3}$$

$$= -\frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{ee^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} - \frac{ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

Mathematica [A] time = 0.607766, size = 213, normalized size = 0.74

$$\frac{e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\left(4c^2d+e\right)\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+3e^{\frac{2a}{b}}\left(4c^2d+e\right)\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{a+b\,\mathrm{c}\cdots}{}\right.\right.}{24c^3\sqrt{a+b\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (3*(4*c^2*d + e)*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcC
osh[c*x]] + Sqrt[3]*e*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b
*ArcCosh[c*x]))/b] + 3*(4*c^2*d + e)*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x]
)/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] + Sqrt[3]*e*E^((6*a)/b)*Sqrt[a/
b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b])/(24*c^3*E^((3*a)/
b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.127, size = 0, normalized size = 0.

$$\int \left(e x^{2}+d\right)\frac{1}{\sqrt{a +b \,\mathrm{arccosh}\left(c x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e x^{2} + d}{\sqrt{b \operatorname{arcosh}\left(c x\right) + a}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex^2}{\sqrt{a+b\operatorname{acosh}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.563 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rubi [A] time = 0.104439, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5658, 3308, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\cosh^{-1}(cx)\right)}{bc}\\
&= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\cosh^{-1}(cx)\right)}{2bc}+\frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\cosh^{-1}(cx)\right)}{2bc}\\
&= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(cx)}\right)}{bc}+\frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(cx)}\right)}{bc}\\
&= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}\end{aligned}$$

Mathematica [A] time = 0.111263, size = 100, normalized size = 1.14

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right)}{2c\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt
[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)])/(2*c*E^(
a/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.001, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\mathrm{arccosh}\left(cx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{arcosh}\left(cx\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.564 $\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Rubi [A] time = 0.0608199, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Mathematica [A] time = 0.150542, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]),x]
```

```
[Out] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Maple [A] time = 0.251, size = 0, normalized size = 0.

$$\int \frac{1}{ex^2 + d} \frac{1}{\sqrt{a + b\operatorname{arccosh}(cx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex^2 + d)\sqrt{b\operatorname{arcosh}(cx) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)*sqrt(b*arccosh(c*x) + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\,(cx)}\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*acosh(c*x))*(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.565 $\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{(d+ex^2)^2\sqrt{a+b\cosh^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Rubi [A] time = 0.0584369, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \int \frac{1}{(d+ex^2)^2\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Mathematica [A] time = 0.27549, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]),x]
```

```
[Out] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Maple [A] time = 0.38, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^{2}+d\right)^{2}} \frac{1}{\sqrt{a+b\operatorname{arccosh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^{2}+d\right)^{2}\sqrt{b\operatorname{arcosh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arccosh(c*x) + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.566 $\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=358

$$\frac{\sqrt{\pi}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}ee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b}}{}\right.}{4b^{3/2}c^3}$$

```
[Out] (-2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) - (2*e*x
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (d*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) + (e*E^((3*
a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2
)*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(
a/b)) + (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*
E^(a/b)) + (e*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(4*b^(3/2)*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.831471, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5707, 5656, 5781, 3307, 2180, 2204, 2205, 5666}

$$\frac{\sqrt{\pi}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}ee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b}}{}\right.}{4b^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```



```
[Out] (-2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) - (2*e*x
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (d*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) + (e*E^((3*
a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2
)*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(
a/b)) + (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*
E^(a/b)) + (e*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
```

```
(4*b^(3/2)*c^3*E^((3*a)/b))
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = \int \left(\frac{d}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{ex^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\right)dx$$

$$= d\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx + e\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2cd)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b} -$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx\right.}{bc}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc} +$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2d)\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos\right.}{b^2c}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{ee^{a/b}\sqrt{\pi}}{}$$

Mathematica [A] time = 1.91394, size = 268, normalized size = 0.75

$$e^{-\frac{3a}{b}}\left(e^{\frac{4a}{b}}\left(-\left(4c^2d+e\right)\right)\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+e^{\frac{2a}{b}}\left(4c^2d+e\right)\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{a+b}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-((4*c^2*d + e)*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcC
osh[c*x]]) + Sqrt[3]*e*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcCosh[c*x]))/b] + (4*c^2*d + e)*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])
/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - E^((3*a)/b)*(2*(4*c^2*d + e)*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + Sqrt[3]*e*E^((3*a)/b)*Sqrt[a/b + ArcC
osh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] + 2*e*Sinh[3*ArcCosh[c*x]]
))/(4*b*c^3*E^((3*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)\left(a+b\text{arccosh}\left(cx\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\left(b\,\text{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{(a + b\operatorname{acosh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(a + b*acosh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.567 $\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

```
[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rubi [A] time = 0.4163, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5656, 5781, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-3/2),x]
```


```
[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```


Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2c)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc} + \frac{\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx, x, \cosh$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2c} + \frac{2\,\mathrm{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}$$

$$= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

Mathematica [A] time = 0.252886, size = 132, normalized size = 1.1

$$\frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)-2e^{a/b}\sqrt{\frac{cx-1}{cx+1}}(cx\right.}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-3/2),x]
```

```
[Out] (-2*E^(a/b)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - E^((2*a)/b)*Sqrt[a/b + A
rcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt[-((a + b*ArcCosh[c*x])/b
)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)])/(b*c*E^(a/b)*Sqrt[a + b*ArcCosh[c
*x]])
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\mathrm{arccosh}\,(cx)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arcosh}(cx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{acosh}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.568 $\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Rubi [A] time = 0.0694119, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx=\int \frac{1}{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.164607, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int \frac{1}{e x^{2}+d}\left(a+b \operatorname{arccosh}\left(c x\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(e x^{2} + d\right)} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{acosh}(cx))^{\frac{3}{2}}\left(d + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**(3/2)*(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.569 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Rubi [A] time = 0.0641523, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)),x]

[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx=\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.288801, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Maple [A] time = 0.392, size = 0, normalized size = 0.

$$\int \frac{1}{\left(e x^{2}+d\right)^{2}} (a+b \operatorname{arccosh}(c x))^{-\frac{3}{2}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e x^{2}+d\right)^{2} (b \operatorname{arcosh}(c x)+a)^{\frac{3}{2}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```